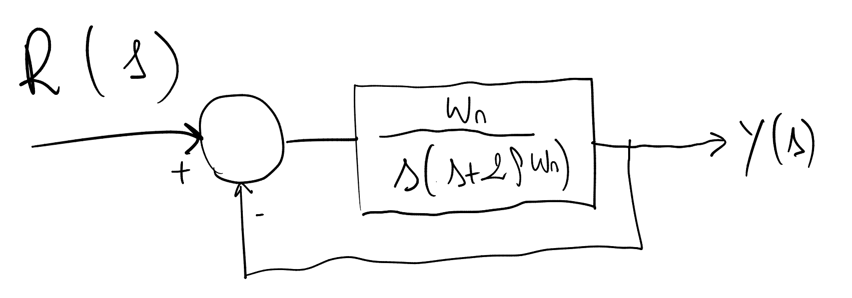
Sistemas de 2° Orden



ρ = coef. de amortiguamiento $\qquad W_n$ = frec. nat de oscilacione.

Le puse ρ pero ni puta idea de cual es. debería ser una ζ

$$G = \frac{W_n^2}{s(s+2\rho W_n)} \qquad \frac{Y}{R} = \frac{G}{1+GH}$$

$H=1$

$$\frac{Y}{R(s)} = \frac{W_n^2}{s^2+2\rho W_n s + W_n^2}$$

$$P_{1,2} = -\rho W_n \pm j W_n\sqrt{1-\rho^2}$$

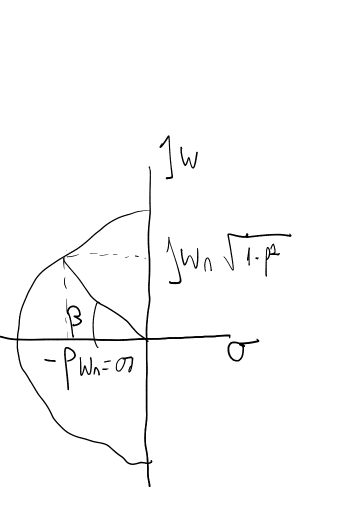
$0<\rho<1$

$$\beta = \text{ATAN}\left(\frac{\cancel{W_n}\sqrt{1-\rho^2}}{\rho \cancel{W_n}}\right)$$

