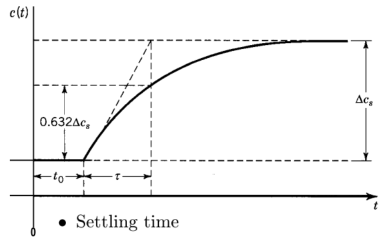
Ziegler-Nichols:

Controller Type	Proportional Gain, K_c'	Integral Time, τ_I'	Derivative Time, τ_D'
Proportional-only, P	$\frac{K_{cu}}{2}$	—	—
Proportional-integral, PI	$\frac{K_{cu}}{2.2}$	$\frac{T_u}{1.2}$	—
Proportional-integral-derivative, PID	$\frac{K_{cu}}{1.7}$	$\frac{T_u}{2}$	$\frac{T_u}{8}$

Fórmula del PID:

$$G_c(s) = \frac{M(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Controller Type	Proportional Gain, K_c'	Integral Time, τ_I'	Derivative Time, τ_D'
Proportional-only, P	$\frac{1}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	—	—
Proportional-integral, PI	$\frac{0.9}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	$3.33t_0$	—
Proportional-integral-derivative, PID	$\frac{1.2}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	$2.0t_0$	$\frac{1}{2}t_0$



Recall 2^{nd}–order underdamped sustem

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

Complex poles $-\sigma_d \pm j\omega_d$,

$$\text{where } \begin{cases} \sigma_d &= \zeta\omega_n, \\ \omega_d &= \sqrt{1-\zeta^2}\,\omega_n. \end{cases}$$

From the geometry,

$$\tan\theta = \frac{\sqrt{1-\zeta^2}}{\zeta} \Rightarrow$$

$$\boxed{\cos\theta = \zeta.}$$

- Settling time

$$T_s \approx 4/(\zeta\omega_n);$$

- Damped osc. frequency

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

- Overshoot %OS

$$\%OS = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

$$\tan\theta = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

Inversa de una matriz de 2x2:

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Reglas para root locus:

Rule 1: # branches = # poles

Rule 2: symmetrical about the real axis

Rule 3: real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros

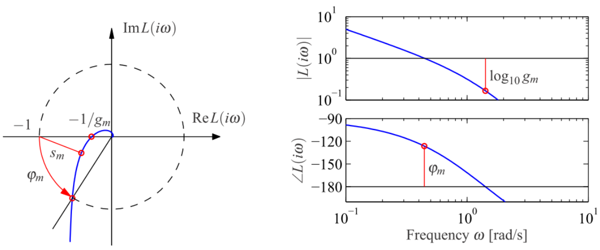
Rule 4: RL begins at poles, ends at zeros

Rule 5: Asymptotes: real-axis intercept σ_a,angles θ_a

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}} \qquad \theta_a = \frac{(2m+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}} \qquad m = 0, \pm1, \pm2, \ldots$$

Rule 6: Real-axis break-in and breakaway points

Found by setting $K(\sigma) = -\frac{1}{G(\sigma)H(\sigma)}$ (σ real) and solving $\frac{dK(\sigma)}{d\sigma} = 0$ for real σ.



(a) Nyquist plot

(b) Bode plot

$$g_m = \frac{1}{|L(i\omega_{pc})|} \qquad \varphi_m = \pi + \arg L(i\omega_{gc})$$

Theorem 9.2 (Nyquist's stability theorem)**.** *Consider a closed loop system with the loop transfer function $L(s)$ that has P poles in the region enclosed by the Nyquist contour. Let N be the net number of clockwise encirclements of -1 by $L(s)$ when s encircles the Nyquist contour Γ in the clockwise direction. The closed loop system then has $Z = N + P$ poles in the right half-plane.*

$$\begin{array}{llllll}
s^n & a_0 & a_2 & a_4 & a_6 & \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 & \cdots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 & \cdots \\
s^{n-4} & d_1 & d_2 & d_3 & d_4 & \cdots \\
\vdots & \vdots & \vdots & & & \\
s^2 & e_1 & e_2 & & & \\
s^1 & f_1 & & & & \\
s^0 & g_0 & & & &
\end{array}$$

