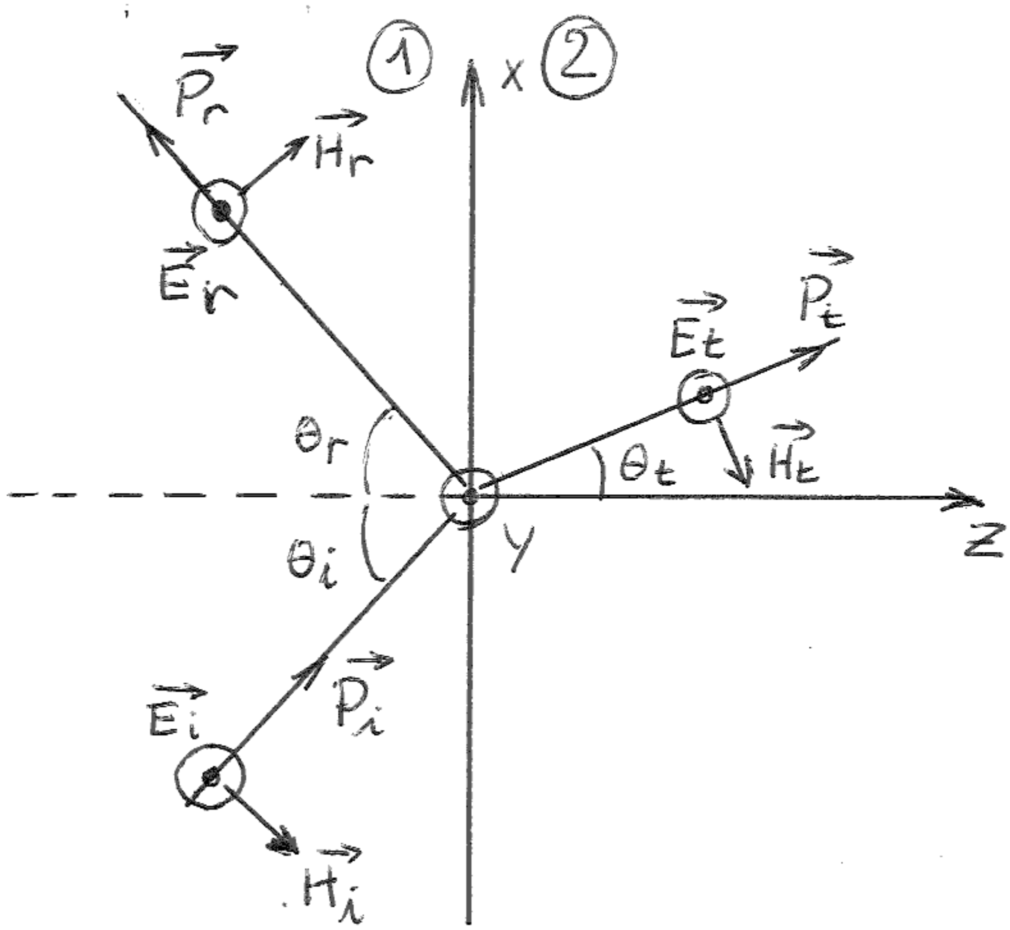
INCIDENCIA OBLICUA SOBRE UNA INTERFAZ DIELECTRICA

POLARIZACION PERPENDICULAR



$z=0$

$\mu_1, \epsilon_1, \sigma_1 = 0$ $\mu_2, \epsilon_2, \sigma_2 = 0$

LOS VECTORES DE POYNTING SON:

$$\vec{P_i} = P_i(\hat{x}\,\mathrm{sen}\theta_i + \hat{z}\cos\theta_i)$$

$$\vec{P_r} = P_r(\hat{x}\,\mathrm{sen}\theta_r - \hat{z}\cos\theta_r)$$

$$\vec{P_t} = P_t(\hat{x}\,\mathrm{sen}\theta_t + \hat{z}\cos\theta_t)$$

$\vec{E_i}$ Y $\vec{H_i}$ TIENEN LAS MISMAS EXPRESIONES QUE EN EL CASO DIELECT. PERF - COND. PERF.