$$\beta = \text{ATAN}\left(\frac{\sqrt{1-\rho^2}}{\rho}\right)$$

$$\beta = \text{ACOS}\left(\frac{\rho W_n}{W_n}\right)$$

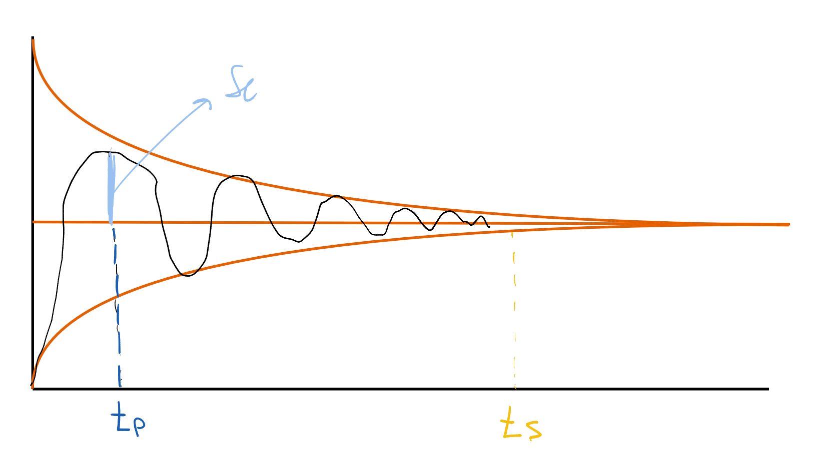
$$\beta = \text{ACOS}(\rho)$$



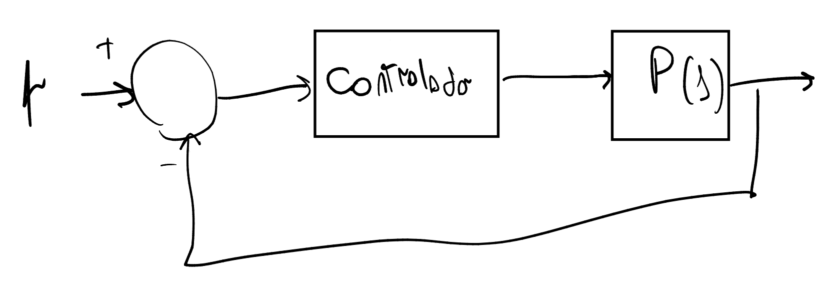
Con $\rho=0 \qquad p_{1,2} = \pm j W_n$

Con $\rho=1 \qquad p_{1,2} = -W_n$

(Ahora hizo la antitransf de esta transferencia)
(Y la grafica) (La genérica)



$$t_p = \frac{\pi}{W_d} \qquad S_c = e^{-\frac{\sigma_d \pi}{W_d}} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \qquad t_s = \frac{4}{\sigma_d}$$



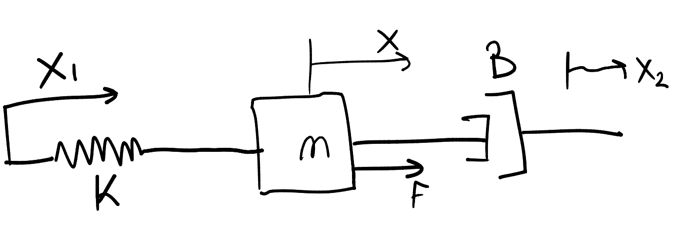
Si P(s) tiene 5 polos el controlador tiene que tener 5 polos también, pero yo uso una señal senoidal amortiguada, con el método anterior puedo encontrar 2 polos. Los otros 3 polos los defino alejados (aprox 5 veces más en módulo)

Sistemas de Traslación

Consideramos los cuerpos como si fueran puntuales.

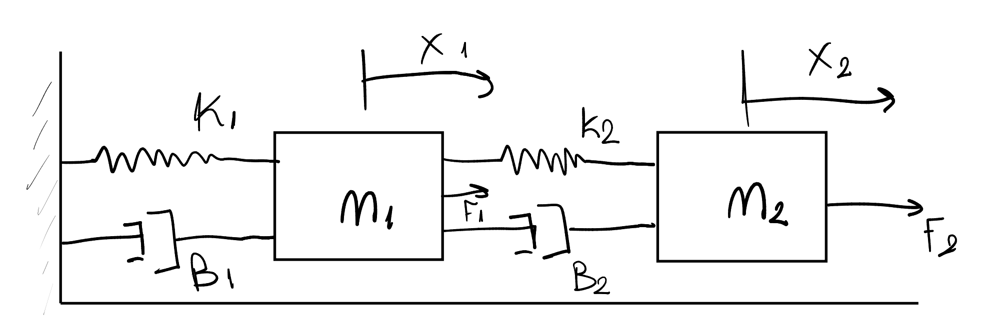
$$\sum F = m\frac{dV_p}{dt}$$

$$\sum F^{ext} - \sum F^{\text{Relación de vínculo}} = m\ddot{x}$$



$$F - K(X-X_1) - B(\dot{X}-\dot{X}_2) = m\ddot{X} \quad (1)$$

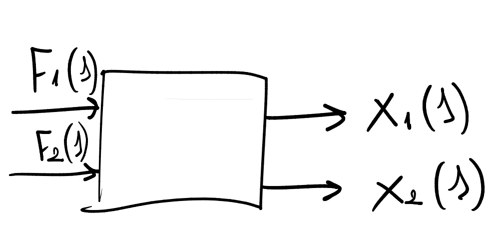
Incognitas
$x(t) \quad x_1(t) \quad x_2(t)$



$$\begin{cases} F_1 - K_1(x_1-0) - B_1(\dot{x}_1-0) - K_2(x_1-x_2) - B_2(\dot{x}_1-\dot{x}_2) = m_1\ddot{x}_1 \\ F_2 - K_2(x_2-x_1) - B_2(\dot{x}_2-\dot{x}_1) = m_2\ddot{x}_1 \end{cases}$$

$$\begin{cases} F_1 + K_2 x_2 + B_2 x_2 = (K_1+K_2)x_1 + (B_1+B_2)\dot{x}_1 + m_1\ddot{x}_1 & (1) \\ F_2 + K_2 x_1 + B_2\dot{x}_1 = K_2 x_2 + B_2\dot{x}_2 + m_2\ddot{x}_2 & (2) \end{cases}$$

