

SOLUTIONS MANUAL for INSTRUCTORS

**DEVICE ELECTRONICS for
INTEGRATED CIRCUITS**

THIRD EDITION

by

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This manual contains solutions for essentially all problems to DEVICE ELECTRONICS for INTEGRATED CIRCUITS, Third Edition. The only problems omitted from consideration are those few that call for an analysis that has already been outlined as part of the text material.

A number of the problems may prove useful for discussion by the instructor to reinforce and expand upon the main topics in the book. Problems in this category include:

- | | |
|------------|--|
| Chapter 1: | 1.3, 1.9, 1.12, 1.13, 1.16, 1.17, A1.1 |
| 2: | 2.3, 2.7, 2.9, 2.16, 2.20, 2.21 |
| 3: | 3.4, 3.7, 3.11, 3.12, 3.13, 3.17 |
| 4. | 4.5, 4.9, 4.10, 4.11 |
| 5. | 5.4, 5.7, 5.10, 5.14, 5.20, 5.24 |
| 6. | 6.3, 6.4, 6.5, 6.8, 6.14, 6.19 |
| 7. | 7.4, 7.6, 7.10, 7.12, 7.16, 7.30 |
| 8. | 8.5, 8.6, 8.14, 8.16 |
| 9. | 9.1, 9.4, 9.10, 9.13, 9.24, 9.25, 9.26 |
| 10. | 10.4, 10.6, 10.7, 10.9, 10.13 |

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CHAPTER 1

1.1

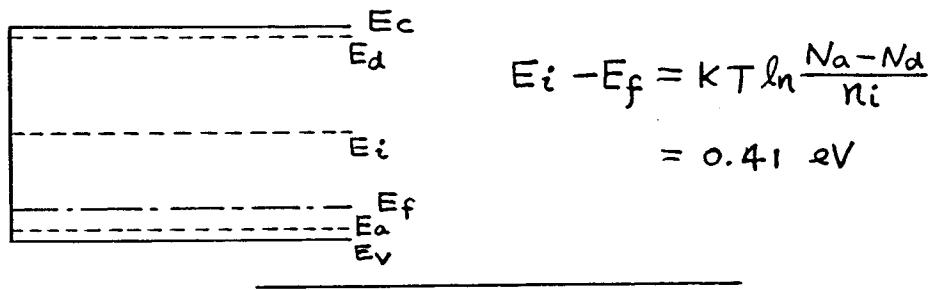
- (a) $n \approx N_d - N_a = 10^{16} \text{ cm}^{-3}$, $g = 1.6 \times 10^{-19} \text{ coul}$, from Table 1.1,
 $\mu_n = 1194 \text{ cm}^2/\text{volt}\cdot\text{sec}$ $\therefore \rho = (n g \mu_n)^{-1} = 0.52 \Omega\cdot\text{cm}$

(b)

	<u>atomic weight</u>	<u>density (cm⁻³)</u>
P	31	10^{16}
Si	28	5×10^{22} (from Table 1.3)
$\therefore P/Si$	$\frac{31 \times 10^{16}}{28 \times 5 \times 10^{22}} = 2.21 \times 10^{-7}$	

- (c) P-type $p \approx N_a - N_d = 9 \times 10^{16} \text{ cm}^{-3}$, $N_a + N_d = 1.1 \times 10^{17} \text{ cm}^{-3}$, from Fig. 1.16
 $\mu_p = 310 \text{ cm}^2/\text{volt}\cdot\text{sec}$ $\therefore \rho = (p g \mu_p)^{-1} = 0.22 \Omega\cdot\text{cm}$

(d)



1.2

- (a) $p \approx N_a - N_d = 10^{16} \text{ cm}^{-3}$, $n = n_i^2/p = 2.1 \times 10^4 \text{ cm}^{-3}$, from Eq.(1.1.27)
 $E_i - E_f = kT \ln \frac{P}{n_i} = 0.35 \text{ eV}$, the Fermi level is 0.35 eV below
 E_i , or $0.56 - 0.35 = 0.21 \text{ eV}$ above E_v the edge of the valence
band.

- (b) $n \approx N_d - N_a = 1 \times 10^{15} \text{ cm}^{-3}$, $p = n_i^2/n = 2.1 \times 10^5 \text{ cm}^{-3}$, from Eq.(1.1.26)
 $E_f - E_i = kT \ln \frac{n}{n_i} \approx 0.29 \text{ eV}$, or $E_c - E_f = 0.56 - 0.29 = 0.27 \text{ eV}$

1.3

Arsenic : 10^{16} cm^{-3} , $E_c - E_d = 0.049 \text{ eV}$

Boron : 10^{15} cm^{-3} , $E_a - E_v = 0.045 \text{ eV}$

At 300°K $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$, $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$

- (a) Eq. (1.1.21) can be written: $n = N'_c \left(\frac{T}{300}\right)^{3/2} \exp[-(E_c - E_f)/kT]$
where N'_c is the effective density of states at 300°K .
When $n = \frac{N_d}{2}$, $E_f \approx E_d$. With these substitutions, the equation

for n can be written: $\ln \frac{2N_c'}{N_d} + \frac{3}{2} \ln \frac{T}{300} = \frac{E_c - E_d}{KT}$

Rearranging, we get: $T = \frac{E_c - E_d}{K} \frac{1}{\ln \frac{2N_c'}{N_d} + \frac{3}{2} \ln \frac{T}{300}}$

Since the right-hand side is only a weak function of temperature, the solution may be obtained iteratively, starting with an arbitrary guess of 100°K for T.

$$\therefore T = \frac{0.049}{8.62 \times 10^{-5}} \frac{1}{\ln \frac{2 \times 2.8 \times 10^{19}}{1 \times 10^{16}} + \frac{3}{2} \ln \frac{T}{300}}$$

$$= \frac{568}{8.63 + \frac{3}{2} \ln \frac{T}{300}}$$

by $T_1 = 100$, $T_2 = 81.4$, $T_3 = 85.1$, $T_4 = 84.3$, $T_5 = 84.4$

$$\therefore T = 84.4 \text{ °K}$$

Similarly for boron: $T = \frac{E_a - E_v}{K} \frac{1}{\ln \frac{2N_v}{N_a} + \frac{3}{2} \ln \frac{T}{300}}$

$$\therefore T = \frac{0.045}{8.62 \times 10^{-5}} \frac{1}{\ln \frac{2 \times 1.04 \times 10^{19}}{10^{15}} + \frac{3}{2} \ln \frac{T}{300}}$$

$$= \frac{522}{9.94 + \frac{3}{2} \ln \frac{T}{300}}$$

by $T_1 = 100$, $T_2 = 63.0$, $T_3 = 68.7$, $T_4 = 67.5$, $T_5 = 67.8$, $T_6 = 67.7$

$$\therefore T = 67.7 \text{ °K}$$

(b) From Table 1.4, $n_i = 3.87 \times 10^{16} T^{3/2} \exp(-\frac{7014}{T})$

$$T = \frac{7014}{\ln \frac{3.87 \times 10^{16}}{n_i} + \frac{3}{2} \ln T}$$

Solve iteratively as in part (a); for $n_i = 10N_d = 10^{17} \text{ cm}^{-3}$

$$T = \frac{7014}{\frac{3}{2} \ln T - 0.95}$$

by $T_1 = 100$, $T_2 = 1177$, $T_3 = 726$, $T_4 = 785$, $T_5 = 775$, $T_6 = 777$

$$\therefore T = 777 \text{ °K} = 504 \text{ °C} \text{ for } n_i = 10 N_d$$

For $n_i = 10 N_d = 10^{16}$

$$T = \frac{7014}{\frac{3}{2} \ln T + 1.35} \quad \text{by } T_1 = 800, T_2 = 617, T_3 = 638, T_4 = 635$$

$$\therefore T = 635 \text{ °K} = 362 \text{ °C}$$

(c) Arsenic: $n \approx N_d - N_a = 10^{16} \text{ cm}^{-3} > n_i$, $P = \frac{n_i^2}{n} = 2.1 \times 10^4 \text{ cm}^{-3}$

Boron: $P \approx N_a - N_d = 10^{15} \text{ cm}^{-3} > n_i$, $n = n_i^2/P = 2.1 \times 10^5 \text{ cm}^{-3}$

(d) Arsenic: $E_f - E_i = KT \ln \frac{n}{n_i} = 0.35 \text{ eV}$, $E_i - E_v = 0.56 \text{ eV}$

$$\therefore E_f - E_v = 0.91 \text{ eV}$$

$$\text{Boron : } E_i - E_f = KT \ln \frac{P}{n_i} = 0.29 \text{ eV}, E_i - E_v = 0.56 \text{ eV}, E_f - E_v = 0.27 \text{ eV}$$

$$\text{Arsenic + Boron : } n \approx N_d - N_a = 9 \times 10^{15} \text{ cm}^{-3}, E_f - E_i = KT \ln \frac{n}{n_i} = 0.347 \text{ eV}$$

$$\therefore E_f - E_v = 0.907 \text{ eV}$$

1.4

From Fig. 1.15, 5 Ω-cm, $n \Rightarrow N_d = 9 \times 10^{14} \text{ cm}^{-3}$. $Pn = n_i^2$, at 27°C (300 K) $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$
and $P = \frac{(1.45 \times 10^{10})^2}{9 \times 10^{14}} \approx 2.3 \times 10^5 \text{ cm}^{-3}$

For 100°C, from Table 1.4,

$$n_i = 3.87 \times 10^{16} (373)^{3/2} \exp(-7014/373) = 1.9 \times 10^{12} \text{ cm}^{-3}$$

$$\text{Since } N_d \gg n_i, n = N_d \approx 9 \times 10^{14} \text{ cm}^{-3} \text{ and } P = n_i^2/n = 4.0 \times 10^9 \text{ cm}^{-3}$$

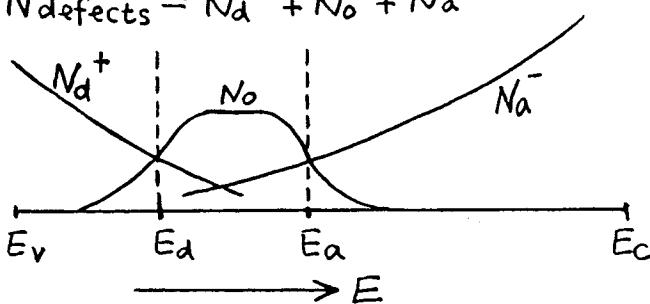
For 500°C, from Table 1.4,

$$n_i = 3.87 \times 10^{16} (773)^{3/2} \exp(-7014/773) = 9.5 \times 10^{16} \text{ cm}^{-3}$$

$$\text{Since } n_i \gg N_d, P \approx n_i = 9.5 \times 10^{16} \text{ cm}^{-3}$$

1.5

$$(a) N_{\text{defects}} = N_d^+ + N_o + N_a^-$$



N_d^+ dominates in heavily doped p-type material.

N_a^- dominates in n-type material.

$$(b) P - n + N_d^+ + N_a^- - N_d^{\text{deep}} - N_a^{\text{deep}} = 0 \text{ for charge neutrality}$$

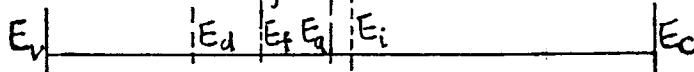
In n-type material $n + N_a^- \approx N_d^+$, so increasing N_a^- decreases n . Similarly, $P + N_d^{\text{deep}} \approx N_a^-$, increasing N_d^{deep} in heavily doped p-type material decreases P .

(c) E_f should be between E_a and E_d for charge neutrality, ($P - n = N_a^- - N_d^+$), Since E_f is below the middle of bandgap, the sample is p-type.

$$\text{However } P \text{ and } n \ll N_{\text{defect}} \text{ so } N_a^- \approx N_d^+ \text{ and } N_{\text{defect}} e^{\frac{E_f - E_a}{KT}} \approx N_{\text{defect}} e^{\frac{E_d - E_f}{KT}}$$

$$\text{so that } E_f \approx \frac{E_d + E_a}{2}, E_f - E_v = \frac{E_d - E_v}{2} + \frac{E_a - E_v}{2} = \frac{0.27 + 0.51}{2} = 0.39 \text{ eV}$$

The sample is p-type and the Fermi level is approximately midway between the defect levels.



(d) Since $N_{\text{shallow}} > N_{\text{defects}}$, the Fermi level is above E_i for this n-type sample and \bar{N}_A dominates the charge state of the defects. ($\bar{N}_A \approx N_{\text{defects}}$)
 $\therefore n-p \approx N_d \text{ phos} - \bar{N}_A = 2 \times 10^{17} - 5 \times 10^{16} = 1.5 \times 10^{17} \text{ cm}^{-3}$
 Since $n-p > n_i \therefore n \approx 1.5 \times 10^{17} \text{ cm}^{-3}$,
 $p = \frac{n_i^2}{n} = \frac{(1.45 \times 10^{10})^2}{1.5 \times 10^{17}} \approx 1.40 \times 10^3 \text{ cm}^{-3}$
 $E_c - E_f = kT \ln \frac{N_c}{n} = 0.026 \text{ eV} \ln \frac{2.8 \times 10^{19}}{1.5 \times 10^{17}} \approx 0.136 \text{ eV}$

1.6

$$\mu_1 = 800 \text{ cm}^2/\text{V-sec}, \mu_2 = 200 \text{ cm}^2/\text{V-sec}$$

$$\frac{1}{\mu_T} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{800} + \frac{1}{200} \therefore \mu_T = 160 \text{ cm}^2/\text{V-sec}$$

1.7

The number of atoms per unit weight is N_0/A , where N_0 is Avogadro's number and A is the atomic weight. If ρ_m is the mass density, the number of atoms per unit volume is

$N = \rho_m N_0 / A$. If each atom contributes Z valence electrons, the electron concentration is $n = Z \rho_m N_0 / A$

with $Z=0.9$, $\rho_m=2.7 \text{ g/cm}^3$, $N_0=6.022 \times 10^{23} (\text{g-mole})^{-1}$, $A=27$
 then $n=5.42 \times 10^{22} \text{ electrons/cm}^3$

$$\mu_n = 1/(pnq) \text{ with } p=2.8 \times 10^{-6} \text{ sc-cm}, \mu_n = 41.2 \text{ cm}^2/\text{V-sec}$$

$$\tau = \frac{m^* \mu_n}{q} = \frac{(9.11 \times 10^{-31} \text{ kg})(41.2 \times 10^{-4} \text{ m}^2/\text{V-sec})}{1.6 \times 10^{-19} \text{ coul}} = 4.12 \times 10^{-3} \text{ m}^2/\text{V-sec}$$

For silicon $m^*=0.26 m_0$ (conductivity effective mass)

$$\tau = \frac{m^* \mu_n}{q} = \frac{0.26 \times (9.11 \times 10^{-31} \text{ kg})(1420 \times 10^{-4} \text{ m}^2/\text{V-sec})}{1.6 \times 10^{-19} \text{ coul}} = 2.00 \times 10^{-13} \text{ sec}$$

$$\therefore \tau_{\text{Si}} = 8.51 \tau_{\text{Ar}}$$

1.8

$$V_{\text{th}} \approx 2.3 \times 10^7 \text{ cm/sec} \therefore V_d \approx 2.3 \times 10^6 \text{ cm/sec}$$

The electron travels a distance L in a time $T = L/V_d$
 If τ is the average time between collisions, the average number of collisions in traveling the distance L is $C = \frac{T}{\tau} = L/\tau V_d$
 With $L=1 \mu\text{m}$ and $\tau = \frac{m^* \mu_n}{q} = 2.1 \times 10^{-13} \text{ sec}$, then $C \approx 207$ collisions

The applied voltage is $V_a = \mathcal{E}L$, with $\mu_n = 1417 \text{ cm}^2/\text{V-sec}$, then
 $\mathcal{E} = \frac{V_a}{\mu_n} = 1623 \text{ Volts/cm}$ and $V_a = 162 \text{ mV}$.

1.9

The sample is biased in the intermediate region of the velocity - field curve where the velocity is not a linear function of the field but is approaching its saturation velocity. Thus, doubling the field results in less than a doubling of velocity and, hence, of current.

1.10

$$\mu_n = 1000 \text{ cm}^2/\text{V-sec}, D_n = \left(\frac{kT}{q}\right) \mu_n = 25.8 \text{ cm}^2 \text{ s}^{-1}$$

$$\frac{dn}{dx} = -\frac{10^{17} \text{ cm}^{-3} - 6 \times 10^{16} \text{ cm}^{-3}}{2 \times 10^{-4} \text{ cm}} = -2 \times 10^{20} \text{ cm}^{-4}$$

The electron diffusion current density is

$$J_n = q D_n \frac{dn}{dx} = -825.6 \text{ amps/cm}^2$$

1.11

(a) Si replacing Ga \Rightarrow extra electron \Rightarrow donor $N_d = 0.05 \times 10^{10} = 5 \times 10^8 / \text{cm}^3$
 Si replacing As \Rightarrow one less electron \Rightarrow acceptor $N_a = 0.95 \times 10^9 = 9.5 \times 10^9 / \text{cm}^3$

(b) $p \approx N_a - N_d = (9.5 - 0.5) \times 10^9 = 9.0 \times 10^9 / \text{cm}^3 \gg n_i = 9.0 \times 10^6$ for GaAs @ 300 K.

$$n = \frac{n_i^2}{p} = \frac{81 \times 10^{12}}{9 \times 10^9} = 9 \times 10^3 / \text{cm}^3$$

$$\therefore E_f - E_v = kT \ln \frac{N_v}{p} = kT \ln \frac{7 \times 10^{18}}{9 \times 10^9} = 0.026 \times 20.5 = 0.53 \text{ eV}$$

$\therefore E_f = 0.53 \text{ eV above } E_v$.

$$(c) \sigma = n q \mu_n + p q \mu_p = (9 \times 10^3)(1.6 \times 10^{-19})(8800) + (9 \times 10^9)(1.6 \times 10^{-19})(400)$$

$$\approx p q \mu_p = 5.8 \times 10^{-7} / \Omega \cdot \text{cm}$$

1.12

$$(a) -\frac{dp}{dt} = \nabla \cdot J = \nabla \cdot \sigma \mathcal{E} = \frac{\sigma}{\epsilon} \nabla \cdot D = \frac{\sigma}{\epsilon} p \quad \therefore \frac{dp}{p} = -\frac{\sigma}{\epsilon} dt$$

$$\ln p - \ln p_0 = -\frac{\sigma}{\epsilon} t \quad \therefore p = p_0 e^{-t/(\frac{\sigma}{\epsilon})} = p_0 e^{-t/\tau_{rel}}$$

$$(b) \text{Intrinsic Si : } \sigma = q n_i (\mu_n + \mu_p), \mu_n = 1417 \frac{\text{cm}^2}{\text{V.sec}}, \mu_p = 471 \frac{\text{cm}^2}{\text{V.sec}}, \therefore \sigma = 4.38 \times 10^{-6} (\Omega \cdot \text{cm})^{-1}$$

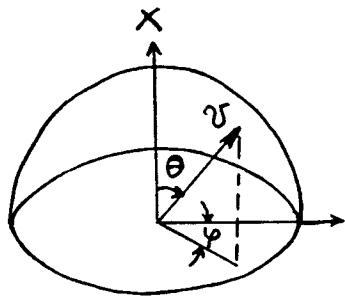
$$\therefore \tau_{rel} = \frac{\epsilon}{\sigma} = 2.4 \times 10^{-7} \text{ sec} = 0.24 \mu\text{sec}$$

$$N_d = 10^{16} \text{ cm}^{-3}, \mu_n \approx 1190 \text{ cm}^2/\text{V-sec}, \sigma = 1.90 (\Omega \cdot \text{cm})^{-1}$$

$$\therefore \tau_{\text{rel}} = \frac{E}{\sigma} = 5.4 \times 10^{-13} \text{ sec} = 0.54 \text{ psec}$$

$$\text{SiO}_2 \quad \tau_{\text{rel}} = \frac{3.9 \times 8.85 \times 10^{-14}}{10^{-16}} = 3450 \text{ sec} = 57.5 \text{ min}$$

1.13



Let each direction and velocity increment contain dN_v particles

$$d^3n_{\theta\phi v} = \frac{1}{4\pi} dN_v \sin\theta d\theta d\phi$$

The average v in one direction is

$$\langle v \rangle = \frac{1}{n_0} \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{v}{4\pi} dN_v \sin\theta d\theta d\phi$$

$$= \frac{1}{n_0} \int_0^\infty v dN_v \quad \text{where } n_0 = \int d^3n_{\theta\phi v}$$

The x -direction component of $\langle v \rangle$ is

$$\langle v_x \rangle = \langle v \cos\theta \rangle = \frac{1}{n_0} \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{v}{4\pi} dN_v \cos\theta \sin\theta d\theta d\phi$$

$$= \frac{1}{4n_0} \int_0^\infty v dN_v = \frac{1}{4} \langle v \rangle$$

$$\text{and } J_x = -g n_0 \langle v_x \rangle = -\frac{g n_0 \langle v \rangle}{4}$$

1.14

$$\lambda = \frac{hc}{E_g}$$

$$\lambda (\mu\text{m}) = \frac{1.24}{E_g (\text{eV})}$$

Material	$E_g (\text{eV})$	$\lambda (\mu\text{m})$	Range
Ge	0.67	1.85	Infrared
Si	1.124	1.10	Infrared
GaAs	1.42	0.87	(Near) infrared
SiO ₂	~9	~0.14	(Vacuum) ultraviolet

1.15

$$\frac{D}{\mu} = \frac{1}{8} \frac{dE_f}{d(\ln n)} . \text{ For a nondegenerate semiconductor,}$$

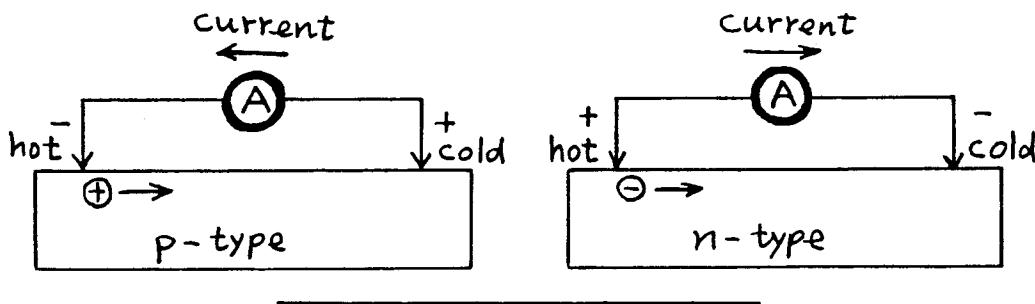
$$n \approx N_c \exp[-(E_c - E_f)/kT] \quad \therefore E_f - E_c = kT \ln \frac{n}{N_c} = kT [\ln n - \ln N_c]$$

$$\therefore \frac{dE_f}{d(\ln n)} = kT , \text{ if } E_c \text{ and } N_c \text{ are independent of doping.}$$

$$\text{Hence : } \frac{D}{\mu} = \frac{1}{8} \frac{dE_f}{d(\ln n)} = \frac{kT}{8} .$$

1.16

In a volume of an extrinsic semiconductor which is hotter than the surrounding material, the carriers have a higher thermal velocity than those in the unheated material. There is thus a net flux of carriers out of the heated volume by diffusion. The magnitude of the flux increases as the temperature difference (and thus the velocity difference) increases. For n-type material there is a net flux of electrons out of the heated region and thus a net current into the region. For p-type material there is a net flux of holes out of the heated region and thus a net current out of the region. In other words, the majority carriers at the hot end tend to move to the cold end.

1.17

$x \backslash t$	0	1	2	3	4	5	6	7	8	9	10
0	1024										
1		512									
2	512		256								
3		384		128							
4	384		256		64						
5		320		160		32					
6	320		240		96		16				
7		280		168		56		8			
8	280		224		112		32		4		
9		252		168		72		18		2	
10	252		210		120		45		10		1

The distribution is symmetrical about $x = 0$

t	w	w^2	w^2/t
0	0	0	—
2	4	16	8
4	5.33	28.44	7.11
6	6.22	38.71	6.45
8	7	49	6.125
10	7.73	59	6.9

t is the time unit

w is the half width

Example : For $t = 8$ half maximum is 140 so that $2 \leq \frac{W}{2} \leq 4$.

The straight line drawn between the known values at $x=2$ and $x=4$ is given by $y = -56x + 336$.

Letting $y = 140$ we find $x = 3.5 \approx \frac{W}{2}$ so that $W \approx 7$.

The other values of W shown can be found in a similar manner.

Examination of the values of W shows that the rate of diffusion slows as the distribution spreads and the gradients are reduced. The values of W^2/t indicate that for $t \gg 1$, $W^2 \propto t$, so that $W \propto (t)^{1/2}$ and $\frac{dW}{dt} \propto (t)^{-\frac{1}{2}}$.

1.18

(a) From Table 1.4,

$$E_g(\text{eV}) = 1.16 - \frac{7.02 \times 10^{-4} T^{3/2}}{T + 1108}$$

At $T = 300 \text{ K}$,

$$E_g = 1.16 - \frac{(7.02 \times 10^{-4})(300)^{3/2}}{300 + 1108} = 1.157 \text{ eV}$$

On the other hand, Table 1.3 lists 1.124 eV for E_g .

(b) From Table 1.3, at $T = 300 \text{ K}$,

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$$

$$N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$$

From Eq. (1.1.25),

$$n_i^2 = N_c N_v \exp\left[-\frac{(E_c - E_v)}{KT}\right]$$

or

$$n_i = (N_c N_v)^{1/2} \exp\left[-\frac{E_g}{2KT}\right]$$

At 300K, using $E_g = 1.157 \text{ eV}$ from Table 1.4,

$$\begin{aligned} n_i &= [(2.8 \times 10^{19})(1.04 \times 10^{19})]^{1/2} \exp\left[-\frac{1.157}{2(0.026)}\right] \\ &= 3.707 \times 10^9 \text{ cm}^{-3} \end{aligned}$$

At 300K, using $E_g = 1.124 \text{ eV}$ from Table 1.3,

$$\begin{aligned} n_i &= [(2.8 \times 10^{19})(1.04 \times 10^{19})]^{1/2} \exp\left[-\frac{1.124}{2(0.026)}\right] \\ &= 6.993 \times 10^9 \text{ cm}^{-3} \end{aligned}$$

(c) From Table 1.4,

$$n_i (\text{cm}^{-3}) = 3.87 \times 10^{16} T^{3/2} \exp\left[-\frac{7014}{T}\right]$$

At $T = 300 \text{ K}$,

$$\begin{aligned} n_i &= (3.87 \times 10^{16})(300)^{3/2} \exp\left[-\frac{7014}{300}\right] \\ &= 1.411 \times 10^{10} \text{ cm}^{-3} \end{aligned}$$

1.19

We replace the force term \vec{F} in Eq. (1.3.1) by

$$\vec{F} = m^* \frac{d\vec{V}}{dt} + m^* \frac{\vec{V}}{\tau}$$

where the second term represents scattering. Also adding electric force term, we get

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) \vec{V} = q(\vec{E} + \vec{V} \times \vec{B}).$$

With the standard Hall geometry of Fig. 1.20

under DC condition ($d/dt = 0$), we solve for velocity components.

For electrons,

$$V_{nx} = \mu_n \frac{-Ex + w_{cn} \tau_n E_y}{1 + (w_{cn} \tau_n)^2}$$

$$V_{ny} = \mu_n \frac{-w_{cn} \tau_n Ex - E_y}{1 + (w_{cn} \tau_n)^2}$$

$$V_{nz} = 0, \text{ where } \mu_n = \frac{q \tau_n}{m_n^*}, w_{cn} = \frac{q B_z}{m_n^*}$$

For holes,

$$V_{px} = \mu_p \frac{Ex + w_{cp} \tau_p E_y}{1 + (w_{cp} \tau_p)^2}$$

$$V_{py} = \mu_p \frac{-w_{cp} \tau_p Ex + E_y}{1 + (w_{cp} \tau_p)^2}$$

$$V_{pz} = 0, \text{ where } \mu_p = \frac{q \tau_p}{m_p^*}, w_{cp} = \frac{q B_z}{m_p^*}$$

We require the transverse current to be zero.

$$J_y = J_{ny} + J_{py} = -q_n V_{ny} + q_p V_{py}$$

$$\cong q(\mu_n^2 n - \mu_p^2 p) E_x B_z + q(\mu_n n + \mu_p p) E_y$$

with the approximation $w_{cn} \ll \frac{1}{\tau_n}$ and $w_{cp} \ll \frac{1}{\tau_p}$.

Now $J_y = 0$ implies

$$ExBz = \frac{\mu_p p + \mu_n n}{\mu_p^2 p - \mu_n^2 n} E_y \quad \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

The longitudinal current is

$$J_x = J_{nx} + J_{px} = -q_n v_{nx} + q_p v_{px}$$

$$\cong q(\mu_p p + \mu_n n) E_x + q(\mu_p^2 p - \mu_n^2 n) E_y B_z$$

$$J_x B_z \cong q(\mu_p p + \mu_n n) E_x B_z$$

$$= q \frac{(\mu_p p + \mu_n n)^2}{\mu_p^2 p - \mu_n^2 n} E_y \quad (\text{replace } ExBz \text{ from (1)}) \quad \dots \dots \dots \quad (2)$$

We neglected the term involving B_z^2 in (2).

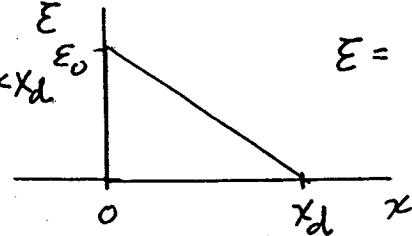
$$R_H = \frac{E_y}{J_x B_z} = \frac{\mu_p p^2 - \mu_n^2 n}{q(\mu_p p + \mu_n n)^2}$$

To see that this result is consistent with the simpler theory of sec. 1.3, we can let $p=0$ for a sample containing electrons only and find $R_H = -\frac{1}{qn}$. For a sample containing holes only, $n=0$ gives $R_H = \frac{1}{qp}$.

APPENDIX PROBLEMS

A 1.1 FOR $0 < x < x_d$ $\rho = -\rho_1$ $\frac{dE}{dx} = -\frac{\rho_1}{\epsilon_s}$

(a) $\therefore E = \epsilon_0 - \frac{\rho_1 x}{\epsilon_s}$
where $\epsilon_0 = \frac{\rho_1 x_d}{\epsilon_s}$



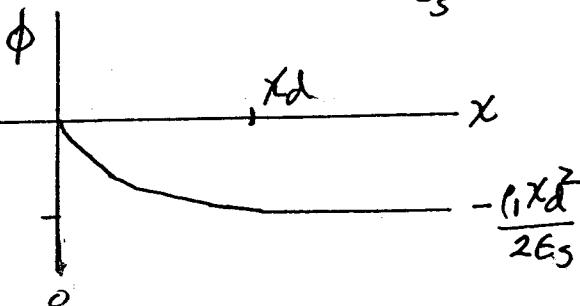
$$\begin{cases} E = 0, x < 0 \\ 0, x > x_d \end{cases}$$

(b) $\phi = - \int E dx$

$$\phi = -\epsilon_0 x + \frac{\rho_1 x^2}{2\epsilon_s} \quad 0 < x < x_d \text{ or } \phi = -\frac{\rho_1}{\epsilon_s} \left(x_d x - \frac{x^2}{2} \right)$$

Take $\phi(x < 0) = 0$

$$\phi(x_d) = -\frac{\rho_1 x_d^2}{2\epsilon_s}$$



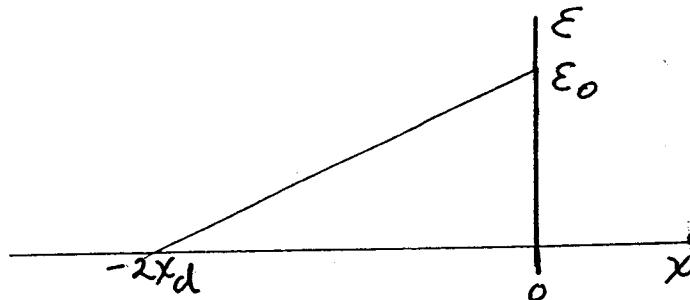
(c) $\phi(0) - \phi_{x_d} = \frac{\rho_1 x_d^2}{2\epsilon_s}$

(d) $E = 0 \begin{cases} x < -2x_d \\ x > 0 \end{cases}$

$$E(x) = \frac{\rho_1}{\epsilon_s} \left(x_d + \frac{x}{2} \right)$$

$$-2x_d < x < 0$$

$$E(0) = \frac{\rho_1 x_d}{\epsilon_s}$$

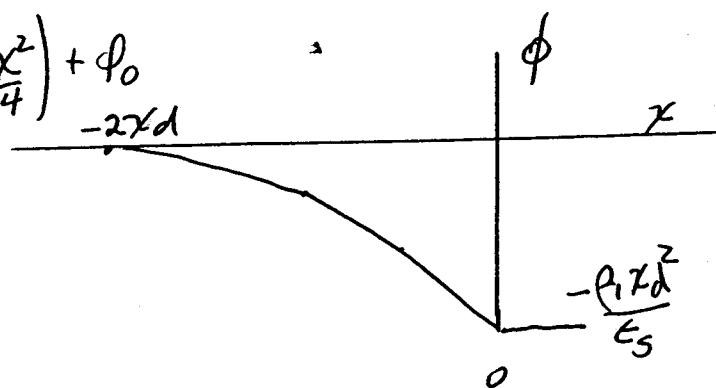


$$\phi = - \int E dx = -\frac{\rho_1}{\epsilon_s} \left(x_d x + \frac{x^2}{4} \right) + \phi_0$$

IF $\phi_0 = 0 (x \leq -2x_d)$

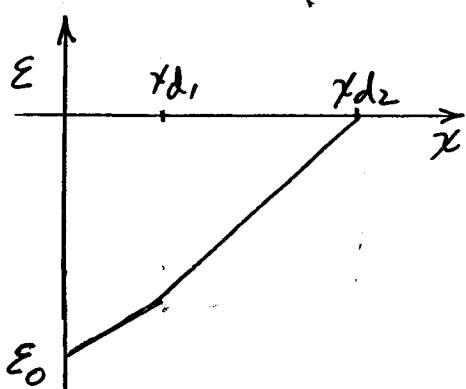
$$\phi = -\frac{\rho_1}{\epsilon_s} \left(x_d + \frac{x}{2} \right)^2$$

$$\therefore \phi(-2x_d) - \phi(x=0) = \frac{\rho_1 x_d^2}{\epsilon_s}$$



A1.2(a) For $x < 0$, $\epsilon = 0$.

$$\text{For } 0 < x < x_{d_1}, \frac{d\epsilon}{dx} = \frac{\rho_1}{\epsilon_s} \quad \therefore \epsilon = \epsilon_0 + \frac{\rho_1 x}{\epsilon_s}$$

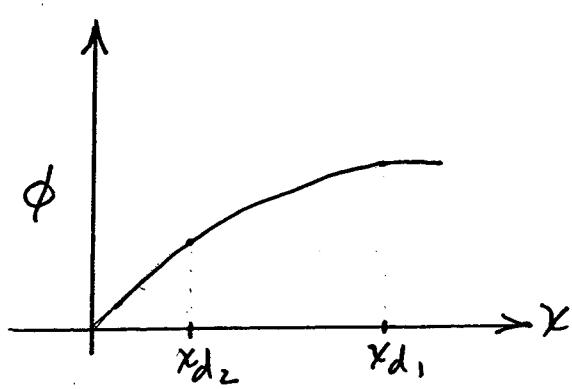


$$\text{By Gauss' Law} \quad \epsilon_0 = -\frac{1}{\epsilon_s} [\rho_1 x_{d_1} + 2\rho_1 (x_{d_2} - x_{d_1})]$$

$$\epsilon_0 = -\frac{\rho_1}{\epsilon_s} [2x_{d_2} - x_{d_1}]$$

$$\text{and } \epsilon(x_{d_1}) = \epsilon_0 x + \frac{\rho_1 x_{d_1}}{\epsilon_s}$$

$$\therefore \epsilon(x_{d_1}) = -\frac{\rho_1}{\epsilon_s} [2x_{d_2} - x_{d_1}, -x_{d_1}] \\ = -\frac{2\rho_1}{\epsilon_s} [x_{d_2} - x_{d_1}]$$

FOR $x_{d_1} < x < x_{d_2}$ 

$$\epsilon = \epsilon(x_{d_1}) + \frac{2\rho_1}{\epsilon_s} (x - x_{d_1})$$

$$= -\frac{2\rho_1}{\epsilon_s} [x_{d_2} - x_{d_1} - (x - x_{d_1})]$$

$$= -\frac{2\rho_1}{\epsilon_s} [x_{d_2} - x]$$

(b) $\phi = 0 \quad x < 0$

$$\phi = -\int \epsilon dx \quad \text{For } 0 < x < x_{d_1}$$

(c) $\phi(0) - \phi(x_{d_1})$

$$\phi = \epsilon_0 x - \frac{\rho_1 x^2}{2\epsilon_s}$$

$$= -\frac{\rho_1}{\epsilon_s} (2x_{d_2}x_{d_1} - \frac{3}{2}x_{d_1}^2)$$

$$= -\frac{\rho_1}{\epsilon_s} \left[2x_{d_2}x - x_{d_1}x - \frac{x^2}{2} \right]$$

 $\phi(0) - \phi(x_{d_2})$

$$\phi(x=x_{d_1}) = \frac{\rho_1}{\epsilon_s} \left[2x_{d_2}x_{d_1} - \frac{3}{2}x_{d_1}^2 \right]$$

$$= -\frac{\rho_1}{\epsilon_s} \left(x_{d_2}^2 - \frac{x_{d_1}^2}{2} \right)$$

FOR $x_{d_1} < x < x_{d_2}$

$$\phi = \phi(x_{d_1}) - \int_{x_{d_1}}^x \epsilon dx$$

$$= \frac{\rho_1}{\epsilon_s} \left(2x_{d_2}x - \frac{x_{d_1}^2}{2} - x^2 \right)$$

$$\phi(x_{d_2}) = \frac{\rho_1}{\epsilon_s} \left(x_{d_2}^2 - \frac{x_{d_1}^2}{2} \right)$$

A1.3

$$(a) \quad \epsilon = 0 \quad x < -x_{d1}$$

$$\frac{d\epsilon}{dx} = -\frac{\rho_1}{\epsilon_s} \quad -x_{d1} < x < 0$$

$$\therefore \epsilon = -\frac{\rho_1}{\epsilon_s} (x + x_{d1})$$

$$-x_{d1} < x < 0$$

$$\text{at } x=0; \epsilon = -\epsilon_0 = -\frac{\rho_1 x_{d1}}{\epsilon_s}$$

$$\text{also; } \epsilon_0 = -\frac{\rho_2 x_{d2}}{\epsilon_s}$$

$$\epsilon = 0 \text{ for } x_{d2} < x < \infty$$

$$(b) \quad \text{Take } \phi(x < -x_{d1}) = 0$$

$$\text{Then for } -x_{d1} < x < 0$$

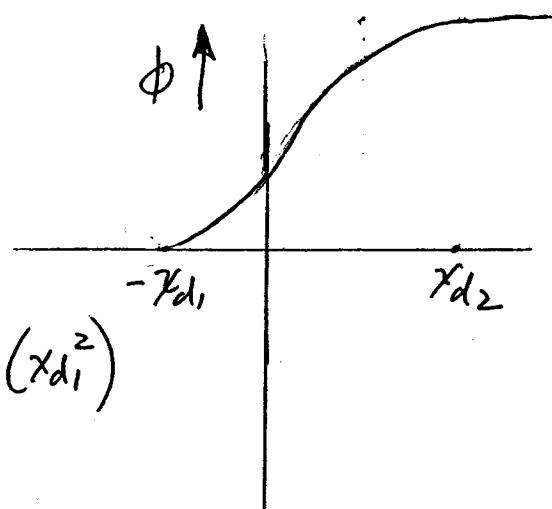
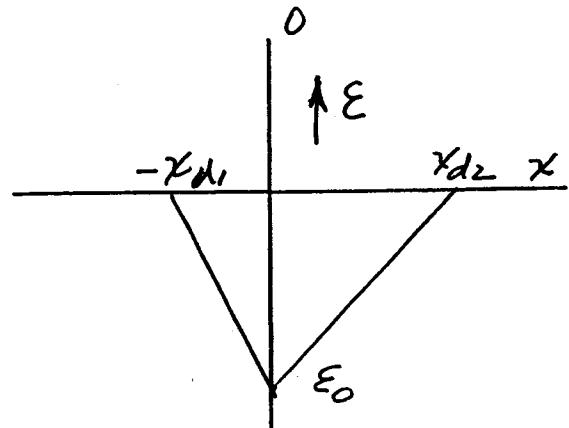
$$\phi = \frac{\rho_1}{\epsilon_s} \left(\frac{x^2}{2} + x_{d1}x \right) + \frac{\rho_1}{2\epsilon_s} (x_{d1}^2)$$

$$\text{For } 0 < x < x_{d2}$$

$$\phi = \frac{\rho_1 x_{d1}^2}{2\epsilon_s} + \frac{\rho_2 x^2}{2\epsilon_s}$$

$$(c) \quad \phi(x_{d2}) - \phi(-x_{d1}) = \frac{1}{2\epsilon_s} (\rho_1 x_{d1}^2 + \rho_2 x_{d2}^2)$$

(d) Problem A1.1 can be constructed as a special case of the negative of the summed charges in A1.1.(a) and (d). The graphs of ϵ and ϕ superpose to the negatives of the graphs in this problem and the expressions for ϕ and ϵ compare similarly.



A 1.4FOR $x < 0, \epsilon = 0$ (a) Charge at $x=0 = -\rho x_d$ In Oxide; $\epsilon_{ox} = -\frac{\rho x_d}{\epsilon_s}$ constantAt $x=x_0$, ϵ changes such that $\epsilon_{ox}\epsilon_{ox} = \epsilon_s\epsilon_{os}$ For $x_0 < x < x_d \quad \epsilon = \epsilon_{os} + \frac{\rho_1 x}{\epsilon_s}$ (b) $\phi = 0 \quad x < 0$

$$\phi = \epsilon_{ox}x \quad 0 < x < x_{ox}$$

$$\phi = \epsilon_{ox}x_{ox} + \frac{\rho(x-x_{ox})^2}{2\epsilon_s} \quad x_{ox} < x < x_d$$

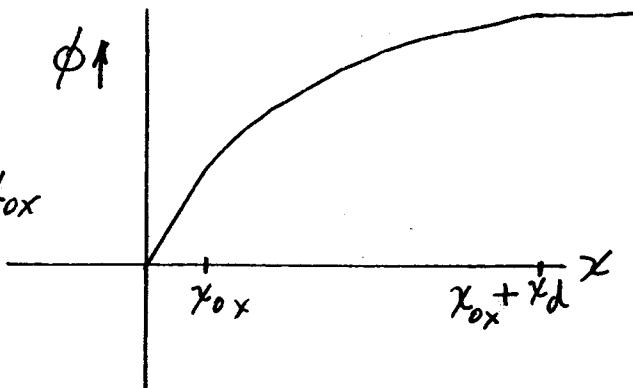
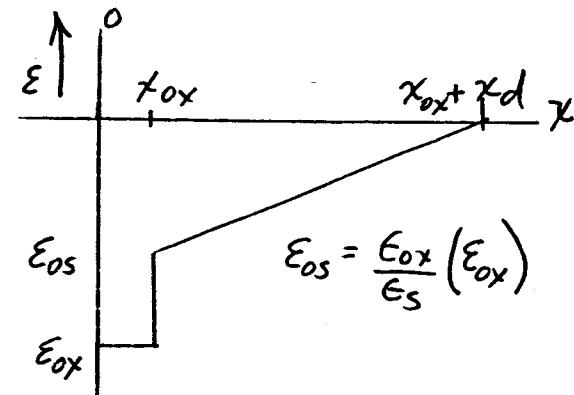
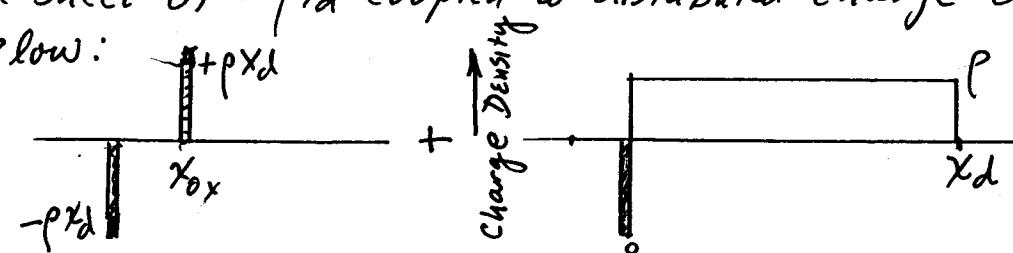
$$(c) \Delta\phi_0 = \epsilon_{ox}x_{ox}$$

$$= -\frac{\rho x_d x_{ox}}{\epsilon_{ox}}$$

$$(d) \Delta\phi_d = \epsilon_{ox}x_{ox} - \frac{\rho x_d^2}{2\epsilon_s} - \epsilon_{ox}x_{ox}$$

$$= -\frac{\rho x_d^2}{2\epsilon_s}$$

(e) A convenient decomposition is into 2 charge sheets of $-\rho_1 x_d$ and $+\rho x_d$ on either side of x_{ox} plus a sheet of $-\rho x_d$ coupled to distributed charge as shown below:



CHAPTER 2

2.1

(a) 1 mg P. At. weight $30.97 \text{ g} / 6.023 \times 10^{23} \text{ atoms} \Rightarrow 1.94 \times 10^{19} \text{ atoms}$

10 kg Si. At. weight $28.09 \text{ g} / 6.023 \times 10^{23} \text{ atoms} \Rightarrow 2.14 \times 10^{26} \text{ atoms}$

Initial $\frac{C_{\text{Phos}}}{C_{\text{Si}}} \text{ in melt} = 9.07 \times 10^{-8}$

in solid $= 0.3 \times 9.07 \times 10^{-8} = 2.72 \times 10^{-8}$

$$C_{\text{Si}} = 5.00 \times 10^{22} \text{ cm}^{-3} \quad \therefore C_{\text{Phos}} = 1.36 \times 10^{15} \text{ cm}^{-3}$$

(b) As crystal is pulled, P concentration in liquid increases.

Let N = number of atoms in liquid

Between t and $t + \Delta t$, N_{Phos} changes by ΔN_{Phos}

$$\Delta N_{\text{Phos}} = -R \Delta t m \frac{N_{\text{Phos}}(t)}{N_{\text{Si}}(t)} \quad N_{\text{Si}} \text{ changes by } \Delta N_{\text{Si}}$$

$$\text{where } R = \text{rate of silicon solidification} = -\frac{\Delta N_{\text{Si}}}{\Delta t}$$

$$\Delta N_{\text{Phos}} = +\frac{\Delta N_{\text{Si}}}{\Delta t} \Delta t m \frac{N_{\text{Phos}}(t)}{N_{\text{Si}}(t)}$$

$$\frac{\Delta N_{\text{Phos}}}{N_{\text{Phos}}(t)} = +m \frac{\Delta N_{\text{Si}}}{N_{\text{Si}}(t)}$$

$$\ln \frac{N_{\text{Phos}}(t)}{N_{\text{Phos}}(0)} = m \ln \frac{N_{\text{Si}}(t)}{N_{\text{Si}}(0)} = \ln \left[\frac{N_{\text{Si}}(t)}{N_{\text{Si}}(0)} \right]^m$$

$$N_{\text{Phos}}(t) = N_{\text{Phos}}(0) \left[\frac{N_{\text{Si}}(t)}{N_{\text{Si}}(0)} \right]^m$$

after 50% of Si solidified (i.e. after 5 kg)

$$N_{\text{Phos}} = 1.94 \times 10^{19} \left(\frac{1}{2} \right)^0.3 = 1.58 \times 10^{19} \text{ atoms}$$

$$\frac{C_{\text{Phos}}}{C_{\text{Si}} \text{ liquid}} = \frac{1.58 \times 10^{19}}{1.07 \times 10^{26}} = 1.47 \times 10^{-7}$$

$$\frac{C_{\text{Phos}}}{C_{\text{Si}} \text{ solid}} = 0.3 \times 1.47 \times 10^{-7}$$

$$C_{\text{Phos}} \text{ solid} = (0.3 \times 1.47 \times 10^{-7})(5.00 \times 10^{22} \text{ cm}^{-3}) = 2.21 \times 10^{15} \text{ cm}^{-3}$$

2.2

The "clumps" or precipitates act as "gettering" sites as described on page 64. These sites attract impurities that might otherwise affect the electrical properties of devices.