$$\vec{E_i} = \hat{y} E_{i_1} e^{-j\beta_1(x\,\mathrm{sen}\theta_i + z\cos\theta_i)}$$

$$\vec{H_i} = \frac{E_{i1}}{Z_1}\left(-\hat{x}\cos\theta_i + \hat{z}\,\mathrm{sen}\theta_i\right) e^{-j\beta_1(x\,\mathrm{sen}\theta_i + z\cos\theta_i)}$$

LOS CAMPOS REFLEJADOS SON:

$$\vec{E_r} = \hat{y} E_{r_1} e^{-j\beta_1(x\,\mathrm{sen}\theta_i - z\cos\theta_i)}$$

$$\vec{H_r} = \frac{E_{r1}}{Z_1}\left(\hat{x}\cos\theta_i + \hat{z}\,\mathrm{sen}\theta_i\right) e^{-j\beta_1(x\,\mathrm{sen}\theta_i - z\cos\theta_i)}$$

TIENEN LA MISMA FORMA QUE EL CASO DIEL. PERF - COND PERF.

LOS CAMPOS TRANSMITIDOS TENDRÁN LA MISMA FORMA QUE LOS CAMPOS INCIDENTES

$$\vec{E_t} = \hat{y} E_{t_2} e^{-j\beta_2(x\,\mathrm{sen}\theta_t + z\cos\theta_t)}$$

$$\vec{H_t} = \frac{E_{t2}}{Z_2}\left(-\hat{x}\cos\theta_t + \hat{z}\,\mathrm{sen}\theta_t\right) e^{-j\beta_2(x\,\mathrm{sen}\theta_t + z\cos\theta_t)}$$

APLICANDO LAS CONDICIONES DE CONTORNO

$$E_{tang_1} = E_{tang_2} \quad \text{EN } Z = 0.$$

$$H_{tang_1} = H_{tang_2}$$

$$E_{i1} e^{-j\beta_1 x\,\mathrm{sen}\theta_i} + E_{r1} e^{-j\beta_1 x\,\mathrm{sen}\theta_i} = E_{t2} e^{-j\beta_2 x\,\mathrm{sen}\theta_t}$$

$$-\frac{E_{i1}}{Z_1}\cos\theta_i e^{-j\beta_1 x\,\mathrm{sen}\theta_i} + \frac{E_{r1}}{Z_1}\cos\theta_i e^{-j\beta_1 x\,\mathrm{sen}\theta_i} = \frac{E_{t2}}{Z_2}(-\cos\theta_t) e^{-j\beta_2 x\,\mathrm{sen}\theta_t}$$

SE DEBEN SATISFACER LAS SIG. CONDICIONES:

$$\begin{cases} e^{-j\beta_1 x \operatorname{sen}\theta_i} = e^{-j\beta_2 x \operatorname{sen}\theta_t} \quad ① \Rightarrow \beta_1 \operatorname{sen}\theta_i = \beta_2 \operatorname{sen}\theta_t \\ E_{i1} + E_{r1} = E_{t2} \quad ② \\ -\dfrac{E_{i1}}{Z_1}\cos\theta_i + \dfrac{E_{r1}}{Z_1}\cos\theta_i = -\dfrac{E_{t2}}{Z_2}\cos\theta_t \quad ③ \end{cases}$$

TOMANDO LAS DOS ÚLTIMAS, ③ $\times \dfrac{Z_2}{\cos\theta_t}$

$$\begin{cases} E_{i1} + E_{r1} = E_{t2} \\ -\dfrac{E_{i1}}{Z_1}\dfrac{\cos\theta_i}{\cos\theta_t} Z_2 + \dfrac{E_{r1}}{Z_1}\dfrac{\cos\theta_i}{\cos\theta_t} Z_2 = -E_{t2} \end{cases}$$