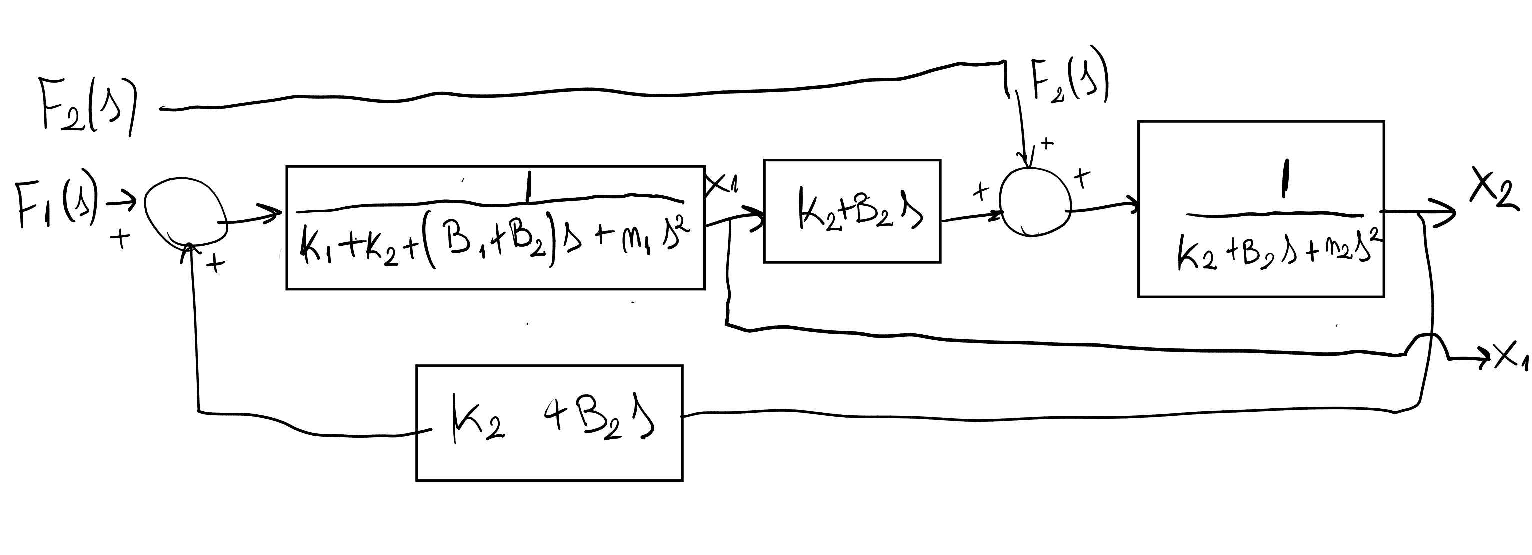
$$\begin{cases} F_1(s) + (K_2+B_2 s)X_2(s) = \left[(K_1+K_2)+(B_1+B_2)s + m_1 s^2\right]X_1(s) & (\text{I}) \\ F_2(s)\,(K_2+B_2 s)X_1(s) = (K_2+B_2 s+m_2 s^2)X_2(s) & (\text{II}) \end{cases}$$