2.3

From Figure 2.8(a) at 1100°C

$X_{\text{ox}} \sim 0.13 \mu\text{m}$ grown in dry O_2

(a) \therefore with HCl present $X_{\text{ox}} = 0.14 \mu\text{m}$

(b) From Fig. 2.8(b), $T = 0.14 \text{ hr}$

$$(t + T) = 2.14 \text{ hr} \Rightarrow X_{\text{ox}} = 0.82 \mu\text{m}$$

(c) From Fig. 2.8(a), $T = 68 \text{ hr}$

$$(t + T) = 74 \text{ hr} \Rightarrow X_{\text{ox}} \approx 0.85 \mu\text{m} \quad (\text{hardly a change})$$

2.4

From Fig. 2.8(a), $T = 0.9 \text{ hr}$ For (111)-oriented Si
 2.0 hr. to grow 300 nm SiO_2
 \therefore time for additional 100 nm is $1.1 \text{ hr} = 66 \text{ min.}$

2.5

From Fig. 2.8 (b)

Assume (111) Si.

(a) 1000°C , 1 atm, $B = 0.35 \text{ } (\mu\text{m})^2 \text{ hr}^{-1}$
 $B/A = 1.05 \text{ } (\mu\text{m}) \text{ hr}^{-1}$

(b) 1000°C , 10 atm, $B = 3.2 \text{ } (\mu\text{m})^2 \text{ hr}^{-1}$
 (estimate curve) $B/A = 18 \text{ } (\mu\text{m}) \text{ hr}^{-1}$

(c) 800°C , 1 atm, $B = 0.045 \text{ } (\mu\text{m})^2 \text{ hr}^{-1}$
 $B/A = 0.04 \text{ } (\mu\text{m}) \text{ hr}^{-1}$

(d) 800°C , 10 atm, $B = 0.45 \text{ } (\mu\text{m})^2 \text{ hr}^{-1}$
 (estimate curve), $B/A = 0.35 \text{ } (\mu\text{m}) \text{ hr}^{-1}$

From Equation (2.3.6) we have :

$$X_{0X} = X = \frac{A}{2} \left[\sqrt{1 + \frac{(t+\tau)}{A^2/4B}} - 1 \right]$$

which may be solved for :

$$\frac{X^2}{B} + \frac{X}{B/A} = (t + \tau) = t \text{ for an initially grown oxide}$$

Using the values in (a) to (d) above with $X = 1 \mu\text{m}$, we find :

(a) $t = 3.81 \text{ hr.}$

(b) $t = 0.37 \text{ hr.} = 22 \text{ min.}$

(c) $t = 47.2 \text{ hr.}$

(d) $t = 5.1 \text{ hr.}$

2.6From Eq. (2.3.5) (let $X_{ox} \rightarrow X$)

$$\frac{dX}{dt} = \frac{k_s C^*/N_{ox}}{1 + k_s/h + k_s X/D}$$

$$\text{or } \left(\frac{k_s}{D}\right) X \frac{dx}{dt} + (1 + k_s/h) \frac{dx}{dt} = N_{ox} k_s C^*$$

$$\text{let } M = k_s/D, \quad N = (1 + k_s/h), \quad P = k_s C^*/N_{ox}$$

$$\therefore M \frac{dx}{dt} + N \frac{dx}{dt} = P$$

$$\text{or } (X + N/M) dx = P/M dt$$

$$\therefore \frac{X^2}{2} + \frac{N}{M} X = \frac{P}{M} (t + \tau), \text{ where } \frac{P}{M} \tau \text{ is the integration constant.}$$

$$\text{Hence } \frac{X^2}{2} + \frac{N}{M} X + \frac{1}{2} \frac{N^2}{M^2} = \frac{P}{M} (t + \tau) + \frac{1}{2} \left(\frac{N}{M}\right)^2$$

$$\left(\frac{X}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{N}{M}\right)^2 = \frac{P}{M} (t + \tau) + \frac{1}{2} \left(\frac{N}{M}\right)^2$$

$$\frac{X}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{N}{M} = \sqrt{\frac{P}{M} (t + \tau) + \frac{1}{2} \left(\frac{N}{M}\right)^2}$$

$$X = \sqrt{2} \sqrt{\frac{P}{M} (t + \tau) + \frac{1}{2} \left(\frac{N}{M}\right)^2} - \frac{N}{M}$$

$$= \frac{N}{M} \left[\sqrt{1 + \frac{(t + \tau)}{N^2/2MP}} - 1 \right]$$

$$\text{Now } \frac{N}{M} = \frac{D}{k_s} (1 + k_s/h) = D \left[\frac{1}{k_s} + \frac{1}{h} \right]$$

$$= \frac{A}{2} \text{ (from Eq. (2.3.7))}$$

$$\text{and } \frac{N^2}{2MP} = \frac{k_s^2 \left(\frac{1}{k_s} + \frac{1}{h} \right)^2 N_{ox}}{2(k_s/D) k_s C^*} = \frac{D \left(\frac{1}{k_s} + \frac{1}{h} \right)^2 N_{ox}}{2C^*}$$

From Eqs. (2.3.7) and (2.3.8)

$$\frac{A^2}{4B} = \frac{4D^2 \left[\frac{1}{k_s} + \frac{1}{h} \right]^2}{4(2DC^*/N_{ox})} = \frac{D \left(\frac{1}{k_s} + \frac{1}{h} \right)^2 N_{ox}}{2C^*}$$

and the results are the same.

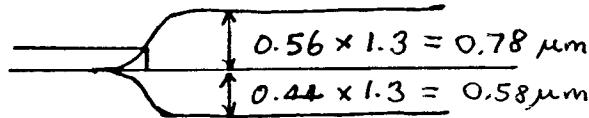
2.7

From Fig. 2.8 (b), approximately $1.3 \mu\text{m}$ SiO_2 grown in 8 hr. @ 1000°C .

44% below original silicon surface

56% above original silicon surface

$\therefore 0.56 \times 1.3 = 0.73 \mu\text{m}$ is height of top of oxide above original silicon surface.

2.8

Using Table 1.1,

$$\text{Molecular density of } \text{Si}_3\text{N}_4 = 1.48 \times 10^{22} \text{ cm}^{-3}$$

$$\text{Molecular density of } \text{SiO}_2 = 2.2 \times 10^{22} \text{ cm}^{-3}$$

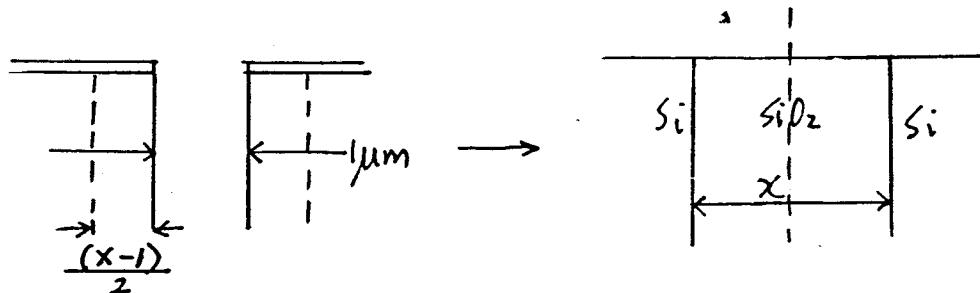
1 molecules of Si_3N_4 converts to 3 molecules of SiO_2

$$\therefore 1 \text{ nm of } \text{Si}_3\text{N}_4 \text{ is equivalent to } = \frac{1 \times 3 \times 1.48 \times 10^{22}}{2.2 \times 10^{22}} \text{ nm}$$

$$= 2.02 \text{ nm of } \text{SiO}_2.$$

2.9

(a) Let X = width of oxide groove



$0.44 \mu\text{m}$ of Si oxidizes to $1 \mu\text{m}$ SiO_2

$$0.44X = X - 1; \quad X = 1.79 \mu\text{m}$$

(b) The grown oxide $X_{ox} = \frac{1.79}{2} = 0.89 \mu\text{m}$

$$\text{From Eq. (2.3.6), } X_{ox} = \sqrt{\frac{A^2}{4} + Bt} - \frac{A}{2}$$

$$\text{or } t = \frac{1}{B} [X_{ox}^2 + AX_{ox}]$$

From Fig. 2.7(a), for (100) Si, $B/A \approx 3(\mu\text{m}) \text{ hr}^{-1}$

From Fig. 2.7(b), $B = 0.5 (\mu\text{m})^2 \text{ hr}^{-1}$ ($A = 0.1667 \mu\text{m}$)

$$\therefore t = \frac{1}{.5} [(0.89)^2 + 0.1667 (.89)]$$

$$= 1.88 \text{ hr.}$$

2.10

$$N' = 3 \times 10^{16} \text{ cm}^{-2} \text{ of P}$$

$$L' = \sqrt{2(AR_p)^2 + 4Dt}, \quad C_s = \frac{N'}{L' \sqrt{\pi}} \exp \left[\frac{-R_p}{L'} \right]^2$$

From Fig. 2.9, concentration-enhanced oxidation becomes important when $C_s \gtrsim 10^{20} \text{ cm}^{-3}$

(a) After implantation, $L' = 3.82 \times 10^{-8} \text{ cm}$

$$C_s = 1.91 \times 10^{19} \text{ cm}^{-3}$$

\therefore conc-enhanced diffusion not important

(b) 1 hr. at 1000°C ($D = 1.7 \times 10^{-13} \text{ cm}^2 \text{ s}^{-1}$ from Fig. 2.20)

and $L' = 2.98 \times 10^{20} \text{ cm}^{-3}$ and conc-enhanced diffusion is important.

(c) for 150 keV

$$(i) L' = 9.05 \times 10^{-8} \text{ cm}; \quad C_s = 6.8 \times 10^{18} \text{ cm}^{-3}$$

conc-enhanced diffusion not important.

(ii) after 1 hr. at 1000°C

$$L' = 5.03 \times 10^{-5} \text{ cm}; \quad C_s = 2.96 \times 10^{20} \text{ cm}^{-3}$$

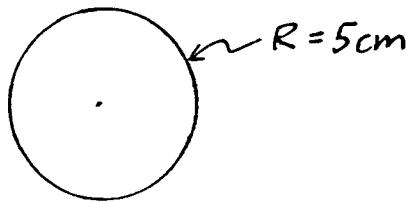
and conc-enhanced diffusion is important.

2.11

The oxidation injects interstitials into the silicon surface. In the single-crystal silicon, these cause enhanced diffusion while, in the polycrystalline silicon, they are lost to vacancies and cannot enter the single crystal below to affect the diffusion.

2.12

$$\frac{\Delta X}{X} = 9 \times 10^{-6} \text{ per } {}^\circ\text{C}$$



$$\begin{aligned} X &= \text{radius} = 5 \text{ cm} \\ \therefore \Delta X &= 5 \times 9 \times 10^{-6} \\ &= 45 \times 10^{-6} \text{ cm} \\ &= .45 \mu\text{m}. \end{aligned}$$

2.13

$$C = C_s \operatorname{erfc} \frac{X}{2\sqrt{Dt}}$$

$$C_s = 10^{21} / \text{cm}^3 ; D = 10^{-13} \text{ cm}^2 / \text{sec} @ 975 {}^\circ\text{C}$$

$$(a) \rho = 0.3 \Omega\text{-cm p-type} \Rightarrow N_A = 6.3 \times 10^{16} \text{ cm}^{-3} \text{ (Fig. 1.15)}$$

$$C_j/C_s = 6.3 \times 10^{16} / 10^{21} = 6.3 \times 10^{-5}$$

$$X_j/L = 2.83 \quad (\text{Fig. 2.21})$$

$$X_j = (2.83)(2)\sqrt{(10^{-13})(1800)} = 7.6 \times 10^{-5} \text{ cm} = 0.76 \mu\text{m}$$

$$(b) \rho = 20 \Omega\text{-cm p-type} \Rightarrow N_A = 6.8 \times 10^{14} \text{ cm}^{-3}$$

$$C_j/C_s = 6.8 \times 10^{14} / 10^{21} = 6.8 \times 10^{-7}$$

$$X_j/L = 3.5$$

$$X_j = (3.5)(2)\sqrt{(10^{-13})(1800)} = 9.4 \times 10^{-5} = 0.94 \mu\text{m}$$

but extending erfc line linearly (on log scale) is reasonable in this range.

2.14

Deposition + drive-in \Rightarrow approximately Gaussian profile.

$$C(x,t) = \frac{N}{\sqrt{\pi D t}} \exp\left[-\frac{x^2}{4Dt}\right]$$

$$C_b = 5 \times 10^{15} \text{ cm}^{-3}, x_j = 2 \mu\text{m}, N = 10^{15} \text{ cm}^{-2}, D = 2 \times 10^{-13} \text{ cm}^2/\text{sec}$$

$$(a) \text{ at } x_j, C = C_b = 5 \times 10^{15} = \frac{10^{15}}{\sqrt{\pi \times 2 \times 10^{-13}}} \frac{1}{\sqrt{t}} \exp\left[-\frac{4 \times 10^{-8}}{8 \times 10^{-13} t}\right]$$

$$\text{or } 3.96 \times 10^{-6} \sqrt{t} = \exp\left[-5 \times 10^4 / t\right]$$

assuming $t_{\text{drive-in}} \gg t_{\text{deposition}}$, solve iteratively

$$t = -5 \times 10^4 \times \frac{1}{\ln(3.96 \times 10^{-6} \sqrt{t})}$$

$$\text{by } t_1 = 1000, t_2 = 5570, t_3 = 6150, t_4 = 6190 \text{ sec}$$

$$\therefore t = 1 \text{ hr. } 43 \text{ min}$$

$$(b) C = 1.6 \times 10^{19} \exp\{-2.02 x^2\} \quad (x \text{ in } \mu\text{m})$$

$$(c) \frac{1}{R_D} = N q \mu$$

$$N = [10^{15} \text{ cm}^{-2} \text{ acceptors}$$

$$- (5 \times 10^{15} \text{ cm}^{-3})(2 \times 10^{-4} \text{ cm}) \text{ donors}]$$

$$\simeq 10^{15} \text{ cm}^{-2}$$

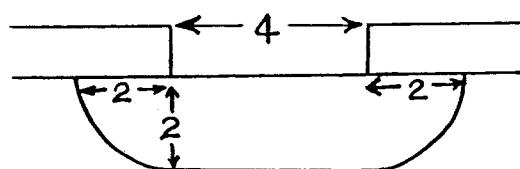
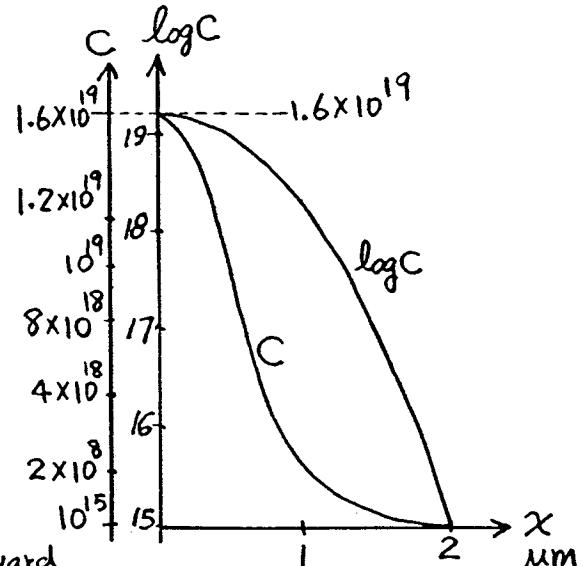
approximation: use μ_p at $C_s/2$,

$$\text{then } \mu_p = 90 \text{ cm}^2/\text{V-sec}$$

$$\therefore \frac{1}{R_D} = 1.44 \times 10^{-2}, R_D = 69.4 \text{ } \square$$

\therefore need 7.2 squares

Assume sideways diffusion \simeq downward diffusion



$\therefore W_{\text{eff}} = 8 \mu\text{m}$ (this is probably somewhat too large)

$$R = 500 \Omega = \frac{L}{W} R_D, L = \frac{WR}{R_D} = \frac{8 \times 500}{69.4} = 58 \mu\text{m}$$

$$A = LW = 8 \times 58 = 464 \mu\text{m}^2 = 4.6 \times 10^{-6} \text{ cm}^2$$

(d) Since the diffusion is deeper, more of the acceptors introduced will be compensated by donors from the starting wafer, increasing the resistance. However, the peak acceptor concentration will be lower so that the mobility increases. The latter effect is usually dominant, and the resistance generally decreases as the drive-in diffusion is lengthened (within reasonable limits).

2.15

$$I = 1 \text{ mA} \quad N_D = 10^{12} \text{ cm}^{-2} / 10^{-4} \text{ cm} = 10^{16} \text{ cm}^{-3}$$

$$\bar{\mu} = 1200 \text{ cm}^2/\text{V-sec} \quad (\text{Fig. 1.16})$$

$$R_D = [N' q \bar{\mu}]^{-1} = [(10^{12} \text{ cm}^2)(1.6 \times 10^{-19} \text{ C})(1200 \text{ cm}^2/\text{V-sec})]^{-1} \\ = 5.21 \text{ k}\Omega = 5.21 \times 10^3 \Omega$$

$$V = R_D \frac{I}{4.53} = (5.21 \times 10^3 \Omega) \left(\frac{10^{-3} \text{ A}}{4.53} \right) = 1.15 \text{ V}$$

2.16

$$N' = 10^{12} \text{ /cm}^2, \quad R_p = 290 \text{ nm}, \quad \Delta R_p = 70 \text{ nm}$$

$$L_{\text{imp}} = \sqrt{2 \Delta R_p} = 9.90 \times 10^{-6} \text{ cm}$$

$$L_{\text{diff}} = \sqrt{2 \Delta R_p^2 + 4 D t} = \left[2(70 \times 10^{-7})^2 + 4(2 \times 10^{-4})(7200) \right]^{1/2} = 2.60 \times 10^{-5} \text{ cm}$$

$$N_D = 9.2 \times 10^{14} \text{ /cm}^3 \quad (\text{Fig. 1.15})$$

$$(a) C_p = N' / (\sqrt{\pi} L_{\text{imp}}) \quad (\text{Eq. 2.5.2})$$

$$= 10^{12} / (\sqrt{\pi} \times 9.9 \times 10^{-6}) = 5.70 \times 10^{16} \text{ /cm}^3$$

$$C_j/C_p = 9.2 \times 10^{14} / 5.7 \times 10^{16} = 1.61 \times 10^{-2}$$

$$\text{For Gaussian } X/L = 2.0 \quad (\text{Fig. 2.21})$$

$$X = 2 L_{\text{imp}} = 2.0 \times 10^{-5} \text{ cm} = 200 \text{ nm} \text{ on each side of peak}$$

\therefore p-type region $\sim 400 \text{ nm wide.}$

$$(b) C_p \approx N' / (\sqrt{\pi} L_{\text{diff}}) = 10^{12} / (\sqrt{\pi} \times 2.6 \times 10^{-5}) = 2.17 \times 10^{16} \text{ /cm}^3$$

2.17

From Eqs. (2.5.1) and (2.5.2),

$$C(x) = \frac{N'}{\sqrt{2\pi} \Delta R_p} \exp\left[-\frac{(x-R_p)^2}{2(\Delta R_p)^2}\right]$$

$$C(x_j) = C_B \Rightarrow x_j = R_p + \sqrt{2} \Delta R_p \left[\ln\left(\frac{N'}{\sqrt{2\pi} \Delta R_p C_B}\right) \right]^{1/2}$$

For 60 keV As, $R_p = 0.0375 \mu\text{m}$ (Fig. 2.18) = $37.5 \times 10^{-7} \text{ cm}$

$$\Delta R_p = 0.0140 \mu\text{m} \text{ (Fig. 2.18)} = 14 \times 10^{-7} \text{ cm}$$

$$\begin{aligned} \therefore x_j &= 0.0375 + 0.0140 \sqrt{2 \ln\left[\frac{10^{15}}{\sqrt{2\pi} \times 14 \times 10^{-7} \times 10^{16}}\right]} \\ &= 0.1 \mu\text{m} = 100 \text{ nm}. \end{aligned}$$

2.18

$$R = \rho \frac{L}{A} = \frac{(500 \times 10^{-6} \Omega \text{ cm})(0.1 \text{ cm})}{(5 \times 10^{-4})(0.5 \times 10^{-4}) \text{ cm}^2} = 2 k\Omega = 2000 \Omega$$

$$\begin{aligned} C &= \frac{\epsilon_0 A}{d} = \frac{(3.9)(8.854 \times 10^{-14} \text{ F/cm}) (0.01 \times 0.05 \text{ cm}^2)}{100 \times 10^{-7} \text{ cm}} \\ &= 17.3 \text{ pf} = 1.73 \times 10^{-11} \text{ f} \end{aligned}$$

$$\tau = RC = 3.45 \times 10^{-8} \text{ sec} = 34.5 \text{ nsec.}$$

2.19

$$(a) n \approx N_d = 10^{16} \text{ cm}^{-3} \Rightarrow \mu_n = 1100 \text{ cm}^2/\text{V-sec}$$

$$p \approx N_a = 10^{16} \text{ cm}^{-3} \Rightarrow \mu_p = 450 \text{ cm}^2/\text{V-sec}$$

$$t = 6.55 \mu\text{m}$$

$$R_\square = \frac{P}{t} = \frac{1}{g(\mu_n n + \mu_p p)t} = \begin{cases} 867 \Omega/\square \text{ for n-type} \\ 2120 \Omega/\square \text{ for p-type} \end{cases}$$

$$R = R_\square \frac{L}{W} \quad \therefore \frac{L}{W} = \frac{R}{R_\square}$$

(b) $1 \mu\text{W}$ can be dissipated in $1 \mu\text{m}^3$

So that $10 \text{ mW} = 10^4 \mu\text{W}$ can be dissipated in $10^4 \mu\text{m}^3 \therefore$

$$WLt = 10^4 \mu\text{m}^3 \Rightarrow WL = 1527 \mu\text{m}^2$$

$$\text{From part (a) we have } r = \frac{L}{W} \Rightarrow W^2 r = 1527 \mu\text{m}^2$$

$$(c) \alpha = \frac{1}{P} \frac{\partial P}{\partial T} \text{ and } P = \frac{1}{g n \mu}$$

For an extrinsic semiconductor at room temperature $\frac{\partial n}{\partial T} \approx 0$

$$\text{Therefore } \frac{\partial P}{\partial T} = - \frac{1}{g n \mu_n^2} \frac{\partial \mu_n}{\partial T}$$

So that $\alpha = - \frac{1}{\mu_n} \frac{\partial \mu_n}{\partial T}$. From the text $\mu \propto T^{-m}$ where $m \sim 2.5$

$$\therefore \alpha = \frac{m}{T} \text{ at } 300^\circ\text{K} \quad \alpha = \frac{2.5}{300} = 0.008\%/\text{K} = 0.8\%/\text{C}$$

$$\text{or from table 1.4, } \frac{\Delta \mu_n}{\Delta T} = -11.6 \quad \frac{\Delta \mu_p}{\Delta T} = -4.3$$

$$\alpha_n \approx \frac{11.6}{1100} \times 100 = 1.05\%/\text{C} \quad \alpha_p \approx \frac{4.3}{450} \times 100 = 0.96\%/\text{C}$$

$R (\Omega)$	$\frac{L}{W} (\text{n-type})$	$\frac{L}{W} (\text{p-type})$
100	0.115	0.0472
1,000	1.15	0.472
10,000	11.5	4.72

$R (\Omega)$	$n\text{-type} (\mu\text{m})$		$p\text{-type} (\mu\text{m})$	
	W	L	W	L
100	115	13.3	180	8.49
1,000	36.4	42	56.9	26.8
10,000	11.5	133	18	84.9

2.20

$$g = g_1 + g_2 = g\mu_n N_{d_1}' + g\mu_n N_{d_2}'$$

$$\left. \begin{aligned} N_{d_1} &= 10^{18} \text{ cm}^{-3} \Rightarrow \mu_n = 279 \\ N_{d_2} &= 10^{17} \text{ cm}^{-3} \Rightarrow \mu_n = 727 \end{aligned} \right\} \text{from Table 1.1 assuming P}$$

$$N_{d_1}' = 10^{18} \times 10^{-4}; \quad N_{d_2}' = 2 \times 10^{17} \times 10^{-4}$$

$$\begin{aligned} g &= 1.6 \times 10^{-19} \times 10^{-4} [2 \times 10^{17} \times 727 + 279 \times 10^{18}] \\ &= 6.79 \times 10^{-3} \Omega^{-1} \end{aligned}$$

$$(a) R_\square = \frac{1}{g} = 147 \Omega / \square$$

$$\text{for } 3 \times 10^{17} \text{ cm}^{-3} = N_d, \mu_n \sim 500 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$g = g_N \mu_n = 3 \times 10^{17} \times 4 \times 10^{-4} \times 1.6 \times 10^{-19} \times 500$$

$$g = 9.6 \times 10^{-3} \Omega^{-1}; \quad R_\square = 104 \Omega / \square$$

We need to increase R_\square . Therefore, we need to add acceptors in order to compensate the n-type Si.

To obtain $R_\square = 147 \Omega / \square$

$$\Rightarrow g = 6.79 \times 10^{-3} \Rightarrow (N_d - N_a) \mu_n = 4.24 \times 10^{16}$$

$$\text{and } (N_d - N_a) \mu_n = \frac{4.24 \times 10^{16}}{4 \times 10^{-4}} = 1.06 \times 10^{20}$$

To add more acceptors, μ_n will be lower than $500 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$

$$\text{Guess } \mu_n \approx 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\text{Then, } (N_d - N_a) = 2.35 \times 10^{17} \text{ cm}^{-3}$$

$$\therefore \text{added } N_a = 3 \times 10^{17} - 2.35 \times 10^{17} = 6.5 \times 10^{16} \text{ cm}^{-3}$$

$$\text{check: } \mu_n \text{ with } |N_d| + |N_a| = 3.65 \times 10^{17}$$

agrees reasonably with $\mu_n \approx 450$

This is an implant dose of $2.6 \times 10^{13} \text{ cm}^{-2}$ B atoms.

2.21

$$(a) \frac{1}{R_\square} = \int_0^{x_i} \sigma(x) dx = \int_0^{x_i} P q \mu dx$$

$P = N_a - N_d \approx N_a$, assuming $N_a \gg N_d$ throughout most of diffused layer. $\mu = \mu(P) = \mu(x)$, let $\mu_p = \bar{\mu}_p$ (a weighted average value) in order to remove μ from the integral.

The value of the expression under the integral decreases rapidly with distance so that the upper limit x_i can be replaced by ∞ . Then

$$\frac{1}{R_\square} \approx g \bar{\mu}_p N_s \int_0^\infty \operatorname{erfc}\left(\frac{x}{\lambda}\right) dx$$

$$(b) \text{Integrate by parts } \int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

$$\text{let } v = \frac{x}{\lambda}, u = \operatorname{erfc}\left(\frac{x}{\lambda}\right) = \operatorname{erfc}(v)$$

$$\int_0^\infty \operatorname{erfc}\left(\frac{x}{\lambda}\right) dx = \int_0^\infty \lambda \operatorname{erfc} v v dv = \lambda \int_0^\infty u dv$$

$$= \lambda v \operatorname{erfc} v \Big|_0^\infty - \lambda \int_0^\infty v \left(-\frac{2}{\sqrt{\pi}} e^{-v^2} dv\right) = 0 + \frac{\lambda}{\sqrt{\pi}} \int_0^\infty 2v e^{-v^2} dv$$

$$= \frac{\lambda}{\sqrt{\pi}} e^{-v^2} \Big|_0^\infty = \frac{\lambda}{\sqrt{\pi}}$$

$$\therefore \frac{1}{R_\square} = \frac{g \bar{\mu}_p N_s \lambda}{\sqrt{\pi}} \quad \therefore R_\square = \frac{\sqrt{\pi}}{g \bar{\mu}_p N_s \lambda}$$

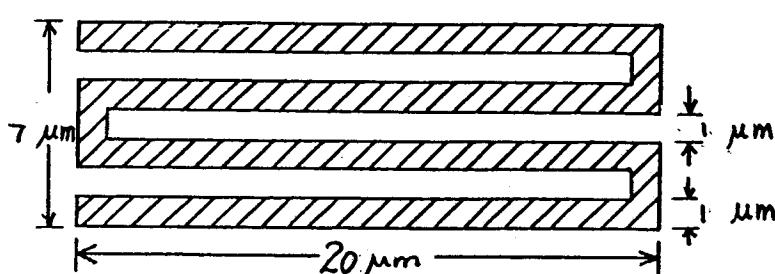
$$(c) R_\square = \frac{\sqrt{\pi}}{g \bar{\mu}_p N_s \lambda}, N_s = 10^{18} \text{ cm}^{-3}, \lambda = 5 \times 10^{-6} \text{ cm}, \bar{\mu}_p = 250 \text{ cm}^2/\text{V-sec}$$

(for $N_a = N_s/2$)

$$L = 200 \mu\text{m}, W = 4 \mu\text{m}$$

$$R_\square = 8860 \Omega/\square, R = R_\square \frac{L}{W} = 443 \text{ k}\Omega$$

(d) Assuming that the entire $70 \times 200 \mu\text{m}^2$ can be used for the resistor and neglecting any contact pad area, the pattern looks as below:

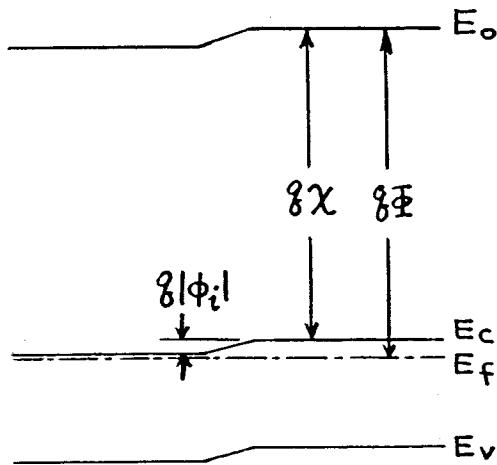


$$R = R_\square (77 + 0.65 \times 6) = 716.8 \text{ k}\Omega$$

straight line squares corner squares

CHAPTER 3

3.1



$$g\chi_s = 4.05 \text{ eV} \quad (\text{from Table 3.1})$$

$$kT = 0.0258 \text{ eV} \quad \text{at } T = 300^\circ\text{K}$$

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3} \quad (\text{from Table 1.3})$$

$$(a) E_{c1} - E_f = kT \ln \frac{N_c}{N_{d1}} \\ = 0.0258 \ln \frac{2.8 \times 10^{19}}{5 \times 10^{18}} = 0.044 \text{ eV}$$

$$E_{c2} - E_f = kT \ln \frac{N_c}{N_{d2}} \\ = 0.0258 \ln \frac{2.8 \times 10^{19}}{8 \times 10^{15}} = 0.211 \text{ eV}$$

Work functions

$$g\Phi_{s1} = g\chi_s + (E_{c1} - E_f) = 4.094 \text{ eV}$$

$$g\Phi_{s2} = g\chi_s + (E_{c2} - E_f) = 4.261 \text{ eV}$$

$$(b) \Phi_i = 4.261 - 4.094 = 0.167 \text{ V}$$

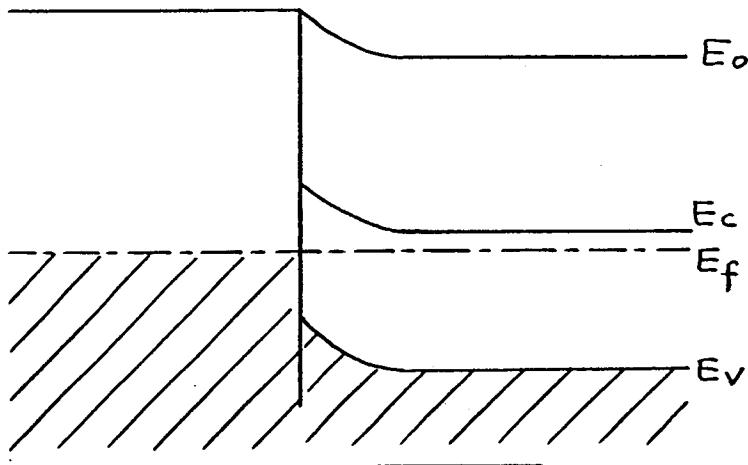
3.2

(a) Fig.(b). E_o cannot be discontinuous since a discontinuity would permit work to be extracted from the system.

Fig.(c). The electron affinity $g\chi = E_o - E_c$ cannot vary in the semiconductor. It is a property of the band configuration.

Fig.(d). The Fermi level must be constant at thermal equilibrium.

(b)

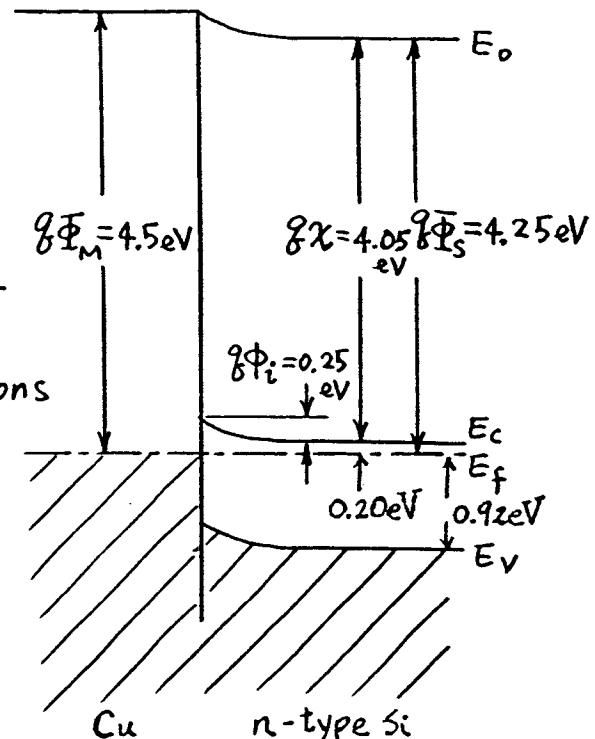


3.3

(a) $\phi_i = 0.25 \text{ V}$ (Silicon to copper)

(b)

(i) The built-in electric field is directed from the silicon toward the copper. If hole-electron pairs are generated within the space-charge-region, the holes move toward the copper and the electrons move toward the silicon bulk. This represents a current in the diode from silicon toward the copper.



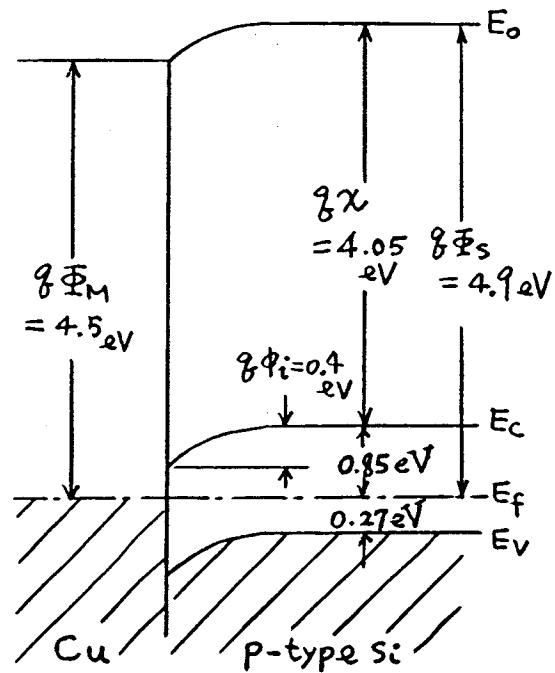
(ii) If the diode is an open circuit, the net current generated by the light must be reduced to zero. This occurs when the built-in field is reduced to zero. The collected holes and electrons raise the potential of the metal with respect to the silicon, thus forward biasing the junction. The bias at the junction can be measured externally and its maximum value is the built-in voltage $\phi_i = 0.25 \text{ V}$.

(c) $g\Phi_S = 4.9 \text{ eV}$, then:

(d) Part (a) refers to a Schottky barrier to n-type silicon.



Part (c) refers to a Schottky barrier to p-type silicon.



3.4

Since $\frac{1}{C^2}$ increases as V_a is made negative, the metal-silicon contact is back-biased. Hence, the silicon is n-type.

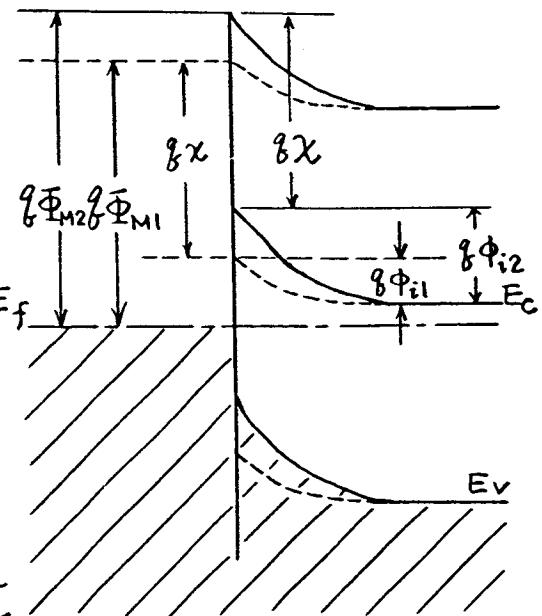
- (a) The intercept of $\frac{1}{C^2}$ versus V_a represents ϕ_i , the built-in potential. Thus, ϕ_i is greater for E_f metal 2. Now $\phi_i = \Phi_M - \Phi_s$, as seen in prob. 3.1 $\Phi_s = X + \frac{E_c - E_f}{q}$ is only changed slightly by the change by factor of 5 in resistivity of the silicon. Hence ϕ_i probably varies directly with Φ_M , and therefore Φ_M for metal 2 is probably greater. From the accompanying energy-band diagram, this means that $\Phi_{M2} > \Phi_{M1}$.

- (b) The resistivity can be inferred from $\frac{1}{C^2}$ behavior at a given total reverse bias.

Resistivity	N_d	x_d	C	$\frac{1}{C^2}$
$1\Omega\text{-cm}$	greater	lesser	greater	lesser
$5\Omega\text{-cm}$	lesser	greater	lesser	greater

Hence, metal 2 contacts the $1\Omega\text{-cm}$ silicon and metal 1 contacts the $5\Omega\text{-cm}$ silicon.

- (c) "Probably" is used for the reason stated in (a) and also because surface states can act to modify the basic Schottky theory as in section 3.5.

3.5

Platinum to silicon, $g\Phi_M = 5.3\text{ eV}$ $N_d = 10^{16}\text{ cm}^{-3}$ $A = 10^{-5}\text{ cm}^2$ as in problem 3.1 $E_c - E_f = kT \ln \frac{N_c}{N_d} = 0.205\text{ eV}$

$$\therefore g\Phi_s = 4.05 + 0.205 = 4.255\text{ eV}$$

$$\therefore \phi_i = -(\Phi_s - \Phi_M) = 1.045\text{ V (Si to Pt)}$$

$$C = \frac{K'}{(\phi_i - V_a)^{\frac{1}{2}}} \quad \text{where} \quad K' = A \left(\frac{8\epsilon_s N_d}{2} \right)^{\frac{1}{2}} = 2.88 \times 10^{-13} \text{ F-V}^{\frac{1}{2}}$$

$$(a) \text{ If } V_a = 0, C = 2.82 \times 10^{-13} = 0.282 \text{ pF.}$$

$$(b) \text{ For a 25\% reduction, } \frac{C}{C(0)} = 0.75 \Rightarrow \left(\frac{\phi_i}{\phi_i - V_a} \right)^{\frac{1}{2}} = 0.75$$

$$\therefore V_a = \phi_i \left(1 - \frac{1}{0.75^2} \right) = -0.813\text{ V}$$

3.6

(a) From Fig. 3.8 and Eq. (3.2.12) the total energy is

$$E_2 = \frac{-g^2}{16\pi\epsilon_s x} - g|\epsilon| x$$

The first term E_1 represents the energy from the image force attraction.

$$E_1 = \int_{\infty}^x \frac{g^2 dx}{4\pi\epsilon_s (2x)^2} = \frac{-g^2}{16\pi\epsilon_s x}$$

negative because work is extracted from the system in establishing the condition.

A field tending to remove electrons from the metal will reduce the potential energy by $\Delta E = - \int_0^x g|\epsilon| dx = -g|\epsilon| x$

The maximum value of E_2 is reached at $x = x_m$, where

$$\frac{dE_2}{dx} = \frac{g^2}{16\pi\epsilon_s x_m^2} - g|\epsilon| = 0 \quad \text{or} \quad x_m = \sqrt{\frac{g}{16\pi\epsilon_s |\epsilon|}}$$

at x_m , $E_1 = \sqrt{\frac{g^3 |\epsilon|}{16\pi\epsilon_s}}$ and the reduction owing to $|\epsilon|$ is:

$$g|\epsilon|x_m = \sqrt{\frac{g^3 |\epsilon|}{16\pi\epsilon_s}}. \quad \text{Hence, } g\Delta\phi = 2\sqrt{\frac{g^3 |\epsilon|}{16\pi\epsilon_s}} = \sqrt{\frac{g^3 |\epsilon|}{4\pi\epsilon_s}}$$

which is Eq. (3.2.13).

(b) For $\epsilon = 10^5 \text{ V/cm}$, $x_m = 1.73 \times 10^{-7} \text{ cm}$, $g\Delta\phi = 0.0346 \text{ eV}$

3.7

(a) From Eq. (3.3.18), we have $\phi(x) = \frac{8N_d x_d x}{\epsilon_s} - \frac{8N_d x^2}{2\epsilon_s}$
Using the condition $\phi(l) = \phi_B$, we get

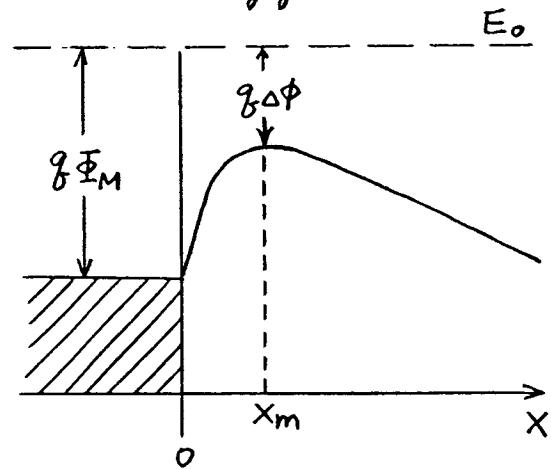
$$\phi_B + \frac{8N_d l^2}{2\epsilon_s} = \frac{8N_d x_d l}{\epsilon_s} \quad \text{and since } x_d = \sqrt{\frac{2\epsilon_s (\phi_i - V_a)}{8N_d}},$$

$$\phi_B + \frac{8N_d l^2}{2\epsilon_s} = l \sqrt{\frac{2g(\phi_i - V_a)}{\epsilon_s}} \sqrt{N_d}$$

$$0.65 + 7.726 \times 10^{-20} N_d = 1.243 \times 10^{-9} \sqrt{N_d}$$

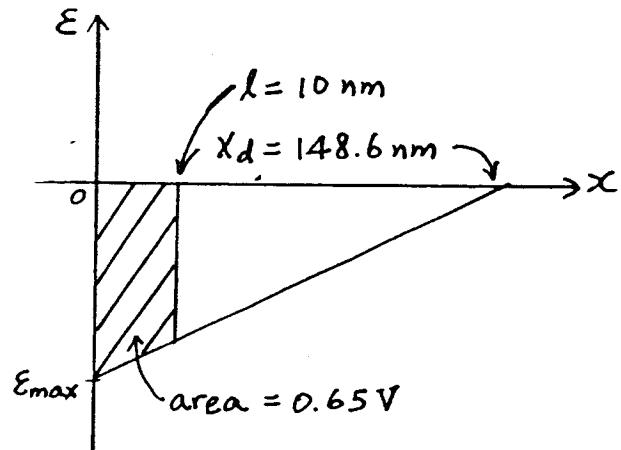
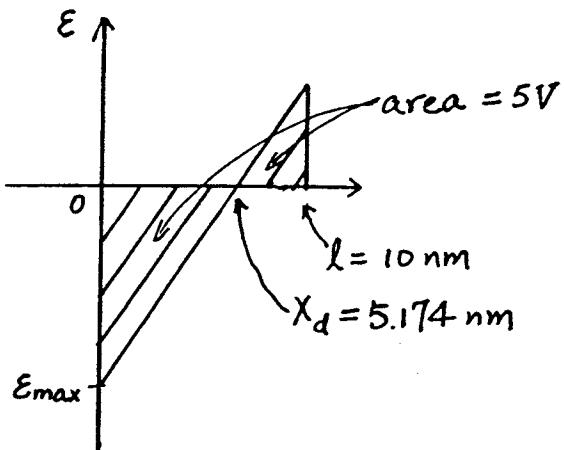
$$\therefore 5.969 \times 10^{-39} N_d^2 - 1.445 \times 10^{-18} N_d + 0.4225 = 0$$

Solving this quadratic equation for N_d , we obtain



$$N_d = 2.929 \times 10^{17} \text{ cm}^{-3} \text{ or } 2.417 \times 10^{20} \text{ cm}^{-3}$$

The solution $N_d = 2.417 \times 10^{20} \text{ cm}^{-3}$ is spurious because it gives $X_d = \sqrt{\frac{2\epsilon_s(\phi_i - V_a)}{qN_d}} = 5.174 \text{ nm}$ and corresponds to the situation below left



The correct answer is $N_d = 2.929 \times 10^{17} \text{ cm}^{-3}$, $X_d = 148.6 \text{ nm}$, which corresponds to the situation above right. Thus $N_d \leq 2.929 \times 10^{17} \text{ cm}^{-3}$ in order to avoid the onset of efficient tunneling. We could have also obtained the approximate value of N_d by

$$|E_{max}|l \approx \phi_i,$$

$$|E_{max}| \approx \frac{0.65 \text{ V}}{10 \text{ nm}} = 6.5 \times 10^5 \text{ V/cm}$$

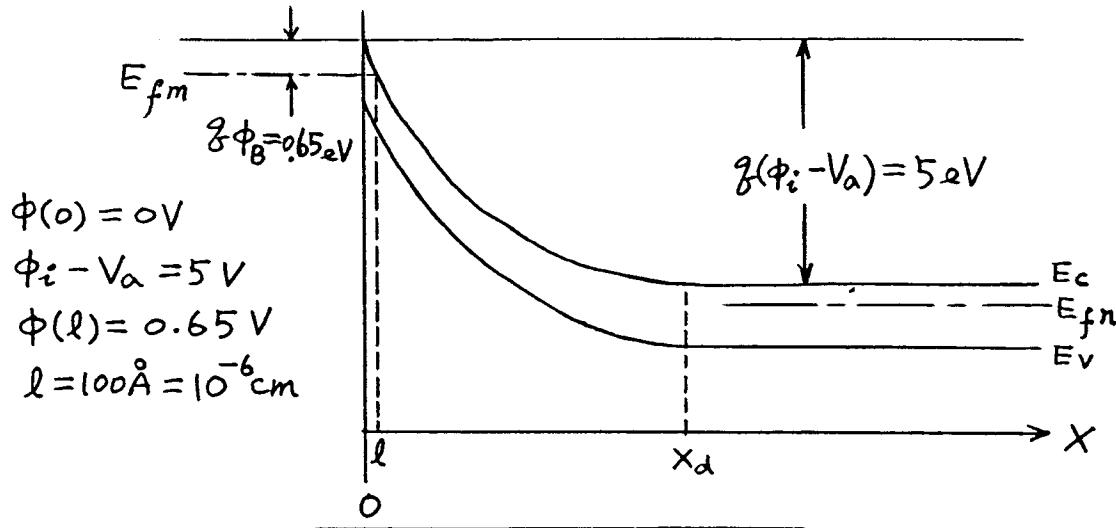
$$= \frac{qN_d X_d}{\epsilon_s} = \sqrt{\frac{2qN_d(\phi_i - V_a)}{\epsilon_s}}$$

$$N_d \approx 2.73 \times 10^{17} \text{ cm}^{-3}$$

(b) The condition $N_d \leq 2.929 \times 10^{17} \text{ cm}^{-3}$, in turn, requires $\rho \geq 0.04 \Omega \cdot \text{cm}$ (from Fig. 1.15) for the epitaxial layer

resistivity in order that Schottky diodes can be made. In practice, resistivity values of roughly 1 Ω -cm are used.

(C)

3.8

Derivation follows the steps outlined in the text.

