Q1 - Cauchy

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The cauchy distribution itself does not have an expected value. However, we will try to simulate its powers to see if we can get one. The formula used on each one will be

$$\hat{\theta} = \frac{1}{m}\sum_{i=1}^{m} g(Y_i)$$

where $\hat{\theta}$ is an estimation for

$$\theta = E[g(Y)] = \int g(y)f(y)dy$$

where $f(Y)$ is the cauchy density sorted.

[1]:
```
m <- 1000
means.g1 <- numeric(m)  # for Y
means.g2 <- numeric(m)  # for Y^2
for (i in 1:m) {
    X <- rcauchy(10)
    Y <- sort(X)
    g1 <- Y
    g2 <- Y^2
    means.g1[i] <- mean(g1)
    means.g2[i] <- mean(g2)
}

rbind(means.g1, means.g2)
rbind(var(means.g1), var(means.g2))
```

A matrix: 2 × 1000 of type dbl

means.g1	-4.133566	-5.47515	1.368941	21.08782	-0.1705071	-1.314
means.g2	192.483919	269.58552	19.227310	5428.62950	8.9026782	14.563

A matrix: 2 × 1 of type dbl

2.982993e+03
4.027452e+11

The extremely high variance of the results tells us there is no convergence in any of the random variables.