$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}}_{\mathcal{H}(s)} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix}$$

$$X_1(s) = G_{11}(s)F_1(s) + G_{12}(s)F_2(s)$$
$$X_2(s) = G_{21}(s)F_1(s) + G_{22}(s)F_2(s)$$

$$G_{12}(s) = \left.\frac{X_1(s)}{F_2(s)}\right|_{F_1=0}$$

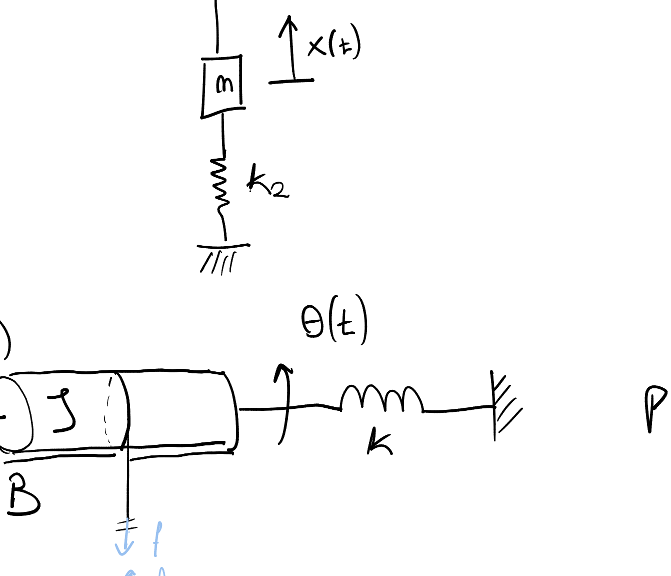


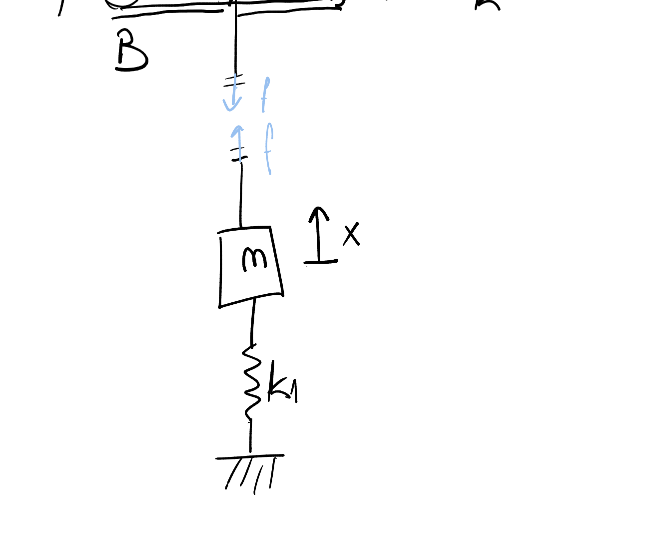
$$\tau = J\ddot{\theta} \qquad \sum \tau^{ext} - \sum \tau_0 = J\ddot{\theta}$$