3.9

To obtain Eq (3.3.17) from Eq (3.3.16), we require

$$K(\phi_i - V_a)^{\frac{1}{2}} \left(e^{\frac{8V_a}{kT}} - 1 \right) = J_s' \left(e^{\frac{8V_a}{nKT}} - 1 \right) \quad (1)$$

where K , a constant independent of Eq.(1), can be written:

$$K' \left(1 - \frac{V_a}{\phi_i} \right)^{\frac{1}{2}} \left(e^{\frac{8V_a}{kT}} - 1 \right) = J_s' \left(e^{\frac{8V_a}{nKT}} - 1 \right) \quad (2)$$

where $K' = K \sqrt{\phi_i}$ is a new constant. Now

$$\left(1 - \frac{V_a}{\phi_i} \right)^{\frac{1}{2}} = \exp \left[\frac{1}{2} \ln \left(1 - \frac{V_a}{\phi_i} \right) \right] \quad (3)$$

Rewriting Eq.(2) under forward bias, and using (3). We have

$$K' \exp \left[\frac{8V_a}{kT} + \frac{1}{2} \ln \left(1 - \frac{V_a}{\phi_i} \right) \right] = J_s' \exp \left[\frac{8V_a}{nKT} \right] \quad (4)$$

Hence, if $J_s' = K'$, Eq.(4) implies :

$$\frac{1}{n} = 1 + \frac{kT}{2eV_a} \ln\left(1 - \frac{V_a}{\phi_i}\right) \quad \dots \quad (5)$$

If $V_a/\phi_i \ll 1$ for a low forward bias, $\ln\left(1 - \frac{V_a}{\phi_i}\right) \approx -\frac{V_a}{\phi_i}$. Hence, from Eq. (5)

$$n = \frac{1}{\left(1 - \frac{1}{2} \frac{kT}{e\phi_i}\right)} \approx 1 + \frac{1}{2} \frac{kT}{e\phi_i} \quad (\text{if } \frac{kT}{2e\phi_i} \ll 1), \quad \left(\frac{1}{1-x} \approx 1+x \text{ for } x \ll 1\right)$$

which was to be proven.

Ex. If $\phi_i = 600mV$, then at $T = 300^{\circ}\text{K}$, $n = 1.0215$

3.10

$$I = I_0 \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]$$

$$(a) I_0 = 1\text{pA} = 10^{-12}\text{A} \quad (b) I_0 = 1\text{nA} = 10^{-9}\text{A} \quad (c) I_0 = 1\mu\text{A} = 10^{-6}\text{A}$$

$$T = 150^{\circ}\text{K} \quad T = 300^{\circ}\text{K} \quad T = 450^{\circ}\text{K}$$

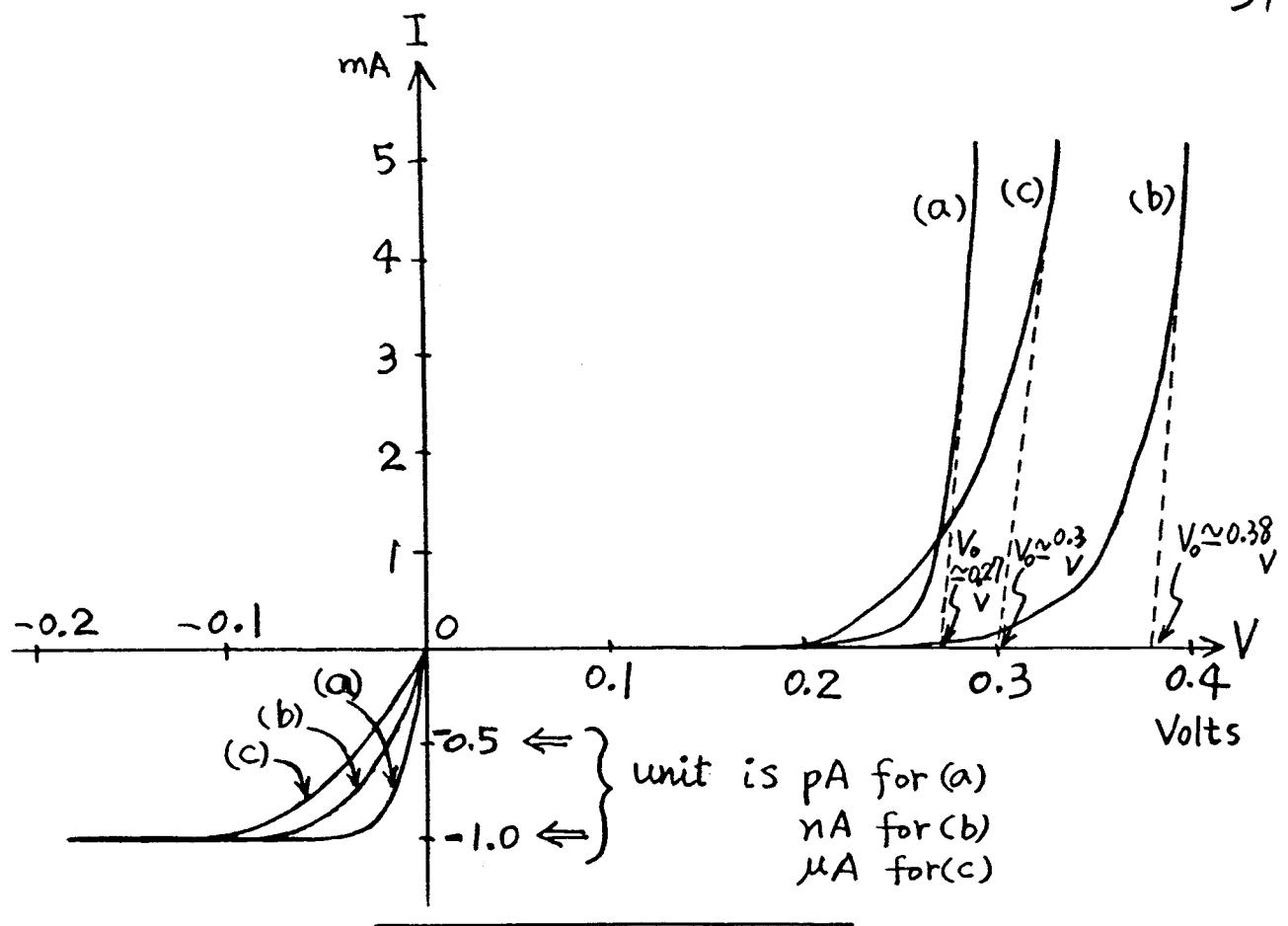
V	I	V	I	V	I
0	0	0	0	0	0
0.1	2.32×10^{-9}	0.1	4.7×10^{-8}	0.1	1.2×10^{-5}
0.2	5.4×10^{-6}	0.2	2.28×10^{-6}	0.2	1.7×10^{-4}
0.25	2.6×10^{-4}	0.3	1.09×10^{-4}	0.26	8.1×10^{-4}
0.27	1.23×10^{-3}	0.35	7.6×10^{-4}	0.28	1.4×10^{-3}
0.28	2.67×10^{-3}	0.38	2.4×10^{-3}	0.3	2.3×10^{-3}
0.29	5.79×10^{-3}	0.39	3.6×10^{-3}	0.32	3.8×10^{-3}
-0.1	-1.0×10^{-12}	0.4	5.2×10^{-3}	0.33	4.95×10^{-3}
-0.03	-0.9×10^{-12}	-0.1	-0.98×10^{-9}	-0.1	-0.92×10^{-6}
-0.015	-0.68×10^{-12}	-0.03	-0.69×10^{-9}	-0.03	-0.54×10^{-6}
		-0.015	-0.44×10^{-9}	-0.015	-0.32×10^{-6}

(Fig. on the next page)

(d) For (a) $V_0 \approx 0.27V$

For (b) $V_0 \approx 0.38V$

For (c) $V_0 \approx 0.3V$

3.11

Excess free electron density

$$n_x = n_s \exp\left[-\frac{8|\phi|}{kT}\right]$$

$$= n_s \exp\left[\frac{8\phi}{kT}\right]$$

Poisson's Equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon} \quad \dots \quad (1)$$

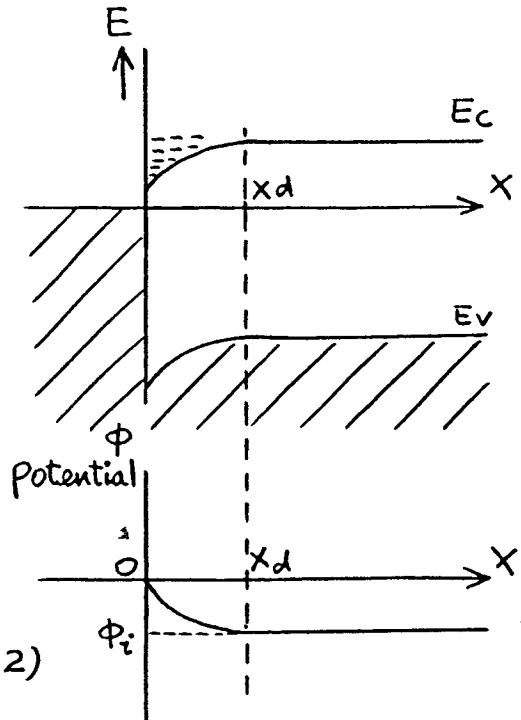
$$= \frac{8}{\epsilon_s} n_s \exp\left(\frac{\phi}{V_t}\right)$$

$$\text{where } V_t = \frac{kT}{8}$$

$$\text{Since } \frac{d^2\phi}{dx^2} = \epsilon \frac{d\epsilon}{d\phi}$$

$$\text{we have } \epsilon \frac{d\epsilon}{d\phi} = k_1 \exp\left(\frac{\phi}{V_t}\right) \quad (2)$$

$$\text{where } k_1 = \frac{8n_s}{\epsilon_s}$$



Eq.(2) can be separated and integrated to find

$$\frac{\varepsilon^2}{2} = K_1 V_t \exp\left(\frac{\phi}{V_t}\right) + K_2 \quad \dots \quad (3)$$

But $\varepsilon \rightarrow 0$ as $-\phi$ becomes much greater than V_t ,

$$\therefore K_2 \text{ must be zero. Hence } \varepsilon = \sqrt{\frac{2n_s k T}{\varepsilon_s}} \exp\left(\frac{\phi}{2V_t}\right) \dots \quad (4)$$

Since $\varepsilon = -\frac{d\phi}{dx}$ we can use Eq.(4) to find

$$\exp\left(\frac{-\phi}{2V_t}\right) = \left(\frac{\varepsilon_s k T}{2\varepsilon_s k T}\right)^{1/2} x + K_3 = \frac{x}{\sqrt{2} L_D} + K_3 \quad \dots \quad (5)$$

where $L_D = \left(\frac{\varepsilon_s k T}{g^2 n_s}\right)^{1/2}$ = Debye length at $x=0$

and K_3 is an integration constant.

Since $\phi=0$ at $x=0$, $\therefore K_3=1$ and Eq.(5) becomes

$$\exp\left(\frac{\phi}{2V_t}\right) = \left(1 + \frac{x}{\sqrt{2} L_D}\right)^{-1} \quad \dots \quad (6)$$

Since $\rho = \frac{g}{\varepsilon_s} n_s \exp\left(\frac{\phi}{V_t}\right)$, we have from Eq.(6)

$$\rho(x) = \frac{g n_s}{\varepsilon_s} \frac{1}{\left(1 + \frac{x}{\sqrt{2} L_D}\right)^2} \text{ which is Eq.(3.4.2)}$$

Also, using Eq.(5) in Eq.(4) we have Eq.(3.4.4)

To obtain Eq.(3.4.5), let $\phi = -|\phi|$ in Eq.(6) and solve for $x = X_d$.

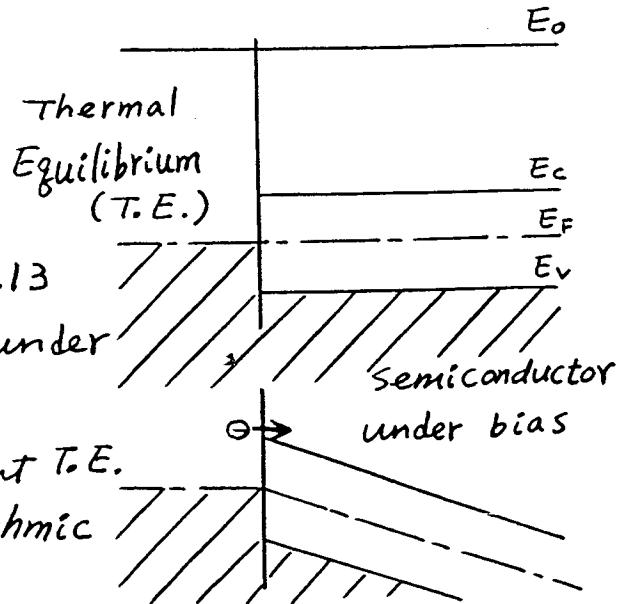
3.12

- (a) The T.E. electron flux from Semiconductor into metal and vice-versa, because of diffusion is

$$J = g n_0 V_{th}/4 \text{ from Problem 1.13}$$

The flow in the semiconductor under bias is $J = g n_0 V = g \mu n_0 E$

If less than the diffusion flux at T.E. this will be supplied by an "ohmic contact". Thus,



$g\mu n_0 \epsilon \leq g n_0 V_{th}/4$, therefore $\epsilon \leq V_{th}/4\mu$

$$(b) I = g A \mu n_0 V_{th}/4\mu = g A n_0 V_{th}/4 = 40 \times 10^{-3} A_{mp} = 40 mA_{mp}$$

$$(c) I = 40mA \exp\left[-g \frac{(\Phi_M - \chi - \phi_n)}{kT}\right] = 4.58 \times 10^{-13} A$$

3.13 Let the total space charge/area be Q'_s , where

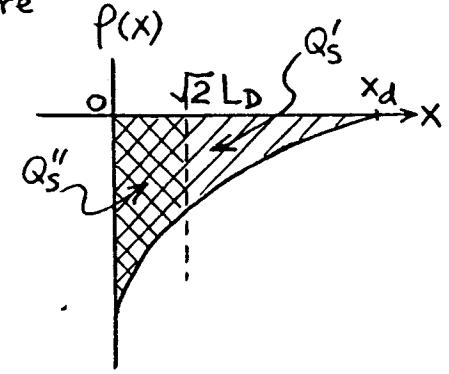
$$Q'_s = \int_0^{x_d} p(x) dx, \text{ From Eq.(3.4.2)}$$

$$Q'_s = -g n_s \int_0^{x_d} \frac{dx}{(1 + x/\sqrt{2} L_D)^2}$$

$$= g n_s \left[\frac{\sqrt{2} L_D}{1 + x/\sqrt{2} L_D} \right] \Big|_0^{x_d}$$

$$= \sqrt{2} g n_s L_D \left[\frac{1}{1 + x_d/\sqrt{2} L_D} - 1 \right]$$

$$= \frac{-g n_s x_d}{1 + x_d/\sqrt{2} L_D} \quad \text{total space charge/area}$$



Let the space charge/area between $x=0$ and $x=\sqrt{2} L_D$ be Q''_s where:

$$Q''_s = \sqrt{2} g n_s L_D \left[\frac{1}{1 + x/\sqrt{2} L_D} \right] \Big|_0^{\sqrt{2} L_D} = -\frac{\sqrt{2} g n_s L_D}{2}$$

$$\text{From Eq.(3.4.5)} x_d = \sqrt{2} L_D \left[\exp\left(\frac{g \phi_i}{2kT}\right) - 1 \right]$$

$$\therefore Q'_s = \frac{-\sqrt{2} g n_s L_D}{\frac{\sqrt{2} L_D}{x_d} + 1} = \frac{-\sqrt{2} g n_s L_D}{\frac{1}{\exp(g \phi_i / 2kT) - 1} + 1} \approx -\sqrt{2} g n_s L_D$$

for $|\phi_i| > \frac{kT}{g}$ as is to be expected

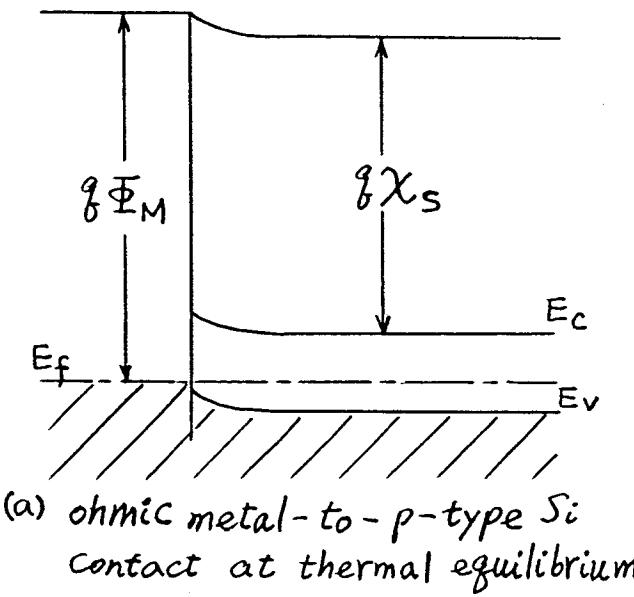
$$\therefore Q'_s = 2 Q''_s$$

3.14

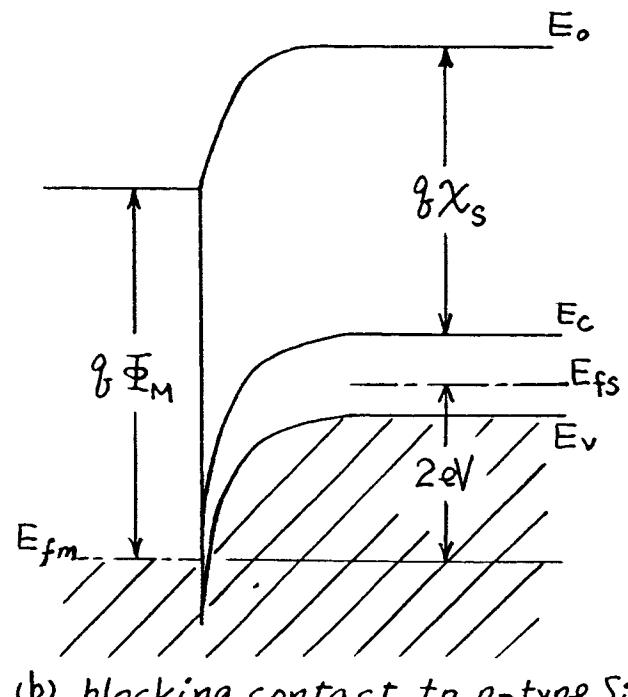
$$D\tau_r = D \cdot \frac{\epsilon_s}{8} = D \cdot \frac{\epsilon_s}{g \mu n} = \frac{kT}{g} \frac{\epsilon_s}{g n} \quad (\text{Einstein relation})$$

$$(D\tau_r)^{1/2} = \left(\frac{kT \epsilon_s}{g^2 n} \right)^{1/2} = L_D \quad \text{as in Eq.(3.4.3)} \quad \left(\frac{D}{\mu} = \frac{kT}{g} \right)$$

3.15



(a) ohmic metal-to-p-type Si contact at thermal equilibrium

(b) blocking contact to p-type Si
2 V reverse bias

3.16

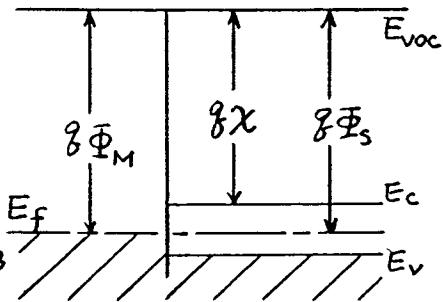
For a neutral contact ($\Phi_M = \Phi_S$)

$$E_C - E_F = g\Phi_M - g\chi = 0.45 \text{ eV}.$$

Since this value is less than $\frac{E_g}{2} = 0.56 \text{ eV}$, the silicon would be n-type.

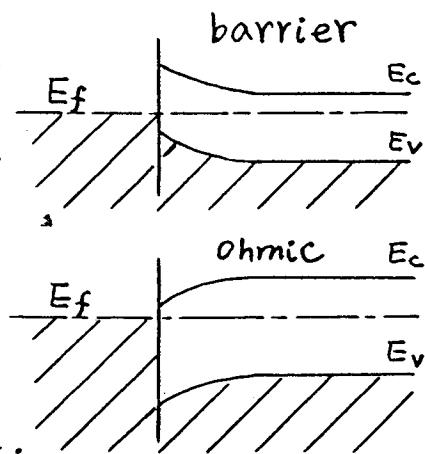
$$\therefore N_d = N_c \exp[-(E_C - E_F)/kT]$$

$$= (2.8 \times 10^9) \exp\left[\frac{-0.45}{0.026}\right] = 7.45 \times 10^{11} \text{ cm}^{-3}$$

n-Silicon

If $N_d > 7.45 \times 10^{11} \text{ cm}^{-3}$ $\Phi_S < \Phi_M$ and we obtain a Schottky-barrier diode.

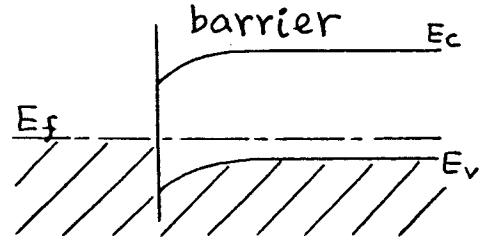
If $N_d < 7.45 \times 10^{11} \text{ cm}^{-3}$ $\Phi_S > \Phi_M$ and we obtain a Schottky ohmic contact. However, the doping concentration is so small that it would be impractical. For an ohmic contact one would dope very heavily and rely on the formation of a barrier that would transmit currents by tunneling.



P-silicon

$$\Phi_s > 4.05 + 0.55 = 4.61 \text{ eV}$$

Thus $\Phi_s > \Phi_M$ for all values of doping and we obtain a Schottky-barrier to P-silicon.

3.17

$$\text{We have } -\frac{1}{4\pi L^{1/2}} \cdot \frac{1}{C^{3/2}} \cdot \frac{dC}{dV} = 2.2 \times 10^5 \text{ Hz/V}$$

$$\text{From Eq.(3.2.10)} \quad \frac{dC}{dV} = -\frac{C^3}{8\epsilon_s A^2 N(x_d)}$$

$$\text{Combining these equations gives : } N(x_d) = \frac{1}{4\pi L^{1/2} g \epsilon_s A^2 (2.2 \times 10^5)} \cdot C^{3/2}$$

$$\text{Using } C = \frac{A \epsilon_s}{x_d},$$

$$N(x_d) = \frac{\epsilon_s^{1/2}}{4\pi L^{1/2} g A^{1/2} (2.2 \times 10^5)} \cdot \frac{1}{x_d^{3/2}} = \frac{1.629 \times 10^9}{x_d^{3/2}}$$

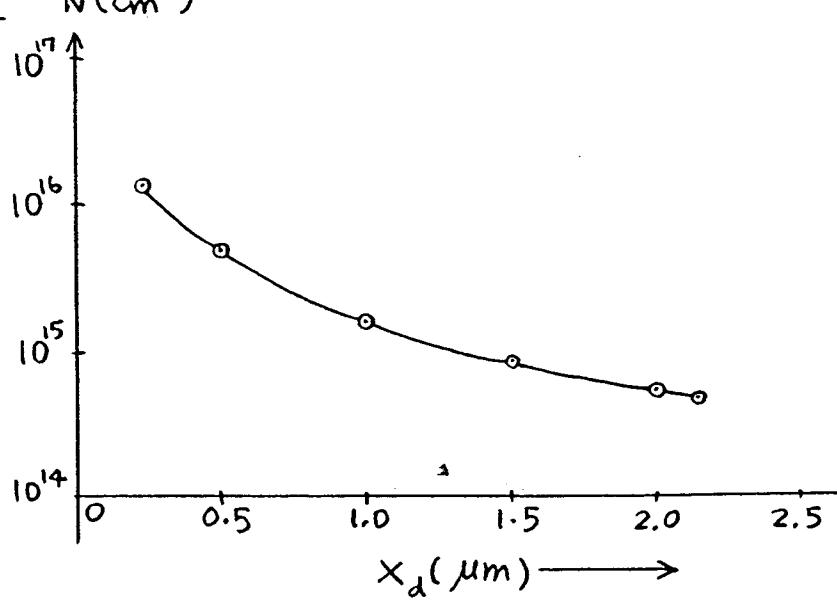
$$\text{where } A = 10^{-3} \text{ cm}^2, L = 2 \times 10^{-3} \text{ m}$$

$$\text{At } 0 \text{ Volt, } C = 41.8 \text{ pF} \Rightarrow x_d = \frac{A \epsilon_s}{C} = 0.247 \mu\text{m}$$

$$\therefore N(x_d) = 1.32 \times 10^{16} \text{ cm}^{-3}$$

$$\text{At } 5 \text{ Volts, } C = 4.65 \text{ pF} \Rightarrow x_d = 2.23 \mu\text{m} \quad \therefore N(x_d) = 4.89 \times 10^{14} \text{ cm}^{-3}$$

$N(x_d) (\text{cm}^{-3})$	$x_d (\mu\text{m})$	$N (\text{cm}^{-3})$
1.32×10^{16}	0.247	10^{17}
4.6×10^{15}	0.5	10^{16}
1.63×10^{15}	1.0	10^{15}
8.85×10^{14}	1.5	10^{14}
5.75×10^{14}	2.0	10^{14}
4.89×10^{14}	2.23	10^{14}



CHAPTER 4

4.1

(a) From Eq. (4.2.10) $\phi_i = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = 0.72 \text{ Volts}$ $(N_a = 1 \times 10^{15} \text{ cm}^{-3}, N_d = 2 \times 10^{17} \text{ cm}^{-3})$

(b) From Eq. (4.3.1)

$$x_d = \left[\frac{2\epsilon_s}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) (\phi_i - V_a) \right]^{\frac{1}{2}} = \begin{cases} 0.97 \mu\text{m} @ V_a = 0 \text{ V} \\ 3.73 \mu\text{m} @ V_a = -10 \text{ V} \end{cases}$$

$$\text{From Eq. (4.3.3)} \quad E_{\max} = \frac{2(\phi_i - V_a)}{x_d} = \begin{cases} 1.48 \times 10^4 \text{ V/cm} @ V_a = 0 \text{ V} \\ 5.75 \times 10^4 \text{ V/cm} @ V_a = -10 \text{ V} \end{cases}$$

4.2

One-sided step junction : $E_{\max} = \frac{q}{\epsilon_s} N x_d$, $x_d = \frac{E_{\max} \epsilon_s}{q N_a}$

$$E_{\max} = \frac{2(\phi_i - V_a)}{x_d}, \quad \phi_i - V_a = \frac{x_d E_{\max}}{2}, \quad V_a = \phi_i - \frac{x_d E_{\max}}{2}$$

where $\phi_i = \frac{1}{2} E_g + kT \ln \frac{N}{n_i}$

	$N(\text{cm}^{-3})$	$E_{\max}(\text{V/cm})$ (From Fig. 4.14)	$x_d(\mu\text{m})$	$\phi_i(\text{V})$	$V_a(\text{V})$
(a)	10^{15}	3.1×10^5	20.1	0.85	-311
(b)	10^{16}	4.2×10^5	2.7	0.91	-55.8
(c)	10^{17}	6.2×10^5	0.40	0.97	-11.4
(d)	10^{18}	1.3×10^6	0.084	1.03	-4.43

4.3

From Eq. (4.1.5), $E_x = -\frac{kT}{q} \frac{1}{n} \frac{dn}{dx}$, $n \approx N = N_0 e^{-x/\lambda}$

$$\frac{dn}{dx} \approx \frac{dN}{dx} = -\frac{N}{\lambda}, \quad N_0 = 10^{18} \text{ cm}^{-3}, \quad \lambda = 0.4 \mu\text{m} = 4 \times 10^{-5} \text{ cm}$$

$$\therefore E_x = \frac{kT}{q\lambda} = 648 \text{ V/cm}$$

For an abrupt junction with $N_a = 1 \times 10^{18} \text{ cm}^{-3}$ and $N_d = 1 \times 10^{15} \text{ cm}^{-3}$

$$E_{\max} = \frac{2(\phi_i - V_a)}{x_d}, \quad \text{where } x_d = \left[\frac{2\epsilon_s}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \phi_i \right]^{\frac{1}{2}} = 0.99 \mu\text{m}$$

$$\text{with } V_a = 0, \quad \phi_i = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = 0.759 \text{ V}$$

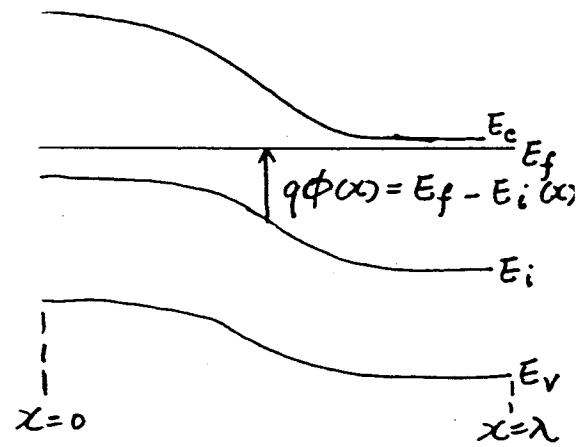
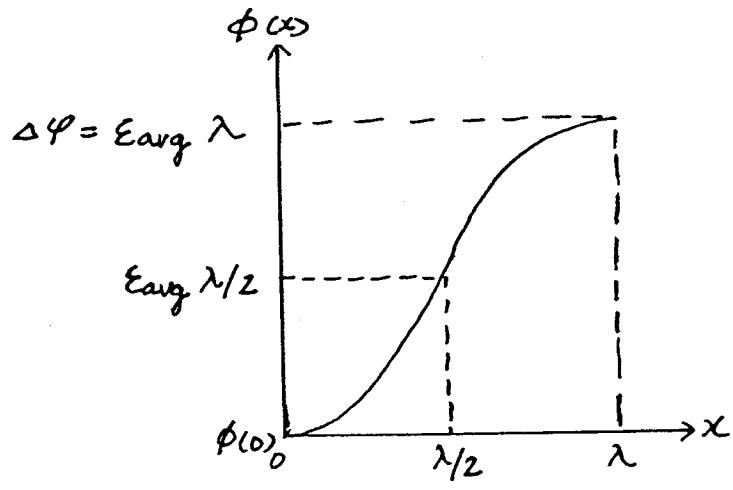
$$\therefore E_{\max} = 1.53 \times 10^4 \text{ V/cm}$$

The exponential impurity distribution case has much weaker built-in field.

4.4

$$\epsilon = \epsilon_{\text{avg}} (1 - \cos \frac{2\pi}{\lambda} x) \quad 0 \leq x \leq \lambda$$

$$\phi(x) = \epsilon_{\text{avg}} \left(x - \frac{\lambda}{2\pi} \sin \frac{2\pi}{\lambda} x \right) + \phi(0) \quad 0 \leq x \leq \lambda$$



4.5

(a) From Poisson's Eq. $\frac{d\phi}{dx} = \frac{\rho}{\epsilon}$ and $\phi = -\int \epsilon dx$

Region	$x_1 < x < x_2$	$x_2 < x < x_3$	$x_3 < x < x_4$
$\rho(x)$	$8Nd$	0	$-8Na$
$\epsilon(x)$	$\frac{8Nd}{\epsilon}(x-x_1)$	$\epsilon_{max} = \frac{8Nd}{\epsilon}(x_2-x_1)$ $= \frac{8Na}{\epsilon}(x_4-x_3)$	$\frac{8Na}{\epsilon}(x_4-x)$
$\phi(x)$	$-\frac{8Nd}{2\epsilon}(x-x_1)^2$	$-\phi_n - \frac{8Nd}{\epsilon}(x_2-x_1)(x-x_2)$	$-\phi_n - \phi_o - \phi_p + \frac{8Na}{2\epsilon}(x_4-x)^2$

where $\phi_n = \frac{8Nd}{2\epsilon}(x_2-x_1)^2$, $\phi_o = \frac{8Nd}{\epsilon}(x_2-x_1)(x_3-x_2)$, and $\phi_p = \frac{8Na}{2\epsilon}(x_4-x_3)^2$

Need to find the depletion region widths: x_2-x_1 and x_4-x_3

$$\phi_i = \phi_n + \phi_o + \phi_p = \frac{kT}{q} [\ln n(x_1) - \ln n(x_4)]$$

$$= \frac{kT}{q} [\ln Nd - \ln \frac{n_i^2}{Na}]$$

$$= \frac{kT}{q} [\ln Nd + \ln Na - \ln n_i^2]$$

$$= \frac{kT}{q} \ln \frac{NdNa}{n_i^2} \quad \text{just as for}$$

$$= 0.637 V \quad \text{PN junction}$$

$$= \frac{8Nd}{2\epsilon}(x_2-x_1)^2 + \frac{8Nd}{\epsilon}(x_2-x_1)(x_3-x_2)$$

$$+ \frac{8Na}{2\epsilon}(x_4-x_3)^2$$

By charge neutrality :

$$8Nd(x_2-x_1) = 8Na(x_4-x_3)$$

$$\phi_i = \frac{8Nd}{2\epsilon}(x_2-x_1)^2 + \frac{8Nd^2}{2\epsilon Na}(x_2-x_1)^2$$

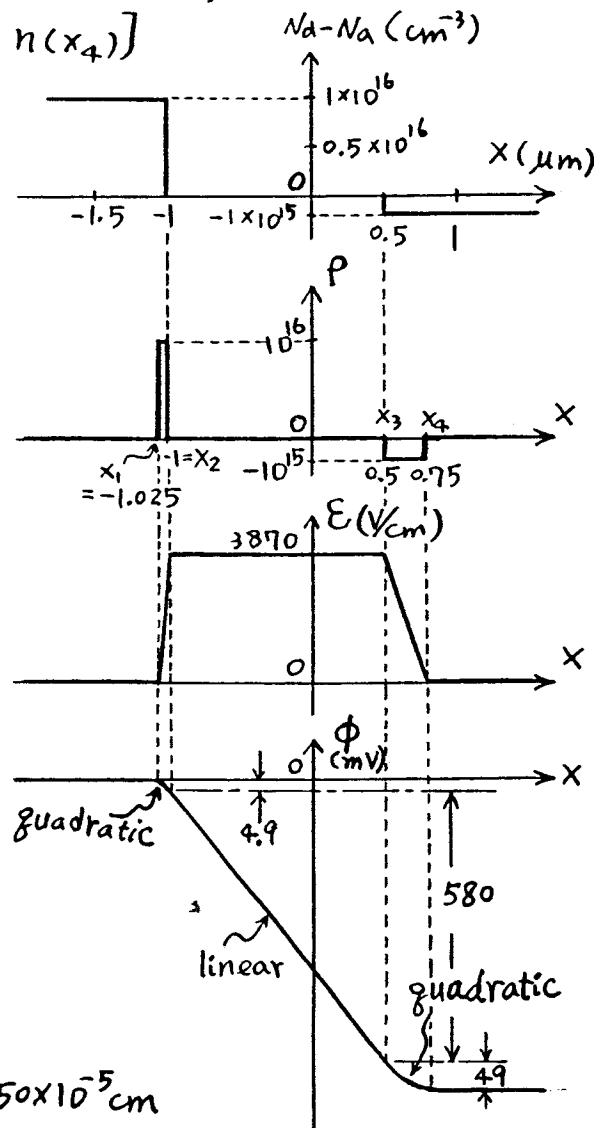
$$+ \frac{8Nd}{\epsilon}(x_2-x_1)(x_3-x_2)$$

$$[7.73 \times 10^8 + 7.73 \times 10^9](x_2-x_1)^2$$

$$+ 2.32 \times 10^5(x_2-x_1) - 0.637 = 0$$

$$\therefore (x_2-x_1)^2 + 2.73 \times 10^{-5}(x_2-x_1) - 7.50 \times 10^{-11} = 0$$

$$\therefore x_2-x_1 = 2.50 \times 10^{-6} \text{ cm}, \quad x_4-x_3 = 2.50 \times 10^{-5} \text{ cm}$$



$$(b) E_{\max} = \frac{8Nd}{\epsilon} (x_2 - x_1) = 3870 \text{ V/cm}$$

$$\text{For pn junction, Eq (4.3.3) } E_{\max} = \frac{2\phi_i}{x_d}$$

$$\text{Eq. (4.3.1)} \quad x_d = \left[\frac{2\epsilon_s}{8} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \phi_i \right]^{\frac{1}{2}} = 9.52 \times 10^{-5} \text{ cm}$$

and $E_{\max} = 13400 \text{ V/cm}$ (3.5 times E_{\max} for pin junction)

(c) Much of the built-in voltage is dropped across the intrinsic region, so that the depletion regions in the n- and p-type material need not extend as far as in a p-n junction. The maximum field, which increases linearly with the extent of the space charge region in the doped material is, therefore, reduced. Alternatively, we may say that the total depletion-region width is the intrinsic region plus some depleted regions in the n- and p-type material. This longer depletion region decreases the maximum field.

$$(d) C = \left| \frac{dQ}{dV_a} \right| = \frac{8Nd}{\epsilon} \left| \frac{d(x_2 - x_1)}{dV_a} \right|. \text{ From part (a)}$$

$$a(x_2 - x_1)^2 + b(x_2 - x_1) - (\phi_i - V_a) = 0$$

$$x_2 - x_1 = \frac{-b + \sqrt{b^2 + 4a(\phi_i - V_a)}}{2a}, \quad \frac{d(x_2 - x_1)}{dV_a} = \frac{1}{4a} \frac{-4a}{\sqrt{b^2 + 4a(\phi_i - V_a)}}$$

$$\text{where } a = \frac{8Nd}{2\epsilon} \left(1 + \frac{N_d}{N_a} \right), \quad b = \frac{8Nd}{\epsilon} (x_3 - x_2)$$

$$\therefore C = \frac{1}{\left[\left(\frac{x_3 - x_2}{\epsilon} \right)^2 + \frac{2}{8\epsilon} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) (\phi_i - V_a) \right]^{\frac{1}{2}}}$$

The capacitance has the same functional dependence as for a p-n junction except for the added term under the square root because of the wider depletion region. This added term corresponds to the intrinsic region and, therefore, does not vary with voltage. The other term corresponds to depletion in the doped regions and, consequently, is voltage dependent. The capacitance of the pin structure is much lower because the depletion region is wider.

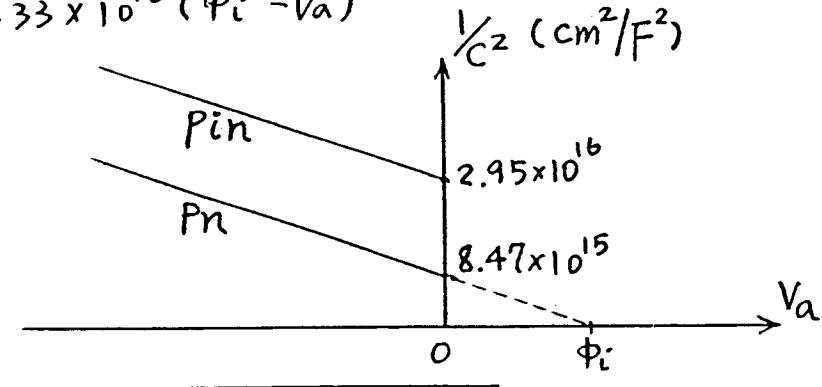
For a p-n junction the above expression reduces to

$$C = \left[\frac{8\epsilon}{2 \left(\frac{1}{N_a} + \frac{1}{N_d} \right) (\phi_i - V_a)} \right]^{\frac{1}{2}} \text{ which is Eq.(4.3.8)}$$

$$\text{For p-n, } \frac{1}{C^2} = \frac{(x_3 - x_2)^2}{\epsilon^2} + \frac{2}{q\epsilon} \left(\frac{1}{N_d} + \frac{1}{N_a} \right) (\phi_i - V_a)$$

$$= 2.10 \times 10^{16} + 1.33 \times 10^{16} (\phi_i - V_a)$$

$$\text{For p-n, } \frac{1}{C^2} = 1.33 \times 10^{16} (\phi_i - V_a)$$



4.6

(a) In equilibrium, at the edge of the neutral regions

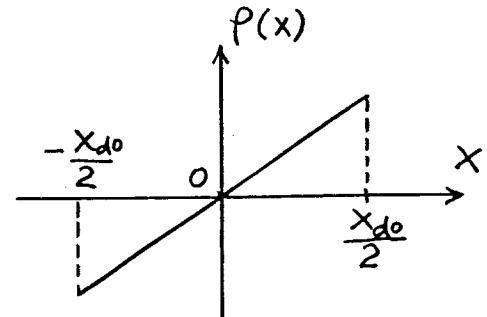
$$n\left(\frac{x_{d0}}{2}\right) = \frac{\alpha x_{d0}}{2} = n_i e^{\left(E_f - E_i\left(\frac{x_{d0}}{2}\right)\right)/kT}$$

$$\Rightarrow E_i\left(\frac{x_{d0}}{2}\right) = E_f - kT \ln \frac{\alpha x_{d0}}{2n_i}$$

$$p\left(-\frac{x_{d0}}{2}\right) = \frac{\alpha x_{d0}}{2} = n_i e^{\left[E_i\left(-\frac{x_{d0}}{2}\right) - E_f\right]/kT}$$

$$\Rightarrow E_i\left(-\frac{x_{d0}}{2}\right) = E_f + kT \ln \frac{\alpha x_{d0}}{2n_i}$$

$$\phi_i = -[E_i\left(\frac{x_{d0}}{2}\right) - E_i\left(-\frac{x_{d0}}{2}\right)]/q = 2 \frac{kT}{q} \ln \frac{\alpha x_{d0}}{2n_i}$$



$$(b) p(x) = q(N_d - N_a) = q\alpha x, \frac{d\epsilon}{dx} = \frac{8\alpha x}{\epsilon_s} \Rightarrow \epsilon(x) = \frac{8\alpha x^2}{2\epsilon_s} + C$$

$$\text{Boundary condition } \epsilon\left(\pm \frac{1}{2}x_d\right) = 0 \Rightarrow C = -\frac{8\alpha x_d^2}{8\epsilon_s}$$

$$\therefore \epsilon(x) = \frac{8\alpha}{\epsilon_s} \left(\frac{x^2}{2} - \frac{x_d^2}{8} \right)$$

$$\frac{d\phi}{dx} = -\epsilon(x) \Rightarrow \phi(x) = -\frac{8\alpha}{\epsilon_s} \left(\frac{x^3}{6} - \frac{x_d^2 x}{8} \right) + C'$$

$$\text{Boundary condition } \phi(0) = 0 \Rightarrow C' = 0$$

$$\text{Now } \phi_i - V_a = \phi\left(\frac{x_d}{2}\right) - \phi\left(-\frac{x_d}{2}\right) = \frac{8\alpha x_d^3}{12\epsilon_s}$$

$$\therefore x_d = \left[\frac{12\epsilon_s(\phi_i - V_a)}{8\alpha} \right]^{\frac{1}{3}}$$

Substituting the equation for x_d into the equation for $E(x)$, gives :

$$E(x) = \frac{8ax^2}{2\epsilon_s} - \frac{1}{2} \left[\frac{8a}{\epsilon_s} \right]^{1/3} \left[\frac{3}{2}(\phi_i - V_a) \right]^{2/3}$$

(c) We know that for an arbitrarily doped junction

$$C = \frac{\epsilon_s A}{x_d}, \text{ using the result for } x_d \text{ from part (b)}$$

$$C = \frac{\epsilon_s A}{\left[\frac{12\epsilon_s(\phi_i - V_a)}{8a} \right]^{1/3}}$$

4.7

$$C = \frac{\epsilon A}{x_d}, \text{ for } C \propto \frac{1}{V}, x_d \propto V$$

The depletion region width in an abrupt step junction varies approximately as $x_d \propto V^{1/2}$. The depletion region in a linearly graded junction, in which the doping increases away from the junction, varies less rapidly with voltage ($x_d \propto V^{1/3}$).

For a more rapid variation than $V^{1/2}$, the doping must decrease away from the junction. This type of profile is called retrograde.

4.8

$$x_0 = 10^{-4} \text{ cm}, \lambda_a = 10^{-4} \text{ cm}, \lambda_d = 2 \times 10^{-4} \text{ cm}$$

$$N_{d0} = 10^{18} \text{ cm}^{-3}$$

(a) at junction $x = x_0$

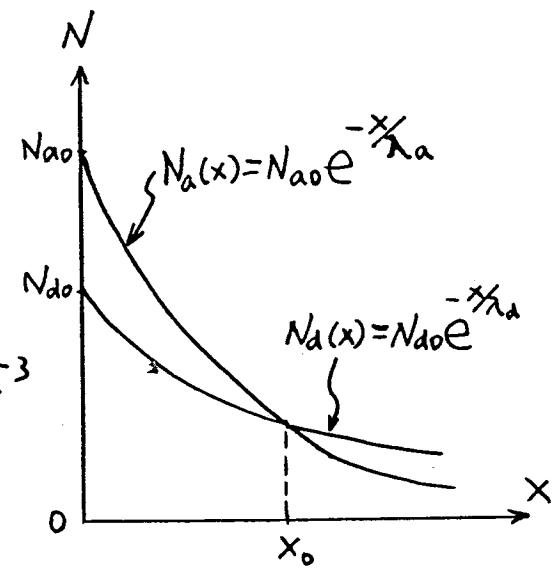
$$N_{d0} e^{-\frac{x_0}{\lambda_d}} = N_{a0} e^{-\frac{x_0}{\lambda_a}}$$

$$N_{d0} = N_{a0} e^{x_0 \left(\frac{1}{\lambda_d} - \frac{1}{\lambda_a} \right)} \approx 6.1 \times 10^{17} \text{ cm}^{-3}$$

(b) $N_d - N_a \approx \alpha(x - x_0)$

$$\text{where } \alpha = \frac{d}{dx} (N_d - N_a) \Big|_{x=x_0}$$

$$= -\frac{N_{d0}}{\lambda_d} e^{-\frac{x_0}{\lambda_d}} + \frac{N_{a0}}{\lambda_a} e^{-\frac{x_0}{\lambda_a}} \approx 1.83 \times 10^{21} \text{ cm}^{-4}$$



Space charge density

$$P(x) = q(N_d - N_a) \approx qa(x - x_0)$$

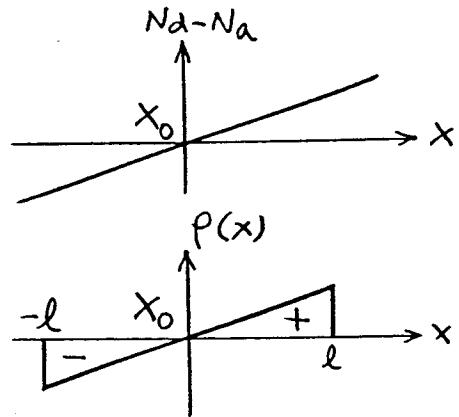
$$(c) x_d = \left(\frac{12\epsilon_s}{qa} \right)^{\frac{1}{3}} (\phi_i - V_a)^{\frac{1}{3}} \quad \text{Eq. (4.3.2)}$$

$$\epsilon_{max} = \frac{3(\phi_i - V_a)}{2x_d} \quad \text{Eq. (4.3.4)}$$

At thermal equilibrium $V_a = 0$,
for $\phi_i = 0.7 \text{ V}$,

$$x_d = \left(\frac{12 \times 11.7 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 1.84 \times 10^{21}} \right)^{\frac{1}{3}} (0.7)^{\frac{1}{3}} \approx 3.1 \times 10^{-5} \text{ cm.}$$

$$\epsilon_{max} = \frac{3 \times 0.7}{2 \times 3.1 \times 10^{-5}} \approx 3.4 \times 10^4 \text{ V/cm}$$



4.9

$$(a) \frac{\Delta(1/C_d^2)}{\Delta(\phi_i - V_a)} = 8.33 \times 10^{25} \text{ F}^{-2} \text{ V}^{-1}$$

For a one-sided step junction

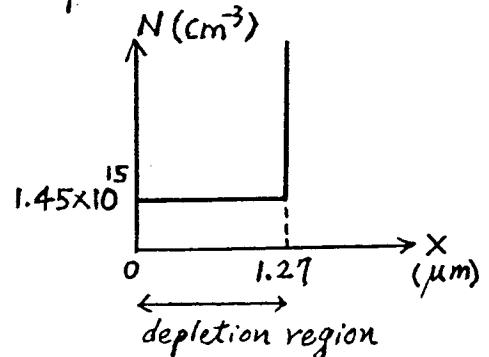
$$C_d = A \left(\frac{8\epsilon_s N}{2} \right)^{\frac{1}{2}} (\phi_i - V_a)^{-\frac{1}{2}}, N = \frac{2C_d^2(\phi_i - V_a)}{A^2 q \epsilon_s} = \frac{2}{A^2 q \epsilon_s} \left(\frac{1/C_d^2}{\phi_i - V_a} \right)^{-1} = 1.45 \times 10^{15} \text{ cm}^{-3}$$

$$(b) x_d = \frac{\epsilon_s A}{C_d}, C_{dmin} = (1.5 \times 10^{26})^{\frac{1}{2}} = 81.6 \text{ fF}$$

$$\therefore x_{dmax} = 1.27 \mu\text{m}$$

$$(c) \phi_i = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = 0.8 \text{ V}$$

$$N' = \frac{n_i^2}{N} e^{q\phi_i/kT} = 3.34 \times 10^{18}$$



$N'/N = 2300$, so the assumption of one-sided step junction is good.