$$b_1 = \frac{a_1a_2 - a_0a_3}{a_1} \qquad b_2 = \frac{a_1a_4 - a_0a_5}{a_1} \qquad b_3 = \frac{a_1a_6 - a_0a_7}{a_1} \qquad \vdots$$

$$c_1 = \frac{b_1a_3 - a_1b_2}{b_1} \qquad c_2 = \frac{b_1a_5 - a_1b_3}{b_1} \qquad c_3 = \frac{b_1a_7 - a_1b_4}{b_1} \qquad \vdots$$

$$d_1 = \frac{c_1b_2 - b_1c_2}{c_1} \qquad d_2 = \frac{c_1b_3 - b_1c_3}{c_1} \qquad \vdots$$

$f(t) \quad (t \geq 0)$	$\mathcal{L}[f(t)]$	Region of Convergence
1	$\frac{1}{s}$	$\sigma > 0$
$\delta_D(t)$	1	$\lvert\sigma\rvert < \infty$
t	$\frac{1}{s^2}$	$\sigma > 0$
$t^n \quad n \in \mathbb{Z}^+$	$\frac{n!}{s^{n+1}}$	$\sigma > 0$
$e^{\alpha t} \quad \alpha \in \mathbb{C}$	$\frac{1}{s-\alpha}$	$\sigma > \Re\{\alpha\}$
$te^{\alpha t} \quad \alpha \in \mathbb{C}$	$\frac{1}{(s-\alpha)^2}$	$\sigma > \Re\{\alpha\}$
$\cos(\omega_o t)$	$\frac{s}{s^2+\omega_o^2}$	$\sigma > 0$
$\sin(\omega_o t)$	$\frac{\omega_o}{s^2+\omega_o^2}$	$\sigma > 0$
$e^{\alpha t}\sin(\omega_o t + \beta)$	$\frac{(\sin\beta)s + \omega_o^2\cos\beta - \alpha\sin\beta}{(s-\alpha)^2+\omega_o^2}$	$\sigma > \Re\{\alpha\}$
$t\sin(\omega_o t)$	$\frac{2\omega_o s}{(s^2+\omega_o^2)^2}$	$\sigma > 0$
$t\cos(\omega_o t)$	$\frac{s^2-\omega_o^2}{(s^2+\omega_o^2)^2}$	$\sigma > 0$
$\mu(t) - \mu(t-\tau)$	$\frac{1-e^{-s\tau}}{s}$	$\lvert\sigma\rvert < \infty$

$f(t)$	$\mathcal{L}[f(t)]$	Names
$\sum_{i=1}^{l} a_i f_i(t)$	$\sum_{i=1}^{l} a_i F_i(s)$	Linear combination
$\frac{dy(t)}{dt}$	$sY(s) - y(0^-)$	Derivative Law
$\frac{d^k y(t)}{dt^k}$	$s^kY(s) - \sum_{i=1}^{k} s^{k-i} \left.\frac{d^{i-1}y(t)}{dt^{i-1}}\right\rvert_{t=0^-}$	High order derivative
$\int_{0^-}^{t} y(\tau)d\tau$	$\frac{1}{s}Y(s)$	Integral Law
$y(t-\tau)\mu(t-\tau)$	$e^{-s\tau}Y(s)$	Delay
$ty(t)$	$-\frac{dY(s)}{ds}$	
$t^k y(t)$	$(-1)^k\frac{d^kY(s)}{ds^k}$	
$\int_{0^-}^{t} f_1(\tau)f_2(t-\tau)d\tau$	$F_1(s)F_2(s)$	Convolution
$\lim_{t\to\infty} y(t)$	$\lim_{s\to 0} sY(s)$	Final Value Theorem
$\lim_{t\to 0^+} y(t)$	$\lim_{s\to\infty} sY(s)$	Initial Value Theorem
$f_1(t)f_2(t)$	$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	Time domain product
$e^{at}f_1(t)$	$F_1(s-a)$	Frequency Shift