SUMANDO

$$E_{i1}\left(1 - \frac{\cos\theta_i}{\cos\theta_t}\frac{Z_2}{Z_1}\right) + E_{r1}\left(1 + \frac{\cos\theta_i}{\cos\theta_t}\frac{Z_2}{Z_1}\right) = 0$$

$$E_{r1} = E_{i1}\left(\frac{-Z_1\cos\theta_t + Z_2\cos\theta_i}{Z_1\cos\theta_t + Z_2\cos\theta_i}\right)$$

POR LO TANTO:

$$\boxed{\Gamma_\perp = \frac{E_{r1}}{E_{i1}} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_1\cos\theta_t + Z_2\cos\theta_i}}$$

AHORA SE HACE ③ $\times Z_1/\cos\theta_i$

$$\begin{cases} E_{i1} + E_{r1} = E_{t2} \\ -E_{i1} + E_{r1} = -E_{t2}\dfrac{\cos\theta_t}{\cos\theta_i}\dfrac{Z_1}{Z_2} \end{cases}$$

RESTANDO SE OBTIENE.

$$E_{i1}\cdot 2 = E_{t2}\left(1+\frac{\cos\theta_t}{\cos\theta_i}\frac{Z_1}{Z_2}\right)$$

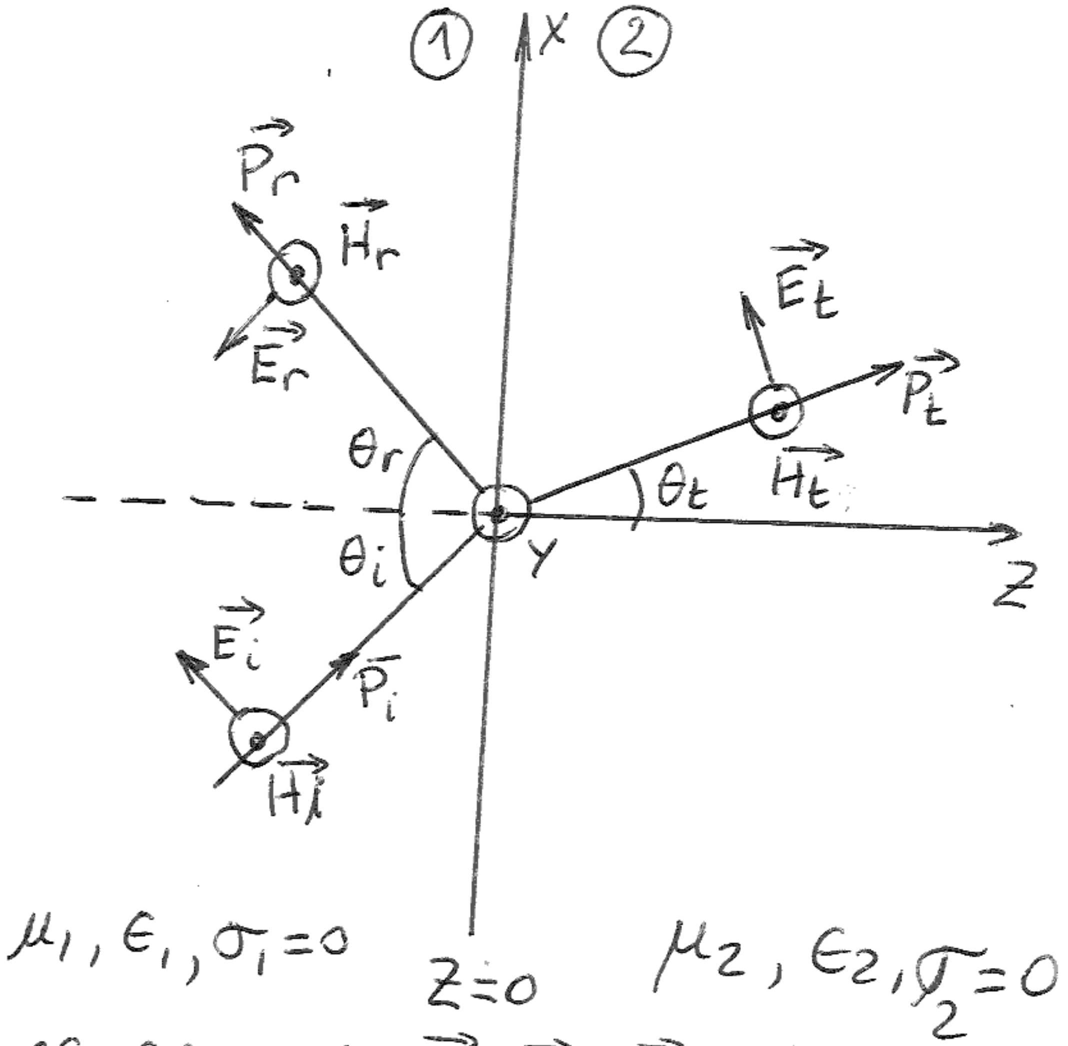
$$E_{t2} = \frac{2E_{i1}}{1+\frac{\cos\theta_t}{\cos\theta_i}\frac{Z_1}{Z_2}}$$