4.10

$$(a) (i) E_f - E_i = kT \ln \frac{N_d}{n_i} = 0.026 \text{ eV} \ln \frac{5 \times 10^{15}}{1.45 \times 10^{10}} \approx 0.33 \text{ eV.}$$

$$(ii) E_i - E_f = kT \ln \frac{N_a}{n_i} \approx 0.36 \text{ eV.} \quad (10 \Omega \cdot \text{cm p-type} \Rightarrow 1.4 \times 10^{16} \text{ cm}^{-3})$$

$$(b) (i) \phi_i = \phi_B - \frac{E_c - E_f}{q} = \phi_B - \frac{(E_c - E_i) - (E_f - E_i)}{q}$$

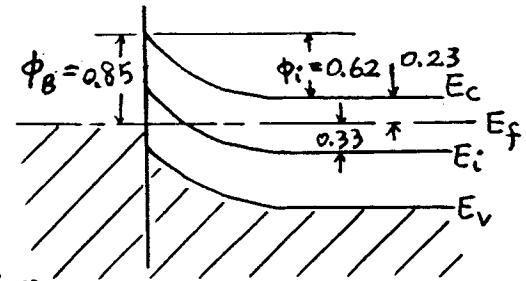
$$= 0.85 - (0.56 - 0.33) = 0.62 \text{ V}$$

(ii) Ideal Schottky

$$\Phi_B = \Phi_M - \chi = 5.3 - 4.05 = 1.25 \text{ V}$$

$\neq 0.85 \text{ V}$ measured

The actual Pt-Si contact is not ideal. Surface states on a transition layer account for the difference between the theoretical and measured barrier heights.



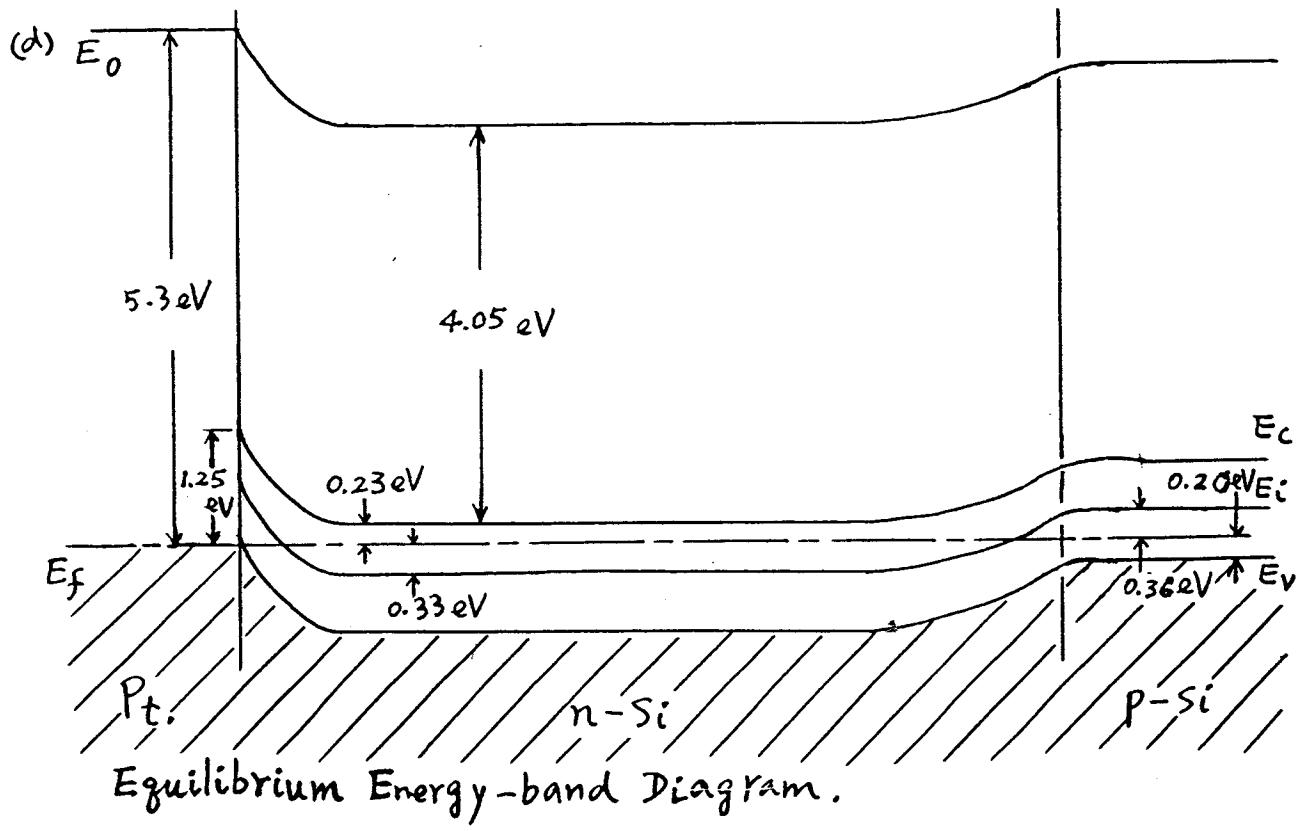
$$(c) \Sigma_{\max} = \frac{q N_d}{\epsilon_s} x_d, \text{ For } x_d = t_{epi} = 2.5 \times 10^{-4} \text{ cm.}$$

$$\Sigma_{\max} = \frac{1.6 \times 10^{19} \times 5 \times 10^{15} \times 2.5 \times 10^{-4}}{11.7 \times 8.85 \times 10^{-14}} \approx 1.93 \times 10^5 \text{ V/cm} < 3 \times 10^5 \text{ V/cm,}$$

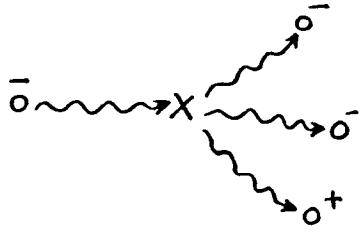
breakdown field.

\therefore total depletion is possible.

$$V_R = \frac{1}{2} \Sigma_{\max} x_d - \phi_i \approx 23.5 \text{ V} \quad (\text{reverse voltage})$$



4.11

Before collision	<u>energy</u> $E_0 = \frac{1}{2}mv_0^2$	<u>momentum</u> mv_0
After collision each particle has	(Kinetic energy) mv_f $E_f = \frac{1}{2}mv_f^2$	\vec{o} 

By conservation of energy

$$E_0 = \text{potential energy change} + \text{kinetic energy} = E_g + 3E_f$$

$$\text{By conservation of momentum } mv_0 = 3mv_f \text{ so } v_f = \frac{1}{3}v_0$$

$$\therefore \frac{1}{2}mv_0^2 = E_g + \frac{3}{2}mv_f^2 = E_g + \frac{3}{2}m\left(\frac{v_0^2}{9}\right) \quad \therefore mv_0^2 = 3E_g \text{ and}$$

$$E_0 = \frac{1}{2}mv_0^2 = \frac{3}{2}E_g.$$

4.12

Zener breakdown depends on tunneling

$$T = \exp\left(-\frac{B}{E}\right) = \exp\left(-\frac{4\sqrt{2m^*}Eg^{3/2}}{3g\hbar\epsilon}\right)$$

The tunneling probability decreases rapidly as the bandgap increases, \therefore tunneling is more likely for Ge because of its smaller bandgap.

4.13

$$I = gANv\theta, A = 10^{-5} \text{ cm}^2, N = 10^{22} \text{ cm}^{-3}, v = 10^7 \text{ cm/sec}, I = 10^{-2} \text{ A},$$

$$\theta = \frac{I}{gANv} = 6.3 \times 10^{-8} \quad \theta = e^{-\frac{8BL}{Eg}}, \quad \frac{8BL}{Eg} = -\ln\theta$$

$$\therefore L = -\frac{Eg}{8B} \ln\theta, \text{ where } B = \frac{4\sqrt{2m^*}Eg^{3/2}}{3g\hbar} = \sqrt{\frac{m^*}{m_0}} \frac{4\sqrt{2m_0}}{3g\hbar} Eg^{3/2}$$

$$\therefore B = \sqrt{0.26} \times 8.10 \times 10^7 \text{ V/cm} = 4.1 \times 10^7 \text{ V/cm}$$

$$\therefore L = -\frac{1.1 \text{ V}}{4.1 \times 10^7 \text{ V/cm}} \ln(6.3 \times 10^{-8}) = 4.4 \text{ nm}$$

$$\text{The corresponding field is } \mathcal{E} = \frac{Eg}{8L} = \frac{1.1}{4.4 \times 10^{-7}} = 2.5 \times 10^6 \text{ V/cm}$$

in agreement with the order-of-magnitude given in the text.

4.14

$$g_D = \frac{I_D}{V_D} = G_0 \left\{ 1 - \left[\frac{2\epsilon_s}{8N_d t^2} (\phi_i - V_G) \right]^{\frac{1}{2}} \right\} V_D , \quad G_0 = \frac{W}{L} g \mu_n N_a t$$

$$\phi_i = \frac{kT}{q} \ln \frac{N_a N_a}{n_i^2} , \quad \frac{n_i^2(T)}{n_i^2(300)} = \frac{\exp[-E_g/kT]}{\exp[-E_g/300k]} = \exp \left[\frac{-E_g}{k} \left(\frac{1}{T} - \frac{1}{300} \right) \right]$$

$$\begin{aligned} \therefore \phi_i &= \frac{kT}{q} \left[\ln(N_a N_a) - \ln n_i^2(300) - \frac{E_g}{k} \left(\frac{1}{T} - \frac{1}{300} \right) \right] \\ &= \frac{kT}{q} \left[\ln \frac{N_a N_a}{n_i^2(300)} \right] + \frac{E_g}{q} - \frac{E_g T}{8 \times 300} \end{aligned}$$

E_g varies slowly with temperature ; generally $|\phi_i| < |V_G|$ and ϕ_i appears in a square root so that its influence on g_D is small . The major temperature dependence is, therefore, contained in μ_n .

$$\frac{\partial g_D}{\partial T} \simeq \frac{W}{L} g N_a t \left\{ 1 - \left[\frac{2\epsilon_s}{8N_d t^2} (\phi_i - V_G) \right]^{\frac{1}{2}} \right\} V_D \frac{d\mu_n}{dT}$$

$$\mu_n = \mu_{n300} \frac{T^{-3/2}}{(300)^{-3/2}} \quad \frac{d\mu_n}{dT} = \mu_{n300} \times (300)^{3/2} \times \left(-\frac{3}{2} T^{-\frac{5}{2}} \right) \\ = \mu_{n300} \left(\frac{300}{T} \right)^{5/2} \left(-\frac{1}{200} \right)$$

$$\text{and } \frac{\partial g_D}{\partial T} = -\frac{\mu_{n300}}{200} \left(\frac{300}{T} \right)^{5/2} \frac{W}{L} g N_a t \left\{ 1 - \left[\frac{2\epsilon_s}{8N_d t^2} (\phi_i - V_G) \right]^{\frac{1}{2}} \right\} V_D \\ = -\frac{1}{200} \left(\frac{300}{T} \right)^{5/2} g_{D300}$$

4.15

$$I_{D\text{sat}} = G \left[\frac{8N_d t^2}{6\epsilon_s} - (\phi_i - V_G) \left\{ 1 - \frac{2}{3} \left[\frac{2\epsilon_s(\phi_i - V_G)}{8N_d t^2} \right]^{\frac{1}{2}} \right\} \right] = K G$$

$$g = K \frac{dG}{dV_D} , \quad G = \frac{W}{L} g \mu_n N_a t , \quad L = L(V_D) , \quad \begin{array}{c} \xrightarrow{L} \\ \text{---} \\ \text{n}^+ \quad \text{P}^+ \quad \text{n}^+ \\ \downarrow \quad \downarrow \quad \downarrow \\ \text{n} \end{array} \quad \begin{array}{c} V_G \\ \xrightarrow{V_D} \end{array}$$

$$g = K g \mu_n N_a t W \left(-\frac{1}{L^2} \right) \frac{dL}{dV_D} , \quad L = L_0 - x_d , \quad \begin{array}{c} \leftarrow L \rightarrow \leftarrow x_d \\ \leftarrow L_0 \rightarrow \end{array}$$

For a one-sided step junction

$$V_D - V_{D\text{sat}} = \frac{8N_d}{2\epsilon_s} x_d^2 \quad (\text{This is a weak approximation when } V_D \approx V_{D\text{sat}})$$

$$x_d = \left(\frac{2\epsilon_s}{8N_d} [V_D - V_{D\text{sat}}] \right)^{\frac{1}{2}} , \quad \text{and} \quad L = L_0 - \left(\frac{2\epsilon_s}{8N_d} [V_D - V_{D\text{sat}}] \right)^{\frac{1}{2}}$$

$$\frac{dL}{dV_D} = -\frac{1}{2} \sqrt{\frac{2\epsilon_s}{8N_d}} (V_D - V_{D\text{sat}})^{-\frac{1}{2}}$$

Substituting expressions for L and $\frac{dL}{dV_D}$ in the expression for g and using the equation for $I_{D\text{sat}}$, we obtain:

$$g = \frac{I_{D\text{sat}} L_0 \sqrt{\frac{\epsilon_s}{2qN_A}}}{\sqrt{V_D - V_{D\text{sat}}} \left(L_0 - \sqrt{\frac{2\epsilon_s}{qN_A} (V_D - V_{D\text{sat}})} \right)^2}$$

Note that this expression fails when $V_D = V_{D\text{sat}}$.

4.16

- (a) At V_T , the channel conductance $\rightarrow 0$ as the depletion region extends across d . (Although for $r_i < d/2$, the reach-through could be right at the source, this is a very poor design which would have virtually no transconductance.)

For a linearly graded junction, $x_d = \left[\frac{12\epsilon_s(\phi_i - V_a)}{8a} \right]^{1/3}$.

Since the junction is symmetric,

x_d is equally divided between the p- and n-type regions, so that at $V_a = V_T$ $w/2 = d/2$ or $w = d$.

$\therefore V_T = -\frac{8aw^3}{12\epsilon_s} + \phi_i$ (This assumes the continual validity of the linear-graded junction).

- (b) $dV = I_D dR \quad dR = \frac{\rho dr}{2\pi r(d-W(v))}$, where ρ = resistivity.

$$\text{and } W(v) = \left[\frac{12\epsilon_s(v(r) - V_G + \phi_i)}{8a} \right]^{1/3}$$

$$\therefore \int_0^{V_D} [d-W(v)] dV = \frac{I_D \rho}{2\pi} \int_{r_i}^{r_2} \frac{dr}{r}$$

- (c) From part (b), we have: $\frac{2\pi}{\rho} \int_0^{V_D} [d-W(v, V_G, V_D)] dV = I_D \ln \frac{r_2}{r_i}$

$$\text{or } f(V_G, V_D) = I_D \ln \frac{r_2}{r_i}, \therefore \frac{\partial f}{\partial V_G} \times \frac{1}{\ln(\frac{r_2}{r_i})} = \frac{\partial I_D}{\partial V_G} = g_m$$

$$\text{and } \frac{g_{ma}}{g_{mb}} = \frac{[\ln(\frac{r_2}{r_i})]_b}{[\ln(\frac{r_2}{r_i})]_a}, \text{ and } g_m \Big|_{\frac{60}{10}} = 10^{-2} \frac{[\ln(\frac{40}{10})]}{[\ln(\frac{60}{10})]} = 7.74 \times 10^{-3} \text{ mho}$$

CHAPTER 55.1

$0.6 \Omega\text{-cm} \Rightarrow n_0 \approx N_D = 8 \times 10^{15} \text{ cm}^{-3}$ and $p_0 = \frac{n_i^2}{n_0} = 2.6 \times 10^4 \text{ cm}^{-3}$.
 with $N_t = 10^{15} \text{ cm}^{-3}$, $\sigma = 10^{-15} \text{ cm}^2$, $V_{th} = 10^7 \text{ cm/sec}$, $\Rightarrow \tau_0 = 1 \times 10^{-7} \text{ sec}$.

with $E_t = E_i$ Eq.(5.2.10) becomes

$$U = \frac{pn - n_i^2}{(p+n+2n_i)\tau_0}$$

$$(a) \text{ with } p=n=0, U = \frac{-n_i^2}{2n_i\tau_0} = -\frac{n_i}{2\tau_0} = -7.25 \times 10^{16} \text{ cm}^{-3}\text{-sec}^{-1}$$

i.e. $G = 7.25 \times 10^{16} \text{ cm}^{-3}\text{-sec}^{-1}$.

$$(b) \text{ with } n \approx n_0 \text{ and } p \ll p_0, pn - n_i^2 = pn - p_0 n_0 \approx n_0(p - p_0) \approx -n_0 p_0$$

and $n \gg p + 2n_i \therefore U = \frac{-n_0 p_0}{n_0 \tau_0} = -\frac{p_0}{\tau_0} = -2.6 \times 10^{11} \text{ cm}^{-3}\text{-sec}^{-1}$

i.e. $G = 2.6 \times 10^{11} \text{ cm}^{-3}\text{-sec}^{-1}$.

5.2

$$\frac{\partial P'}{\partial t} = G - R = G_s - U$$

$$(a) U = \frac{N_t V_{th} \sigma^2 (pn - n_i^2)}{\sigma(p+n_i) + \sigma(n+n_i)} \approx \frac{N_t V_{th} \sigma n_0 P'}{n_0 + 2n_i} = N_t V_{th} \sigma P'$$

assuming low-level injection (check at end of problem)

$$U = (10^{15})(10^7)(10^{-14}) P' \frac{1}{\text{cm}^3} \frac{\text{cm}}{\text{sec}} \text{ cm}^2 = 10^8 P' / \text{sec}$$

$$\text{In steady state } \frac{\partial P'}{\partial t} = 0 \quad G_s = U = \frac{10^{21}}{\text{cm}^3 \text{sec}} = \frac{10^8 P'}{\text{sec}} = \frac{10^{21}}{\text{cm}^3 \text{sec}}$$

$\therefore P' = 10^{13} \text{ cm}^{-3} \approx P, \quad n \approx 10^{16} \text{ cm}^{-3}$ (low-level injection)

$$(b) \text{ Transient decay } G_s = 0 \quad t > 0, \quad \frac{\partial P'}{\partial t} = -U = -10^8 P'$$

$$P' = P'(t=0) e^{-10^8 t} = 10^{13} e^{-10^8 t} \text{ cm}^{-3} = 10^{13} e^{-t/10^8} \text{ cm}^{-3}$$

$$\therefore \tau_p = 10^{-8} \text{ sec.}$$

5.3

$$n_1 = 10^{18} \text{ cm}^{-3}, \quad W = 1 \times 10^{-4} \text{ cm}, \quad D_n = 7 \text{ cm}^2/\text{sec}, \quad M_n = \frac{8D_n}{kT} = 271 \frac{\text{cm}^2}{\text{V-sec}}$$

$$(a) \frac{dE}{dx} = \frac{P}{\epsilon_s} = -\frac{8n(x)}{\epsilon_s} = -\frac{8n_1}{\epsilon_s} (1 - \frac{x}{W}) \quad \therefore E(x) = -\frac{8n_1}{\epsilon_s} \left(x - \frac{x^2}{2W} \right) + C$$

$$\text{Boundary Condition } E(W) = 0 \quad \therefore E(x) = \frac{8n_1}{\epsilon_s} \left(\frac{x^2}{2W} - x + \frac{W}{2} \right)$$

$$E(0) = \frac{8n_1 W}{2\epsilon_s} = 7.726 \times 10^6 \text{ V/cm}$$

$$J_n(x) = q\mu_n n(x) E(x) + qD_n \frac{dn}{dx}, \quad n = n_i(1 - \frac{x}{W}), \quad n(0) = n_i$$

$$\begin{aligned} \frac{dn}{dx} &= -\frac{n_i}{W} \quad \therefore J_n(0) = q\mu_n n_i E(0) - \frac{qD_n n_i}{W} \\ &= \frac{q^2 n_i^2 \mu_n W}{2E_s} - \frac{qD_n n_i}{W} = 3.35 \times 10^8 A/cm^2 \end{aligned}$$

(b) Since from Table 1.3 the breakdown field of Si is $3 \times 10^5 V/cm$ the charge configuration is not reasonable.

(c) If the current at $x=0$ can only be $10^5 A/cm^2$, then $E(0)$ should be $E(0) = \frac{1}{q\mu_n n_i} [J_n(0) + \frac{qD_n n_i}{W}] = 2565 V/cm$.

(d) $\rho = -q(n_i - N_{do})(1 - \frac{x}{W})$. The result of part (a) can be used provided $n_i \rightarrow n_i - N_{do}$ in the expression for $E(x)$

$$E(x) = \frac{q(n_i - N_{do})}{2E_s} \left(\frac{x^2}{2W} - x + \frac{W}{2} \right); \quad E(0) = \frac{q(n_i - N_{do})W}{2E_s}$$

$$J_n(0) = \frac{q^2 \mu_n n_i (n_i - N_{do}) W}{2E_s} - \frac{q D_n n_i}{W}$$

We require $J_n(0) = 10^5 A/cm^2$, then

$$N_{do} = n_i - \frac{2E_s}{q^2 \mu_n n_i} [J_n(0) + \frac{q D_n n_i}{W}] = 10^{18} cm^{-3} - 3.32 \times 10^{10} cm^{-3} \approx 10^{18} cm^{-3}$$

$$\frac{n_i - N_{do}}{n_i} = \frac{3.32 \times 10^{10} cm^{-3}}{10^{18} cm^{-3}} = 3.32 \times 10^{-8}$$

The relative deviation from electrical neutrality is extremely small

5.4

$$(a) \frac{q^2}{4\pi E_s r} = \frac{3}{2} kT$$

$$r = \frac{q^2}{6\pi E_s kT} = 3.15 \times 10^{-7} cm = 3.15 nm$$

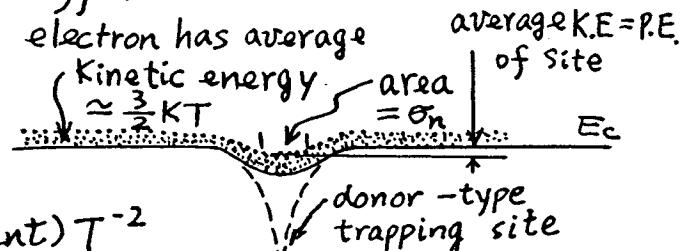
$$\therefore \delta_n = \pi r^2 = 3.12 \times 10^{-13} cm^2$$

$$(b) \sigma_n = (\text{constant}) r^2 = (\text{constant}) T^{-2}$$

$$V_{th} \propto T^{1/2}, \sigma_n \propto T^{-2} \therefore \text{Capture rate} = \frac{n'}{T} = n' N_c V_{th} \sigma_n$$

$\rightarrow \propto T^{-3/2}$ for extrinsic Si. N_t is constant.

Energy Picture:



5.5

For very high level injection $p, n \gg n_i$ and $p, n \gg n_i e^{(E_i - E_e)/kT}$

From Eq. (5.2.9a) with $P \approx n$

$$T_J \approx \frac{N_t V_{th} \delta_n \delta_p n^2}{\sigma_p n + \delta_n n} = \frac{N_t V_{th} \delta_n \delta_p n}{\delta_p + \delta_n} \approx \frac{n}{\tau_{eff}}$$

$$\tau_{\text{eff}} \approx \frac{\tau_p + \tau_n}{N_t V_{\text{th}} \tau_n \tau_p} , \quad \tau_{\text{low}} = \tau_0 , \quad \tau_{\text{on}} = \frac{1}{N_t V_{\text{th}} \tau_n} , \quad \tau_{\text{op}} = \frac{1}{N_t V_{\text{th}} \tau_p}$$

$$\tau_{\text{eff}} \approx \frac{1}{N_t V_{\text{th}} \tau_n} + \frac{1}{N_t V_{\text{th}} \tau_p} = \tau_{\text{on}} + \tau_{\text{op}} \quad \text{if } \tau_n \neq \tau_p$$

If $\tau_n = \tau_p$ $\tau_{\text{on}} = \tau_{\text{op}} = \tau_0$ and $\tau_{\text{eff}} = 2\tau_0$

5.6

$D_p \frac{d^2 P'_n}{dx^2} - \frac{P'_n}{\tau_p} = 0$ Eq.(5.3.10) . For short base diode , recombination in the bulk can be neglected , i.e. $\frac{P'_n}{\tau_p} = 0$, so that $\frac{d^2 P'_n}{dx^2} = 0$ $P'_n(x) = A + Bx$

The boundary conditions are $P'_n(x_n) = P_{n0} (e^{\frac{qV_a}{kT}} - 1)$ and

$$P'_n(W_B) = 0 \quad \therefore A = \frac{P_{n0} (e^{\frac{qV_a}{kT}} - 1) W_B}{W_B - x_n} , \quad B = - \frac{P_{n0} (e^{\frac{qV_a}{kT}} - 1)}{W_B - x_n}$$

Hence , $P'_n(x) = P_{n0} (e^{\frac{qV_a}{kT}} - 1) \frac{W_B - x}{W_B - x_n} = P_{n0} (e^{\frac{qV_a}{kT}} - 1) \left(1 - \frac{x - x_n}{W_B}\right)$

(where $W_B' = W_B - x_n$) \Rightarrow linear variation [Eq. (5.3.17)]

5.7

In the steady state , Eq. (5.1.3b) Reduces to

$$\frac{dJ_p}{dx} = g(G_p - R_p) = -g \frac{P - P_n}{\tau_p} = -\frac{g P'_n}{\tau_p}$$

Integrating this equation across the n-type neutral bulk gives ,

$$J_p(W_B) - J_p(x_n) = -\frac{g}{\tau_p} \int_{x_n}^{W_B} P'_n(x) dx . \quad \text{For a long-base diode } J_p(W_B) \rightarrow 0$$

so that $J_p(x_n) = \frac{g}{\tau_p} \int_{x_n}^{W_B} P'_n(x) dx$

Under reverse bias Eq. (5.3.12) reduces to $P'_n(x) = -P_{n0} e^{-\frac{(x-x_n)}{L_p}}$

$$\text{so that } J_p(x_n) = -\frac{8P_{n0}}{\tau_p} \int_{x_n}^{W_B} e^{-\frac{(x-x_n)}{L_p}} dx = \frac{8P_{n0} L_p}{\tau_p} \left[e^{-\frac{(W_B-x_n)}{L_p}} - 1 \right]$$

For a long base diode $(W_B - x_n)/L_p \gg 1$ so that $e^{-\frac{(W_B-x_n)}{L_p}} \ll 1$

$$\therefore J_p(x_n) = -\frac{8P_{n0} L_p}{\tau_p} = -\frac{8P_{n0} D_p}{L_p} \quad \text{where we have used } L_p = \sqrt{D_p \tau_p}$$

A similar integration for electrons on the p-side gives

$$J_n(-x_p) = -\frac{8N_{p0} D_n}{L_n}$$

50

For the ideal diode, there is no generation in the space-charge zone, and the total reverse current density is

$$J_t = J_p(x_n) + J_n(-x_p) = -g \left[\frac{P_{no} D_p}{L_p} + \frac{N_{po} D_n}{L_n} \right]$$

Using $P_{no} = \frac{n_i^2}{N_d}$ and $N_{po} = \frac{n_i^2}{N_a}$ then

$J_t = -g n_i^2 \left[\frac{D_p}{N_d L_p} + \frac{D_n}{N_a L_n} \right]$ which is a limiting reverse-bias current expression obtained from Eq.(5.3.15).

5.8

$$P'_n(x) = A e^{-\frac{x-x_n}{L_p}} + B e^{\frac{x-x_n}{L_p}}$$

For long base diode $B=0$, $P'_n(x) = P_{no} (e^{\frac{8V_a}{kT}} - 1) e^{-\frac{x-x_n}{L_p}}$ Eq.(5.3.12)

$$\langle x \rangle = \frac{\int_0^\infty x P'_n(x) dx}{\int_0^\infty P'_n(x) dx} = \frac{\int_0^\infty x e^{-\frac{x-x_n}{L_p}} dx}{\int_0^\infty e^{-\frac{x-x_n}{L_p}} dx}, \text{ let } y = x - x_n$$

$$\langle x \rangle = \frac{\int_{-x_n}^\infty y e^{-\frac{y}{L_p}} dy + x_n \int_{-x_n}^\infty e^{-\frac{y}{L_p}} dy}{\int_{-x_n}^\infty e^{-\frac{y}{L_p}} dy}$$

$$= \frac{-L_p^2 e^{x_n/L_p} \left(\frac{x_n}{L_p} - 1 \right) + x_n L_p e^{x_n/L_p}}{L_p e^{x_n/L_p}} = \frac{-L_p x_n + L_p^2 + x_n L_p}{L_p} = L_p$$

5.9

$$(a) \gamma = \frac{J_p(x_n)}{J_t} = \frac{J_p(x_n)}{J_n(-x_p) + J_p(x_n)} = \frac{1}{\frac{J_n(-x_p)}{J_p(x_n)} + 1}$$

From Eq.(5.3.13) and Eq.(5.3.14)

$$J_p(x_n) = \frac{8 D_p n_i^2}{N_d L_p} \left(e^{\frac{8V_a}{kT}} - 1 \right) \text{ and } J_n(-x_p) = \frac{8 D_n n_i^2}{N_a L_n} \left(e^{\frac{8V_a}{kT}} - 1 \right)$$

$$\therefore \gamma = \frac{1}{\frac{D_n N_a L_n}{D_p N_d L_p} + 1} = \frac{N_d}{N_a} \sqrt{\frac{D_n \tau_p}{D_p \tau_n}} + 1$$

(b) For $\rho_n = 0.001 \Omega\text{-cm}$ $N_d = 8 \times 10^{19} \text{ cm}^{-3}$, for $\rho_p = 15 \Omega\text{-cm}$, $N_a = 1.5 \times 10^{16} \text{ cm}^{-3}$

μ_n (the mobility of electrons in the p-region) = $1100 \text{ cm}^2/\text{V}\cdot\text{sec}$

μ_p (the mobility of holes in the n-region) = $75 \text{ cm}^2/\text{V}\cdot\text{sec}$

$$D_n/D_p = \mu_n/\mu_p = 14.67 \therefore \tau_p/\tau_n = 0.1$$

$$\therefore \gamma = \left[\frac{8 \times 10^{19}}{1.5 \times 10^{16}} \sqrt{(14.67)(0.1)} + 1 \right]^{-1} = 1.55 \times 10^{-4}$$

5.10

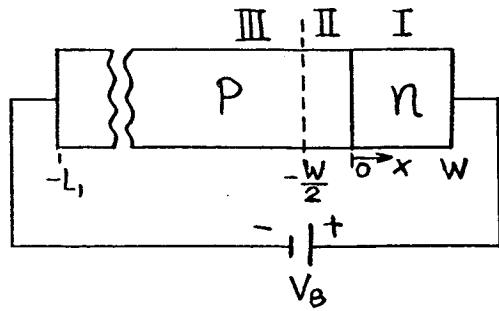
(a) The solution has the form

$$P = A_1 + A_2 x \quad \text{in region I}$$

$$n = B_1 + B_2 x \quad \text{in region II}$$

$$n = C_1 e^{\frac{x+w/2}{L}} + C_2 e^{-\frac{x+w/2}{L}} + C_3 \quad \text{in region III}$$

The boundary conditions are



(1) Since reverse bias $|V_B| >> \phi_i \therefore n=0$ at $x=0$ and $P=0$ at $x=0$

(2) Infinite recombination at $x=w \therefore P(w) = P_{no} = \frac{n_i^2}{N_o}$

(3) $L_1 >> L$ gives $n(-L_1) = n_{po} = \frac{n_i^2}{N_o}$

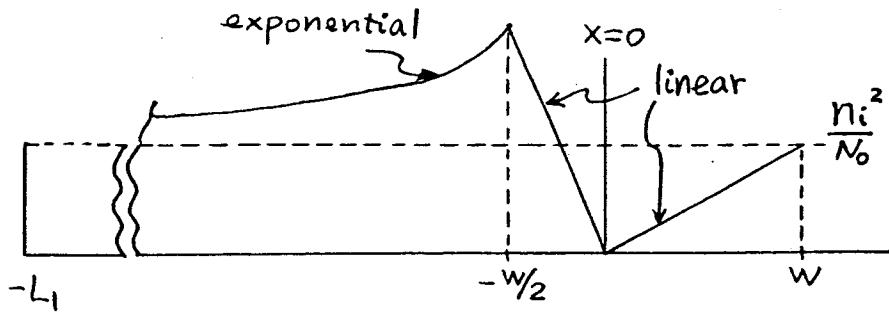
(4) The light is applied to a plane sheet of negligible width at $x = -\frac{w}{2}$; \therefore the number of carriers diffusing outward from $x = -\frac{w}{2}$ equals the number of carriers created.

$$\therefore \left. \frac{\partial n}{\partial x} \right|_{x=-\frac{w}{2}^+} - \left. \frac{\partial n}{\partial x} \right|_{x=-\frac{w}{2}^-} = -\frac{G_o}{D} \quad \text{where } G_o \text{ has units of electron-hole pairs/cm}^2\text{-sec.}$$

Applying the boundary conditions we get:

$$P = \frac{n_i^2}{N_o w} x \quad \text{for region I} ; \quad n = \frac{-\left(\frac{G_o L}{D} + \frac{n_i^2}{N_o}\right)}{L + w/2} x \quad \text{for region II}$$

$$n = \frac{\left(\frac{G_o WL}{2D} - \frac{n_i^2 L}{N_o}\right)}{L + w/2} e^{\frac{x+w/2}{L}} + \frac{n_i^2}{N_o} \quad \text{for region III}$$



(b)

The current resulting from the light arises from the term for n in region II that is proportional to G_o . That is: $n_e = \frac{-G_o L x}{D(L+w/2)} \left(\frac{w}{2} \times c(x) \right)$

$$\therefore J_L = -g D n_e \left. \frac{dn}{dx} \right|_{x=0} = \frac{g G_o L}{L+w/2} \quad \text{which} \rightarrow g G_o \text{ for } L \gg w/2.$$

(c) When the light beam is removed, the p-region can be represented as a long-base diode. $\therefore n = \frac{n_i^2}{N_o} - \frac{n_i^2}{N_o} e^{x/L}$ and

$$J_{DARK} = -\frac{g D n_i^2}{N_o} \left(\frac{1}{L} + \frac{1}{w} \right)$$

5.11

(a) From Fig. 1.15 $\rho = 0.2 \Omega\text{-cm}$ n-type $\Rightarrow N_d = 3 \times 10^{16} \text{ cm}^{-3}$
 $\rho = 1 \Omega\text{-cm}$ p-type $\Rightarrow N_a = 1.5 \times 10^{16} \text{ cm}^{-3}$

$$\phi_i = \frac{kT}{q} \ln \frac{N_d N_a}{N_i^2} = 0.738 \text{ V.}$$

$$(b) n_p(-x_p) = \frac{n_i^2}{N_a} e^{\frac{8V_a}{kT}} = 1.37 \times 10^{14} \text{ cm}^{-3}$$

$$P_n(x_n) = \frac{n_i^2}{N_d} e^{\frac{8V_a}{kT}} = 6.83 \times 10^{13} \text{ cm}^{-3}$$

(c) In p-type Si $N_a = 1.5 \times 10^{16} \text{ cm}^{-3}$, $M_n = 1150 \frac{\text{cm}^2}{\sqrt{\text{sec}}}$, $\tau_n = 10^{-6} \text{ s}$, $D_n = 29.9 \frac{\text{cm}^2}{\text{sec}}$

$$L_n = 5.47 \times 10^{-3} \text{ cm} \therefore J_n = \frac{8D_n n_i^2}{N_a L_n} (e^{\frac{8V_a}{kT}} - 1) e^{\frac{x+x_p}{L_n}} = 0.119 e^{\frac{x+x_p}{L_n}} \text{ A/cm}^2$$

$$J_p = J_t - J_n$$

In n-type Si $N_d = 3 \times 10^{16} \text{ cm}^{-3}$, $M_p = 400 \frac{\text{cm}^2}{\sqrt{\text{sec}}}$, $\tau_p = 10^{-8} \text{ s}$, $D_p = 10.4 \frac{\text{cm}^2}{\text{sec}}$

$$L_p = 3.22 \times 10^{-4} \text{ cm} \therefore J_p = \frac{8D_p n_i^2}{N_d L_p} (e^{\frac{8V_a}{kT}} - 1) e^{-(x-x_n)/L_p}$$

$$= 0.353 e^{-(x-x_n)/L_p} \text{ A/cm}^2 \quad J_n = J_t - J_p.$$

$$J_t = J_n(-x_p) + J_p(x_n) = 0.119 + 0.353 = 0.472 \text{ A/cm}^2$$

$$(d) \text{ at } x=x_1, J_p = J_n = \frac{1}{2} J_t$$

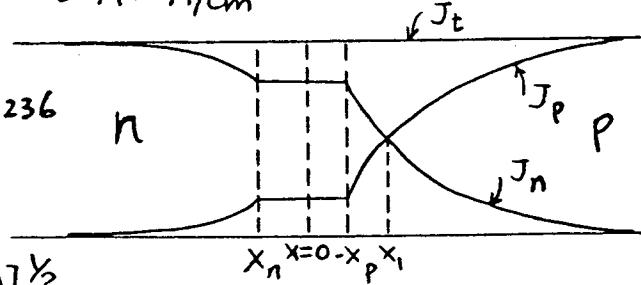
$$\therefore J_p = 0.353 e^{-(x_1-x_n)/L_p} = \frac{0.472}{2} = 0.236$$

$$\therefore x_1 - x_n = 1.30 \mu\text{m} \quad x_1 = 1.30 + x_n$$

x_n can be found from :

$$x_n + x_p = \left[\frac{2\epsilon_s}{q} (\phi_i - V_a) \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right] \frac{V_a}{2} = 0.138 \mu\text{m}$$

$$\therefore x_n = \left(\frac{N_a}{N_a + N_d} \right) (x_n + x_p) \quad \therefore x_n = 0.046 \mu\text{m} \quad \therefore x_1 = 1.35 \mu\text{m}$$

5.12

Reasoning :

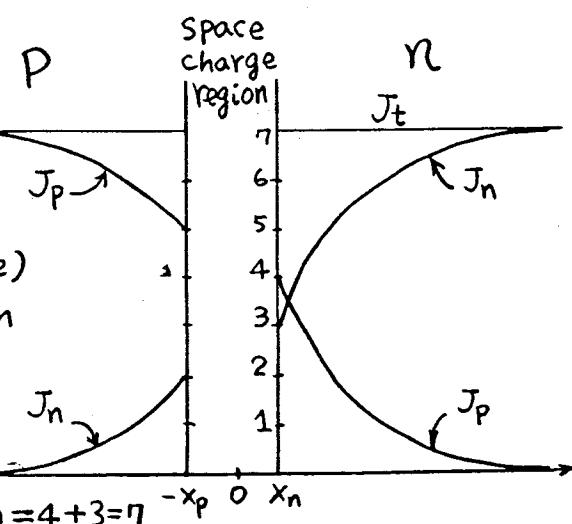
(1) At the p-type boundary of the space charge region, let $J_n = 2$

(2) $\therefore J_n$ at the n-type edge =

$2 \left(\text{to supply injection through S.C. zone} \right) + \frac{1}{2} \times 2 \left(\text{to supply recombination in the S.C. zone.} \right); J_n = 3$

(3) At the n-side of the S.C. zone

$J_p = 2 \times 2 = 4$. Hence the total current at x_n is $J_p(x_n) + J_n(x_n) = 4 + 3 = 7$.



(4) J_p in the p-region at the edge of the S.C. zone is $J_p(x_n) + 1$ (to account for recombination) or 5 units of current.

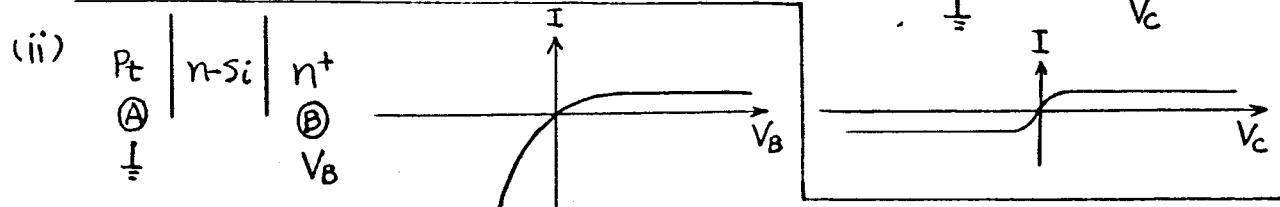
Thus, $J_t =$ Total recombination current

$$= \text{Recombination in neutral region (ideal diode component)} \\ + \text{Recombination in Space-Charge zone}.$$

5.13

(a) (i) $V_c > 0$ forward biased p-n junction diode in series with reverse biased Schottky diode.

$V_c < 0$ forward biased Schottky diode in series with reverse-biased p-n junction diode.



Schottky diode characteristic.

(b) As $|V_c|$ increases and the n- region is depleted, the barrier toward current flow is decreased at the junction opposite the reverse-biased junction from which depletion takes place. The current then increases and becomes space charge limited under this punch through condition.

5.14

$$\frac{J_f}{J_r} = e^{\frac{qV_a}{kT}} - 1 = 10^4 \simeq e^{\frac{0.58}{kT}} \quad \frac{0.58}{kT} = \ln 10^4 \quad \therefore T = 629 K = 356^\circ C$$

Limited to operation below $356^\circ C$ if rectification ratio requirement dominates.

$$I_s = A_g n_i^2 \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right) \quad N_a \gg N_d \quad \therefore I_s \simeq A_g n_i^2 \frac{D_p}{N_d L_p}$$

$$\text{Major temperature dependence in } n_i^2 = N_c N_{V_3} e^{-E_g/kT}$$

$$L_p = \sqrt{D_p \tau_p} = (12 \times 10^{-7})^{1/2} = 1.1 \times 10^{-3} \text{ cm}$$

$$n_i^2 = \frac{I_s}{A_g} \frac{L_p N_d}{D_p} = 5.7 \times 10^{-27} \text{ cm}^{-6}, \quad \frac{E_g}{kT} = \ln \frac{N_c N_{V_3}}{n_i^2} = 24.7, \quad T = \frac{E_g}{k(24.7)} = 524 K = 251^\circ C$$

K = Boltzmann's constant. Limited to operation below $251^\circ C$ by reverse saturation current requirement. Overall limit : $T = 251^\circ C$

5P

5.15

Using Eq. 5.3.20 in Eq. (5.2.10) (for the case $\bar{G}_n = \bar{G}_P$), we obtain

$$U = \frac{n_i^2 (e^{\frac{8V_a}{kT}} - 1) / \tau_0}{\left[P + \frac{n_i^2 e^{\frac{8V_a}{kT}}}{P} + 2n_i \cosh(\frac{E_t - E_i}{kT}) \right]} = \frac{n_i^2 (e^{\frac{8V_a}{kT}} - 1) / \tau_0}{D(P)}$$

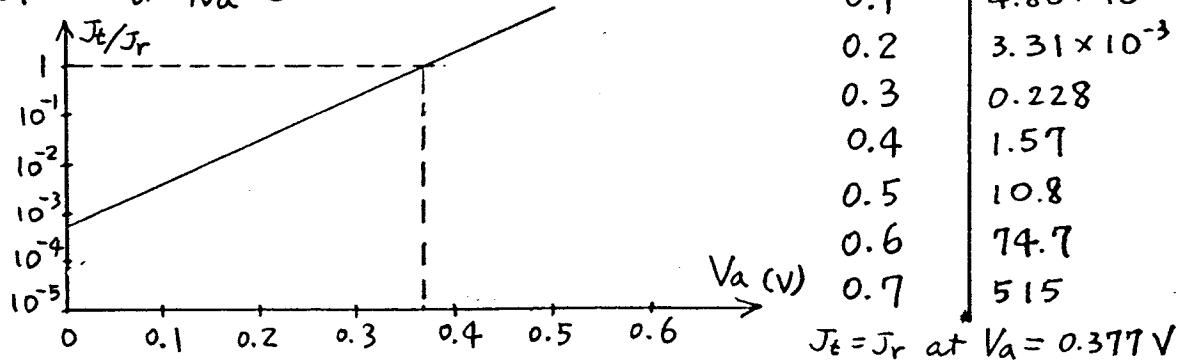
$$\frac{dU}{dP} = -\frac{n_i^2 (e^{\frac{8V_a}{kT}} - 1) / \tau_0}{[D(P)]^2} \left[1 - \frac{n_i^2 e^{\frac{8V_a}{kT}}}{P^2} \right]$$

$$\text{For } U_{\text{maximum}} \quad \frac{dU}{dP} = 0 = 1 - \frac{n_i^2 e^{\frac{8V_a}{kT}}}{P^2} \quad \text{or } P = n_i e^{\frac{8V_a}{2kT}}$$

5.16 A similar calculation for n yields the same result; $\therefore n = P$

$$N_A = 10^{16} \text{ cm}^{-3}, L_n = 60 \mu\text{m}, x_d = 0.25 \mu\text{m}$$

$$\frac{J_t}{J_r} = \frac{2n_i}{x_d} \frac{L_n}{N_A} e^{\frac{8V_a}{2kT}} = 6.96 \times 10^{-4} e^{\frac{V_a}{0.0518}}$$

5.17

$$N_A = 10^{17} \text{ cm}^{-3}, N_D = 10^{18} \text{ cm}^{-3}, \tau_n = 10^{-6} \text{ sec}, \tau_p = 10^{-8} \text{ sec.}$$

$$(a) \frac{J_t}{J_g} = \frac{2n_i}{x_i} \left[\frac{L_n}{N_A} + \frac{L_p}{N_D} \right], \phi_i = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 0.87 \text{ V}$$

$$\text{Let } x_i \approx x_d = \sqrt{\frac{2\varepsilon_s}{q} \frac{N_A + N_D}{N_A N_D} (\phi_i - V_a)} = 0.267 \mu\text{m} \quad \text{for } \phi_i - V_a = 5 \text{ V}$$

$$D_n \text{ (on the p-side)} \approx 19 \text{ cm}^2/\text{sec} \Rightarrow L_n = \sqrt{D_n \tau_n} = 43 \mu\text{m} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Fig. 1.16}$$

$$D_p \text{ (on the n-side)} \approx 4 \text{ cm}^2/\text{sec} \Rightarrow L_p = \sqrt{D_p \tau_p} = 2 \mu\text{m} \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\therefore \frac{J_t}{J_g} = 4.7 \times 10^{-5} \text{ for } \phi_i - V_a = 5 \text{ V}$$

(b) The temperature dependence of the ratio J_t/J_g is dominated by the temperature dependence of n_i . As the temperature increases, the ratio increases; that is, the reverse current from the quasi-neutral region has an increasing tendency to dominate the reverse current generated in space charge region.