$$\tau - k(\theta-\theta_1) = J\ddot{\theta}$$

$$\tau - k(\theta-\theta_1) - B(\dot{\theta}-\dot{\theta}_2) = J\ddot{\theta}$$

3 Incognitas $\theta, \theta_1(t), \theta_2(t)$
1 ec.





Parte Rotacional

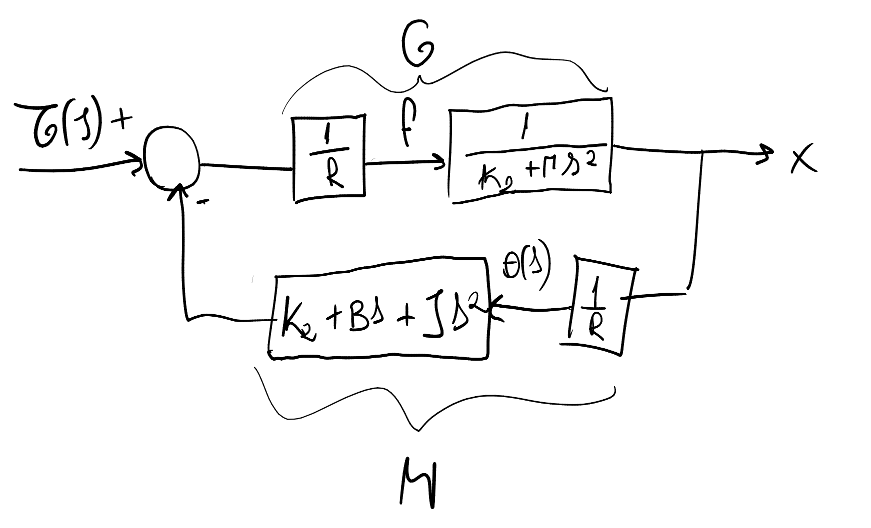
Parte traslacional

$$\begin{cases} \tau(t) - K_1\theta(t) - B\dot{\theta}(t) - Rf = J\ddot{\theta} \end{cases}$$

$$\left\{ f - K_2 x = m\ddot{x}(t) \right.$$

$$\begin{cases} \tau(t) - Rf(t) = K_1\theta(t) + B\dot{\theta}(t) + J\ddot{\theta}(t) & (1) \\ f(t) = k_2 x(t) + m\ddot{x}(t) & (2) \end{cases}$$

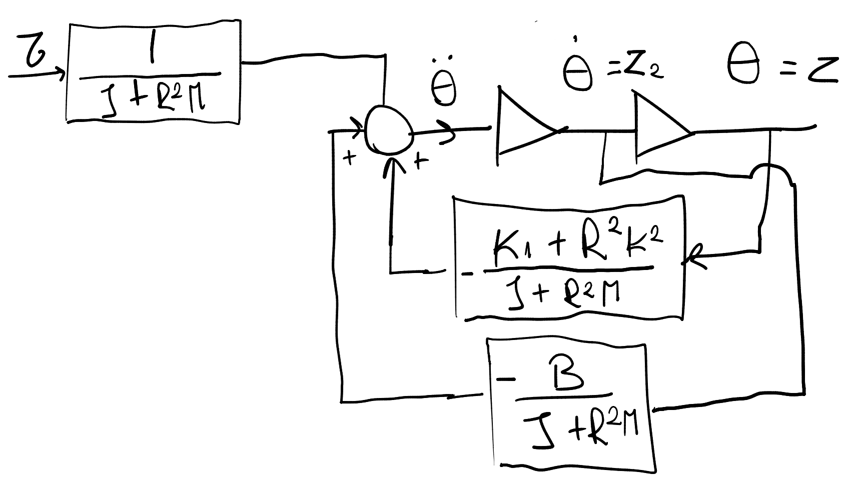
$$\begin{cases} \tau(s) - Rf(s) = (K_1 + Bs + Js^2)\,\theta(s) & (\mathrm{I}) \\ f(s) = (K_2 + Ms^2)\,X(s) & (\mathrm{II}) \end{cases}$$



$$\tau(t) = R(K_2 x + M\ddot{x}) + K_1\theta + B\dot{\theta} + J\ddot{\theta} \qquad x = R\theta$$

$$\tau(t) = R^2 K_2\theta + R^2 M\ddot{\theta} + K_1\theta + B\dot{\theta} + J\ddot{\theta}$$

$$\tau(t) = (K_1 + R^2 K_2)\,\theta(t) + B\dot{\theta}(t) + (J + R^2 M)\,\ddot{\theta}(t)$$



$$Z_1 = \theta \qquad Z_2 = \dot{\theta}$$

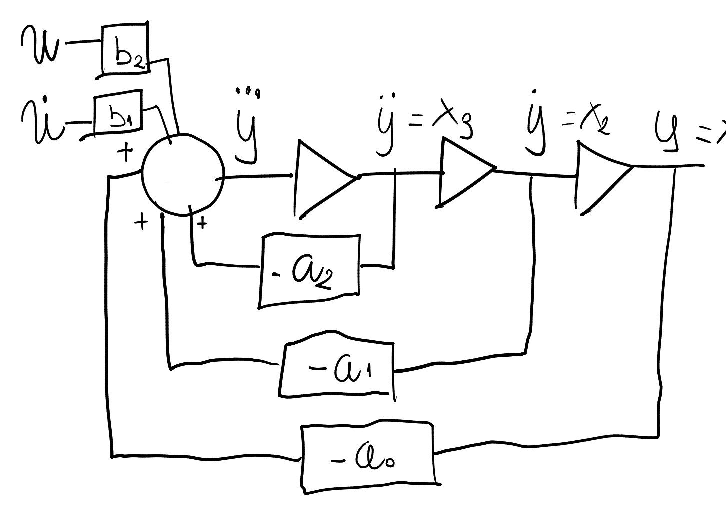
$$\dot{Z}_1 = \dot{\theta} = Z_2 \Rightarrow \boxed{\dot{Z}_1 = Z_2}$$