POR LO TANTO

$$\boxed{T_{\perp}=\frac{E_{t2}}{E_{i1}}=\frac{2\,Z_2\cos\theta_i}{Z_2\cos\theta_i+Z_1\cos\theta_t}}$$

$\Gamma_{\perp}$ Y $T_{\perp}$ SON LOS COEFICIENTES DE REFLEXIÓN Y TRANSMISIÓN RESPECTIVAMENTE

POLARIZACION PARALELA



LOS CAMPOS $\vec{E_i}, \vec{H_i}, \vec{E_r}$ Y $\vec{H_r}$ SON LOS MISMOS QUE SE HAN VISTO EN POL. PARA. DIEL. PERF._ COND. PERF.

$$\vec{E_i} = E_{i1}\left(\hat{x}\cos\theta_i - \hat{z}\,\text{sen}\,\theta_i\right) e^{-j\beta_1(x\,\text{sen}\,\theta_i + z\cos\theta_i)}$$

$$\vec{H_i} = \hat{y}\,\frac{E_{i1}}{Z_1}\cdot e^{-j\beta_1(x\,\text{sen}\,\theta_i + z\cos\theta_i)}$$

$$\vec{E_r} = E_{r1}\left(-\hat{x}\cos\theta_i - \hat{z}\,\text{sen}\,\theta_i\right) e^{-j\beta_1(x\,\text{sen}\,\theta_i - z\cos\theta_i)}$$

$$\vec{H_r} = \hat{y}\,\frac{E_{r1}}{Z_1}\, e^{-j\beta_1(x\,\text{sen}\,\theta_i - z\cos\theta_i)}$$

$$\vec{E_t} = E_{t1}\left(\hat{x}\cos\theta_t - \hat{z}\,\mathrm{sen}\,\theta_t\right) e^{-j\beta_2(x\,\mathrm{sen}\,\theta_t + z\cos\theta_t)}$$

$$\vec{H_t} = \hat{y}\,\frac{E_{t1}}{Z_2}\, e^{-j\beta_2(x\,\mathrm{sen}\,\theta_t + z\cos\theta_t)}$$

LOS CAMPOS TRANSMITIDOS TIENEN LA MISMA FORMA QUE LOS INCIDENTES

APLICANDO LAS CONDICIONES DE CONTORNO

$z=0$ $\quad E_{tang_1} = E_{tang_2}$

$\quad H_{tang_1} = H_{tang_2}$

$$E_{i1}\cos\theta_i\, e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} - E_{r1}\cos\theta_i\, e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} =$$

$$E_{t1}\cos\theta_t\, e^{-j\beta_2 x\,\mathrm{sen}\,\theta_t}$$

$$\frac{E_{i1}}{Z_1} e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} + \frac{E_{r1}}{Z_1} e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} = \frac{E_{t1}}{Z_2} e^{-j\beta_2 x\,\mathrm{sen}\,\theta_t}$$

$$\begin{cases} E_{i1}\cos\theta_i - E_{r1}\cos\theta_i = E_{t1}\cos\theta_t & (1) \\ \dfrac{E_{i1}}{Z_1} + \dfrac{E_{r1}}{Z_1} = \dfrac{E_{t1}}{Z_2} & (2) \end{cases}$$