5.18

$$I = 5 \times 10^{-4} A, N_a = 10^{17} \text{ cm}^{-3}, W_p \approx 3 \times 10^{-4} \text{ cm}, D_n = 19, A = 10^{-5} \text{ cm}^{-2}$$

$$I = JA = \frac{8n_i^2 D_n A}{N_a W_p} (e^{\frac{qV_a}{kT}} - 1), e^{\frac{qV_a}{kT}} - 1 = \frac{IN_a W_p}{8n_i^2 D_n A} = 2.35 \times 10^{12}$$

$$\therefore V_a = 0.741 \text{ V}, \phi_i = \frac{kT}{q} \ln \frac{N_a}{n_i} + 0.56 = 0.969 \text{ V} \quad \therefore \phi_i - V_a = 0.228 \text{ V}$$

$$x_d \approx x_p = \left[\frac{2\epsilon_s}{qN_a} (\phi_i - V_a) \right]^{\frac{1}{2}} = 5.43 \times 10^{-6} \text{ cm} \ll W_p \text{ with } I = 0.5 \text{ mA}$$

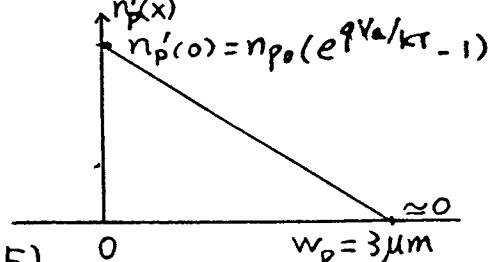
$$= 1.12 \times 10^{-5} \text{ cm} \text{ with } I = 0$$

(a) $\frac{Q_n}{q} = A \int n'_p(x) dx = \frac{W_p}{2} n_{p0} A (e^{\frac{qV_a}{kT}} - 1) \text{ for short-base diode.}$

$$= \frac{W_p}{2} \frac{n_i^2 A}{N_a} (e^{\frac{qV_a}{kT}} - 1)$$

$$= 7.52 \times 10^6 \text{ excess electrons}$$

$$\Rightarrow -1.20 \times 10^{-12} \text{ C}$$



Or from expression similar to Eq. (4.4.5)

$$\text{for electrons } Q_n = -\frac{(W_p - x_p)^2}{2 D_n} J_p A = -1.20 \times 10^{-12} \text{ C}$$

(b) fixed charge $Q_j = -qN_a x_p A = -8.69 \times 10^{-13} \text{ Coul}$

$$\text{mobile charge } \simeq I T_{tr} \simeq -(5 \times 10^{-4}) (8.5 \times 10^{-14}) = -4.3 \times 10^{-17} \text{ Coul.}$$

(where transit time $T_{tr} = \frac{x_p}{V} \simeq 8.5 \times 10^{-14} \text{ sec}$, $V = 1.07 \times 10^7 \text{ cm/sec}$, limiting velocity from Table 1.2)

$$Q_j = -8.69 \times 10^{-13} \text{ Coul with } I = 0.5 \text{ mA}$$

$$Q_j = -qN_a x_p(0) A = -1.79 \times 10^{-12} \text{ Coul with } I = 0$$

(c) $t_r = \frac{\Delta Q}{I} \quad \Delta Q = \Delta Q_p + \Delta Q_j$

$$\Delta Q_p = -1.20 \times 10^{-12} \text{ (electrons flow from n-region)}$$

$$\Delta Q_j = Q_j(v) - Q_j(0) = (-8.69 \times 10^{-13}) - (-1.79 \times 10^{-12}) = 9.21 \times 10^{-13} \text{ Coul.}$$

(holes flow from p-region)

$$t_r = t_{rp} + t_{rj}$$

$$t_{rp} \text{ to charge neutral p-region} = \frac{1.20 \times 10^{-12}}{5 \times 10^{-4}} = 2.40 \text{ nsec}$$

$$t_{rj} \text{ to charge junction} = \frac{9.21 \times 10^{-13}}{5 \times 10^{-4}} = 1.84 \text{ nsec}$$

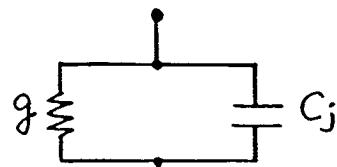
$$\therefore t_r = 2.40 + 1.84 = 4.24 \text{ nsec.}$$

5.19

Schottky diode

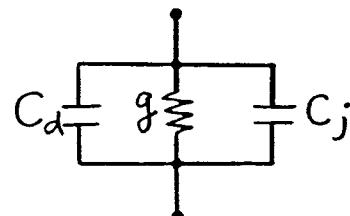
$$C_j = A \left(\frac{8E_s}{2} N_d \right)^{1/2} (\phi_i - V)^{-\frac{1}{2}}$$

$$g = \frac{\partial I}{\partial V}, \text{ where } I = I_s (e^{\frac{8V}{nkT}} - 1)$$

Pn-junction diode

$$g = \frac{\partial I}{\partial V}, \text{ where } I = I_s (e^{\frac{8V}{kT}} - 1)$$

$$C_j = A \left(\frac{8E_s}{2} \frac{N_a N_d}{N_a + N_d} \right)^{1/2} (\phi_i - V)^{-\frac{1}{2}}$$



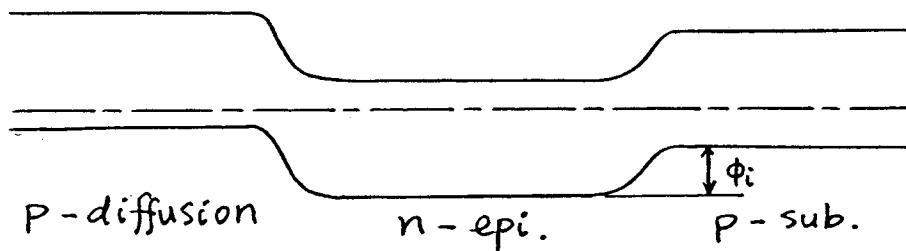
$$C_d = \tau g, \text{ where } \tau \text{ is lifetime or transit time.}$$

The conductance g & junction capacitance C_j appear in both equivalent circuits.

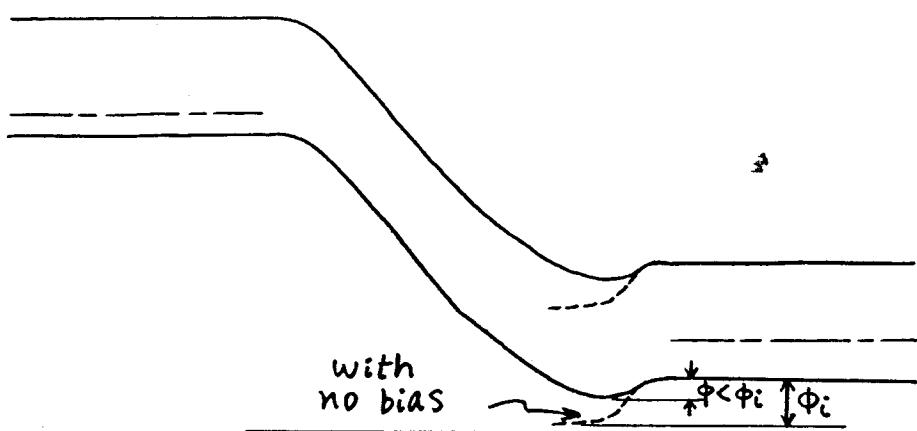
The primary difference is that C_d , the diffusion capacitance resulting from stored minority charge, is present in the pn diode, but not in the Schottky diode. The bias voltage dependence and temperature dependence of g are somewhat different for the Schottky and pn-junction diodes.

5.20

(a)



(b)



5.21

$$r_d = \frac{1}{g_d}, g_d = \frac{dI}{dV} = \frac{8}{kT} AJ_0 e^{\frac{8V_a}{kT}}, C = C_d + C_j$$

$$C_d = \frac{A\bar{g}^2}{kT} (P_{n0}L_p + n_{p0}L_n) e^{\frac{8V_a}{kT}} = \frac{A\bar{g}^2 n_i^2}{kT} \left(\frac{L_p}{N_d} + \frac{L_n}{N_a} \right) e^{\frac{8V_a}{kT}}$$

$$C_j = A \left[\frac{\frac{8eS}{2(\frac{N_d+N_a}{N_a N_d})(\phi_i - V_a)}}{\left(\frac{N_d+N_a}{N_a N_d} \right)} \right]^{\frac{1}{2}}, \quad \phi_i = \frac{kT}{8} \ln \frac{N_a N_d}{n_i^2}$$

$$J_0 = 8n_i^2 \left(\frac{D_p}{N_d L_p} + \frac{D_n}{N_a L_n} \right); \quad A = 10^{-4}, \quad N_d = 10^{18}, \quad N_a = 10^{16}, \quad D_p = 11.5, \quad D_n = 31.0$$

$$\tau_p = \tau_n = 10^{-8}, \quad L_p = 3.39 \times 10^4, \quad L_n = 5.57 \times 10^{-4}$$

$$\therefore J_0 = 1.88 \times 10^{-10}, \quad \phi_i = 0.819 V, \quad g_d = 7.23 \times 10^{-13} e^{\frac{8V_a}{kT}}$$

$$r_d = 1.38 \times 10^{12} e^{\frac{-8V_a}{kT}}, \quad C_d = 7.25 \times 10^{-21} e^{\frac{8V_a}{kT}}, \quad C_j = \frac{2.85 \times 10^{-12}}{\sqrt{\phi_i - V_a}}.$$

	V_a	$r_d (\Omega)$	$C_d (F)$	$C_j (F)$	$C(F) = C_d + C_j$
(a)	0.1	2.95×10^{10}	3.39×10^{-19}	3.36×10^{-12}	3.36×10^{-12}
	0.5	6.14×10^3	1.63×10^{-12}	5.05×10^{-12}	6.68×10^{-12}
	0.7	2.80	3.57×10^{-9}	8.26×10^{-12}	3.58×10^{-9}
(b)	0	1.38×10^{12}	7.25×10^{-21}	3.15×10^{-12}	3.15×10^{-12}
	-5	$\sim 5 \times 10^{95} *$	$\sim 10^{-104}$	1.18×10^{-12}	1.18×10^{-12}
	-20	$\sim 10^{346} *$	$\sim 10^{-354}$	6.25×10^{-13}	6.25×10^{-13}

(* other mechanisms actually limit the resistance, e.g. space-charge-region generation)

$$(c) N_a = 10^{16} \Rightarrow 1.3 \Omega \text{-cm}, \quad R = \frac{PL}{A} = \frac{1.3 \times 0.1}{10^{-4}} = 1300 \Omega$$

5.22

To change the width of the space charge region, majority carriers are rearranged, and the time is related to the dielectric relaxation time. (see problem 1.12) : $\tau = \epsilon_f \sim 10^{-12} \text{ sec}$. Minority carriers must propagate into the neutral region by diffusion with a transit time $\frac{W^2}{2D} \sim \frac{10^{-8}}{20} \sim 10^{-9} \text{ sec}$.

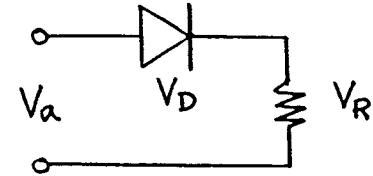
5.23

$$L = \sqrt{DT} \approx \sqrt{12 \times 10^{-6}} = 3.5 \times 10^{-3} = 3.5 \mu\text{m} \gg 1 \mu\text{m}, \text{ for } N_d \geq 10^{16} \text{ cm}^{-3}$$

\therefore holes flowing to buried layer : short-base ($35 \mu\text{m} \gg 1 \mu\text{m}$);
holes flowing laterally : long base ($35 \mu\text{m} < 100 \mu\text{m}$).

5.24

$$A = 10^{-5}, N_d = 5 \times 10^{15}, D_p = 12.0, \rho = 1\text{-}\Omega\text{-cm}, T_p = 10^{-9}. \\ L_p = 1.10 \times 10^{-4} \Rightarrow \text{long-base diode.}$$



$$I = I_0 \left(e^{\frac{qV_D}{kT}} - 1 \right) \simeq \frac{A q n_i^2 D_p}{N_d L_p} \left(e^{\frac{qV_D}{kT}} - 1 \right) \text{ since } N_d \gg N_d \\ = 7.36 \times 10^{-15} \left(e^{\frac{qV_D}{kT}} - 1 \right)$$

$$1.36 \times 10^{14} I = e^{\frac{qV_D}{kT}} - 1 \quad V_D = \frac{kT}{q} \ln(1.36 \times 10^{14} I + 1)$$

$$V_R = RI = \frac{\rho L}{A} I = \frac{1 \times 10 \times 10^{-4}}{10^{-5}} I = 100 I = V_a - V_D$$

$$\text{For } V_R = 0.1 V_a \quad 0.1 V_a = 100 I, \quad V_D = 0.9 V_a \quad \text{and}$$

$$0.9 V_a = \frac{kT}{q} \left[\ln(1.36 \times 10^{14} \times \frac{0.1 V_a}{100} + 1) \right]$$

$$\text{Solve iteratively} \quad V_{a,i+1} = 0.0288 \left[\ln(1.36 \times 10^{14} V_{a,i} + 1) \right]$$

$$\text{Let } V_{a1} = 0.5V, V_{a2} = 0.718, V_{a3} = 0.729, \quad V_a = 0.729V$$

$$V_R = 0.0729V, \quad I = \frac{V_R}{100} = 7.29 \times 10^{-4}, \quad I = 0.73mA.$$

$$\text{Check: } V_D = 0.9 V_a = 0.656V \Rightarrow I = 0.74mA.$$

CHAPTER 6

6.1

Let $N_a(x) = N_{a(0)} e^{-\frac{x}{L}}$ $\Rightarrow \frac{dN_a}{dx} = -\frac{N_a(x)}{L}$. From eq. (6.1.2)

$E_x = \frac{kT}{q} \frac{1}{P} \frac{dP}{dx}$. Assuming quasi-neutrality and low-level injection $p(x) = N_a(x)$, so that $E_x = -\frac{kT}{qL}$.

If $E_x = -4000 \text{ Volts/cm}$, $L = 0.065 \mu\text{m}$, with $N_{a(0)} = 10^{17} \text{ cm}^{-3}$ and $x_B = 0.3 \mu\text{m}$ $N_a(x_B) = 10^{17} e^{-\frac{0.3}{0.065}} = 9.9 \times 10^{14} \text{ cm}^{-3}$

6.2

Using eq. (6.1.10) with $x' = x_B$ gives :

$$\frac{J_n}{q} \int_x^{x_B} \frac{pdx}{D_n} = P(x_B) n(x_B) - P(x) n(x) \quad \dots \dots \dots \quad (1)$$

Under active bias the base-collector junction is reverse biased so that $n(x_B) \approx 0$. Using $I_c = -J_n A_E$ in eq. (1), we thus obtain

$$I_c = \frac{8 A_E N_a(x) n(x)}{\int_x^{x_B} \frac{N_a dx}{D_n}} \quad \dots \dots \dots \quad (2)$$

(Here we assume low-level injection and quasi-neutrality in the base so that $p(x) \approx N_a(x)$)

Define $\tilde{D}_n(x) = \frac{\int_x^{x_B} N_a dx}{\int_x^{x_B} \frac{N_a dx}{D_n}}$, using this definition

$$I_c = \frac{8 A_E \tilde{D}_n(x) N_a(x) n(x)}{\int_x^{x_B} N_a dx}$$

6.3

Let $N_e(x)$ denote the electron density in the case of exponential base doping and $n_c(x)$ denote the density in the case of constant doping. Using the result of Prob. 6.2 with $N_a(x) = N_{a(0)} e^{-\frac{x}{L}}$ gives

$$I_c = \frac{8 A_E \tilde{D}_n(x) N_{a(0)} e^{-\frac{x}{L}} N_e(x)}{N_{a(0)} \int_x^{x_B} e^{-\frac{x'}{L}} dx'} = \frac{8 A_E \tilde{D}(x) e^{-\frac{x}{L}} N_e(x)}{L [e^{-\frac{x}{L}} - e^{-\frac{x_B}{L}}]}$$

Solving for $N_e(x)$, gives :

$$n_e(x) = \frac{I_c L [1 - \exp(-\frac{x-x_B}{L})]}{8 A_E \hat{D}_n(x)}, \text{ using } n_e(0) = \frac{I_c L [1 - \exp(-\frac{-x_B}{L})]}{8 A_E \hat{D}_n(0)}$$

$$\text{then } n_e(x) = n_e(0) \frac{\hat{D}_n(0)}{\hat{D}_n(x)} \left[\frac{1 - \exp(-\frac{x-x_B}{L})}{1 - \exp(-\frac{-x_B}{L})} \right]$$

As $L \rightarrow \infty$ the base doping becomes constant. Hence as $L \rightarrow \infty$
 $n_e(x) \rightarrow n_c(x)$, $\hat{D}_n(x) \rightarrow D_n$, and $\exp(-\frac{x-x_B}{L}) \rightarrow 1 + \frac{x-x_B}{L}$

$$\therefore n_c(x) = \frac{n_c(0)}{x_B} (x_B - x), \text{ where } n_c(0) = \frac{I_c x_B}{8 A_E D_n}$$

Since the exponential doping variation of prob. 6.1 is not very large
 $\hat{D}_n(x) \approx D_n$. If we normalize both densities with respect
to $n_c(0)$; i.e., if we let $\hat{n}(x) = n(x)/n_c(0)$, we obtain

$$\hat{n}_e(x) = \hat{n}_e(0) \left[\frac{1 - \exp(-\frac{x-x_B}{L})}{1 - \exp(-\frac{-x_B}{L})} \right] = \frac{L}{x_B} \left[1 - \exp(-\frac{x-x_B}{L}) \right]$$

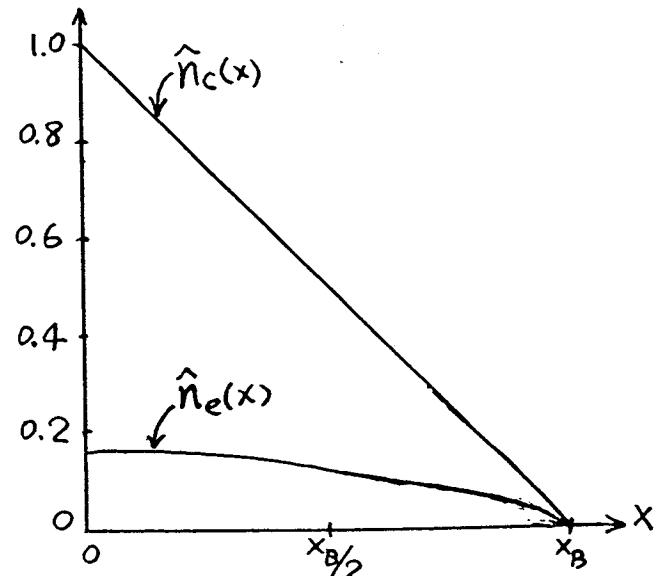
$$\text{and } \hat{n}_c(x) = 1 - \frac{x}{x_B} \quad \text{where } \hat{n}_e(0) = \frac{n_e(0)}{n_c(0)} = \frac{L}{x_B} \left[1 - \exp(-\frac{-x_B}{L}) \right]$$

\hat{n}_c	\hat{n}_e	x
1	0.137	0
0.5	0.054	$x_B/2$
0	0	x_B

$J_{ndiff}(x_B) \propto n(x_B)$ and $n(x_B) \approx 0$

Thus the total current at $x=x_B$
is a diffusion current no matter
how the base is doped. With
all other parameters equal,
the only way to have the same
current in the two cases is

to have the same gradient at $x=x_B$, since
 $J_{ndiff} \propto dn/dx$



6.4

The total stored minority charge is given by $Q_B = -8 A_E \int_0^{x_B} n(x) dx$

For the exponential doping we have from prob. 6.3

$$n_e(x) = \frac{n_{e(0)}}{[1 - e^{\frac{-x_B}{L}}]} \left(1 - e^{\frac{(x-x_B)}{L}} \right)$$

$$\therefore Q_B|_{\text{exp.}} = \frac{-q A_E n_{e(0)}}{\left[1 - e^{\frac{-x_B}{L}} \right]} \int_0^{x_B} \left[1 - e^{\frac{(x-x_B)}{L}} \right] dx = \frac{-q A_E n_{e(0)} L}{\left[1 - e^{\frac{-x_B}{L}} \right]} \left[e^{\frac{-x_B}{L}} - 1 + \frac{x_B}{L} \right]$$

Letting $L \rightarrow \infty$ and expanding the exponential terms gives

$$Q_B|_{\text{constant}} = \frac{-q A_E n_{e(0)} x_B}{2}$$

Assume that base current is due only to base recombination.

If I_c is the same in the two transistors

$$\beta = \frac{I_c}{I_B} \Rightarrow \frac{\beta_c}{\beta_e} = \frac{I_{rB}|_{\text{exp.}}}{I_{rB}|_{\text{const.}}}$$

Since the injected electron density greatly exceeds the equilibrium electron density over most of the base and lifetime is not strongly x dependent, eq. (6.2.4) can be written :

$$I_{rB} = \frac{q A_E}{\tau_n} \int_0^{x_B} n dx = \frac{-Q_B}{\tau_n}$$

$$\therefore \frac{I_{rB}|_{\text{exp.}}}{I_{rB}|_{\text{const.}}} = \frac{Q_B|_{\text{exp.}}}{Q_B|_{\text{const.}}} = \frac{2 n_{e(0)} L}{n_{e(0)} x_B} \frac{\left[e^{\frac{-x_B}{L}} - 1 + \frac{x_B}{L} \right]}{\left[1 - e^{\frac{-x_B}{L}} \right]} = \frac{2 L^2}{x_B^2} \left[e^{\frac{-x_B}{L}} - 1 + \frac{x_B}{L} \right]$$

$$\therefore \frac{\beta_c}{\beta_e} = \frac{2 L^2}{x_B^2} \left[e^{\frac{-x_B}{L}} - 1 + \frac{x_B}{L} \right] = 0.340$$

6.5

(a) From eq. (6.2.1) under forward active bias

$$J_c = J_s \exp \left[\frac{q V_{BE}}{kT} \right] \text{ or } \log_{10} J_c = \log_{10} J_s + \frac{q V_{BE}}{kT} \log_{10} e$$

The current density at $V_{BE}=0$ obtained from the extrapolated line drawn through the $\log_{10} J_c$ versus V_{BE} data for large V_{BE} is thus J_s . From Fig. 6.4 $J_s = 10^{-10} \text{ Amps/cm}^2$

Now $A = 10 \mu\text{m} \times 10 \mu\text{m} = 10^{-6} \text{ cm}^2$, $I_s = 10^{-16} \text{ A}$ and $D_n = 25 \text{ cm}^2 \text{ sec}^{-1}$

$$\text{Using eq. (6.2.2)} \quad Q_{B_0} = \frac{q^2 n_i^2 D_n}{J_s} = 1.346 \times 10^{-6} \text{ C/cm}^2$$

(b) Assuming that total charge = $Q_{B_0} A = 1.346 \times 10^{-12} C$

62

$$q \tilde{N}_B = \frac{Q_{B_0} A}{A X_B}$$

$$X_B = \frac{Q_{B_0}}{q \tilde{N}_B} = \frac{1.346 \times 10^{-6} C/cm^2}{(1.60 \times 10^{-19} C)(10^{17} cm^{-3})} = 0.841 \times 10^{-4} cm = 0.841 \mu m$$

6.6

Take $x=0$ at base-emitter junction plane. From Fig. 6.15

emitter : $x=0 \mu m \quad N_a = 10^{17} cm^{-3}$

$x = 0.13 \mu m \quad N_a = 2 \times 10^{15} cm^{-3}$

base : $x=0 \mu m \quad N_a = 10^{17} cm^{-3}$

$x = 0.13 \mu m \quad N_a = 3.5 \times 10^{16} cm^{-3}$

Collector : $N_d = 3.5 \times 10^{16} cm^{-3}$ a constant

Using exponentials to approximate the profiles in the emitter and base we have

$$\text{emitter: } N_a = 10^{17} e^{-x/\lambda_b} \quad (x \text{ in } \mu m)$$

$$\text{base: } N_a = 10^{17} e^{-x/\lambda_e} \quad (x \text{ in } \mu m)$$

$$\begin{aligned} \frac{Q'_B}{q} &= 10^{17} \int_0^{0.13} (e^{-\frac{x}{\lambda_b}} - e^{-\frac{x}{\lambda_e}}) dx - 3.5 \times 10^{16} \int_0^{0.13} dx \\ &= 10^{17} \left[\lambda_b (1 - e^{-\frac{0.13}{\lambda_b}}) - \lambda_e (1 - e^{-\frac{0.13}{\lambda_e}}) \right] - 3.5 \times 10^{16} (1.3 \times 10^{-5}) \end{aligned}$$

$$= 2.7 \times 10^{10} \text{ atoms/cm}^2$$

From Fig. 6.16 emitter: $x=0 \quad N_d = 10^{17} cm^{-3}$

$x=0.15 \mu m \quad N_d = 3.5 \times 10^{15} cm^{-3}$

base : $x=0 \quad N_a = 10^{17} cm^{-3}$

$x=0.3 \mu m \quad N_a = 3.5 \times 10^{15} cm^{-3}$

Collector : $N_d = 3.5 \times 10^{15} cm^{-3}$, a constant

Using exponentials to approximate the profiles in the emitter and base we have

$$\text{emitter: } N_a = 10^{17} e^{-x/\lambda_b} \quad (x \text{ in } \mu m)$$

$$\text{base: } N_a = 10^{17} e^{-x/\lambda_e} \quad (x \text{ in } \mu m)$$

$$\begin{aligned} \frac{Q'_B}{q} &= 10^{17} \int_0^{0.3} (e^{-\frac{x}{\lambda_b}} - e^{-\frac{x}{\lambda_e}}) dx - 3.5 \times 10^{15} \int_0^{0.3} dx \\ &= 10^{17} \left[\lambda_b (1 - e^{-\frac{0.3}{\lambda_b}}) - \lambda_e (1 - e^{-\frac{0.3}{\lambda_e}}) \right] - 3.5 \times 10^{15} \times 3 \times 10^{-5} \end{aligned}$$

$$= 3.12 \times 10^{11} \text{ atoms/cm}^2$$

From Prob. 6.5

$$\frac{Q_B'}{A} = \frac{Q_B}{A} = 8.413 \times 10^{12} \text{ atoms/cm}^2$$

The lower value of $\frac{Q_B'}{A}$ in the switching transistor is due to the lower resistivity of the epitaxial layer in that structure.

6.7

$$N_d - N_a = \alpha X$$

From Eq. (4.3.2)

$$X_d = \left[\frac{12\epsilon_s(\phi_i - V_a)}{qa} \right]^{1/3}$$

$$Q = -\frac{1}{2} qa \frac{X_d}{2} \frac{X_d}{2}$$

$$= -\frac{qa}{8} \left[\frac{12\epsilon_s(\phi_i - V_a)}{qa} \right]^{2/3}$$

$$Q_v = Q(V) - Q(0)$$

$$= \frac{qa}{8} \left(\frac{12\epsilon_s}{qa} \right)^{2/3} \left[\phi_i^{2/3} - (\phi_i - V_a)^{2/3} \right]$$

$$= \left(\frac{98a\epsilon_s^2 \phi_i^2}{32} \right)^{1/3} \left[1 - \left(1 - \frac{V_a}{\phi_i} \right)^{2/3} \right]$$

$$K_q = \left(\frac{98a\epsilon_s^2 \phi_i^2}{32} \right)^{1/3} = 71.6 \text{ nC/cm}^2$$

Since the junction area is 10^{-5} cm^2 , we have $K_q A = 0.72 \text{ pC}$ for the factor representing total charge at each junction. Initially, $V_a = -6V = -6.88 \times \phi_i$. From Fig. 6.11 the charge stored at the collector.

$$Q_{VC} = -3K_q A = -2.16 \text{ pC}$$

when $V_B = 0V$, $V_a = -3V = -3.44\phi_i$, and,

$Q_{VC} = -1.7 K_q A = -1.22 \text{ pC}$. The charge supplied from the base is therefore 0.94 pC . At the base-emitter junction, the bias change is from $-3V$ to $0V$; hence the stored charge changes from -1.22 pC to 0 C . The total charge supplied from the base is the sum of these charges or 2.16 pC .

6.8

(a) See Fig. 1.

(b) From prob. 6.7with $N_d \gg N_a$

$$Q_{VE} = A [2\epsilon_s N_a]^{1/2} [\phi_i^{1/2} - (\phi_i - V)^{1/2}] \\ = K'_A [\phi_i^{1/2} - (\phi_i - V)^{1/2}]$$

with $A = 2 \times 10^{-6} \text{ cm}^2$

$$N_a = 2 \times 10^{16} \text{ cm}^{-3}$$

$$K'_A = 1.628 \times 10^{-13} \text{ Coul/Volt}^{1/2}$$

$$\phi_i = \frac{E_g}{2} + \frac{kT}{q} \ln \frac{N_a}{n_i} = 0.91 \text{ Volts}$$

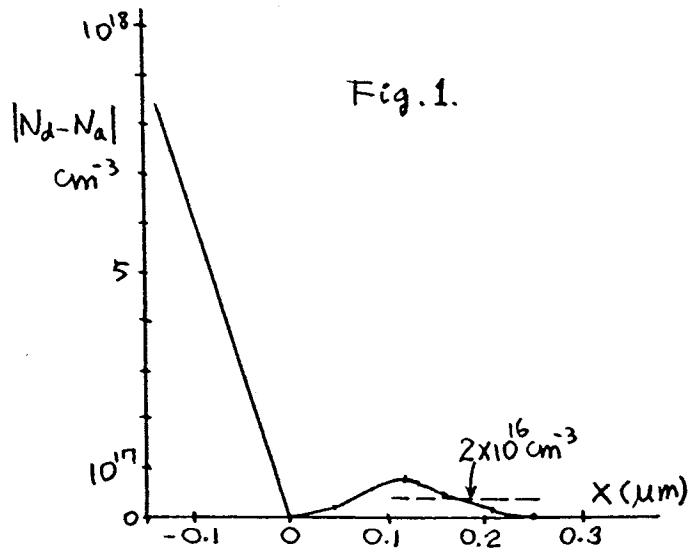


Fig. 1.

Q_{VE} (coul)	$\phi_i - V$ (Volts)	V (Volts)
$+2.815 \times 10^{-14}$	+0.61	+0.3
0	+0.91	0
-3.824×10^{-13}	+10.91	-10
-5.891×10^{-13}	+20.91	-20
-7.5×10^{-13}	+30.91	-30
-8.86×10^{-13}	+40.91	-40
-10×10^{-13}	+50.91	-50

6.9

The data of prob. 6.8 are plotted in Fig. 2.

$$\text{If } N = N_0 e^{-x/\ell}$$

$$\text{then } \log_{10} N = \log_{10} N_0 - \frac{x}{\ell} \log_{10} e$$

The slope S of the semi-log plot is $S = -\frac{\log_{10} e}{\ell}$

From the data

$$S = \frac{16.6 - 15.8}{(0.12 - 0.21) 10^{-4}} = -8.89 \times 10^4$$

$$\therefore \ell = \frac{\log_{10} e}{8.89 \times 10^4} = 4.89 \times 10^{-6} \text{ cm} \\ = 0.0489 \mu\text{m}$$

$$\text{From Prob. 6.1 } \Sigma = \frac{-KT}{8\ell} = -5.3 \text{ KV/cm}$$

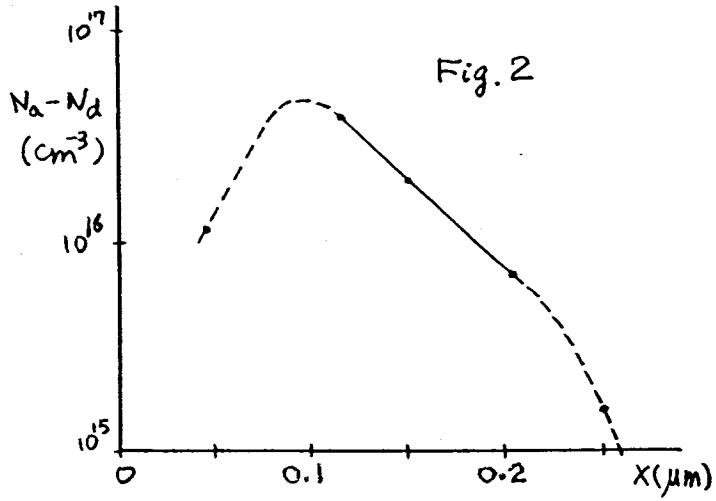
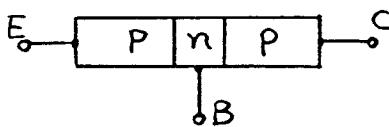


Fig. 2

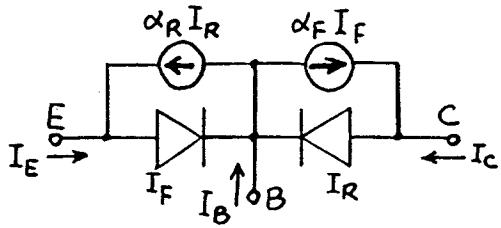
6.10

$$I_E = I_F - \alpha_R I_R$$

$$I_C = I_R - \alpha_F I_F$$

I_F and I_R are again given by

Eq.(6.4.9) except $V_{BE} \rightarrow V_{EB}$ and
 $V_{BC} \rightarrow V_{CB}$



6.11

Under active bias eq. (6.4.9) gives $I_R = -I_{CS}$, So that
 $I_C = \alpha_F I_F + I_{CS}$ and $I_E = -I_F - \alpha_R I_{CS}$

Combining these equations gives

$$I_B = -(I_C + I_E) = I_F(1 - \alpha_F) - I_{CS}(1 - \alpha_R) \quad \text{or}$$

$$I_F = \frac{I_B}{1 - \alpha_F} + I_{CS} \frac{1 - \alpha_R}{1 - \alpha_F} \quad \text{Using this result}$$

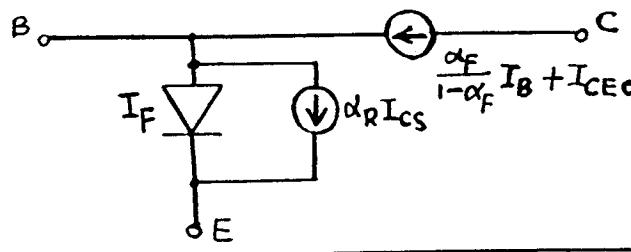
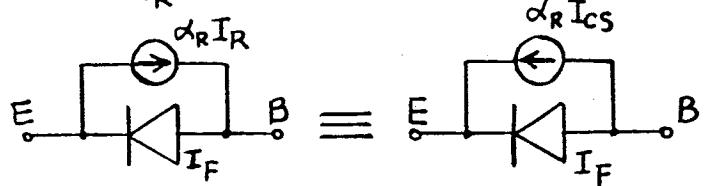
$$I_C = \frac{\alpha_F}{1 - \alpha_F} I_B + I_{CS} \left[1 + \frac{\alpha_F(1 - \alpha_R)}{1 - \alpha_F} \right] = \frac{\alpha_F}{1 - \alpha_F} I_B + I_{CS} \left[\frac{1 - \alpha_F \alpha_R}{1 - \alpha_F} \right]$$

Using eq. (6.4.16) this can be written

$$I_C = \frac{\alpha_F}{1 - \alpha_F} I_B + I_{CEO} \quad \therefore \quad \begin{array}{c} \text{B} \\ \text{---} \\ \text{C} \end{array} \xrightarrow{\alpha_F I_F} \begin{array}{c} \text{B} \\ \text{---} \\ \text{C} \end{array} \equiv \begin{array}{c} \text{B} \\ \text{---} \\ \text{C} \end{array} \xrightarrow{\frac{\alpha_F I_B}{1 - \alpha_F} + I_{CEO}}$$

We also have

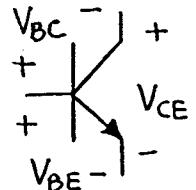
Thus, for a base-driven transistor we have

6.12

$$V_{CE} = V_{BE} - V_{BC}$$

For forward active bias eq. (6.4.9) give

$$V_{BE} = \frac{kT}{q} \ln \frac{I_F}{I_{ES}}, \quad V_{BC} = \frac{kT}{q} \ln \frac{I_R}{I_{CS}} \quad \therefore \quad V_{CE} = \frac{kT}{q} \ln \frac{I_F I_{CS}}{I_R I_{ES}}$$



Adding eqs.(6.4.10a + 6.4.10b) give $-(I_E + I_C) = I_B = I_F(1 - \alpha_F) + I_R(1 - \alpha_R) \quad \dots (1)$

Substitution of $I_R = \alpha_F I_F - I_C$ into (1) and solving for I_F gives

$$I_F = \frac{I_B}{1 - \alpha_F \alpha_R} \left[1 + \frac{I_C}{I_B} (1 - \alpha_R) \right] \quad \dots \quad (2)$$

Substitution of (2) into $I_R = \alpha_F I_F - I_C$ gives

$$I_R = \frac{I_B}{1 - \alpha_F \alpha_R} \left[\alpha_F - \frac{I_C}{I_B} (1 - \alpha_F) \right] \quad \dots \quad (3)$$

$$\text{Using (2) and (3)} \quad V_{CE} = \frac{kT}{q} \ln \left\{ \frac{1 + \frac{I_C}{I_B} (1 - \alpha_R)}{\alpha_R \left[1 - \frac{I_C}{I_B} \left(\frac{1 - \alpha_F}{\alpha_F} \right) \right]} \right\}$$

$$\text{with } \frac{I_C}{I_B} = 10, \alpha_F = 0.985, \alpha_R = 0.72 \Rightarrow V_{CE} = 0.048 \text{ Volts}$$

6.13

From eq.s (6.4.9) and (6.4.10) under forward active bias

$$I_C = \alpha_F I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) + I_{CS} \quad \dots \quad (1) \quad I_E = -I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) - \alpha_R I_{CS} \quad \dots \quad (2)$$

$$\text{Now } I_B = -(I_E + I_C) = (1 - \alpha_F) I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) - I_{CS} (1 - \alpha_R) \quad \dots \quad (3)$$

With the emitter open $I_E = 0$ eq.(2) requires $I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) = -\alpha_R I_{CS}$
and eq.(1) gives $I_{CBO} = -\alpha_F \alpha_R I_{CS} + I_{CS} = I_{CS} (1 - \alpha_F \alpha_R)$

With the base open $I_B = 0$, eq.(3) requires $I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) = I_{CS} \frac{(1 - \alpha_R)}{(1 - \alpha_F)}$
and eq.(1) gives $I_{CEO} = \alpha_F I_{CS} \frac{(1 - \alpha_R)}{(1 - \alpha_F)} + I_{CS} = I_{CS} \frac{(1 - \alpha_F \alpha_R)}{(1 - \alpha_F)}$

$$\therefore \frac{I_{CEO}}{I_{CBO}} = \frac{1}{1 - \alpha_F}$$

Reason: when the base is open, the electron leakage current forces the B-E to forward bias in order to clear the base of holes. This leads to electron injection and transistor action. When the base is not open, the leakage current of holes passes out the base lead.

6.14

- (a) The transistor is biased with voltage sources across the base-emitter and collector-base.

The light causes a current from collector to base.

Using a simplified Ebers-Moll model.

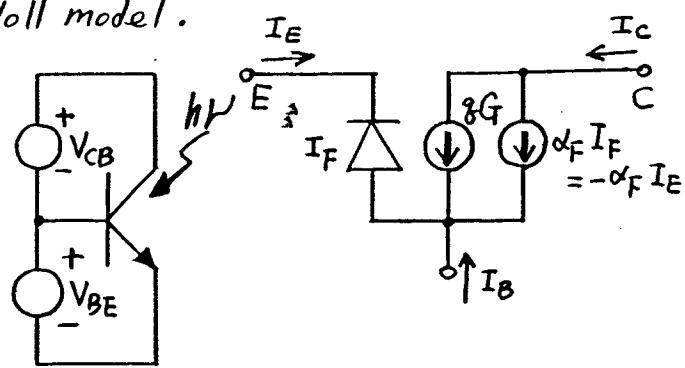
If the current flowing before $t=0$ are
 I_{BO} , $-I_{EO}$ and I_{CO}

Then, after $t=0$,

$$I_B \rightarrow I_{BO} - gG$$

$$I_C \rightarrow I_{CO} + gG = \alpha_F I_{EO} + gG$$

$$I_E \rightarrow -I_{EO}$$



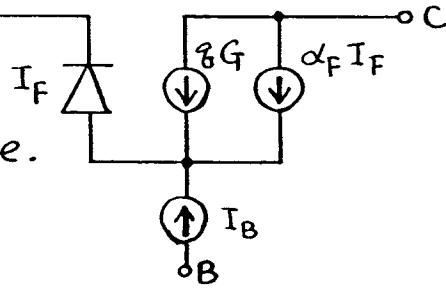
(b) If the base is driven by a current source, then $I_B \rightarrow I_{B0}$
 Hence, the holes arriving at the base will change the
 base-emitter bias. $\therefore E$

To find the new condition,
 see the currents at the base node.

$$I_B + gG + \alpha_F I_F = I_F$$

$$\therefore I_F = \frac{I_B + gG}{1 - \alpha_F} = -I_E$$

$$I_C = gG + \alpha_F I_F = gG + \alpha_F \left(\frac{I_B + gG}{1 - \alpha_F} \right) = gG(1 + \beta_F) + I_E \beta_F$$



6.15

For an n-p-n transistor biased in the active mode, the currents in the base lead are due to hole flow in the base. The causes for base current and the corresponding signs of the current at the base lead are as follows:

- (1.) injection of holes into the emitter (always positive under active bias)
- (2.) recombination in the emitter-base space-charge region (always positive under active bias)
- (3.) recombination in the base itself (always positive under active bias)
- (4.) injection of holes from generation in the collector-base space-charge region (always negative under active bias)
- (5.) injection of holes from the collector region (always negative under active bias)

The collector current under active bias has two causes

- (i) collection of electrons emitted from the forward-biased, emitter-base junction (always positive under active bias)
- (ii) generation of electrons in the base and in the collector-base space-charge region (always positive under active bias)

As the temperature is increased, the generation rates for carriers will increase. Hence, if the collector current is held constant, current (i) will have to be reduced in order

to balance the increase in current (ii). To reduce current (i), the injection rate of electrons into the base will be reduced by a lowered emitter-base voltage. This causes a reduction in components (1) to (3) of the base current. Base current components (4) and (5) will, however, increase so that the net base current will be reduced under the conditions stated. Ultimately I_B will change sign as components (4) and (5) become larger than components (1) through (3).

6.16

$$\text{From Eq. (6.2.19)} \quad \gamma = \frac{1}{1 + I_{PE}/I_{nE}}$$

If we define $Q_E = gA_E \int_{-X_E}^{-X_n} N_d(x) dx$, Eq. (6.2.18) under forward active bias becomes

$$I_{PE} = -\frac{g^2 A_E^2 \tilde{D}_{PE} n_i^2 e^{\frac{-gV_{BE}}{kT}}}{Q_E} \quad . \quad \text{Using Eq.(6.2.1) and Eq.(6.2.2)}$$

$$I_{nE} = -\frac{g^2 A_E^2 \tilde{D}_{nB} n_i^2 e^{\frac{-gV_{BE}}{kT}}}{Q_E} \quad \therefore \quad \gamma = \frac{1}{1 + \frac{Q_B \tilde{D}_{PE}}{Q_E \tilde{D}_{nB}}}$$

For a uniformly doped base $Q_B = gA_E N_{AB} X_B$ and $\tilde{D}_{nB} = D_{nB}$

For a uniformly doped emitter ($X_n \ll X_E$)

$$Q_E = gA_E N_{dE} X_E \quad \text{and} \quad \tilde{D}_{PE} = D_{PE} \quad \therefore \quad \gamma = \frac{1}{1 + \frac{N_{AB} D_{PE} X_B}{N_{dE} D_{nB} X_E}}$$

6.17

$$(a) \text{ From Prob. 6.16} \quad \gamma = \frac{1}{1 + \frac{gA_E N_{AB} X_B \tilde{D}_{PE}}{Q_E D_{nB}}}$$

with $A_E = 10^5 \text{ cm}^2$

$$N_{AB} = 4 \times 10^{17} \text{ cm}^{-3}, \quad X_B = 5 \times 10^{-5} \text{ cm}, \quad \tilde{D}_{PE} = 2 \text{ cm}^2 \cdot \text{sec}^{-1},$$

$$Q_{E_0} = 1.28 \times 10^{-9} \text{ Coul}, \quad D_{nB} = 18 \text{ cm}^2 \cdot \text{sec}^{-1}. \quad \Rightarrow \gamma = 0.99722$$

$$(b) \text{ From eq. (6.2.8)} \quad \alpha_T = 1 - \frac{X_B^2}{2 D_{nB} T_n}$$

$$\text{with } \tilde{T}_n = 10^{-6} \text{ sec} \Rightarrow \alpha_T = 0.99993$$

$$(c) \alpha_F = \gamma \alpha_T = 0.99715, \beta_F = \frac{\alpha_F}{1-\alpha_F} = 350$$

If we use the approximation $\alpha_T = 1 \Rightarrow \beta_F = \frac{\gamma}{1-\gamma} = 359$
% error = 2.57 %

6.18

β_F depends on τ_n through α_T $\tau_n = \tau_{no} e^{-t/t_d}$

$$\alpha_T = 1 - \frac{x_B^2}{2D_{nB}\tau_n} = 1 - \frac{x_B^2 e^{t/t_d}}{2D_{nB}\tau_{no}}$$

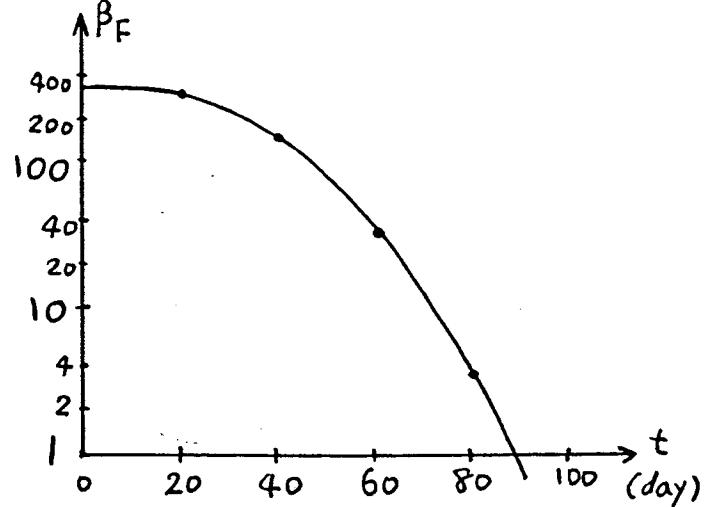
$$\beta_F = \frac{\partial \alpha_T}{1-\gamma \alpha_T} = \frac{\gamma \left[1 - \frac{x_B^2 e^{t/t_d}}{2D_{nB}\tau_{no}} \right]}{1-\gamma \left[1 - \frac{x_B^2 e^{t/t_d}}{2D_{nB}\tau_{no}} \right]} = \frac{1 - \frac{x_B^2 e^{t/t_d}}{2D_{nB}\tau_{no}}}{\gamma - 1 + \frac{x_B^2 e^{t/t_d}}{2D_{nB}\tau_{no}}}$$

From Prob. 6.17 $\gamma = 0.99722, x_B = 5 \times 10^{-5} \text{ cm}, D_{nB} = 18 \text{ cm}^2 \cdot \text{sec}^{-1}$

$$\tau_{no} = 10^{-6} \text{ sec. } t_d = 10 \text{ days} \quad \therefore$$

$$\beta_F = \frac{1 - (6.944 \times 10^{-5}) e^{t/10}}{2.777 \times 10^{-3} + (6.944 \times 10^{-5}) e^{t/10}}$$

t (days)	β_F
0	351.1
20	304
40	152
60	31.5
80	3.78



$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = 1 \Rightarrow \alpha_F = \frac{1}{2}$$

$$\text{But } \alpha_F = \gamma \left[1 - \frac{x_B^2 e^{t_1/t_d}}{2D_{nB}\tau_{no}} \right] = \frac{1}{2} \quad 1 - \frac{1}{2\gamma} = \frac{x_B^2 e^{t_1/t_d}}{2D_{nB}\tau_{no}}$$

$$t_1 = t_d \ln \left[\frac{2D_{nB}\tau_{no}}{x_B^2} \left(1 - \frac{1}{2\gamma} \right) \right] = 88.8 \text{ days}$$

$$\text{at } t_1 = 88.8 \text{ days} \quad \tau_n = 0.139 \text{ nsec.}$$

6.19

(a) For a uniform base under low-level injection, we have to solve

$$\frac{d^2 n_p}{dx^2} - \frac{n_p}{L_n^2} + \frac{n_{po}}{L_n^2} = 0 \quad L_n = (D_{nB} T_n)^{1/2}$$

$$\therefore n_p(x) = A e^{-\frac{x}{L_n}} + B e^{\frac{x}{L_n}} + n_{po}, \text{ Boundary Conditions:}$$

$$(1) \quad x=0, n_p(0) = A + B + n_{po}$$

$$(2) \quad x=x_B, 0 = A e^{-\frac{x_B}{L_n}} + B e^{\frac{x_B}{L_n}} + n_{po}$$

$$\text{Substracting (2) from (1) gives } n_p(0) = A(1 - e^{-\frac{x_B}{L_n}}) + B(1 - e^{\frac{x_B}{L_n}})$$

Using this equation in (1) leads to

$$A = \frac{[n_p(0) - n_{po}]}{2 \sinh(\frac{x_B}{L_n})} e^{\frac{x_B}{L_n}} + n_{po} \quad \text{and} \quad B = -\frac{[n_p(0) - n_{po}]}{2 \sinh(\frac{x_B}{L_n})} e^{-\frac{x_B}{L_n}} + n_{po}$$

$$\therefore n_p(x) = \frac{1}{2 \sinh(\frac{x_B}{L_n})} \left\{ [n_p(0) - n_{po}] e^{\frac{x_B-x}{L_n}} + n_{po} e^{-\frac{x}{L_n}} - [n_p(0) - n_{po}] e^{-\frac{(x_B-x)}{L_n}} - n_{po} e^{\frac{x}{L_n}} \right\} + n_{po}$$

which can be written

$$n_p(x) = \frac{[n_{po}(0) - n_{po}]}{\sinh(\frac{x_B}{L_n})} \sinh\left(\frac{x_B-x}{L_n}\right) + n_{po} \left\{ 1 - \frac{\sinh(\frac{x}{L_n})}{\sinh(\frac{x_B}{L_n})} \right\}$$

$$(b) \frac{dn_p}{dx} = -\frac{1}{L_n} \left\{ \frac{[n_p(0) - n_{po}]}{\sinh(\frac{x_B}{L_n})} \cosh\left(\frac{x_B-x}{L_n}\right) + \frac{n_{po} \cosh(\frac{x}{L_n})}{\sinh(\frac{x_B}{L_n})} \right\}$$

$$J_{nE} = g D_{nB} \left. \frac{dn_p}{dx} \right|_{x=0} = -\frac{g D_{nB}}{L_n \sinh(\frac{x_B}{L_n})} \left\{ [n_p(0) - n_{po}] \cosh\left(\frac{x_B}{L_n}\right) + n_{po} \right\}$$

$$J_{nC} = g D_{nB} \left. \frac{dn_p}{dx} \right|_{x=x_B} = \frac{-g D_{nB}}{L_n \sinh(\frac{x_B}{L_n})} \left\{ [n_p(0) - n_{po}] + n_{po} \cosh(\frac{x_B}{L_n}) \right\}$$

$$J_{rB} = -A_E [J_{nE} - J_{nC}] = \frac{g D_{nB} A_E}{L_n \sinh(\frac{x_B}{L_n})} \left[\cosh\left(\frac{x_B}{L_n}\right) - 1 \right] [n_p(0) - 2 n_{po}]$$

$$\text{Using } n_p(0) = n_{po} e^{\frac{8V_{BE}}{kT}}$$

$$I_{Br} = \frac{g A_E D_{nB} \left[\cosh\left(\frac{x_B}{L_n}\right) - 1 \right]}{L_n \sinh(\frac{x_B}{L_n})} n_{po} \left(e^{\frac{8V_{BE}}{kT}} - 2 \right),$$

$$(c) \gamma = \frac{1}{1 + \frac{I_{pE}}{I_{nE}}}, \text{ from part (b)}$$

$$I_{nE} = A_E J_{nE} = \frac{-g A_E D_{nB} n_{po}}{L_n \sinh(\frac{x_B}{L_n})} \left\{ \left(e^{\frac{8V_{BE}}{kT}} - 1 \right) \cosh\left(\frac{x_B}{L_n}\right) + 1 \right\}$$

Under forward active bias ($V_{BE} \gg \frac{kT}{q}$) this reduces to

$$I_{nE} = \frac{-8A_E D_{nB} n_i^2 \cosh(\frac{x_B}{L_n})}{N_{nB} L_n \sinh(\frac{x_B}{L_n})} e^{\frac{8V_{BE}}{kT}} \quad \text{and by analogy}$$

$$I_{pE} = \frac{-8A_E D_{pE} n_i^2 \cosh(\frac{x_E}{L_p})}{N_{pE} L_p \sinh(\frac{x_E}{L_p})} e^{\frac{8V_{BE}}{kT}}$$

$$\therefore \gamma = \frac{1}{1 + \frac{D_{pE} N_{nB} L_n \tanh(\frac{x_B}{L_n})}{D_{nB} N_{pE} L_p \tanh(\frac{x_E}{L_p})}}$$

with $N_{nB} = 10^{17} \text{ cm}^{-3}$, $D_{nB} = 17.5 \text{ cm}^2/\text{sec}$, $N_{pE} = 10^{19} \text{ cm}^{-3}$

$$D_{pE} = 2 \text{ cm}^2/\text{sec}, \tau_n = 10^{-7} \text{ sec} \Rightarrow L_n = 13.23 \mu\text{m}$$

$$\tau_p = 10^{-9} \text{ sec} \Rightarrow L_p = 0.447 \mu\text{m}, x_E = x_B = 1 \mu\text{m}$$

$$\Rightarrow \gamma = 0.9974$$

$$\alpha_T = \frac{I_{nc}}{I_{nE}} = \frac{1}{\cosh(\frac{x_B}{L_n})} \quad \text{for forward active bias and } x_B \leq L_n$$

$$= 0.99715$$

$$\alpha_F = \gamma \alpha_T = 0.99456 \quad \therefore \beta_F = \frac{\alpha_F}{1 - \alpha_F} = 182.7$$

6.20

$$dt = \frac{dx}{v(x)} \quad \text{so that} \quad \tau_B = \int_0^{x_B} \frac{dx}{v(x)}$$

$$\text{Since } I_c = g n(x) v(x) A, \quad v(x) = \frac{I_c}{g n(x) A} \quad \therefore \tau_B = \frac{8A}{I_c} \int_0^{x_B} n(x) dx$$

For the prototype transistor, we have from Prob. 6.3

$$n_c(x) = \frac{n_c(0)}{x_B} (x_B - x) = \frac{I_c}{8A_E D_n} (x_B - x)$$

$$\therefore \tau_B = \frac{8A_E I_c}{I_c 8A_E D_n} \int_0^{x_B} (x_B - x) dx = \frac{1}{D_n} \left[x_B^2 - \frac{x_B^2}{2} \right]$$

$$\therefore \tau_B = \frac{x_B^2}{2 D_n}$$

CHAPTER 7

7.1

For a constant base doping and low-level injection Eq. (7.1.1) gives $I_c = \frac{8D_n n_i^2 A_E}{N_a x_B} e^{\frac{qV_{BE}}{kT}}$

$$\therefore \frac{\partial I_c}{\partial V_{CB}} = - \frac{8D_n n_i^2 A_E}{N_a x_B^2} e^{\frac{qV_{BE}}{kT}} \cdot \frac{\partial x_B}{\partial V_{CB}} = - \frac{I_c}{x_B} \frac{\partial x_B}{\partial V_{CB}}$$

Eg. (7.1.4) gives $V_A = \frac{x_B}{\partial x_B / \partial V_{CB}}$ so that using Eq. (7.1.3)

$$\frac{\partial I_c}{\partial V_{CB}} = - \frac{I_c}{V_A} = - \frac{I_c}{x_B} \frac{\partial x_B}{\partial V_{CB}} \text{ as above.}$$

7.2

$N_a = 10^{17} \text{ cm}^{-3}$, $N_d = 10^{16} \text{ cm}^{-3}$, $x_B = 2.5 \mu\text{m}$ at $V_{CB} = 0$ Volts.