$$\dot{Z}_2 = \ddot{\theta} = \frac{-(K_1 + R^2 k_2)}{J + R^2 M} Z_1 - \frac{B}{J + R^2 M} Z_2 + \left(\frac{1}{J + R^2 M}\right) u$$

$$y = \theta = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \underbrace{0}_{D} u$$

A y B me da paja copiarlas.

$$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_0 u(t) + b_1\dot{u}(t)$$



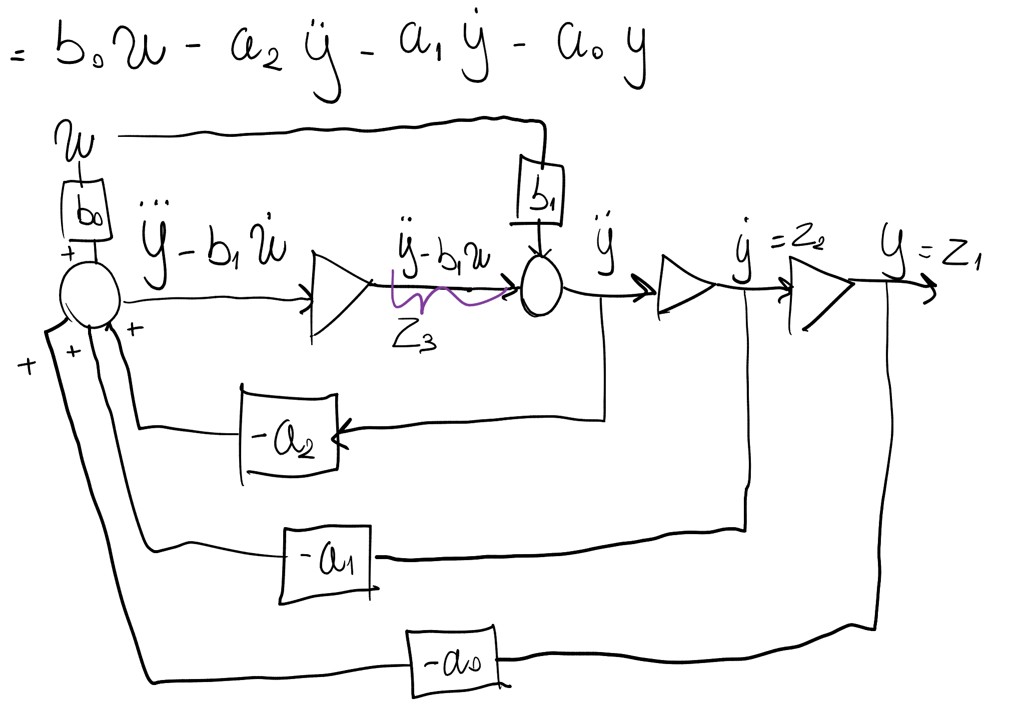
$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -a_2 x_3 - a_1 x_2 - a_0 x_1 + b_0 u + b_1\dot{u}$$