① $\times 1/\cos\theta_t$ Y ② $\times Z_2$

$$\begin{cases} E_{i1}\dfrac{\cos\theta_i}{\cos\theta_t} - E_{r1}\dfrac{\cos\theta_i}{\cos\theta_t} = E_{t1} \\ E_{i1}\dfrac{Z_2}{Z_1} + E_{r1}\dfrac{Z_2}{Z_1} = E_{t1} \end{cases}$$

RESTANDO

$$E_{i1}\left(\frac{\cos\theta_i}{\cos\theta_t} - \frac{Z_2}{Z_1}\right) - E_{r1}\left(\frac{\cos\theta_i}{\cos\theta_t} + \frac{Z_2}{Z_1}\right) = 0.$$

$$E_{r1} = E_{i1}\,\frac{Z_1\cos\theta_i - Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t}$$

POR LO TANTO EL COEF. DE REFLEXIÓN ES:

$$\Gamma_{//} = \frac{-E_{r1}}{E_{i1}} = \frac{-Z_1\cos\theta_i + Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t}$$

SE DEFINIÓ CON MENOS PORQUE E_{i1} Y E_{r1} QUEDARON EN DIRECCIONES OPUESTAS.

EL COEF. DE TRANSMISIÓN RESULTA:

$$T_{//} = \frac{E_{t1}}{E_{i1}} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_t + Z_1\cos\theta_i}$$

ANGULO DE BREWSTER

ES EL ÁNGULO DE REFLEXIÓN NULA

POL. PARALELA

$\Gamma_{//} = 0 \Rightarrow Z_2 \cos\theta_t = Z_1 \cos\theta_i$

COMO $\quad \beta_2 \operatorname{sen}\theta_t = \beta_1 \operatorname{sen}\theta_i \quad$ (SNELL)

$$\operatorname{sen}\theta_t = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\operatorname{sen}\theta_i$$

$$Z_2 \cos\theta_t = Z_1 \cos\theta_i$$

$$Z_2\sqrt{1-\operatorname{sen}^2\theta_t} = Z_1\sqrt{1-\operatorname{sen}^2\theta_i}$$

$$Z_2\sqrt{1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\operatorname{sen}^2\theta_i} = Z_1\sqrt{1-\operatorname{sen}^2\theta_i}$$

$$\left(\cancel{\sqrt{\frac{\mu_2}{\epsilon_2}}}\right)^{\cancel{2}}\left(1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\operatorname{sen}^2\theta_i\right) = \left(\cancel{\sqrt{\frac{\mu_1}{\epsilon_1}}}\right)^{\cancel{2}}\left(1-\operatorname{sen}^2\theta_i\right)$$

$$\frac{\mu_2}{\epsilon_2} - \frac{\mu_1\epsilon_1}{\epsilon_2^2}\operatorname{sen}^2\theta_i = \frac{\mu_1}{\epsilon_1} - \frac{\mu_1}{\epsilon_1}\operatorname{sen}^2\theta_i$$

$$\frac{\mu_2}{\epsilon_2} - \frac{\mu_1}{\epsilon_1} = \operatorname{sen}^2\theta_i\left(\frac{\mu_1\epsilon_1}{\epsilon_2^2} - \frac{\mu_1}{\epsilon_1}\right)$$

$$\sqrt{\frac{\dfrac{\mu_2}{\epsilon_2}-\dfrac{\mu_1}{\epsilon_1}}{\dfrac{\mu_1\epsilon_1}{\epsilon_2^2}-\dfrac{\mu_1}{\epsilon_1}}} = \operatorname{sen}\theta_i$$