For the B-C junction $\phi_i = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = 0.7531$ Volts

From eqs. (4.2.6) and (4.3.1)

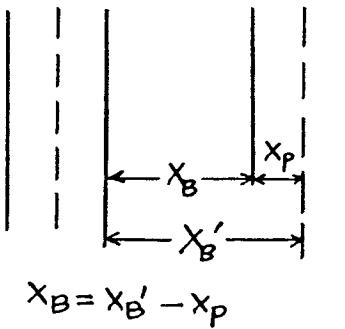
$$x_p = \left[\frac{2\epsilon_s N_d}{8} \frac{1}{N_a} \frac{1}{N_a + N_d} (\phi_i + V_{CB}) \right]^{1/2}$$

$$\therefore \frac{\partial x_B}{\partial V_{CB}} = - \frac{\partial x_p}{\partial V_{CB}} = - \left[\frac{\epsilon_s N_d}{2q N_a (N_a + N_d) (\phi_i + V_{CB})} \right]^{1/2}$$

$$\text{at } V_{CB} = 0 \quad \frac{\partial x_B}{\partial V_{CB}} = -1.976 \times 10^{-6} \text{ cm/Volt}$$

$$V_A|_{V_{CB}=0} = \frac{x_B}{\frac{\partial x_B}{\partial V_{CB}}|_{V_{CB}=0}} = -126.5 \text{ Volts}$$

$$\frac{\partial I_c}{\partial V_{CB}}|_{V_{CB}=0} = \frac{-I_c}{V_A|_{V_{CB}=0}} = \frac{I_c}{126.5}$$



7.3

For a constant doping we have from Prob. 7.1,

$$V_A|_{\text{const.}} = \frac{x_B}{(\partial x_B / \partial V_{CB})_C}$$

For the exponential doping,

$$V_A|_{\text{exp.}} = \frac{N_{A0} \int_0^{x_B} e^{-x/l} dx}{N_{A0} e^{-x_B/l} (\partial x_B / \partial V_{CB})_{\text{exp.}}} = \frac{l (e^{x_B/l} - 1)}{(\partial x_B / \partial V_{CB})_{\text{exp.}}}$$

so that

$$\frac{V_A|_{\text{const.}}}{V_A|_{\text{exp.}}} = \frac{x_B}{l (e^{x_B/l} - 1)} \frac{(\partial x_B / \partial V_{CB})_{\text{exp.}}}{(\partial x_B / \partial V_{CB})_{\text{const}}} \approx \frac{x_B}{l (e^{x_B/l} - 1)}$$

$N_d = 10^{15} \text{ cm}^{-3}$, from Prob. 6.3 and Prob. 6.1,

$x_B = 0.3 \mu\text{m}$, $l = 0.516 \mu\text{m}$,

$$\Rightarrow \frac{V_A|_{\text{const.}}}{V_A|_{\text{exp.}}} = 0.74$$

7.4

$$(a) \varepsilon = \frac{kT}{q} \frac{1}{P} \frac{dP}{dx}$$

$\frac{dP}{dx}$ is constant and $\frac{dP}{dx} < 0$

$$\therefore \varepsilon \propto \frac{-1}{P}$$

$$(c) \varepsilon = \frac{kT}{q} \frac{1}{P} \frac{dP}{dx} \quad \text{Eq. (6.1.2)}$$

$$P = 10^{16} \left(10 - \frac{9x}{x_B} \right) \quad (x \text{ in } \mu\text{m})$$

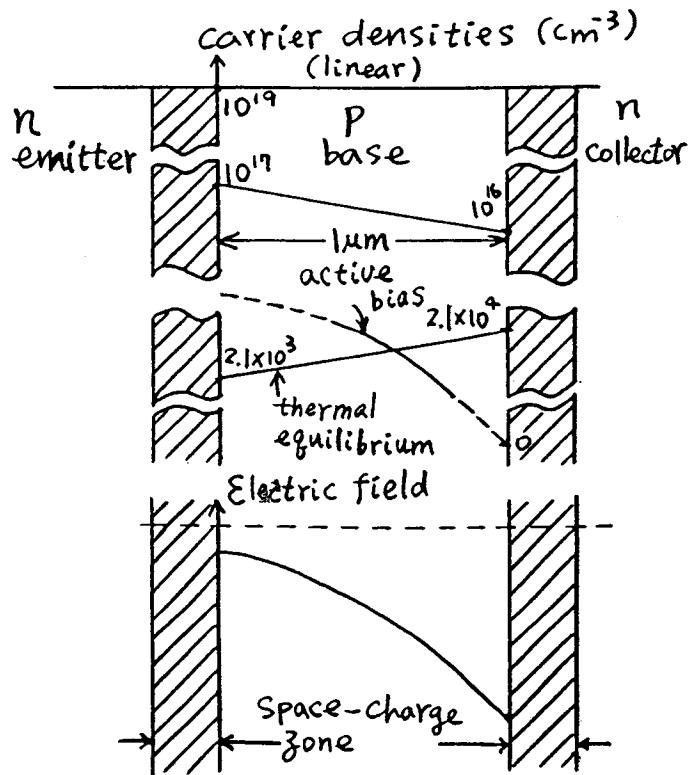
$$\frac{dP}{dx} = \frac{-9x \times 10^{16}}{x_B}$$

$$\therefore \varepsilon = -\frac{kT}{q} \frac{9}{10x_B - 9x} \quad (x_B \text{ in } \mu\text{m})$$

$|\varepsilon_{\text{max}}|$ is at $x = x_B = 1 \mu\text{m}$

$$\therefore \varepsilon_{\text{max}} = -0.0258 \times 9 \times 10^4 \text{ V/cm}$$

$$= -2322 \text{ V/cm}$$



$$(d) V_A = \frac{\int_0^{x_B} P dx}{P(x_B) \frac{d(x_B)}{dV_{CB}}} \text{ (for(i))}, V_B = \frac{\int_0^{x_B} P dx}{P(0) \left. \frac{dX_B}{dV_{EB}} \right|_{x_B=0}} \text{ (for(ii))}$$

(Assume that junctions are one-sided Step junctions)

$$\frac{V_A}{V_B} = \frac{P(0)}{P(x_B)} \times \frac{\left. \frac{dX_B}{dV_{EB}} \right|_{x_B=0} \sim \frac{P(0)}{P(x_B)} \frac{\sqrt{P(x_B)}}{\sqrt{P(0)}}}{\frac{d(x_B)}{dV_{CB}}} = \sqrt{\frac{P(0)}{P(x_B)}} = \sqrt{10} = 3.162$$

7.5

$$\text{From Eq. (7.1.4)} \quad V_A = \frac{\int_0^{x_B} N_a(x) dx}{N_a(x_B) \frac{\partial X_B}{\partial V_{CB}}}$$

For both the prototype and the amplifying transistors, we assume V_{CB} much less than the voltage at which the B-C space-charge region reaches the E-B space-charge region. Thus for both transistors the change in $\int_0^{x_B} N_a(x) dx$ is small.

(a) For the prototype transistor $N_a(x_B)$ is constant and $\frac{\partial X_B}{\partial V_{CB}}$ decreases as V_{CB} increases. Thus V_A increases.

(b) For the amplifying transistor $N_a(x_B)$ increases and $\frac{\partial X_B}{\partial V_{CB}}$ decreases as V_{CB} increases. As V_{CB} increases the increasing base doping reduces the change in X_B , hence reducing the change in $N_a(x_B)$ and $\frac{\partial X_B}{\partial V_{CB}}$. The change in V_A decreases rapidly as V_{CB} increases.

7.6

$$\text{Assume } N_a(x) - N_d(x) = N_{ao} e^{-x/\ell}$$

$$Z_E = 10^5 \text{ cm}, Y_E = 2 \times 10^{-3} \text{ cm}, \beta_F = 50$$

$$x=0 \quad N_a - N_d = 2 \times 10^{18} \text{ cm}^{-3}$$

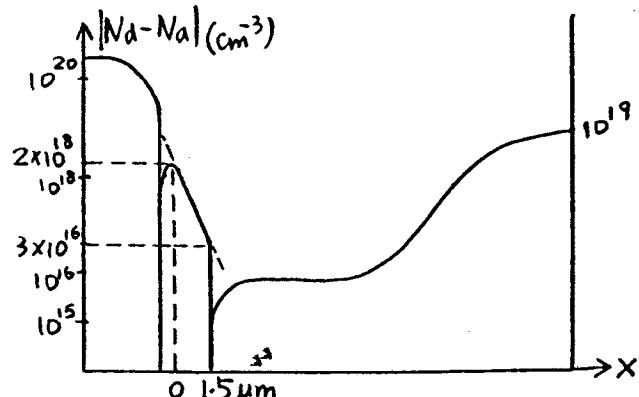
$$x_B = 1.5 \mu\text{m} \quad N_a - N_d = 3 \times 10^{16} \text{ cm}^{-3}$$

$$\therefore \ell = 0.357 \mu\text{m}$$

$$Q_B = g A_E \int_0^{x_B} (N_a - N_d) dx$$

$$= g A_E N_{ao} \int_0^{x_B} e^{-x/\ell} dx = g A_E \ell N_{ao} [1 - e^{-x_B/\ell}] = g A_E (7 \times 10^{13}) \text{ Coul.}$$

$$\text{at } \bar{N} = \frac{Q_B}{g A_E \ell} = 4.66 \times 10^{17} \text{ cm}^{-3} \quad \mu_p \approx 200 \text{ cm}^2/\text{Volt-sec}$$



$$\therefore \frac{1}{\bar{\rho}_B} = \mu_p \frac{Q_B}{A_E X_B} = 8 \mu_p \bar{N} = 14.9 \text{ mhos/cm} \quad \text{and}$$

$$R_B = \frac{1}{6} \frac{\bar{\rho}_B}{X_B} \frac{Y_E}{Z_E} = 1.49 \Omega$$

$$\text{For a voltage drop of } kT/q, I_B = \frac{kT}{8R_B} = 17.4 \text{ mA}$$

$$I_C = \beta_F I_B = 0.87 \text{ amps}$$

7.7

Since β_F is to be a maximum, we neglect recombination in the n-type base. From Prob. 5.12 we have, in arbitrary units,

$$J_C = J_P(x_n) = 4 \text{ and } J_B = J_n(x_n) = 3$$

$$\therefore \beta_F = \frac{J_C}{J_B} = 1.333$$

7.8

(a) With recombination in the base-emitter space-charge region

$$\gamma = \frac{I_{nE}}{I_{nE} + I_{pE} + I_{scr}} = \frac{\gamma_0}{1 + \frac{I_{scr}}{I_{nE} + I_{pE}}} \quad \text{where } \gamma_0 = \frac{I_{nE}}{I_{nE} + I_{pE}}$$

γ_0 is the value of γ when $I_{scr} = 0$

$$\frac{I_{scr}}{I_{nE} + I_{pE}} = 0.1 \quad \text{and from Prob. 6.17} \quad \gamma_0 = 0.99722 \Rightarrow \gamma = 0.90656$$

$$\text{From Prob. 6.17 } \alpha_T = 0.99993 \quad \beta_F = \frac{\gamma \alpha_T}{1 - \gamma \alpha_T} = 9.695$$

(b) If life-time in the space-charge zone varies in the same way as the base lifetime given in Prob. 6.17 we can write (See Eq.(5.3.24))

$$\frac{I_{scr}}{I_{nE} + I_{pE}} = \frac{8 X_i N_i e^{\frac{qV_{BE}}{2kT}}}{2 \tau_0 e^{-t/\tau_0} (I_{nE} + I_{pE})} = 0.1 e^{t/\tau_0} \quad \therefore \gamma = \frac{\gamma_0}{1 + 0.1 e^{t/\tau_0}}$$

Using the result of Prob. 6.18

$$\beta_F = \frac{1 - \frac{X_B^2 e^{t/\tau_0}}{2 D_{nB} \tau_{no}}}{\frac{1}{\gamma_0} - 1 + \left(\frac{0.1}{\gamma_0} + \frac{X_B^2}{2 D_{nB} \tau_{no}} \right) e^{t/\tau_0}}$$

From Prob. 6.18 : $\gamma_0 = 0.99722$, $X_B = 5 \times 10^{-5} \text{ cm}$, $D_{nB} = 18 \text{ cm}^2/\text{sec}$, $\tau_{no} = 10^{-6} \text{ sec}$, $\tau_0 = 10 \text{ days}$

$$\therefore \beta_F = \frac{1 - (6.944 \times 10^{-5}) e^{t/10}}{(2.777 \times 10^{-3}) + (0.1003) e^{t/10}}$$

t (days)	β_F
0	9.7
5	5.94
10	3.630
20	1.344

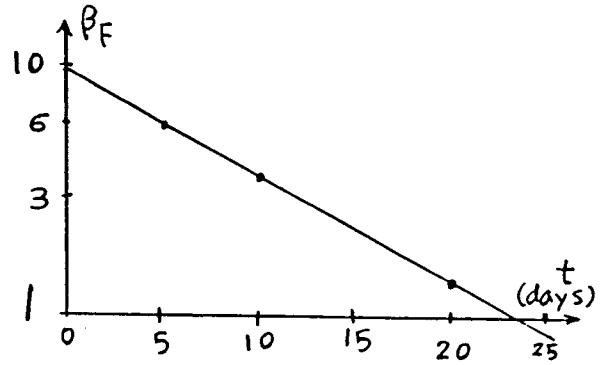
From Prob 6.18

$$\beta_F = 1 \Rightarrow 1 - \frac{1}{2\tau_0} = \frac{x_B^2 e^{t_1/t_0}}{2D_{nB} T_{n0}}$$

$$1 - \frac{1 + 0.1 e^{t_1/t_0}}{2\tau_0} = \frac{x_B^2 e^{t_1/t_0}}{2D_{nB} T_{n0}}, \quad 1 - \frac{1}{2\tau_0} = \left(\frac{0.1}{2\tau_0} + \frac{x_B^2}{2D_{nB} T_{n0}} \right) e^{t_1/t_0}$$

$$t_1 = t_0 \ln \left[\frac{1 - \frac{1}{2\tau_0}}{\frac{0.1}{2\tau_0} + \frac{x_B^2}{2D_{nB} T_{n0}}} \right] = 22.956 \text{ days}$$

$$\text{at } t = 22.956 \text{ days} \quad T_n = 0.1 \mu\text{sec}$$



7.9

$$\text{From eq. (7.1.1)} \quad I_C = \frac{8D_n n_i^2 A_E e^{qV_{BE}/kT}}{\int_0^{x_B} p dx}$$

Now $p = N_a + n'(x) \approx N_a + n(x)$ in high-level injection. Since $n(0)$ dominates the integral over p , we make the approximation

$$\int_0^{x_B} p dx = \int_0^{x_B} (N_a + n'(x)) dx \approx \int_0^{x_B} (N_a + n(x)) dx,$$

$$\text{where } n(x) \approx n(0) \left[1 - \frac{x}{x_B} \right] \quad \{ \text{linear} \}$$

$$\text{Then: } \int_0^{x_B} N_a dx = N_a x_B + \frac{1}{2} n(0) [x_B] = N_a x_B \left[1 + \frac{n(0)}{2N_a} \right]$$

$$\text{Defining } I_F = \frac{8D_n n_i^2 A_E e^{qV_{BE}/kT}}{N_a x_B} \text{ then } I_C = \frac{I_F}{1 + \frac{n(0)}{2N_a}}$$

Eg. (7.2.3) for $n(0)$ can be written

$$n(0) = \frac{N_a}{2} \left[\left(1 + \frac{4I_F}{I_k} \right)^{1/2} - 1 \right] \text{ where } I_k = \frac{8D_n A_E N_a}{x_B}$$

$$\therefore I_C = \frac{I_F}{\left[1 + \frac{1}{4} \left(1 + \frac{4I_F}{I_k} \right)^{1/2} - 1 \right]} \quad \text{Since } I_B \text{ is a result of injected holes into the emitter.}$$

$$\therefore I_B = \frac{I_F}{\beta_{F0}} \quad \therefore \beta_F = \frac{I_C}{I_B} = \frac{\beta_0}{\left[1 + \frac{1}{4}\left(\left(1 + \frac{4I_F}{I_K}\right)^{1/2} - 1\right)\right]} \quad \left[\frac{\beta_F}{\beta_0} = \frac{I_C}{I_F}\right]$$

For the constants given: $I_K = 16 \text{ mA}$

$$I_F \ll I_K \Rightarrow \beta_F \rightarrow \beta_0 \quad ; \quad I_F \gg I_K \Rightarrow \beta_F \rightarrow 2\beta_0 \sqrt{\frac{I_K}{I_F}}$$

At $I_F = I_K$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{\left\{1 + \frac{1}{4}\left(\left(1 + 4\right)^{1/2} - 1\right)\right\}} = \frac{1}{1.31} = 0.76$$

$$\text{and } \frac{I_C}{I_F} = \frac{\beta_F}{\beta_0} \Rightarrow I_C = 0.76 I_K$$

At $I_F = 2I_K$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{\left(1 + \frac{1}{4}(2)\right)} = \frac{1}{1.5} = 0.67$$

$$I_C = 2 \times I_K \times 0.67 = 1.34 I_K$$

At $I_F = 10I_K$,

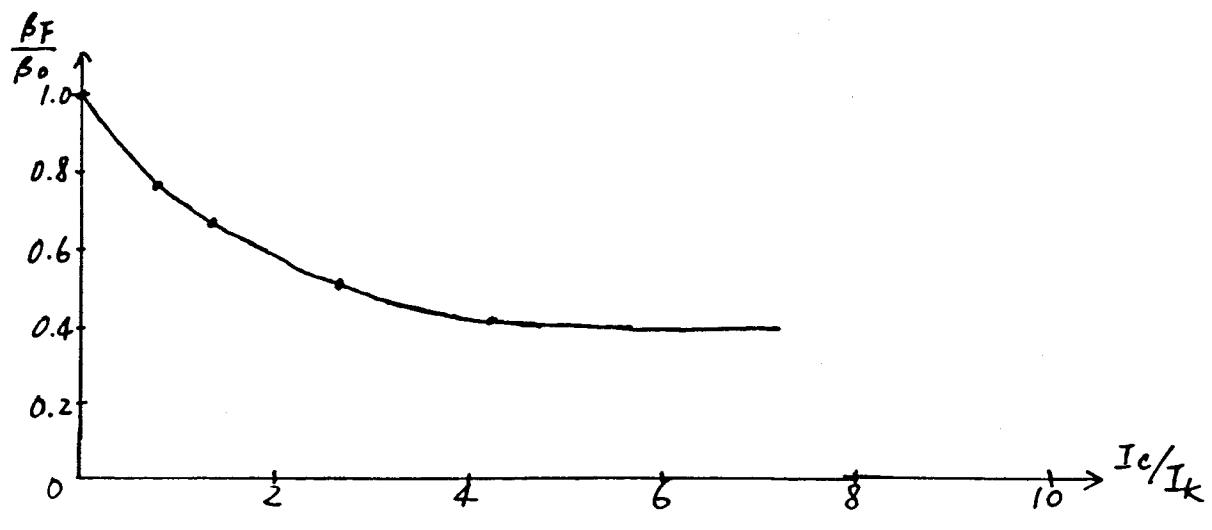
$$\frac{\beta_F}{\beta_0} = \frac{1}{\left(1 + \frac{1}{4}(\sqrt{41} - 1)\right)} = 0.425$$

$$I_C = 10 \times I_K \times 0.425 = 4.25 I_K$$

At $I_F = 5I_K$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{\left(1 + \frac{1}{4}(\sqrt{21} - 1)\right)} = 0.527$$

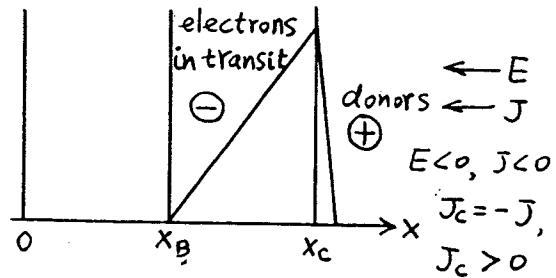
$$I_C = 5 \times I_K \times 0.527 = 2.63 I_K$$



7.10

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_s} \quad J_c = g n v \Rightarrow n = \frac{J_c}{g v}$$

$$\rho = -g n = -\frac{J_c}{v}$$

Case I

$$V = -\mu_n E$$

$$\frac{dE}{dx} = -\frac{J_c}{\epsilon_s V} = \frac{J_c}{\epsilon_s \mu_n E}$$

$$\int_{x_B}^x E dx = \frac{J_c}{\epsilon_s \mu_n} \int_{x_B}^x dx$$

$$\frac{E^2(x)}{2} - \frac{E^2(x_B)}{2} = \frac{J_c}{\epsilon_s \mu_n} (x - x_B)$$

$$E(x_B) = 0$$

$$E(x) = -\sqrt{\frac{2 J_c}{\epsilon_s \mu_n} (x - x_B)}$$

$$V_{CB} + \phi_i = - \int_{x_B}^{x_C} E(x) dx$$

$$V_{CB} + \phi_i = \left(\frac{2 J_c}{\epsilon_s \mu_n}\right)^{1/2} \int_{x_B}^{x_C} (x - x_B)^{1/2} dx$$

$$V_{CB} + \phi_i = \left(\frac{2 J_c}{\epsilon_s \mu_n}\right)^{1/2} \frac{2}{3} x_{CB}^{3/2}$$

$$x_{CB} = \left(\frac{9 \epsilon_s \mu_n}{8 J_c}\right)^{1/3} (V_{CB} + \phi_i)^{2/3}$$

Case II

$$V = V_L$$

$$\frac{dE}{dx} = -\frac{J_c}{\epsilon_s V} = -\frac{J_c}{\epsilon_s V_L}$$

$$\int_{x_B}^x dE = -\frac{J_c}{\epsilon_s V_L} \int_{x_B}^x dx$$

$$E(x) - E(x_B) = -\frac{J_c}{\epsilon_s V_L} (x - x_B)$$

$$E(x_B) = 0$$

$$E(x) = -\frac{J_c}{\epsilon_s V_L} (x - x_B)$$

$$V_{CB} + \phi_i = \frac{J_c}{\epsilon_s V_L} \int_{x_B}^{x_C} (x - x_B) dx$$

$$V_{CB} + \phi_i = \frac{J_c}{\epsilon_s V_L} x_{CB}^2$$

$$x_{CB} = \sqrt{\frac{2 \epsilon_s V_L}{J_c} (V_{CB} + \phi_i)}$$

7.11

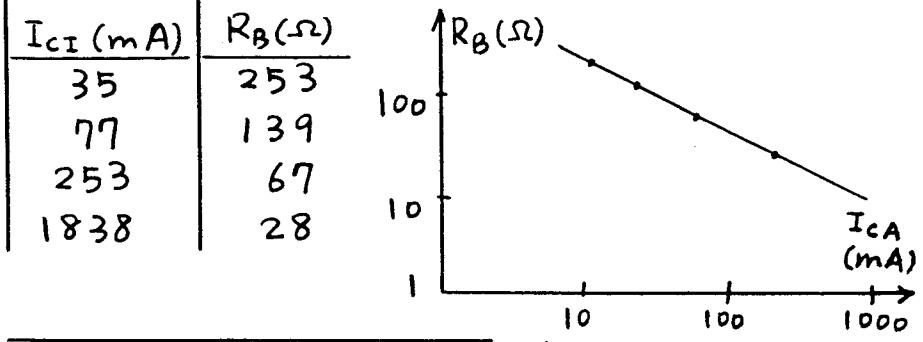
(a) $I_{CA} = I_s \exp\left(\frac{V_{BE} - \frac{I_{CA}}{\beta_F} R_B}{V_t}\right)$ Eg. (7.2.13) — — — — — (1)

$$I_{C_I} = I_S e^{\frac{V_{BE}}{V_t}} \quad \dots \quad (2)$$

$$(2)/(1) \quad \frac{I_{C_I}}{I_{CA}} = e^{\frac{I_{CA} R_B}{\beta_F V_t}} \quad \text{or} \quad R_B = \frac{V_t \beta_F}{I_{CA}} \ln \left(\frac{I_{C_I}}{I_{CA}} \right)$$

(b) With $I_S = 3 \times 10^{-14} \text{ A}$, $V_t = 0.0252 \text{ Volts}$, $\beta_F = 100.0$

$V_{BE} (\text{V})$	$I_{CA} (\text{mA})$	$I_{C_I} (\text{mA})$	$R_B (\Omega)$
0.70	11.25	35	253
0.72	22.4	77	139
0.75	56.2	253	67
0.80	200	1838	28



7.12

$$(a) \frac{R}{8} = 18.75 \Omega, \frac{R}{16} = 9.375 \Omega$$

$$\& (b) \beta_F = 100, I_S = 3 \times 10^{-14} \text{ A}$$

$$V_t = 0.0258 \text{ V}$$

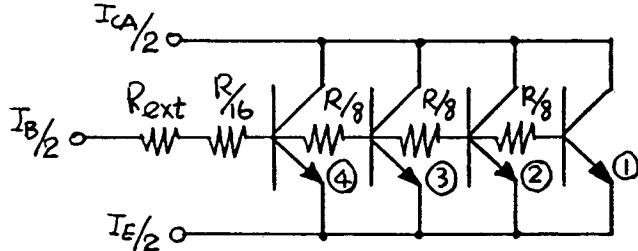
$$I_{C_I} = \frac{I_S}{8} e^{\frac{V_{BE_I}}{V_t}} \quad I_{B_I} = \frac{I_{C_I}}{\beta_F}$$

For $i = 2, 3, 4$

$$I_{C_i} = \frac{I_S}{8} e^{\frac{V_{BE_i}}{V_t}}, \quad V_{BE_i} = V_{BE(i-1)} + I_{B(i-1)} \frac{R}{8}, \quad I_{B_i} = I_{B(i-1)} + \frac{I_{C_i}}{\beta_F}$$

$$V_{BE|total} = V_{BE_4} + I_{B_4} (R_{ext} + \frac{R}{16}), \quad I_{C_I} = I_S e^{\frac{V_{BE|total}}{V_t}}$$

$$I_{CA} = 2(I_{C_1} + I_{C_2} + I_{C_3} + I_{C_4}), \quad R_B = \frac{V_t \beta_F}{I_{CA}} \ln \frac{I_{C_I}}{I_{CA}}$$



$I_{C_I} (\text{A})$	$I_{CA} (\text{A})$	$V_{BE total} (\text{V})$	$I_{C_I} (\text{A})$	$R_B (\Omega)$
10^{-7}	8.0000145×10^{-7}	0.441152	8.0000713×10^{-7}	22.89
10^{-6}	8.000145×10^{-6}	0.500561	8.000713×10^{-6}	22.89
10^{-5}	8.00145×10^{-5}	0.559988	8.00713×10^{-5}	22.889
10^{-4}	8.0145×10^{-4}	0.6196	8.0717×10^{-4}	22.877
10^{-3}	8.148×10^{-3}	0.6811	8.7556×10^{-3}	22.755
10^{-2}	9.836×10^{-2}	0.7646	2.23×10^{-1}	21.45

(c) Assume that the collector current flows entirely in the outer transistor

$$\frac{I_{CA}}{2} = \frac{I_S}{8} e^{\frac{V_{BE_4}}{V_t}} \quad \text{then} \quad I_S e^{\frac{V_{BE_4}}{V_t}} = 4 I_{CA}$$

$$V_{BE1\text{total}} = \frac{I_B}{2} \left(R_{ext.} + \frac{R}{16} \right) + V_{BE1}$$

$$\therefore I_{C1} = I_s e^{\frac{V_{BE1\text{total}}}{V_t}} = I_s e^{\frac{V_{BE1}}{V_t}} \cdot e^{\frac{I_B}{2V_t} \left(R_{ext.} + \frac{R}{16} \right)} = 4 I_{CA} e^{\frac{I_B}{2V_t} \left(R_{ext.} + \frac{R}{16} \right)}$$

$$\ln \frac{I_{C1}}{I_{CA}} = \ln 4 + \frac{I_B}{2V_t} \left(R_{ext.} + \frac{R}{16} \right)$$

$$R_B = \frac{\beta_F V_t}{I_{CA}} \ln \frac{I_{C1}}{I_{CA}} = \frac{V_t}{I_B} \left[\ln 4 + \frac{I_B}{2V_t} \left(R_{ext.} + \frac{R}{16} \right) \right] = \frac{V_t}{I_B} \ln 4 + \frac{R_{ext.}}{2} + \frac{R}{32}$$

$$\text{For large } I_B, R_B \approx \frac{R_{ext.}}{2} + \frac{R}{32} = 14.6875 \Omega$$

$$(d) \text{ For } ① \quad V_{BE1} = V_{BE1}, \quad I_{C1} = \frac{I_s}{8} e^{\frac{V_{BE1}}{V_t}}, \quad I_{B1} = \frac{I_{C1}}{\beta_F}$$

$$\text{For } ② \quad V_{BE2} = V_{BE1} + \frac{I_{B1}R}{8}, \quad I_{C2} = \frac{I_s}{8} e^{\frac{V_{BE2}}{V_t}} = \frac{I_s}{8} e^{\frac{V_{BE1}}{V_t}} \cdot e^{\frac{I_{B1}R}{8V_t}} = I_{C1} e^{\frac{I_{B1}R}{8V_t}}$$

$$\text{Since we are assuming low currents, } \frac{I_{B1}R}{8} \ll V_t$$

Expanding the exponential gives

$$I_{C2} = I_{C1} \left(1 + \frac{I_{B1}R}{8V_t} \right), \quad I_{B2} = I_{B1} + \frac{I_{C2}}{\beta_F} = I_{B1} \left(2 + \frac{I_{B1}R}{8V_t} \right) \approx 2 I_{B1}$$

(since we keep only terms of order I_{B1})

$$\text{For } ③ \quad V_{BE3} = V_{BE2} + \frac{2I_{B1}R}{8} = V_{BE1} + \frac{3I_{B1}R}{8}, \quad I_{C3} = I_{C1} \left(1 + \frac{3I_{B1}R}{8V_t} \right)$$

$$I_{B3} \approx 3 I_{B1}$$

$$\text{For } ④ \quad V_{BE4} = V_{BE3} + \frac{3I_{B1}R}{8} = V_{BE1} + \frac{6I_{B1}R}{8}$$

$$I_{C4} = I_{C1} \left(1 + \frac{6I_{B1}R}{8V_t} \right), \quad I_{B4} \approx 4 I_{B1}$$

$$\frac{I_{CA}}{2} = I_{C1} + I_{C2} + I_{C3} + I_{C4} = I_{C1} \left(4 + \frac{10}{8} \frac{I_{B1}R}{V_t} \right)$$

$$I_{CA} = 8 I_{C1} \left(1 + \frac{5}{16} \frac{I_{B1}R}{V_t} \right)$$

$$V_{BE1\text{total}} = V_{BE4} + 4 I_{B1} \left(R_{ext} + \frac{R}{16} \right)$$

$$= V_{BE1} + \frac{3I_{B1}R}{4} + \frac{I_{B1}R}{4} + 4 I_{B1} R_{ext} = V_{BE1} + I_{B1} (R + 4 R_{ext})$$

$$I_{C1} = I_s e^{\frac{V_{BE1\text{total}}}{V_t}} = 8 I_{C1} \left(1 + \frac{I_{B1}R}{V_t} + \frac{4I_{B1}R_{ext}}{V_t} \right)$$

$$\text{From Prob. 7.11} \quad R_B = \frac{V_T \beta_F}{I_{CA}} \ln \frac{I_{C1}}{I_{CA}} \approx \frac{V_T}{8 I_{B1}} \ln \frac{I_{C1}}{I_{CA}}$$

Using the above results

$$\begin{aligned} \ln \frac{I_{C1}}{I_{CA}} &\approx \ln 8 I_{C1} + \frac{I_{B1}R}{V_t} + \frac{4I_{B1}R_{ext}}{V_t} - \ln 8 I_{C1} - \frac{5}{16} \frac{I_{B1}R}{V_t} \\ &= \frac{I_{B1}}{V_t} \left[4 R_{ext} + \frac{11}{16} R \right] \quad \therefore R_B \approx \frac{R_{ext}}{2} + \frac{11}{128} R = 22.89 \Omega \end{aligned}$$

7.13

$$Q_B = 8A_E \int_0^{x_B} n'(x) dx, \text{ from eq. (6.1.16)} \quad n'(x) = n'(0)(1 - \frac{x}{x_B})$$

$$\therefore Q_B = 8A_E n'(0) \int_0^{x_B} (1 - \frac{x}{x_B}) dx = 8A_E n'(0) \left[x - \frac{x^2}{2x_B} \right] \Big|_0^{x_B} = \frac{8A_E n'(0) x_B}{2}$$

$$\text{Since } \frac{dn'}{dx} = -\frac{n'(0)}{x_B}, \quad I_C = -8A_E D_n \frac{dn'}{dx} = \frac{8A_E n'(0) D_n}{x_B}$$

$$\text{and } \tau_B = \frac{Q_B}{I_C} = \frac{x_B^2}{2D_n}$$

With $x_B = 1\mu m$, $D_n = 34.8 \text{ cm}^2/\text{sec}$ (intrinsic value)

$$\Rightarrow \tau_B = 144 \text{ psec}$$

$$\text{With } \tau_B |_{\text{drift}} = \frac{x_B^2}{\mu_n V}, \quad \tau_B |_{\text{drift}} < \tau_B |_{\text{diff.}} \text{ if } \frac{x_B^2}{\mu_n V} < \frac{x_B^2}{2D_n} \text{ or}$$

$$V > \frac{2D_n}{\mu_n} = \frac{2kT}{q} \quad V > 51.6 \text{ mV}$$

7.14

$$(a) \text{ From eq. (7.3.8)} \quad \tau_B = \frac{x_B^2}{D_n} \left[\int_0^1 \frac{1}{P(y)} \left(\int_y^1 P(\xi) d\xi \right) dy \right]$$

For exponential doping and low-level injection

$$P(x) = N_a(x) = N_{ao} e^{-x/L}. \quad \text{If we let } y = \frac{x}{x_B} \text{ and define } K = \frac{x_B}{L}$$

$$\text{then } P(y) = N_{ao} e^{-Ky}$$

$$\therefore \int_y^1 P(\xi) d\xi = N_{ao} \int_y^1 e^{-K\xi} d\xi = \frac{N_{ao}}{K} [e^{-Ky} - e^{-K}] \quad \text{and}$$

$$\int_0^1 \frac{1}{P(y)} \left(\int_y^1 P(\xi) d\xi \right) dy = \frac{1}{K} \int_0^1 \frac{e^{-Ky} - e^{-K}}{e^{-Ky}} dy = \frac{1}{K} \int_0^1 [1 - e^{K(y-1)}] dy$$

$$= \frac{1}{K} \left[1 - \frac{1}{K} + \frac{e^{-K}}{K} \right] = \frac{1}{K^2} [K - 1 + e^{-K}]$$

$$\therefore \tau_B = \frac{x_B^2}{D_n K^2} [K - 1 + e^{-K}]$$

If we write

$$\tau_B = \frac{x_B^2}{V D_n} \quad \text{then } V = \frac{K^2}{[K - 1 + e^{-K}]},$$

$K \rightarrow 0 \Rightarrow L \rightarrow \infty$ so that the base doping becomes uniform.

$$e^{-K} \approx 1 - K + \frac{K^2}{2} \Rightarrow K - 1 + e^{-K} \approx \frac{K^2}{2} \text{ and } V = 2$$

$$\therefore \tau_B = \frac{x_B^2}{2D_n} \text{ as expected}$$

$$(b) \text{ With } K=20 \quad V = \frac{400}{19+e^{-20}} = 21.05$$

We have $\frac{N_A(0)}{N_A(x_B)} = e^K$, since $\phi_F = \frac{kT}{q} \ln \frac{N_A}{n_i}$ \therefore

$$\phi_F(0) - \phi_F(x_B) = \frac{kT}{q} \ln \frac{N_A(0)}{N_A(x_B)} = \frac{kT}{q} K = 0.516 \text{ eV if } K=20$$

Since this value is close to $Eg/2$, $K=20$ is about as large as can be realized practically.

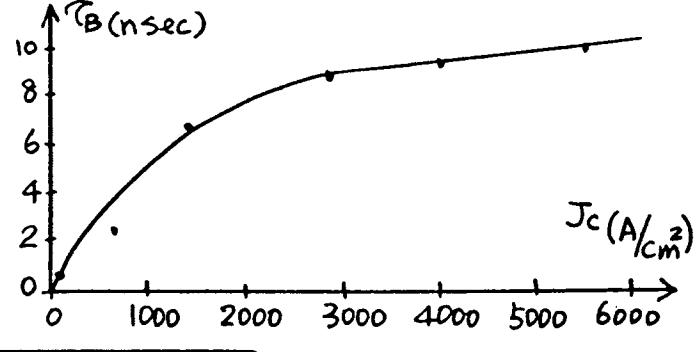
7.15

$$\text{From Eq. (7.3.8)} \quad \tau_B = \frac{x_B^2}{V D_{nB}}$$

For $N_B \sim 10^{17}$, $D_{nB} \sim 18 \text{ cm}^2/\text{sec}$; the E-B junction is at $x = 2.5 \mu\text{m}$

For high-level injection, $V=4$; the C-B junction is at $x = 4.0 \mu\text{m}$

$J_c (\text{A/cm}^2)$	$x_B (\mu\text{m})$	$\tau_B (\text{nsec})$
99.2	1.5	0.3125
695	4	2.22
1380	7	6.8
2770	8	8.89
3930	8.25	9.45
5560	8.5	10



7.16

We assume an npn transistor under forward-active bias.

Since eq. (6.2.20) is valid the base-emitter junction can be treated as a short-base diode. For the prototype transistor

$$n_p'(x) = n_p'(0) \left(1 - \frac{x}{x_B}\right), \text{ so that } Q_B = 8A_E \int_0^{x_B} n_p'(x) dx = \frac{8A_E n_p'(0) x_B}{2}$$

$$\text{Using eq. (5.3.7)} \quad n_p'(0) = \frac{n_i^2}{N_{AB}} \left(e^{\frac{8V_{BE}}{kT}} - 1\right)$$

$$\text{Therefore } Q_B = \frac{8A_E n_i^2 x_B}{2 N_{AB}} \left(e^{\frac{8V_{BE}}{kT}} - 1\right)$$

$$\text{Similarly } Q_E = \frac{8A_E n_i^2 x_E}{2 N_{AE}} \left(e^{\frac{8V_{BE}}{kT}} - 1\right) \quad \therefore \frac{Q_B}{Q_E} = \frac{N_{AB} x_E}{N_{AE} x_B}$$

The base current consists of holes recombining in the base and holes injected into the emitter.

$$I_B = I_{rB} - I_{pE}. \text{ From eq. (6.2.6)} \quad I_{rB} = \frac{g_B}{\tau_n}$$

and from eq. (6.2.10) $I_{PE} = -\frac{2D_{PE}\delta_E}{x_E^2} = -\frac{2D_{PE}N_{AB}\delta_B}{N_{DE}x_Ex_B}$
 $\therefore I_B = \delta_B \left[\frac{1}{\tau_n} + \frac{2D_{PE}N_{AB}}{N_{DE}x_Ex_B} \right]$. But $I_B = \frac{\delta_F}{\tau_{BF}} = \frac{\delta_B + \delta_E}{\tau_{BF}} = \frac{\delta_B}{\tau_{BF}} \left(1 + \frac{N_{AB}x_E}{N_{DE}x_B} \right)$

Comparing the two equations for I_B , we find

$$\tau_{BF} = \frac{\left[1 + \frac{N_{AB}x_E}{N_{DE}x_B} \right]}{\left[\frac{1}{\tau_n} + \frac{2D_{PE}N_{AB}}{N_{DE}x_Ex_B} \right]}$$

If $N_{DE} \gg N_{AB}$, i.e. if $\gamma \rightarrow 1$
then $\tau_{BF} \rightarrow \tau_n$

7.17

For both npn and pnp transistors we use the conditions :

- (1) Current is positive into the device
- (2) a component of charge (such as Q_F) is positive if the junction controlling the charge is forward-biased.

In going from an npn transistor to a pnp transistor, the roles of electrons and holes are reversed. For an equivalent bias the charge components in the pnp transistor have the same sign, but the terminal currents have the opposite sign.

Hence eq. (7.4.11) can be used for a pnp transistor if $i_C \rightarrow -i_C$, $i_B \rightarrow -i_B$ and $i_E \rightarrow -i_E$. Using these substitutions leads to the equations given in the problem statement.

7.18

We use the conventions for current and charge given in Prob. 7.17. The charge Q_{VE} is positive when the E-B junction is forward-biased. Thus $dQ_{VE}/dt > 0$ if the forward bias of the E-B junction is increased. For an npn transistor this requires a flow of holes into the base to reduce the space-charge layer width. Since this represents a positive base current and $dQ_{VE}/dt > 0$, the sign of dQ_{VE}/dt should be positive in the equation for i_B . For a pnp transistor dQ_{VE}/dt is still positive, but now electrons flow into the base instead of holes. Since this represents a

negative base current, the sign of dQ_{VE}/dt should be negative in the equation for i_B . The current flowing to the emitter side of the space-charge region is the opposite of that flowing to the base side. Hence in the equation for i_E , the sign of dQ_{VE}/dt is negative for an n-p-n transistor and positive for a p-n-p transistor. Similar considerations apply to the C-B junction and Q_{VC} .