No puede depender de la derivada de la entrada

$$\dddot{y} - b_1\dot{u} = b_0 u - a_2\ddot{y} - a_1\dot{y} - a_0 y$$



$$\dot{Z}_1 = Z_2$$

$$\dot{Z}_2 = Z_3 + b_1 u$$

$$\dot{Z}_3 = -a_2(Z_3 + b_1 u) - a_1 Z_2 - a_0 Z_1 + b_0 u$$

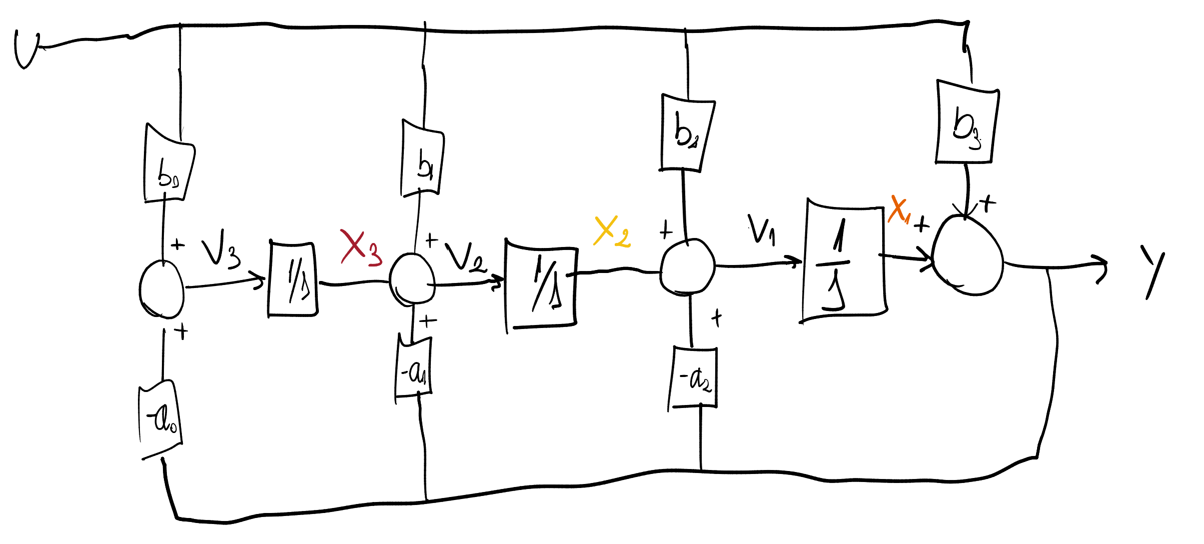
$$\dot{Z}_3 = -a_2 Z_3 - a_1 Z_2 - a_0 Z_1 + (b_0 - a_2 b_1) u$$

$$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_3\dddot{u} + b_2\ddot{u} + b_1\dot{u} + b_0 u$$

$$s^3 Y + a_2 s^2 Y + a_1 s Y + a_0 Y = b_3 s^3 U + b_2 s^2 U + b_1 s U + b_0 U$$

$$s^3 Y = s^3 b_3 U + s^2(b_2 U - a_2 Y) + s(b_1 U - a_1 Y) + (b_0 U - a_0 Y)$$

$$Y = b_3 U + \frac{1}{s}\underbrace{(b_2 U - a_2 Y) + \frac{1}{s^2}\underbrace{(b_1 U - a_1 Y)\ \frac{1}{s^3}\underbrace{(b_0 U - a_0 Y)}_{V_3}}_{V_2}}_{V_1}$$



$$Y = X_1 + b_3 U$$

$$\dot{X}_1 = X_2 - a_2(X_1 + b_2 U) + b_2 U$$

$$\dot{X}_1 = -a_2 X_1 + X_2 + (b_2 - a_2 b_3) U$$

$$\dot{X}_2 = -a_1(X_1 + b_2 U) + X_3 + b_1 U$$

$$\dot{X}_2 = X_3 - a_1 X_1 + (b_1 - a_1 b_3) U$$

$$\dot{X}_3 = -a_0(X_1 + b_2 U) + b_0 U$$

$$\dot{X} = -a_0 X_1 + (b_0 - a_0 b_3) U$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -a_2 & 1 & 0 \\ -a_1 & 0 & 1 \\ -a_0 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_2 - a_2 b_3 \\ b_1 - a_1 b_3 \\ b_0 - a_0 b_3 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \underbrace{b_0}_{D} u$$