$$\sqrt{\frac{\dfrac{\mu_2\epsilon_1-\mu_1\epsilon_2}{\epsilon_1\epsilon_2}}{\dfrac{\mu_1\epsilon_1^2-\mu_1\epsilon_2^2}{\epsilon_1\epsilon_2^2}}} = \operatorname{sen}\theta_i$$

$$\sqrt{\frac{\dfrac{\mu_2\epsilon_1-\mu_1\epsilon_2}{\cancel{\epsilon_1}\epsilon_2}}{\dfrac{\mu_1(\epsilon_1^2-\epsilon_2^2)}{\cancel{\epsilon_1}\epsilon_2\epsilon_2}}} = \operatorname{sen}\theta_i$$

$$\sqrt{\frac{\epsilon_2(\mu_2\epsilon_1-\mu_1\epsilon_2)}{\mu_1(\epsilon_1^2-\epsilon_2^2)}} = \operatorname{sen}\theta_i = \operatorname{sen}\theta_{Brewster}$$

si $\mu_1=\mu_2=\mu_0$.

$$\boxed{\sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}} = \operatorname{sen}\theta_B}$$

ES EL CASO MAS COMÚN.

POL. PERP.

$\Gamma_{\perp} = 0 \Rightarrow Z_2 \cos\theta_i - Z_1 \cos\theta_t$

SE LLEGA A OBTENER:

$$\sqrt{\frac{\mu_2(\epsilon_1\mu_2 - \epsilon_2\mu_1)}{\epsilon_1(\mu_2^2 - \mu_1^2)}} = \operatorname{sen}\theta_B$$

si $\mu_1 = \mu_2 \rightarrow$ NO TIENE SOLUCIÓN

PERO SI $\epsilon_1 = \epsilon_2$ y $\mu_1 \neq \mu_2$.

$$\boxed{\sqrt{\frac{\mu_2}{\mu_1 + \mu_2}} = \operatorname{sen}\theta_B.}$$

CASO POCO COMUNMENTE ENCONTRADO

MUCHAS VECES SE DICE QUE EL ÁNGULO DE BREWSTER ESTÁ ASOCIADO A LA POL. PARALELA.

ÁNGULO DE REFLEXIÓN TOTAL

$\Gamma_{\perp} = 1 \ , \ \Gamma_{\parallel} = -1$

SE DEFINE LA REF. TOTAL A PARTIR DE LA LEY DE LA REFRACCIÓN:

$$\beta_2 \operatorname{sen}\theta_t = \beta_1 \operatorname{sen}\theta_i$$

$$\operatorname{sen}\theta_t = \frac{\beta_1}{\beta_2} \operatorname{sen}\theta_i = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \operatorname{sen}\theta_i$$

si $\theta_t = \pi/2$

$$\boxed{\operatorname{sen}\theta_i = \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}}$$

PARA $\mu_1 \epsilon_1 \geq \mu_2 \epsilon_2$

SE PUEDE OBSERVAR QUE EL ÁNGULO CRÍTICO ES INDEPENDIENTE DE LA POL.

SI SE DESEA QUE LA REFLEXIÓN SEA NULA EN CASO DE POLARIZACIÓN PERPENDICULAR DIELECT. PERF (1) Y DIEL. PERF (2) NO EXISTIRÁ EL ÁNGULO θ_i.

SIN EMBARGO PARA "POL. PARALELA."

$\Gamma_{\parallel} = 0$

$Z_2 \cos\theta_t = Z_1 \cos\theta_i \Rightarrow \boxed{\theta_i = \operatorname{arctg}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$

θ_i SE DENOMINA "ÁNGULO DE BREWSTER"

ESTE TEMA SE PUEDE VER EN LA PAG. 363 DEL LIBRO DE INGENIERÍA ELECTROMAGNÉTICA TOMO II

EL COEFICIENTE DE REFLEXIÓN