7.19

We assume that $dQ_{VC}/dt \ll Q_F/\tau_F$ so that the current dQ_{VC}/dt can be neglected in the equation for i_C . We neglect the current dQ_{VE}/dt in the equation for i_B . e.g. (7.4.6) thus give

$$i_C = Q_F/\tau_F, \quad i_B = \frac{Q_F}{\tau_{BF}} + \frac{dQ_F}{dt} + \frac{dQ_{VC}}{dt}.$$

$$V_{CC} = i_C R_L + V_{CB} + V_{BE}$$

Since V_{BE} is approximately constant differentiation with respect to time gives

$$\frac{dV_{CB}}{dt} = -R_L \frac{di_C}{dt}$$

Using the equation for i_C gives $\frac{dV_{CB}}{dt} = -\frac{R_L}{\tau_F} \frac{dQ_F}{dt}$

If we define $C_{JC} = -\frac{dQ_{VC}}{dV_{CB}}$, we can write

$$\frac{dQ_{VC}}{dt} = \frac{dQ_{VC}}{dV_{CB}} \frac{dV_{CB}}{dt} = \frac{C_{JC} R_L}{\tau_F} \frac{dQ_F}{dt}$$

$$\therefore i_B = \frac{Q_F}{\tau_{BF}} + \frac{dQ_F}{dt} \left(1 + \frac{C_{JC} R_L}{\tau_F}\right) \text{ or } \tau_{BF} i_B = Q_F + \frac{dQ_F}{dt} \tau_{BF} \left(1 + \frac{C_{JC} R_L}{\tau_F}\right)$$

This equation has a solution of the form

$$Q_F = A + B t^{-\frac{1}{\tau_{BF}'}} \quad \text{where} \quad \tau_{BF}' = \tau_{BF} \left(1 + \frac{C_{JC} R_L}{\tau_F}\right)$$

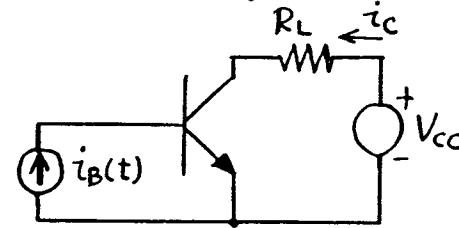
7.20

Under DC (i.e. static) Conditions, all time derivatives are zero.

e.g. (7.4.11) reduces to :

$$I_E = -Q_F \left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) + \frac{Q_R}{\tau_R}, \quad I_C = \frac{Q_F}{\tau_F} - Q_R \left(\frac{1}{\tau_R} + \frac{1}{\tau_{BF}}\right), \quad I_B = \frac{Q_F}{\tau_{BF}} + \frac{Q_R}{\tau_{BR}}$$

We also have the relationships (7.4.2) and (7.4.12)



$$Q_F = Q_{F0} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right), \quad Q_R = Q_{R0} \left(e^{\frac{qV_{BC}}{kT}} - 1 \right)$$

Further if we utilize the relationships given in the problem, we find :

$$\alpha_F I_{ES} = Q_{F0} \left(\frac{\tau_F + \tau_{BF}}{\tau_F \tau_{BF}} \right) \left(\frac{\tau_{BF}}{\tau_F + \tau_{BF}} \right) = \frac{Q_{F0}}{\tau_F}$$

$$\alpha_R I_{CS} = Q_{R0} \left(\frac{\tau_R + \tau_{BR}}{\tau_R \tau_{BR}} \right) \left(\frac{\tau_{BR}}{\tau_R + \tau_{BR}} \right) = \frac{Q_{R0}}{\tau_R}$$

$$I_{ES}(1-\alpha_F) = Q_{F0} \left(\frac{\tau_F + \tau_{BF}}{\tau_F \tau_{BF}} \right) \left(\frac{\tau_F}{\tau_F + \tau_{BF}} \right) = \frac{Q_{F0}}{\tau_{BF}}$$

$$I_{CS}(1-\alpha_R) = Q_{R0} \left(\frac{\tau_R + \tau_{BR}}{\tau_R \tau_{BR}} \right) \left(\frac{\tau_R}{\tau_R + \tau_{BR}} \right) = \frac{Q_{R0}}{\tau_{BR}}$$

Finally, upon substitution we find that :

$$I_E = -I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) + \alpha_R I_{CS} \left(e^{\frac{qV_{BC}}{kT}} - 1 \right)$$

$$I_C = -I_{CS} \left(e^{\frac{qV_{BC}}{kT}} - 1 \right) + \alpha_F I_{ES} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right)$$

These are the Ebers-Moll relationships defined by eqs (6.4.7)

7.21

From eq. (7.4.15)

$$\tau_{slow} = \frac{\frac{1}{\tau_{fast}}}{\left(\frac{1}{\tau_F \tau_{BR}} + \frac{1}{\tau_R \tau_{BF}} + \frac{1}{\tau_{BF} \tau_{BR}} \right)} = \frac{\left(\frac{1}{\tau_F} + \frac{1}{\tau_R} + \frac{1}{\tau_{BR}} + \frac{1}{\tau_{BF}} \right)}{\left(\frac{1}{\tau_F \tau_{BR}} + \frac{1}{\tau_R \tau_{BF}} + \frac{1}{\tau_{BF} \tau_{BR}} \right)}$$

Multiplying top and bottom by $\tau_{BF} \tau_{BR}$ leads to

$$\tau_{slow} = \frac{\left(\frac{\tau_{BF}}{\tau_F} + 1 \right) \tau_{BR} + \left(\frac{\tau_{BR}}{\tau_R} + 1 \right) \tau_{BF}}{\left(\frac{\tau_{BF}}{\tau_F} + \frac{\tau_{BR}}{\tau_R} + 1 \right)}, \quad \text{Using } \beta_F = \tau_{BF}/\tau_F \text{ and } \beta_R = \tau_{BR}/\tau_R$$

$$\text{then } \tau_{slow} = \frac{(\beta_F + 1) \tau_{BR} + (\beta_R + 1) \tau_{BF}}{1 + \beta_F + \beta_R}$$

7.22

(a) From eq.(7.4.1) $Q_F = \tau_F I_C$, using eq.(7.1.1) and assuming

low-level injection $Q_F = \frac{C_F \cdot 8 \cdot D_n \cdot n_i^2 \cdot A_E \cdot e^{8V_{BE}/kT}}{\int_0^{x_B} N_a(x) dx}$

Normalizing $N_a(x)$ with respect to $N_a(0)$ and x with respect to x_B

then $Q_F = \frac{C_F \cdot 8 \cdot D_n \cdot n_i^2 \cdot A_E \cdot e^{8V_{BE}/kT}}{N_a(0) \cdot x_B \int_0^1 \frac{N_a(x)}{N_a(0)} dx'}$

Since $n_p(0) = n_i^2 / N_a(0)$ and for forward-active bias

$$Q_F = Q_{F0} e^{8V_{BE}/kT} \therefore Q_{F0} = \frac{C_F \cdot 8 \cdot D_n \cdot A_E \cdot n_p(0)}{x_B \int_0^1 \frac{N_a(x')}{N_a(0)} dx'}$$

For a uniform base $\int_0^1 \frac{N_a(x)}{N_a(0)} dx' = 1 \therefore Q_{F0} = \frac{C_F \cdot 8 \cdot D_n \cdot A_E \cdot n_p(0)}{x_B}$

(b) Using eqs (7.4.2) and (7.4.12)

$$Q_F = Q_{F0} (e^{8V_{BE}/kT} - 1), \quad Q_R = Q_{R0} (e^{8V_{BC}/kT} - 1)$$

$$\therefore V_{BE} = \frac{kT}{8} \ln \left[\frac{Q_F + Q_{F0}}{Q_{F0}} \right] \quad V_{BC} = \frac{kT}{8} \ln \left[\frac{Q_R + Q_{R0}}{Q_{R0}} \right]$$

$$V_{CE} = V_{BE} - V_{BC} \quad \therefore V_{CE} = \frac{kT}{8} \ln \left[\frac{Q_{R0}(Q_F + Q_{F0})}{Q_{F0}(Q_R + Q_{R0})} \right] = \frac{-kT}{8} \left[\frac{Q_{F0}(Q_R + Q_{R0})}{Q_{R0}(Q_F + Q_{F0})} \right]$$

7.23

(a) $V_{CB} = 0 \Rightarrow Q_R = 0$. In the steady-state eq. (7.4.11) gives

$$Q_F = i_C C_F. \text{ With } i_C = 2 \text{ mA, } C_F = 12 \text{ nsec} \Rightarrow Q_F = 2.4 \times 10^{-10} \text{ coul}$$

(b) In the steady-state eq. (7.4.11) gives

$$i_C = \frac{Q_F}{C_F} - \frac{Q_R}{C_R} \left(1 + \frac{C_R}{C_{BR}} \right) \Rightarrow Q_F = i_C C_F + Q_R \frac{C_F}{C_R} \left(1 + \frac{C_R}{C_{BR}} \right)$$

$$i_B = \frac{Q_F}{C_{BF}} + \frac{Q_R}{C_{BR}} \Rightarrow Q_F = C_{BF} i_B - Q_R \frac{C_{BF}}{C_{BR}}$$

Using $\beta_F = \frac{C_{BF}}{C_F}$ and $\beta_R = \frac{C_{BR}}{C_R}$ then

$$Q_F = i_C C_F + Q_R \frac{C_F}{C_R} \left(1 + \frac{1}{\beta_R} \right) \quad \text{and} \quad Q_F = C_F \beta_F i_B - \frac{Q_R \beta_F C_F}{\beta_R C_R}$$

$$\text{Eliminating } Q_F \text{ gives } Q_R = \frac{\beta_R C_R [\beta_F i_B - i_C]}{(1 + \beta_R + \beta_F)}$$

$$\text{With } i_B = 0.5 \text{ mA, } i_C = 2 \text{ mA, } \beta_F = 100, \beta_R = 10, C_R = 36 \text{ nsec, } C_F = 12 \text{ nsec}$$

$$\Rightarrow Q_R = 1.56 \times 10^{-10} \text{ coul.}$$

Solving for Q_F gives $Q_F = \frac{\beta_F \tau_F [i_C + (1 + \beta_R) i_B]}{[1 + \beta_R + \beta_F]} = 8.11 \times 10^{-11} \text{ Coul.}$

- (c) The total charge in case (b) is 9.88 times the total charge in case (a)

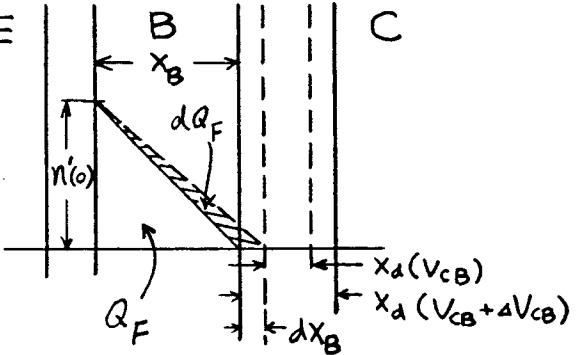
7.24

It is required to show that an increment dV_{CB} produces physical effects that have a polarity consistent with a capacitor between the collector and the base.

Under active bias Q_F and i_C are positive [Eq. (7.4.6)].

An increase in V_{CB} causes Q_F to be reduced as can be seen in the accompanying sketch for a prototype transistor.

The increment in Q_F , dQ_F will flow out of the collector since it consists of electrons being transported across the base to the collector. Since dQ_F consists of electrons, there will be a flow of positive current into the collector that is proportional to dV_{CB} . This behavior is consistent with the flow into a capacitor between the collector and the base, because a positive dV_{CB} causes a positive increment in current across the collector-base junction and a decrease in stored base charge.



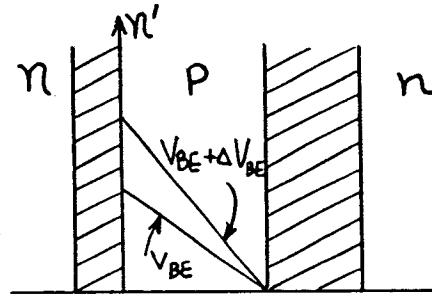
7.25

The resistors and capacitors in the equivalent circuit of Fig. 7.27 are insensitive to the bias direction, and are therefore equally valid for pnp and for npn transistors. The generator and current flow should, however, be investigated for the two cases.

For npn transistors, i_C is positive under active bias.

A positive V_{BE} increases Q_F (which consists of electrons in the base) and the collector current i_c .

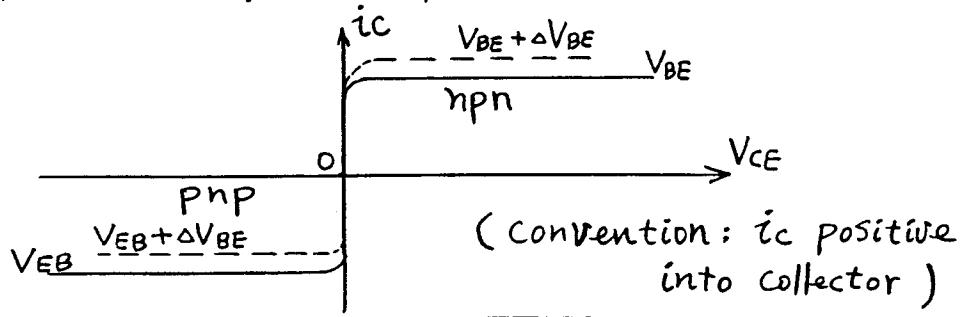
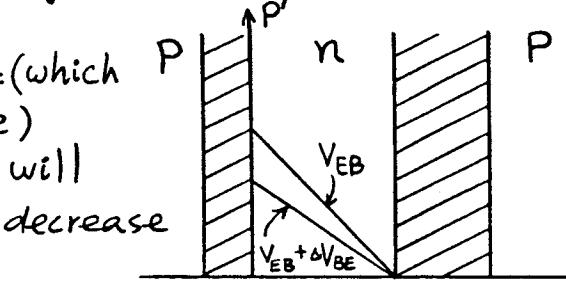
This is consistent with the equivalent circuit of Fig. 7.27.



For p-n-p transistors, i_c is negative under active bias.

A positive V_{BE} will decrease Q_F (which consists of holes in the base) but the collector current i_c will decrease in magnitude. A decrease in the magnitude of i_c is

equivalent to an increase in the current flowing into the collector. Hence, the circuit of Fig. 7.27. applies also for the p-n-p transistor.



7.26

From eq. (7.5.1) $I_c = I_s e^{\frac{V_{BE}}{V_t}}$

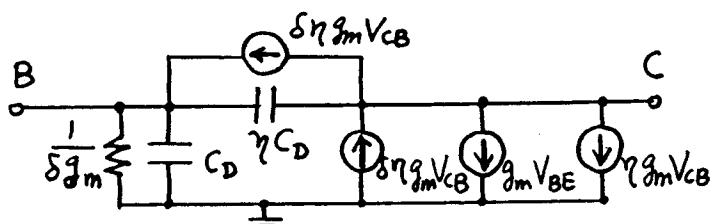
$\therefore I_c + i_c = I_s e^{(\frac{V_{BE} + \Delta V_{BE}}{V_t})} = I_c e^{\frac{V_{BE} + \Delta V_{BE}}{V_t}}$. If $V_{BE} < V_t$ then

$$I_c + i_c = I_c \left(1 + \frac{\Delta V_{BE}}{V_t}\right) \quad \therefore i_c = \frac{I_c}{V_t} \Delta V_{BE} = g_m V_{BE}$$

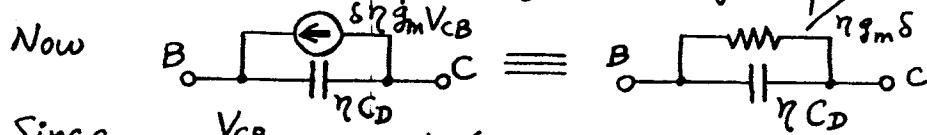
Thus the validity of the expression for g_m requires $V_{BE} < V_t$

7.27

The hybrid π circuit shown in Fig. 7.25 can be drawn :



By using generator splitting (See Fig. 7.26)



$$\text{Since } R = \frac{V_{CB}}{\delta \eta g_m V_{CB}} = \frac{1}{\eta g_m \delta}$$

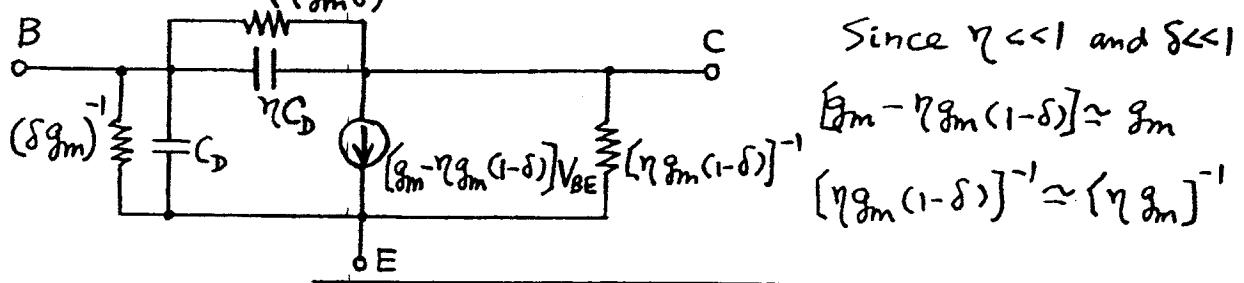
The three current sources between the collector and the emitter are to be combined into one current source and a resistor.

$$\therefore g_m V_{BE} + \eta g_m V_{CB} - \delta \eta g_m V_{CB} = g_m V_{BE} + \eta g_m (1-\delta) V_{CB} = i + \frac{V_{CE}}{R_i}$$

$$\text{Since } V_{CB} = V_{CE} - V_{BE} \quad \therefore$$

$$\begin{aligned} g_m V_{BE} + \eta g_m (1-\delta) (V_{CE} - V_{BE}) &= [g_m - \eta g_m (1-\delta)] V_{BE} + \eta g_m (1-\delta) V_{CE} \\ &= i + \frac{V_{CE}}{R_i} \end{aligned}$$

$$\therefore i = [g_m - \eta g_m (1-\delta)] V_{BE} \quad \text{and} \quad R_i = \frac{1}{\eta g_m (1-\delta)}$$



7.28

Neglecting C_{jc} Fig 7.30 reduces to

$$i_1 = g_m(1+\delta)U_{BE} + (C_{je} + g_m\tau_F) \frac{dU_{BE}}{dt} \quad \text{or}$$

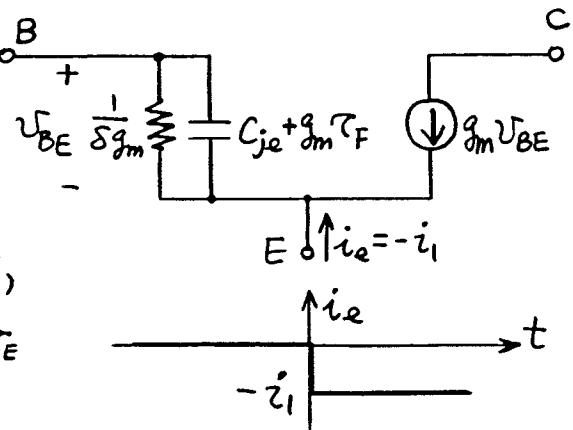
$$\frac{i_1}{g_m(1+\delta)} = U_{BE} + \frac{(C_{je} + g_m\tau_F)}{g_m(1+\delta)} \frac{dU_{BE}}{dt}$$

$$t=0, U_{BE}=U_{BEO}; t \rightarrow \infty, U_{BE} = \frac{i_1}{g_m(1+\delta)}$$

$$\therefore U_{BE} = \frac{i_1(1 - e^{-t/\tau_E})}{g_m(1+\delta)} + U_{BEO}e^{-t/\tau_E}$$

$$\text{where } \tau_E = \frac{\tau_F}{1+\delta} + \frac{C_{je}}{g_m(1+\delta)}$$

$$\text{Since } \delta \ll 1 \text{ and } g_m = \frac{8I_c}{KT} \quad \therefore \tau_E \approx \tau_F + \frac{C_{je}KT}{8I_c}$$

7.29

(a) The hybrid π model of Fig. 7.27 requires the specification of four parameters: g_m , δ , C_D and η .

For both transistors $I_c = 2 \text{ mA}$, $X_B = 0.3 \mu\text{m}$, and $\phi_i + V_{CB} = 10 \text{ Volts}$.

For the uniform-base transistor $N_A = 10^{17} \text{ cm}^{-3}$ ($\Rightarrow D_{nB} = 18 \text{ cm}^2/\text{sec}$) and $\tau_n = 10^{-7} \text{ sec}$. For both transistors $g_m = \frac{8I_c}{KT} = 0.0775 \text{ mhos}$

In prob. 6.4 we assumed $\gamma = 1$ for these transistors. Thus for the uniform-base structure: $\tau_{BF} = \tau_n = 10^{-7} \text{ sec}$ and $\tau_F = \frac{X_B^2}{2D_n} = 2.5 \times 10^{-11} \text{ sec}$
 $\therefore \delta_{\text{const.}} = \frac{1}{\beta_F \text{ const.}} = \frac{\tau_F}{\tau_{BF}} = 2.5 \times 10^{-4}$.

$$\text{From Prob. 6.4} \quad \frac{\beta_F \text{ const.}}{\beta_F \text{ exp.}} = \frac{\delta_{\text{exp.}}}{\delta_{\text{const.}}} = 0.831 \quad \therefore \delta_{\text{exp.}} = 2.0775 \times 10^{-4}$$

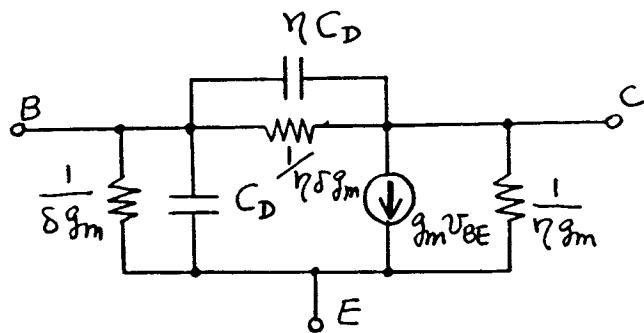
$$C_D = g_m \tau_F \quad \therefore C_D \text{ const.} = 1.9375 \text{ pF}$$

$$\text{Now} \quad \frac{C_D \text{ exp.}}{C_D \text{ const.}} = \frac{\tau_F \text{ exp.}}{\tau_F \text{ const.}}. \quad \text{Since } \gamma = 1, I_c = \frac{Q_F}{\tau_F} = \frac{Q_B}{\tau_F} \Rightarrow \frac{\tau_F \text{ exp.}}{\tau_F \text{ const.}} = \frac{Q_B \text{ exp.}}{Q_B \text{ const.}}$$

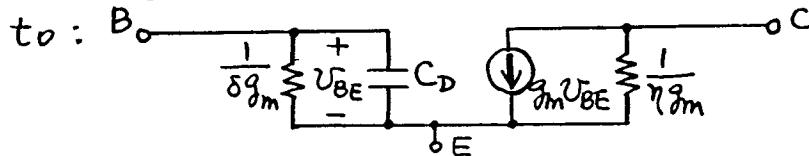
$$\text{From Prob. 6.4} \quad \frac{Q_B \text{ exp.}}{Q_B \text{ const.}} = 0.831 = \frac{C_D \text{ exp.}}{C_D \text{ const.}} \quad \therefore C_D \text{ exp.} = 1.61 \text{ pF}$$

$$\gamma = \frac{KT}{8|V_A|} \therefore \gamma_{\text{exp}} = 2.202 \times 10^{-4}, \quad \gamma_{\text{const.}} = 1.54 \times 10^{-4}$$

element	Constant	exponential
$\frac{1}{8g_m}$	$51.6 \text{ k}\Omega$	$62.1 \text{ k}\Omega$
C_D	1.9375 pF	1.61 pF
γC_D	$2.98 \times 10^{-4} \text{ pF}$	$3.54 \times 10^{-4} \text{ pF}$
$\frac{1}{\gamma g_m}$	$335.15 \text{ M}\Omega$	$282.06 \text{ M}\Omega$
g_m	0.0775 V	0.0775 V
$\frac{1}{\gamma g_m}$	83.787Ω	58.597Ω



(b) Since γC_D is small and $\frac{1}{\gamma g_m}$ is large the circuit reduces



The short-circuit current gain
can be calculated from the circuit:

$$I_i = V_{BE} \delta g_m + j\omega C_D V_{BE}$$

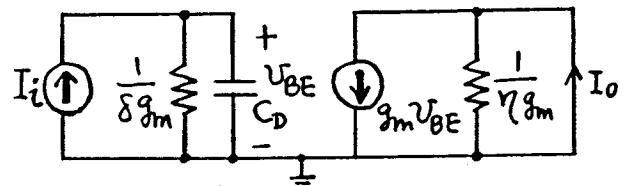
$$\Rightarrow V_{BE} = \frac{I_i}{\delta g_m + j\omega C_D}$$

$$I_o = g_m V_{BE} = \frac{g_m I_i}{\delta g_m + j\omega C_D} = \frac{I_i}{1 + j\omega \frac{C_D}{\delta g_m}}$$

$$\frac{I_o}{I_i} = \frac{1/8}{1 + j\omega \frac{C_D}{\delta g_m}} \quad \text{The low frequency gain is } \frac{1}{8}$$

$$\frac{1}{\delta_{\text{const.}}} = 4,000 \quad \frac{1}{\delta_{\text{exp}}} = 4,810 \quad \therefore \text{The exponentially doped base has the highest gain.}$$

The gain is down 0.707 of its low frequency value at

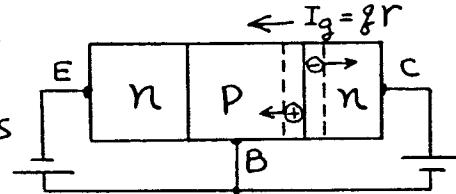


$$f_B = \frac{g_m \delta}{2\pi C_D}, \quad f_{B \text{ const.}} = 1.59 \text{ MHz} \quad f_{B \text{ exp.}} = 1.59 \text{ MHz}$$

The exponential base has the same frequency response as the uniform base. (In practical BJTs, $\delta \neq 1$ and lower β values are typical.)

7.30

(a) In the active mode the collector-base junction is reverse biased. Holes are transported to the base and electrons are transported to the collector.



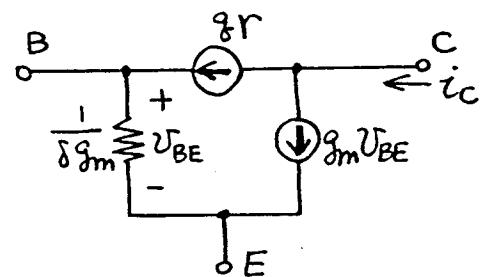
The generation current flows from the collector toward the base so that it represents a positive component of i_C .

$$(b) i_C = g_m V_{BE} + g_f r$$

$$(c) \text{ If } V_{BE} = 0 \Rightarrow i_C = g_f r$$

$$(d) \text{ If } i_B = 0 \Rightarrow V_{BE} = \frac{g_f r}{\delta g_m}$$

$$\therefore i_C = g_m V_{BE} + g_f r = g_f r \left(1 + \frac{1}{\delta}\right) = g_f r (1 + \beta_F)$$



7.31

$$\text{From Fig. P7.31} \quad I_E = I_{BB} - \left(1 + \frac{1}{\beta_F}\right) I_{AA} = I_{BB} - I_{AA} - \frac{I_{AA}}{\beta_F}$$

$$I_C = I_{AA} - \left(1 + \frac{1}{\beta_R}\right) I_{BB} = I_{AA} - I_{BB} - \frac{I_{BB}}{\beta_R}$$

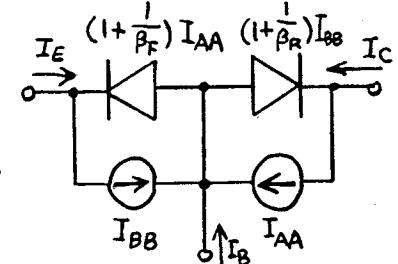
Using the definitions of I_{AA} and I_{BB} given in the problem yields $I_E = I_{ES}' (e^{\frac{V_{BE}}{V_t} - 1}) - I_{ES} (e^{\frac{V_{BE}}{V_t} - 1})$

$$I_C = I_{ES}' (e^{\frac{V_{BE}}{V_t} - 1}) - I_{CS}' (e^{\frac{V_{BC}}{V_t} - 1}) - \frac{I_{ES}'}{\beta_F} (e^{\frac{V_{BE}}{V_t} - 1}) - \frac{I_{CS}'}{\beta_F} (e^{\frac{V_{BC}}{V_t} - 1})$$

where $V_t = \frac{kT}{q}$ and the saturation currents I_{ES}' and I_{CS}' in equations for I_{AA} and I_{BB} are

different from I_{ES} and I_{CS} in eqs. (6.4.1), (6.4.4) and (6.4.7)

Comparing to the E-M model :



$$I_E = \alpha_R I_{CS} (e^{V_{BC}/V_t - 1}) - \alpha_F I_{ES} (e^{V_{BE}/V_t - 1}) - \frac{\alpha_F I_{ES}}{\beta_F} (e^{V_{BE}/V_t - 1})$$

$$I_C = \alpha_F I_{ES} (e^{V_{BE}/V_t - 1}) - \alpha_R I_{CS} (e^{V_{BC}/V_t - 1}) - \frac{\alpha_R I_{CS}}{\beta_R} (e^{V_{BC}/V_t - 1})$$

$\therefore I_{CS}' = \alpha_R I_{CS}$, $I_{ES}' = \alpha_F I_{ES}$.
The reciprocity relationship of E-M model is $\alpha_R I_{CS} = \alpha_F I_{ES}$
 $\therefore I_{CS}' = I_{ES}'$ in the "transport model".

7.32

Using eq. (6.4.1), (6.4.4) + (6.4.8) and the definition $V_t = K^T / g$, then

eq (6.4.7) can be written

$$I_E = I_S (e^{V_{BC}/V_t} - e^{V_{BE}/V_t}) - I_S \frac{(1-\alpha_F)}{\alpha_F} (e^{V_{BE}/V_t - 1})$$

$$I_C = -I_S (e^{V_{BC}/V_t} - e^{V_{BE}/V_t}) - I_S \frac{(1-\alpha_R)}{\alpha_R} (e^{V_{BC}/V_t - 1})$$

Using eq. (7.7.1) and $\beta_F = \frac{\alpha_F}{1-\alpha_F}$ and $\beta_R = \frac{\alpha_R}{1-\alpha_R}$ these become

$$I_E = I_n - \frac{I_S}{\beta_F} (e^{V_{BE}/V_t - 1}) \quad I_C = -I_n - \frac{I_S}{\beta_R} (e^{V_{BC}/V_t - 1})$$

Using $I_B = -(I_C + I_E)$ gives

$$I_B = \frac{I_S}{\beta_F} (e^{V_{BE}/V_t - 1}) + \frac{I_S}{\beta_R} (e^{V_{BC}/V_t - 1})$$

7.33

$$\text{Eq. (7.7.7) gives } Q_B = Q_{B0} + C_{je} V_{BE} + C_{jc} V_{BC} + \frac{Q_{B0}}{Q_B} \tau_F I_{SO} (e^{V_{BE}/V_t - 1})$$

$$\text{Using the definition } g_b = \frac{Q_B}{Q_{B0}} \text{ gives } + \frac{Q_{B0}}{Q_B} \tau_R I_{SO} (e^{V_{BC}/V_t - 1})$$

$$g_b = 1 + \frac{C_{je} V_{BE}}{Q_{B0}} + \frac{C_{jc} V_{BC}}{Q_{B0}} + \frac{\tau_F I_{SO}}{Q_{B0} g_b} (e^{V_{BE}/V_t - 1}) + \frac{\tau_R I_{SO}}{Q_{B0} g_b} (e^{V_{BC}/V_t - 1})$$

Using the definitions

$$I_{KR} = \frac{Q_{B0}}{\tau_R}, \quad I_{KF} = \frac{Q_{B0}}{\tau_F}, \quad |V_A| = \frac{Q_{B0}}{C_{je}} \frac{A_C}{A_E}, \quad |V_B| = \frac{Q_{B0}}{C_{jc}} \frac{A_E}{A_C}$$

$$g_b = 1 + \frac{A_E}{A_C} \frac{V_{BE}}{|V_B|} + \frac{A_C}{A_E} \frac{V_{BC}}{|V_A|} + \frac{1}{g_b} \left[\frac{I_{SO}}{I_{KF}} (e^{V_{BE}/V_t - 1}) + \frac{I_{SO}}{I_{KR}} (e^{V_{BC}/V_t - 1}) \right]$$

$$\text{If } g_1 = 1 + \frac{A_E}{A_C} \frac{V_{BE}}{|V_B|} + \frac{A_C}{A_E} \frac{V_{BC}}{|V_A|} \text{ and } g_2 = \frac{I_{SO}}{I_{KF}} (e^{V_{BE}/V_t - 1}) + \frac{I_{SO}}{I_{KR}} (e^{V_{BC}/V_t - 1})$$

$$\text{then } g_b = g_1 + \frac{g_2}{g_b}$$

7.34

Under active-bias and low-level conditions

$$I_C = I_{SO} e^{\frac{V_{BE}}{V_t}} \text{ and } I_B \approx I_1 e^{\frac{V_{BE}}{n_e V_t}}$$

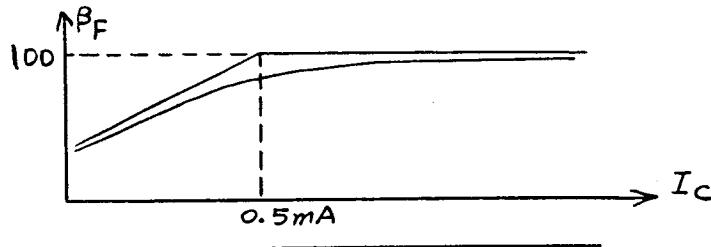
$$\beta_F = \frac{I_C}{I_B} = \frac{I_{SO}}{I_1} e^{\frac{V_{BE}(1 - \frac{1}{n_e})}{V_t}} = \frac{I_{SO}}{I_1} \left(e^{\frac{V_{BE}}{V_t}} \right)^{1 - \frac{1}{n_e}} = \frac{I_{SO}}{I_1} \frac{(I_C)^{1 - \frac{1}{n_e}}}{(I_{SO})^{1 - \frac{1}{n_e}}}$$

$$\therefore \beta_F = \frac{(I_{SO})^{1/n_e} (I_C)^{1 - \frac{1}{n_e}}}{I_1}$$

Since $n_e > 1$, β_F decreases as I_C decreases

$$\log \beta_F = \log \left[\frac{(I_{SO})^{1/n_e}}{I_1} \right] + (1 - \frac{1}{n_e}) \log I_C$$

The value of n_e can be determined from the slope of the low-level asymptote. The value of I_1 can be obtained from the intercept of the asymptote with the β_F axis.

7.35

(a) At intermediate levels and active bias $I_C = I_{SO} e^{\frac{V_{BE}}{V_t}}$

From eq. (7.7.15) the high-level asymptote is $I_C = \sqrt{I_{SO} I_{KF}} e^{\frac{V_{BE}}{2V_t}}$

Equating these expressions gives $e^{\frac{V_{BE}}{V_t}} = I_{KF}/I_{SO}$

at the intersection point. Therefore : $I_C/\text{intersection} = I_{KF}$

(b) In the high-current region $I_C = \sqrt{I_{SO} I_{KF}} e^{\frac{V_{BE}}{2V_t}}$

and $I_B = \frac{I_{SO}}{\beta_{FO}} e^{\frac{V_{BE}}{V_t}}$

$$\therefore \beta_F = \frac{I_C}{I_B} = \beta_{FO} \sqrt{\frac{I_{KF}}{I_{SO}}} e^{-\frac{V_{BE}}{2V_t}} = \frac{\beta_{FO} I_{KF}}{\sqrt{I_{SO} I_{KF}} e^{\frac{V_{BE}}{2V_t}}}$$

$$\therefore \beta_F = \frac{\beta_{FO} I_{KF}}{I_C}$$

7.36 For uniform doping (no field) $\tau_B = \frac{x_B^2}{2 D_{nB}}$ (7.6.2)

With a drift field $\tau_B = \frac{x_B}{\mu n e}$ (7.6.3)

Field for exponential doping $E = \frac{hT}{q \times_B} \ln \frac{N_B E}{N_B C}$ (7.6.4)

Field for heterojunction $E = \frac{\Delta E_g}{x_B}$

Let $x_B = 0.1 \mu m$, $N_B = 5 \times 10^{17}$ for uniform doping

$N_B = 10^{18} \rightarrow 10^{17} cm^{-3}$ for graded doping

$\Delta E_g = 0.2 eV$ for heterojunction

$$(1) \tau_B = \frac{(0.1 \times 10^{-4} cm)^2}{(2)(8.5 \text{ cm}^2/\text{s})} = 5.9 \text{ ps} \text{ for uniform doping}$$

$$(2) E = \frac{0.0259 V}{0.1 \times 10^{-4} cm} \ln \frac{10^{18}}{10^{17}} = 6 \times 10^3 V/cm$$

$$\tau_B = \frac{(0.1 \times 10^{-4} cm)}{(330 \text{ cm}^2/\text{V-s})(6 \times 10^3 V/cm)} = 5.1 \text{ ps (graded)}$$

$$(3) E = \frac{0.2 V}{0.1 \times 10^{-4} cm} = 2 \times 10^4 V/cm$$

$$\tau_B = \frac{(0.1 \times 10^{-4} cm)}{(330 \text{ cm}^2/\text{V-s})(2 \times 10^4 V/cm)} = 1.5 \text{ ps (heterojunction)}$$

CHAPTER 8

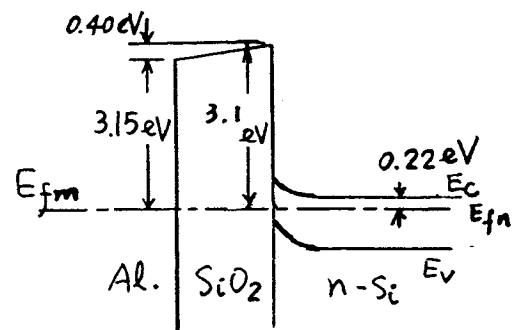
97

8.1

(a) $n = 5 \times 10^{15} \text{ cm}^{-3}$ (from Fig. 1.15)

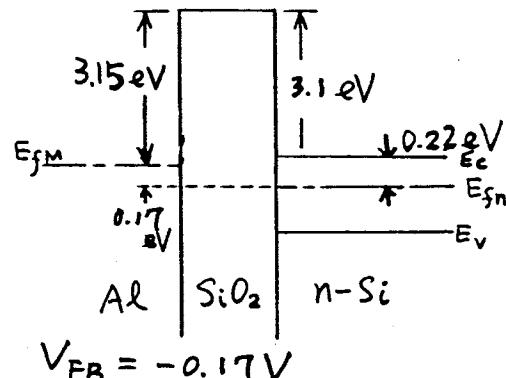
$$E_c - E_f = kT \ln \frac{N_c}{n} = 0.026 \ln \frac{2.8 \times 10^{19}}{5 \times 10^{15}} \approx 0.22 \text{ eV}$$

(i) Thermal Equi.



$$\delta\Phi_{MS} = 3.15 - 3.32 = -0.17 \text{ eV}$$

(ii) Flat band

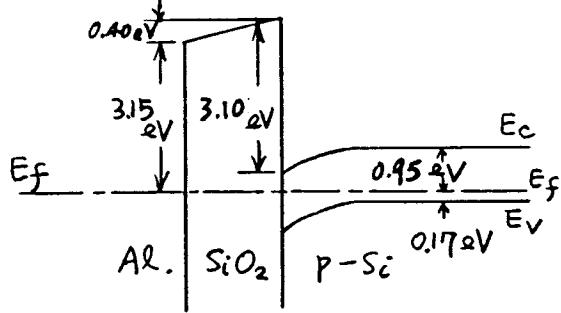


$$V_{FB} = -0.17 \text{ V}$$

(b) $p = 1.3 \times 10^{16} \text{ cm}^{-3}$ (from Fig. 1.15) , $E_f - E_v = kT \ln \frac{N_d}{p} = 0.026 \ln \frac{1.04 \times 10^{19}}{1.3 \times 10^{16}} \approx 0.17 \text{ eV}$

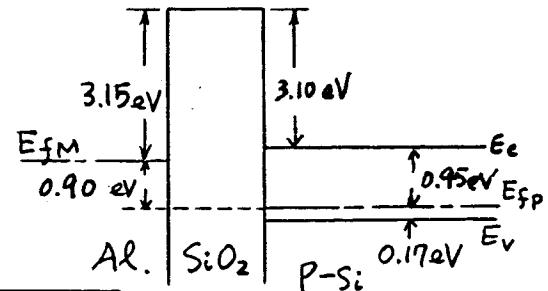
(i) Thermal Equi.

$$\delta\Phi_{MS} = 3.15 - 4.05 = -0.90 \text{ eV}$$



(ii) Flat band

$$V_{FB} = -0.90 \text{ V}$$



8.2

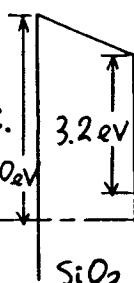
(a)

(i) Thermal Equi.

$$E_g = 4.30 \text{ eV}$$

Al contact

p⁺-Si



$$3.20 + 1.1 = 4.30$$

$$\Phi_{MS} = 0.88 \text{ V}$$

$$0.22 \text{ eV}$$

$$E_c$$

$$E_v$$

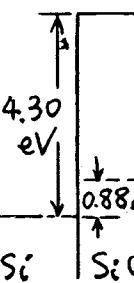
(ii)

Flat band

$$E_f^{\text{pt}}$$

Al contact

p⁺-Si



$$V_{FB} = 0.88 \text{ V}$$

$$0.22 \text{ eV}$$

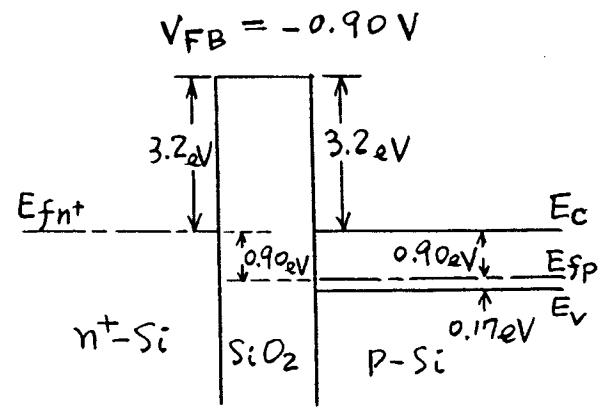
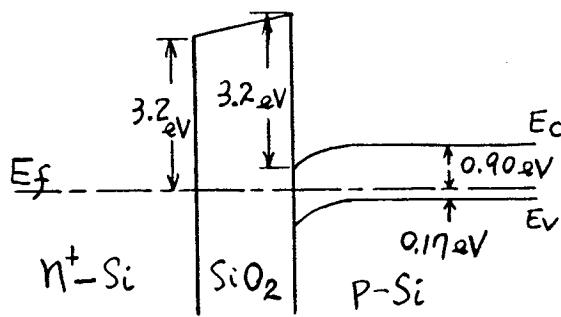
$$E_c$$

$$E_f^{\text{pt}}$$

$$E_v$$

(b) (i) Thermal Equi.

$$\Phi_{MS} = -0.90 \text{ V}$$

8.3

$$\Delta V_{ox} = \Delta \epsilon_{ox} x_{ox}, \Delta V_s = \Delta \epsilon_s x_d$$

$$\Delta \epsilon_s = \frac{\epsilon_{ox} \Delta \epsilon_{ox}}{\epsilon_s}, \therefore \Delta V_s = \frac{\epsilon_{ox} \Delta \epsilon_{ox}}{\epsilon_s} x_d$$

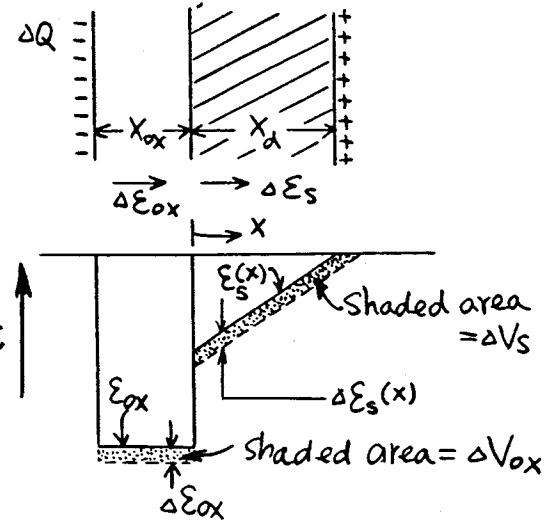
$$\Delta V = \Delta V_{ox} + \Delta V_s$$

$$= \Delta \epsilon_{ox} (x_{ox} + \frac{\epsilon_{ox}}{\epsilon_s} x_d)$$

$$\Delta Q = \epsilon_{ox} \Delta \epsilon_{ox}; \Delta V = \Delta Q \left(\frac{x_{ox}}{\epsilon_{ox}} + \frac{x_d}{\epsilon_s} \right)$$

$$\text{and } C = \frac{\Delta Q}{\Delta V} = \frac{1}{\frac{1}{C_{ox}} + \frac{1}{C_d}}$$

where $C_d = \frac{\epsilon_s}{x_d}$ is a capacitor made in Si with plate-spacing = x_d

8.4

$$V_{Gx} = V_G - V_{FB} = \frac{8N_a}{C_{ox}} x_d + \frac{8N_a}{2\epsilon_s} x_d^2$$

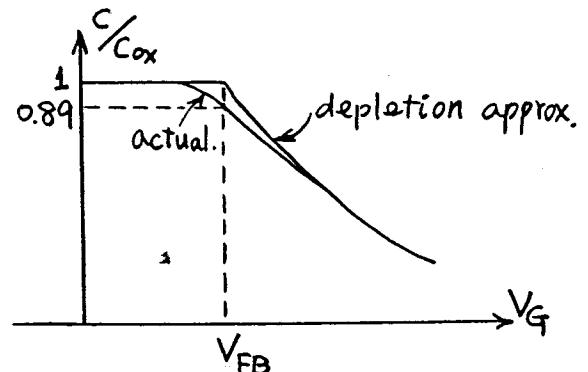
$$x_d^2 + \frac{2\epsilon_s}{C_{ox}} x_d - \frac{2\epsilon_s V_{Gx}}{8N_a} = 0$$

$$\therefore x_d = -\frac{\epsilon_s}{C_{ox}} + \sqrt{\left(\frac{\epsilon_s}{C_{ox}}\right)^2 + \frac{2\epsilon_s V_{Gx}}{8N_a}}$$

$$\frac{C}{C_{ox}} = \frac{1}{1 + \frac{C_{ox}}{\epsilon_s} x_d} = \frac{1}{1 + \frac{2C_{ox}^2 V_{Gx}}{\epsilon_s 8 N_a}}$$

For $\rho = 1 \Omega \cdot \text{cm}$ p-type Si. $N_a = 1.3 \times 10^{16} \text{ cm}^{-3}$, $x_{ox} = 1000 \text{ \AA} = 10^{-5} \text{ cm}$

$$\frac{C_{ox} L_D}{\epsilon_s} = \frac{\epsilon_{ox}}{x_{ox}} \left(\frac{K_T}{8} \cdot \frac{1}{8 N_a \epsilon_s} \right)^{1/2} \approx 0.12$$



$$\text{At } V_G = V_{FB}, \frac{C_{FB}}{C_{ox}} = \frac{1}{1 + \frac{C_{ox} L_D}{\epsilon_s}} \approx 0.89$$

8.5

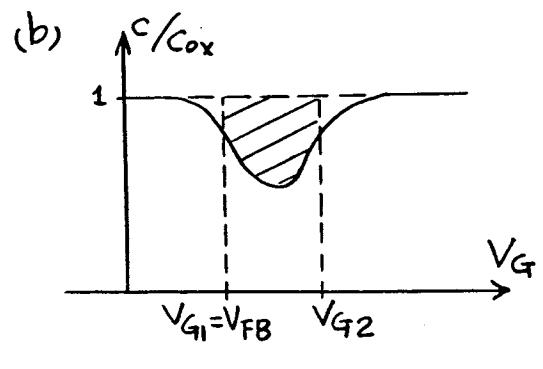
$$(a) Q_s = -C_{ox} (V_G - V_{FB} - \phi_s)$$

$$C = -dQ_s/dV_G = C_{ox} (1 - d\phi_s/dV_G)$$

$$\text{or } d\phi_s/dV_G = 1 - C/C_{ox}$$

Integrating gives

$$\phi_s(V_{G2}) - \phi_s(V_{G1}) = \int_{V_{G1}}^{V_{G2}} \left(1 - \frac{C}{C_{ox}}\right) dV_G$$

8.6

$$\frac{dQ_n}{dt} = \frac{-gN_i}{2\tau_0} (x_d - x_{df}) \quad \text{At all times: } Q_g = -(Q_n - gN_a x_d) \quad \text{or}$$

$$x_d = \frac{Q_g + Q_n}{gN_a} \quad \text{Hence: } \frac{dQ_n}{dt} = \frac{-gN_i}{2\tau_0} \left(\frac{Q_g + Q_n}{gN_a} - x_{df} \right)$$

$$= \frac{-N_i}{2N_a\tau_0} (Q_g + Q_n - gN_a x_{df})$$

$$\text{or } -\left(\frac{2N_a\tau_0}{N_i}\right) \left(\frac{dQ_n}{dt}\right) = Q_g + Q_n - gN_a x_{df}$$

$$\therefore Q_n + \left(\frac{2N_a\tau_0}{N_i}\right) \left(\frac{dQ_n}{dt}\right) = -[Q_g - gN_a x_{df}] \quad \text{as is given}$$

Solution @ let $\frac{2N_a\tau_0}{N_i} \rightarrow \tau_a$

$$\frac{dQ_n}{dt} + \frac{Q_n}{\tau_a} = \frac{-1}{\tau_a} (Q_g - gN_a x_{df})$$

$$\text{Homogeneous: } \frac{dQ_n}{dt} + \frac{Q_n}{\tau_a} = 0 \quad \frac{dQ_n}{Q_n} = -\frac{dt}{\tau_a} \quad \text{or } \ln Q_n = -\frac{t}{\tau_a} + \text{const.}$$

$$\therefore Q_n = A e^{-t/\tau_a}$$

$$\text{Particular: } Q_n = (Q_g - gN_a x_{df})$$

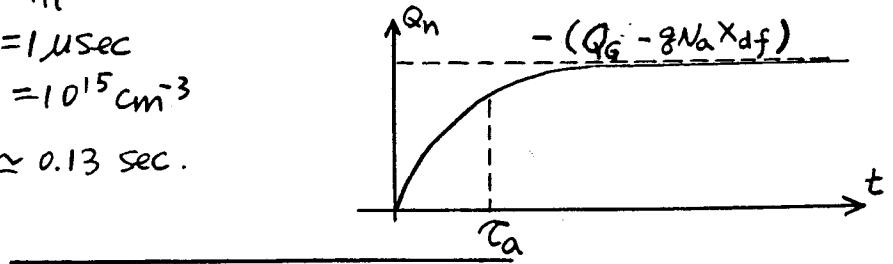
$$\therefore Q_n = (Q_g - gN_a x_{df}) (1 - e^{-t/\tau_a}) \quad [\text{Initial Condition: } Q_n(t=0) = 0]$$

where $\tau_a = \frac{2N_a\tau_0}{N_i}$ is the characteristic time to form the layer.

Example: for $\tau_0 = 1 \mu\text{sec}$

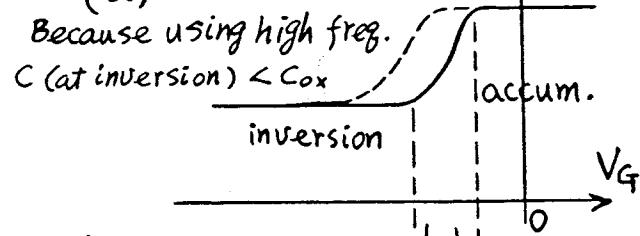
$$N_a = 10^{15} \text{ cm}^{-3}$$

$$\tau_a = \frac{2 \times 10^{15} \times 10^{-6}}{1.45 \times 10^{10}} \approx 0.13 \text{ sec.}$$

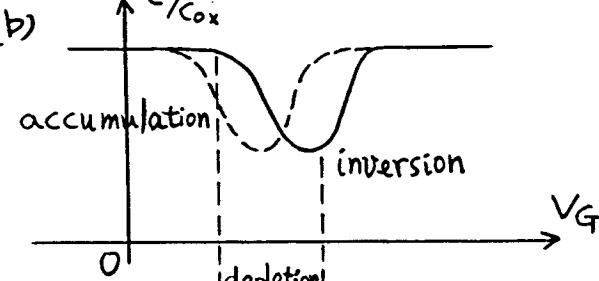


8.7

(a)



(b)

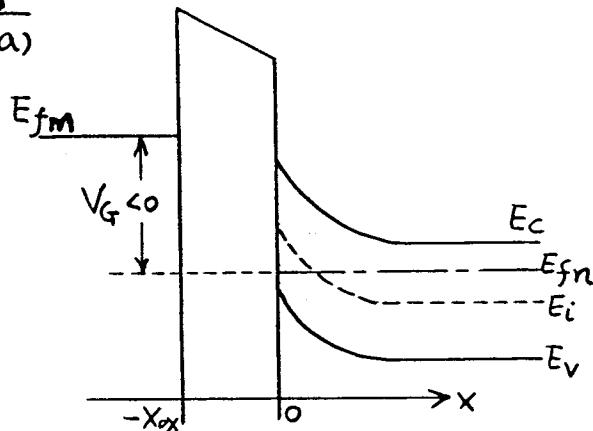


(c) Similar to (a)
but the n⁺ type will make depletion
and inversion at lower voltages than
in part (a)

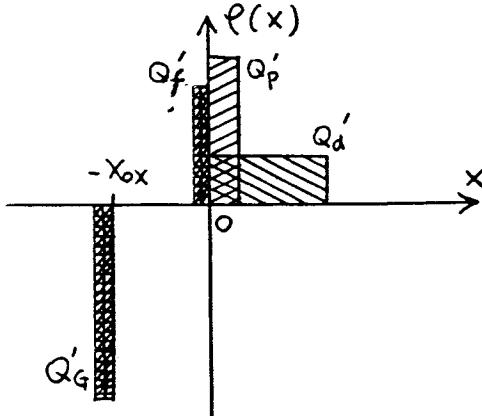
The n⁺ supplies the minority carriers at
inversion to make them follow the
measuring signal even at high freq.

8.8

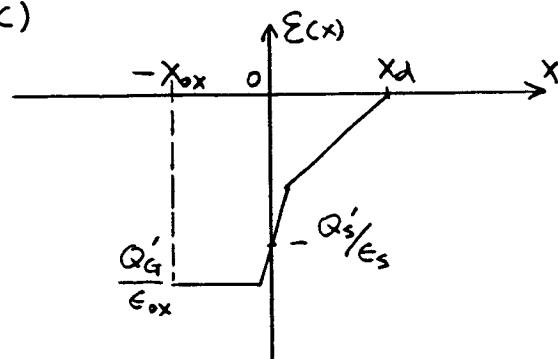
(a)



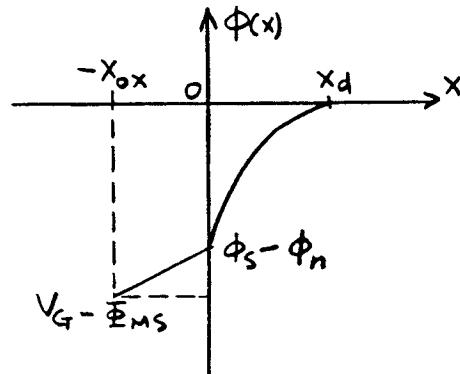
(b)



(c)



(d)

8.9n-type
SiliconV_G - V_{FB}

positive

φ_spositive φ_s > φ_n

Surface charge condition

accumulation

Surface carrier density

n_s > N_d

0

positive φ_s = φ_n

neutral

n_s = N_d

negative

φ_s < φ_n

depletion

n_i < n_s < N_d

more negative

0

intrinsic

n_s = P_s = n_i

more negative

negative φ_s < 0

weak inversion

P_s > n_s, P_s < N_d

more negative

negative φ_s = -φ_n

onset of strong inv.

P_s = N_d

more negative

negative φ_s < -φ_n

Strong inversion

P_s > N_d

8.10

From (8.3.3) $n_s = n_i \exp(-\frac{q\phi_s}{kT})$ also $N_d = n_i \exp(-\frac{q\phi_p}{kT})$

$$10N_d = n_i \exp(-\frac{q\phi_s}{kT}) \therefore 10 = \exp\left[\frac{q}{kT}(\phi_s + \phi_p)\right]$$

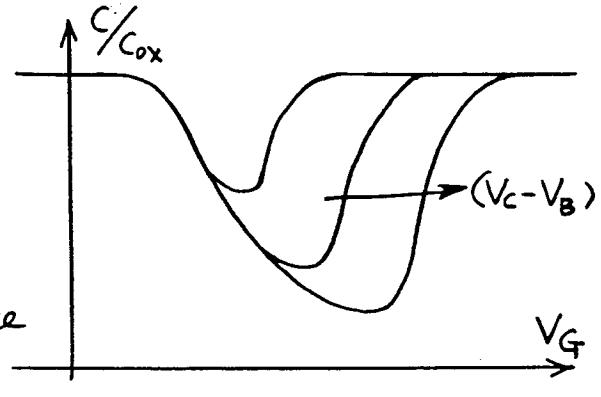
$$\phi_s = -\phi_p + \frac{kT}{q} \ln 10; \text{ If we take } T = 70^\circ F = 21.11^\circ C \Rightarrow \frac{kT}{q} = 0.0253 V$$

$$\text{then } \phi_s = -\phi_p + 58 mV, \text{ (at } T = 300^\circ K \text{ or } 80.6^\circ F \Rightarrow \phi_s = -\phi_p + 59 mV)$$

8.11

As $(V_c - V_B)$ is increased, the turn-on voltage is increased.

The capacitance follows the deep-depletion $C-V_G$ curve until $V_G = V_T$ and then rapidly increases to the oxide capacitance as the surface becomes strongly inverted.

8.12

$$(a) \text{ p-type Si } 1\Omega\text{-cm} \Rightarrow N_d = 1.3 \times 10^{16} \text{ cm}^{-3}$$

$$x_{ox} = 100 \text{ nm} = 10^{-5} \text{ cm} \Rightarrow C_{ox} = \epsilon_{ox}/x_{ox} = 3.45 \times 10^{-8} \text{ F/cm}^2$$

$$\Phi_{MS} = -0.90 \text{ V}, Q_f = 8 \times 10^{-9} \text{ C/cm}^2, Q_f/C_{ox} = 0.23 \text{ V}$$

$$V_{FB} = \Phi_{MS} - Q_s/C_{ox} = -1.13 \text{ V} \quad 2|\phi_p| = 0.71 \text{ V}$$

$$Q_B = -\sqrt{2\epsilon_s q N_d (2|\phi_p|)} = -5.53 \times 10^{-8} \text{ Coul/cm}^2$$

$$Q_B/C_{ox} = -1.6 \text{ V}$$

$$V_T = V_{FB} + 2|\phi_p| - Q_B/C_{ox} = +1.18 \text{ V}$$

$$(b) \text{ n-type Si } 1\Omega\text{-cm} \Rightarrow N_d = 5 \times 10^{15} \text{ cm}^{-3}$$

$$\Phi_{MS} = -0.17 \text{ V}, V_{FB} = -0.40 \text{ V}, 2|\phi_n| = 0.66 \text{ V}, Q_B/C_{ox} = -0.96 \text{ V}$$

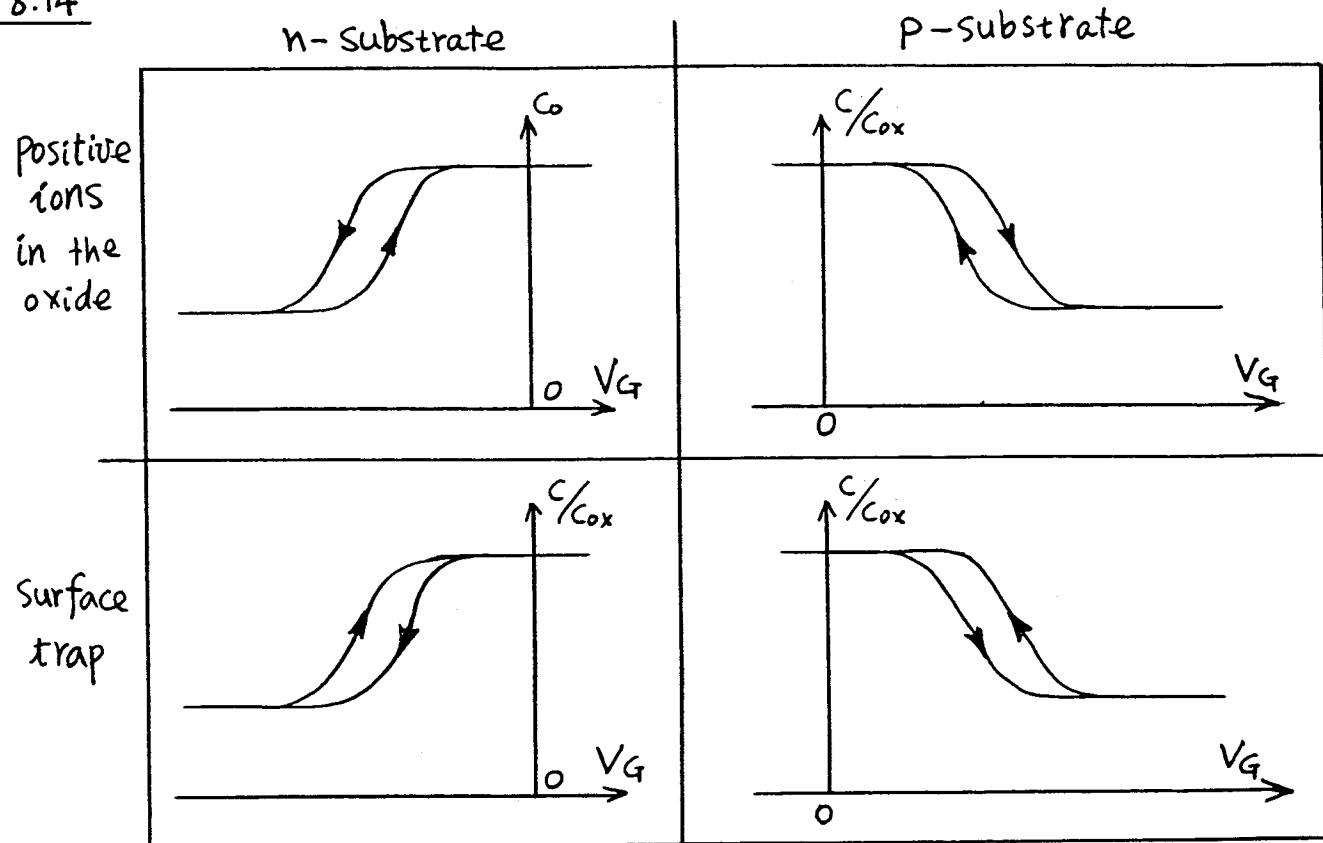
$$V_T = V_{FB} - 2|\phi_n| - Q_B/C_{ox} = -2.02 \text{ V}$$

8.13

Positive oxide charge causes the surfaces of n-regions to be accumulated and the surfaces of p-regions to be depleted or inverted. Since the collector of a double-diffused pnp transistor is a lightly doped p-region, its surface can be inverted causing

Short circuits between adjacent base and isolation regions in an integrated circuit.

8.14



8.15

$$A = 100 \times 100 \mu\text{m}^2 = 10^{-4} \text{ cm}^2$$

$$\text{For MOS } 8 \times 10^6 = \frac{10V}{X_{ox}} \Rightarrow X_{ox} = 1.25 \times 10^{-6} \text{ cm} \therefore C_{ox} = \frac{\epsilon_{ox}}{X_{ox}} A = 27.6 \text{ pF}$$

or 12.5 nm

$$\text{For pn-junction } \phi_B = \frac{2kT}{q} \ln \frac{N_d}{N_i} = 0.699 \text{ V for } N_d = 10^{16} \text{ cm}^{-3}$$

$$W = \sqrt{\frac{2\epsilon_s(\phi_B + V_R)}{qN_d}} = 8.56 \times 10^{-5} \text{ cm for } V_R = 5 \text{ V}$$

$$C = \frac{\epsilon_s}{W} A = 1.21 \text{ pF} \quad C_{MOS}/C_{pn} = 22.8$$

8.16

$$\frac{I_S}{I_F} = \frac{8n_i S_0 A_F}{2} / \frac{q n_i}{2 \tau_0} X_{ds} A_F = \frac{S_0 \tau_0 / X_{ds}}{2} , \tau_0 = 10^{-6} \text{ sec} , X_{ds} = 10^{-4} \text{ cm} , \sigma = 10^{-15} \text{ cm}^{-2}$$

$$\frac{I_S}{I_F} = 2 = \frac{S_0 \tau_0 / X_{ds}}{2} \therefore S_0 = 200 \text{ cm/sec} . \quad S_0 = N_{st} 2 V_{th} \sigma = N_{st} \times 10^7 \times 10^{-15}$$

$\therefore N_{st} = 200 \times 10^9 = 2 \times 10^{10} \text{ cm}^{-2}$

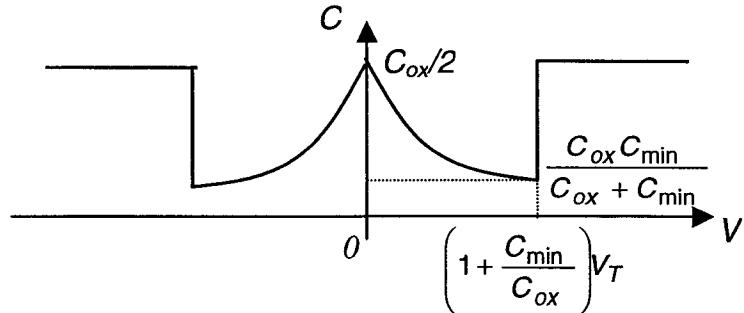
8.17

Let $V = V_1 + V_2$ where V_1 is the voltage across C_1 and V_2 is the voltage across C_2 .

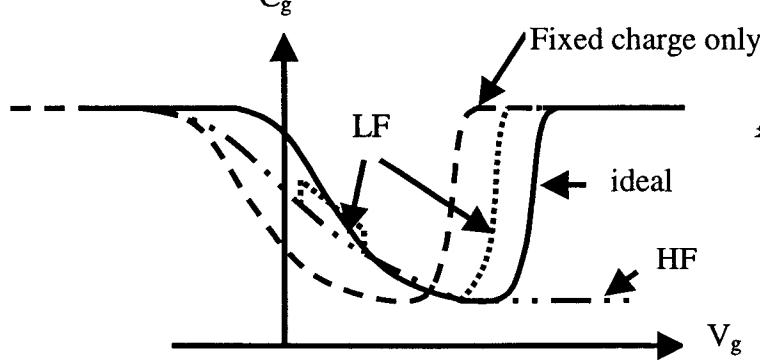
When $V = 0$, both C_1 and C_2 are at the flat-band condition, and the capacitance is given by $C_{ox}/2$.

When $V > 0$, (note that C is symmetric around $V = 0$) C_2 is in accumulation. So, $C_2 = C_{ox}$. The total capacitance will just be the capacitance of C_1 in series with a constant capacitor C_{ox} .

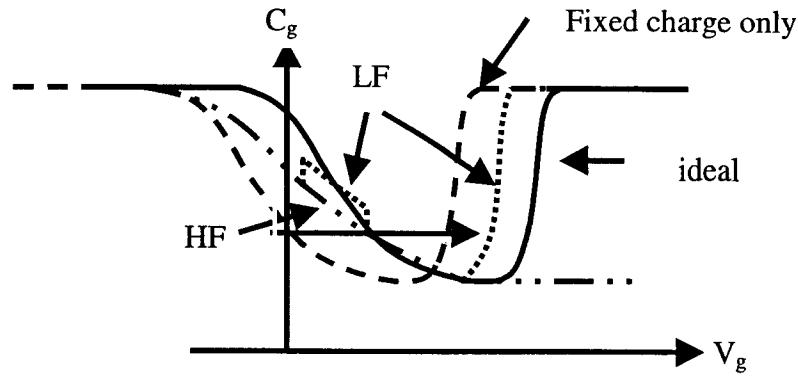
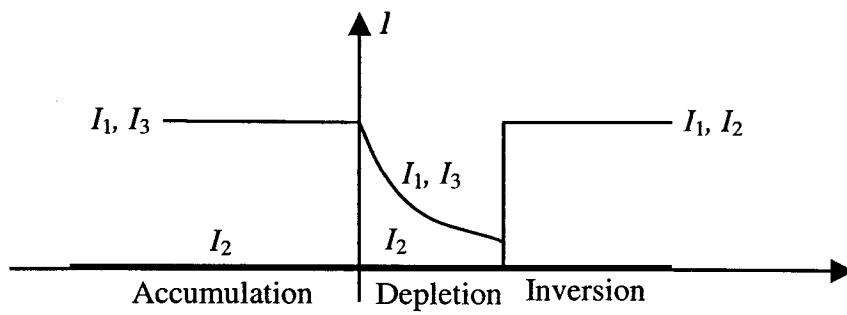
Next, consider what voltage V will give $C_1 = C_{min}$. Since the body is floating, $Q_1 = Q_2$. And we know that $V_1 = V_T$. So, we have $Q_1 = Q_2 \Rightarrow C_{ox}V_2 = C_{min}V_T \Rightarrow V_2 = (C_{min}/C_{ox})V_T$ and the minimum C occurs at $V = V_1 + V_2 = (1 + C_{min}/C_{ox})V_T$. The final result should look like the following:

8.18 Assume positive fixed charge and acceptor type interface traps.

(a)



(b)

8.19

In accumulation: $I_2 = 0$ because n^+ and substrate have the same potential

$$\therefore I_1 = I_3 = \frac{dQ_G}{dt} = \frac{dC_{ox}V_G}{dt} = C_{ox} \frac{dV_G}{dt} = C_{ox}R = \text{constant}$$

In depletion: $I_2 = 0$ for the same reason as in accumulation

$$\therefore I_1 = I_3 = \frac{dC(V_G)V_G}{dt} = V_G \frac{dC(V_G)}{dt} + C(V_G) \frac{dV_G}{dt} = C_{ox} \frac{dV_G}{dt} = V_G \frac{dC(V_G)}{dt} + C(V_G)R$$

Assuming $\frac{dC(V_G)}{dt}$ is a constant K ($K < 0$), we have $I_1 = I_3 \approx KV_G + C(V_G)R$

In inversion: the channel forms and masks I_3 so that $I_3 = 0$

$$\therefore I_1 = I_2 = C_{ox}R = \text{constant}$$

Chapter 9

$$9.1 \quad V_T = \Phi_{MS} - \frac{Q_f}{C_{ox}} + 2\phi_p + \frac{\sqrt{2\epsilon_s q N_A |2\phi_p + V_{SB}|}}{C_{ox}}$$

For NMOSFET, $\Phi_{MS} \approx -0.55 - \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right)$

For PMOSFET, $\Phi_{MS} \approx -0.55 + \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right)$

with $x_{ox}=20\text{nm}$, $V_{SB}=0\text{V}$, we have

$N (\text{cm}^{-3})$	n-channel MOSFET		p-channel MOSFET	
	V_T	Mode	V_T	Mode
10^{15}	-0.29V	depletion	-0.81V	enhancement
10^{16}	0.034V	enhancement	-1.13V	enhancement
10^{17}	1V	enhancement	-2.10V	enhancement
10^{18}	4.14V	enhancement	-5.24V	enhancement

9.2 Solving the 1-D Poisson's equation from the point at V_{Dsat} to the drain, where the charge density is $-N_a$

We have $\frac{d^2V}{dx^2} = \frac{qN_a}{\epsilon_s} \Rightarrow \frac{dV}{dx} = - \int \frac{qN_a}{\epsilon_s} dx = \frac{qN_a}{\epsilon_s} x$

$$\Rightarrow \int_{V_{Dsat}}^{V_D} dV = \frac{qN_a}{\epsilon_s} \int_0^{\Delta L} x dx$$

$$\Rightarrow V_D - V_{Dsat} = \frac{qN_a}{\epsilon_s} \frac{(\Delta L)^2}{2}$$

Therefore $\Rightarrow \Delta L = \sqrt{\frac{2\epsilon_s (V_D - V_{Dsat})}{qN_a}}$

9.3 (a) for $V_{DS} \sim 0V$, $g = -\frac{W}{L}\mu_n Q_n \Rightarrow \frac{Q_n}{q} = \frac{gL}{qW\mu_n} = 4.17 \times 10^{12} \text{ cm}^{-2}$

(b) $Q_n = -C_{ox}(V_{GS} - V_T) \Rightarrow V_{GS} - V_T = -\frac{Q_n}{C_{ox}} = 3.82V$

9.4 (a) current is given by $I_D = -WQ_n v$. Using the long channel model, the device is in the saturation region

$$I_D = \mu_n C_{ox} \frac{W}{2L} (V_G - V_T)^2 \Rightarrow v = \frac{\mu_n C_{ox} (V_G - V_T)^2}{2L Q_n}$$

at the source, $Q_n = C_{ox}(V_G - V_T) \Rightarrow v = \frac{\mu_n (V_G - V_T)}{2L} = 1.26 \times 10^7 \text{ cm/s}$

as Q_n is assumed to be close to zero at the drain in the long channel model, the carrier velocity have to approach infinity.

(b) I_D has to be calculated first.

Using Equation 9.2.3, $\mathcal{E}_{eff} = 0.6 \text{ MV/cm}$

Using Equation 9.2.4 with value given in Table 9.3, $\mu_{eff} = 364 \text{ cm}^2/\text{V-s}$

Taking $v_{sat} = 8 \times 10^6 \text{ cm/s}$ and with Equation 9.2.7, we have $\mathcal{E}_{sat} = 4.39 \times 10^4 \text{ V/cm}$

Therefore, using Equation 9.2.11, $V_{Dsat} = 1.62V$ and the device is also in saturation

At the source,

$$I_D = WC_{ox} (V_G - V_T - V_{Dsat}) v_{sat} \Rightarrow v = \frac{WC_{ox} (V_G - V_T - V_{Dsat}) v_{sat}}{WC_{ox} (V_G - V_T)} = 3.68 \times 10^6 \text{ cm/s}$$

At the drain, the carrier is moving at $v_{sat} = 8 \times 10^6 \text{ cm/s}$

9.5 From Equation 9.1.11, $\Delta V_T = 0.5(\sqrt{0.6+V_{SS}} - \sqrt{0.6})$

V_{SS} (V)	2	3	4	4.5
ΔV_T (V)	0.419	0.561	0.685	0.742
V_T (V)	-1.581	-1.439	-	-1.258

For $V_{SS} = 2\text{V}$, $V_{DS} = 5\text{V}-2\text{V} = 3\text{V}$. The MOSFET is saturated because $V_{DS} > V_{GS} - V_T$

From Equation 9.1.6, $I_D = \mu_n C_{ox} \frac{W}{2L} (V_G - V_T)^2 = 7.5(0 + 1.581)^2 = 18.7\mu\text{A}$

For $V_{SS} = 3\text{V}$, $V_{DS} = 2\text{V} > V_{GS} - V_T$. The MOSFET is saturated

$$\therefore I_D = \mu_n C_{ox} \frac{W}{2L} (V_G - V_T)^2 = 7.5(0 - 1.439)^2 = 15.5\mu\text{A}$$

For $V_{SS} = 4\text{V}$, $V_{DS} = 1\text{V} < V_{GS} - V_T$. The MOSFET is in the linear region

From Equation 9.1.5, $I_D = \mu_n C_{ox} \frac{W}{L} \left(V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS} = 15 \left(0 + 1.5315 - \frac{1}{2} \right) \times 1 = 12.2\mu\text{A}$

For $V_{SS} = 4.5\text{V}$, $V_{DS} = 0.5\text{V} < V_{GS} - V_T$. The MOSFET is in the linear region

$$\therefore I_D = \mu_n C_{ox} \frac{W}{L} \left(V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS} = 15 \left(0 + 1.258 - \frac{0.5}{2} \right) \times 0.5 = 7.56\mu\text{A}$$

9.6 Under these bias conditions, $V_{SB}=5\text{V}$

$$\therefore \Delta V_T = 0.5(\sqrt{0.6+5} - \sqrt{0.6}) = 0.796 \Rightarrow V_T = -1.204\text{V}$$

Since $V_{GD} = 0\text{V}$, $V_{DS} < V_{GS} - V_T$

Thus, with $V_{SS} = 6, 8$ and 10V , the MOSFET is in the linear region

Using $I_D = \mu_n C_{ox} \frac{W}{L} \left(V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS}$

We get

With $V_{SS}=6V$, $I_D = 15 \left(1 + 1.204 - \frac{1}{2} \right) \times 1 = 25.56\mu A$

With $V_{SS}=8V$, $I_D = 15 \left(3 + 1.204 - \frac{3}{2} \right) \times 3 = 121.7\mu A$

With $V_{SS}=10V$, $I_D = 15 \left(5 + 1.204 - \frac{25}{2} \right) \times 5 = 277.8\mu A$

9.7 From the data, we get the following equations from Equation 9.1.10

$$120 = \frac{k}{2} (3 - V_T(0))^2 \left(1 + \frac{4}{V_A} \right) \quad \dots \dots \dots \quad (1)$$

$$130 = \frac{k}{2} (3 - V_T(0))^2 \left(1 + \frac{6}{V_A} \right) \quad \dots \dots \dots \quad (2)$$

$$76.8 = \frac{k}{2} (3 - V_T(4))^2 \left(1 + \frac{4}{V_A} \right) \quad \dots \dots \dots \quad (3)$$

$$270 = \frac{k}{2} (4 - V_T(0))^2 \left(1 + \frac{4}{V_A} \right) \quad \dots \dots \dots \quad (4)$$

Dividing (2) by (1) we get $\frac{130}{120} = \frac{1+6/V_A}{1+4/V_A} \Rightarrow V_A = 20V$

Dividing (4) by (1), we get $\frac{270}{120} = \frac{(4 - V_T(0))^2}{(3 - V_T(0))^2} \Rightarrow V_T(0) = 1V$

Substituting V_A and $V_T(0)$ into (1), we have $120 = \frac{k}{2} (3 - 1)^2 (1 + 0.05 \times 4) \Rightarrow k = 50\mu A/V^2$

From equation (3), $76.8 = \frac{50}{2} [3 - V_T(4)]^2 (1 + 4/20) \Rightarrow V_T(4) = 1.4V$

Then from equation 9.1.11, $\Delta V_T = 0.4 = \gamma (\sqrt{0.6+4} - \sqrt{0.6}) \Rightarrow \gamma = 0.29V^{1/2}$

9.8 (a) $C_{\alpha} = \frac{\epsilon_{\alpha}}{t_{\alpha}} = 6.9 \times 10^{-8} \text{ F/cm}^2$, $\mu_n \frac{W}{L} = 5 \times 10^3 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, $N_a = 2 \times 10^{15} \text{ cm}^{-3}$, $V_{FB} = -0.2 \text{ V}$

$$|\phi_p| = 0.0258 \ln \left(\frac{2 \times 10^{15}}{1.45 \times 10^{10}} \right) = 0.3035 \text{ V} \Rightarrow 2|\phi_p| = 0.6107 \text{ V}$$

Assuming minimal surface states, $V_T = V_{FB} + 2|\phi_p| + \frac{\sqrt{2\epsilon_s q N_A |2\phi_p|}}{C_{\alpha}}$ = 0.702V

Using Equation 9.1.6, we have

$V_{GS} (\text{V})$	2.5	3.5	4.5	5.5
$I_{Dsat} (\text{mA})$	0.558	1.35	2.49	3.97

Differentiating Equation 9.16, set to 0, we obtained the V_{Dsat} expression as

$$V_{Dsat} = V_G - V_{FB} - 2|\phi_p| - \frac{\epsilon_s q N_A}{C_{\alpha}^2} \left[\sqrt{1 + \frac{2C_{\alpha}^2}{\epsilon_s q N_A} (V_G - V_{FB})} - 1 \right] \dots\dots (*)$$

Using Equation 9.1.16 substituting (*) into V_{DS} , we have

$V_{GS} (\text{V})$	2.5	3.5	4.5	5.5
$V_{Dsat} (\text{V})$	1.54	2.43	3.34	4.26
$I_{Dsat} (\text{mA})$	0.47	1.15	2.15	3.46

(b) Matching current calculated with Equation 9.1.16 and 9.1.19 at $V_{GS} = 3.5 \text{ V}$, we have

$$I_D = \frac{5 \times 10^3 \times 6.9 \times 10^{-8}}{2\alpha} (3.5 - 0.702)^2 = 1.15 \times 10^{-3} \Rightarrow \alpha = 1.17$$

Applying $\alpha = 1.17$ to Equation 9.1.18 and 9.1.19, we have

$V_{GS} (\text{V})$	2.5	3.5	4.5	5.5
$V_{Dsat} (\text{V})$	1.54	2.39	3.25	4.1
$I_{Dsat} (\text{mA})$	0.48	1.15	2.13	3.39

9.9

By Gauss' law

$$\epsilon_s = -\frac{q}{\epsilon_s} [N_{ai}x_i + N_a x_d] \quad (1)$$

$$\epsilon_i = -\frac{q}{\epsilon_s} N_a [x_d - x_i] \quad (2)$$

Since $-\int_0^{x_d} \epsilon dx = \phi_s + |\phi_p|$, we have:

$$\frac{\epsilon_i(x_d - x_i)}{2} + \epsilon_i x_i + \frac{(\epsilon_s - \epsilon_i)x_i}{2} = \phi_s + |\phi_p|$$

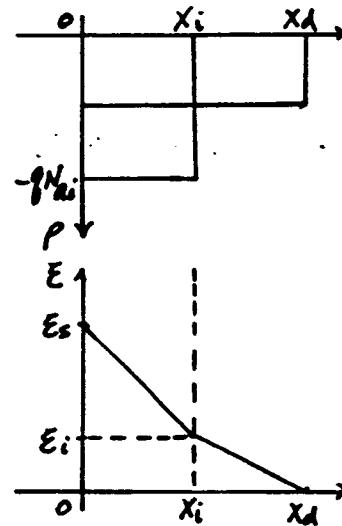
$$\text{or: } \frac{\epsilon_i(x_d + x_i)}{2} + \frac{\epsilon_s x_i}{2} - \frac{\epsilon_i x_i}{2} = \phi_s + |\phi_p|$$

$$\frac{\epsilon_i x_d}{2} + \frac{\epsilon_s x_i}{2} = \phi_s + |\phi_p|$$

$$\frac{q}{\epsilon_s} N_a (x_d - x_i) \frac{x_d}{2} + \frac{q}{\epsilon_s} N_a \left[x_d + x_i - \frac{N_{ai}}{N_a} \right] \frac{x_i}{2} = \phi_s + |\phi_p|$$

$$\therefore \frac{2\epsilon_s [\phi_s + |\phi_p|]}{q N_a} = x_d^2 - x_d x_i + x_d x_i + x_i^2 \frac{N_{ai}}{N_a}$$

$$\text{and } x_d = \sqrt{\frac{2\epsilon_s}{q N_a} (\phi_s + |\phi_p|) - x_i^2 \frac{N_{ai}}{N_a}}$$



9.10

The charge is just

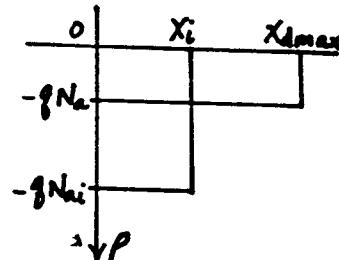
$$Q_d = -q N_{ai} x_i - q N_a x_{d\max}$$

From Eq. (9.1.21)

$$x_{d\max} = \sqrt{\frac{2\epsilon_s}{q N_a} [|\phi_{ps}| + |\phi_p| + V_{SB}] - x_i^2 \frac{N_{ai}}{N_a}}$$

$$\therefore Q_d = -q N_a x_i - \sqrt{2\epsilon_s q N_a [|\phi_{ps}| + |\phi_p| + V_{SB}] - q^2 x_i^2 N_a N_{ai}}$$

$$\text{and } V_T = V_{FB} + V_s + |\phi_p| + |\phi_{ps}| + \frac{q N'}{C_{ox}} + \frac{Q_d}{C_{ox}} \text{ which is Eq. (9.1.23)}$$



$$9.11 \text{ (a)} \quad I_{diff} = WD_n \frac{dQ_n}{dy} = WD_n \frac{d(C_{ox}(V_G - V_T - V(y)))}{dy} = WD_n C_{ox} \frac{dV(y)}{dy}$$

$$\text{(b)} \quad I_{diff} = WD_n \frac{dQ_n}{dy} = WD_n \frac{d(C_{ox}(V_G - V_T - V(y)))}{dy} = WD_n C_{ox} \frac{dV(y)}{dy}$$

$$\text{(c)} \quad \frac{I_{drift}}{I_{diff}} = \frac{V_G - V_T - V(y)}{\frac{kT}{q}} = \frac{V_G - V_T - V(y)}{V_{th}} \quad \text{note: the Einstein's relationship } \frac{D_n}{\mu_n} = \frac{kT}{q} \text{ is used}$$

For strong inversion with $V_G - V_T - V(y) \gg V_{th}$ (around 0.025V at room temperature), the drift current dominate the conductance and the strong inversion equations derived based on the drift mechanism is valid.

9.12

$$\text{(a)} \quad K = K' \left(\frac{W}{L} \right) = 50 \times 10^{-6} \text{ A/V}^2, \quad V_T = 1 \text{ V}$$

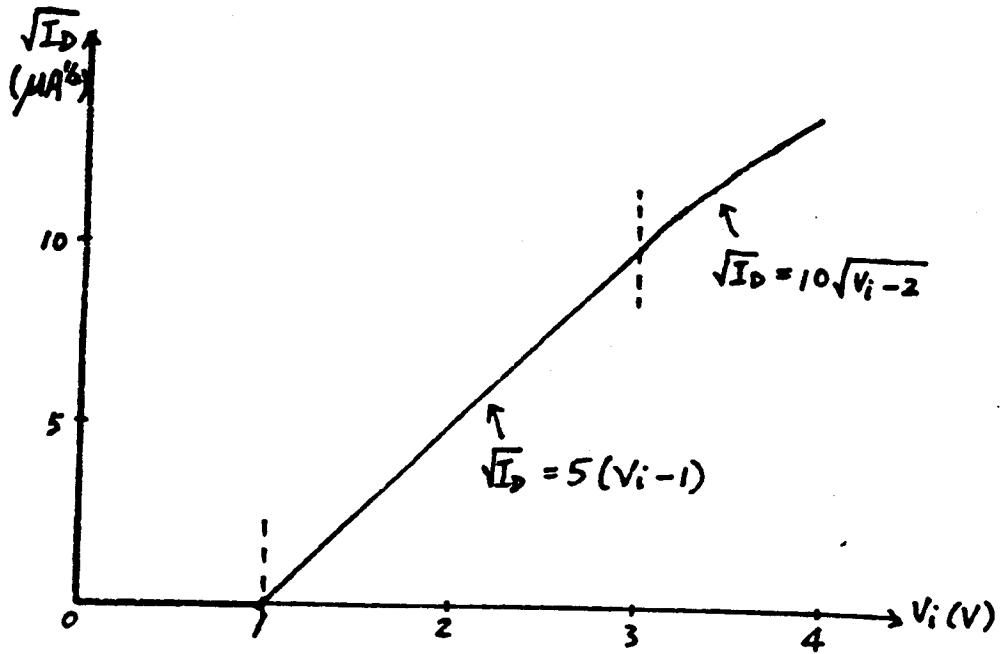
For $V_i \leq 1 \text{ V}$, $I_D = 0$

$$\text{For } 1 \text{ V} \leq V_i \leq 3 \text{ V}, \quad I_D = \frac{k}{2} (V_i - V_T)^2 \text{ because the MOSFET is saturated.}$$

$$= 25 \times 10^{-6} (V_i - 1)^2$$

$$\text{For } 3 \text{ V} \leq V_i \leq 4 \text{ V}, \quad I_D = K \left[(V_i - V_T) V_{DD} - \frac{V_{DD}^2}{2} \right], \text{ the MOSFET is not saturated.}$$

$$= 100 \times 10^{-6} (V_i - 2)$$



(b) & (c)

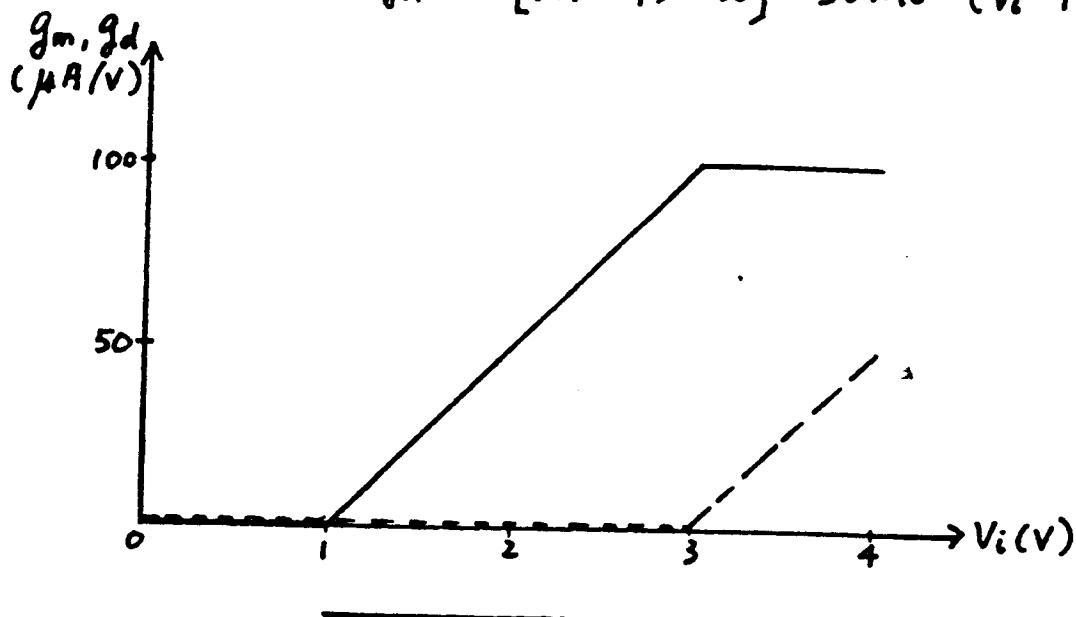
For $V_i \leq 1V$, $g_m = g_d = 0$

For $1V \leq V_i \leq 3V$, $g_m = \frac{\partial I_D}{\partial V_i} = 50 \times 10^{-6} (V_i - 1)$

$$g_d = \frac{\partial I_D}{\partial V_{DS}} = 0$$

For $3V \leq V_i \leq 4V$, $g_m = k V_{DD} = 100 \times 10^{-6} A/V$

$$g_d = k[(V_i - V_T) - V_{DS}] = 50 \times 10^{-6} (V_i - 1 - 2)$$



9.13 from Equation 9.1.3, $Q_n(y) = -C_{\alpha}[V_G - V_T - V(y)]$ and $V(y)$ has to be calculated first

Performing the integration in Equation 9.1.4 to some arbitrary point y , we have

$$\begin{aligned} \int_0^y I_D dy &= \mu W C_{\alpha} \int_0^{V(y)} (V_G - V_T - V) dV \\ \Rightarrow I_D y &= \mu W C_{\alpha} \left[(V_G - V_T) V(y) - \frac{V^2(y)}{2} \right] \\ \therefore V(y) &= (V_G - V_T) \pm \sqrt{(V_G - V_T)^2 - \frac{2I_D y}{\mu W C_{\alpha}}} \quad (\text{negative is taken to give } V(0) = 0) \end{aligned}$$

At saturation, $\therefore I_D = \mu_n C_{\alpha} \frac{W}{2L} (V_G - V_T)^2 \Rightarrow V(y) = (V_G - V_T) \left(1 - \sqrt{1 - \frac{y}{L}} \right)$

Therefore, total Q_n is given by, $Q_n = -W C_{\alpha} (V_G - V_T) \int_0^L \sqrt{1 - \frac{y}{L}} dy = -\frac{2}{3} W L C_{\alpha} (V_G - V_T)$

Finally, $C_{GS} \equiv \left| \frac{\partial Q_n}{\partial V_{GS}} \right| = \frac{2}{3} C_{\alpha} WL$

9.15 At thick x_{α} is very large and approach infinity, $\mathcal{E}_{eff} \rightarrow 0$ as given in Equation 9.2.3

and $\mu_{eff} = \mu_0$ after Equation 9.2.4

Then Equation 9.2.9 becomes $I_D = \mu_0 C_{\alpha} \frac{W}{L} \left(V_G - V_T - \frac{V_{DS}}{2} \right) \frac{V_{DS}}{1 + \frac{V_{DS}}{\mathcal{E}_{sat} L}}$

For large L , $V_{DS}/\mathcal{E}_{sat} L \rightarrow 0$ and Equation 9.2.9 becomes Equation 9.1.5

From Equation 9.2.11 with at large L , we have, $\mathcal{E}_{sat} L \gg V_G - V_T$

$$V_{DSat} = \frac{\mathcal{E}_{sat} L (V_G - V_T)}{\mathcal{E}_{sat} L + (V_G - V_T)} \approx \frac{\mathcal{E}_{sat} L (V_G - V_T)}{\mathcal{E}_{sat} L} = (V_G - V_T)$$

Also, Equation 9.2.10 becomes

$$\begin{aligned}
 I_{Dsat} &= WC_{\alpha} v_{sat} \left((V_G - V_T) - \frac{\mathcal{E}_{sat} L (V_G - V_T)}{\mathcal{E}_{sat} L + (V_G - V_T)} \right) \\
 &= WC_{\alpha} v_{sat} \left(\frac{(V_G - V_T)^2}{\mathcal{E}_{sat} L + (V_G - V_T)} \right) \approx WC_{\alpha} v_{sat} \left(\frac{(V_G - V_T)^2}{\mathcal{E}_{sat} L} \right) \\
 &= \frac{v_{sat}}{\mathcal{E}_{sat}} C_{\alpha} \frac{W}{L} (V_G - V_T)^2 = \mu_0 C_{\alpha} \frac{W}{2L} (V_G - V_T)^2 \quad \text{by using Equation 9.2.7}
 \end{aligned}$$

Similarly, Equation 9.2.14 becomes

$$\begin{aligned}
 I_{Dsat} &= WC_{\alpha} v_{sat} \frac{(V_G - V_T)(V_G - V_T + 2\mathcal{E}_{sat} L)}{(V_G - V_T + \mathcal{E}_{sat} L)^2} \\
 &\approx WC_{\alpha} v_{sat} \frac{2(V_G - V_T) \mathcal{E}_{sat} L}{(\mathcal{E}_{sat} L)^2} = WC_{\alpha} v_{sat} \frac{2(V_G - V_T)}{\mathcal{E}_{sat} L} = C_{\alpha} \frac{W}{L} \left(\frac{2v_{sat}}{\mathcal{E}_{sat}} \right) (V_G - V_T) = \mu_0 C_{\alpha} \frac{W}{L} (V_G - V_T)
 \end{aligned}$$

same as Equation 9.1.37

9.16 Follow the procedures given in the example in section 9.2, the following table is obtained

	V_{DD} (V)	\mathcal{E}_{eff} (MV/cm)	μ_{eff} (cm ² /V-s)	\mathcal{E}_{sat} (V/cm)	V_{Dsat} (V)	I_{Dsat} (mA)	g_{msat} (mA/V)
$L=0.5\mu m$							
NMOS	5	0.744	307.0	5.21×10^4	1.62	49.36	15.8
	3.3	0.556	384.6	4.16×10^4	1.16	26.52	14.8
PMOS	5	0.744	92.78	12.9×10^4	2.58	23.76	8.84
	3.3	0.556	112.0	10.7×10^4	1.75	11.7	7.56
$L=0.02\mu m$							improvement
NMOS	5	0.744	307.0	5.21×10^4	0.102	77.31	56.6%
	3.3	0.556	384.6	4.16×10^4	0.081	46.39	74.9%
PMOS	5	0.744	92.78	12.9×10^4	0.243	56.04	135.9%
	3.3	0.556	112.0	10.7×10^4	0.198	33.18	183.6%

9.17 Old:

$$I_{Dsat(old)} = v_{sat(old)} WC_{\alpha} \frac{(V_{GS} - V_T)^2}{V_{GS} - V_T + \mathcal{E}_{sat(old)} L}$$

New:

$$I_{Dsat(new)} = v_{sat(new)} WC_{\alpha} \frac{(V_{GS} - V_T)^2}{V_{GS} - V_T + \mathcal{E}_{sat(new)} L}$$

Note:

$$\mathcal{E}_{sat} = \frac{2v_{sat}}{\mu_{eff}} \Rightarrow \mathcal{E}_{sat(old)} = 4 \times 10^4 V/cm$$

Therefore:

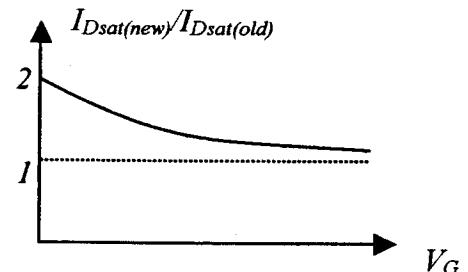
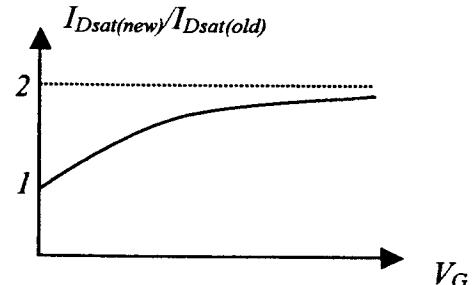
$$\frac{I_{Dsat(new)}}{I_{Dsat(old)}} = \frac{v_{sat(new)}}{v_{sat(old)}} \frac{V_{GS} - V_T + \mathcal{E}_{sat(old)} L}{V_{GS} - V_T + \mathcal{E}_{sat(new)} L}$$

Part (a):

$$\frac{I_{Dsat(new)}}{I_{Dsat(old)}} = 2 \frac{V_{GS} + 4}{V_{GS} + 8}$$

Part (b):

$$\frac{I_{Dsat(new)}}{I_{Dsat(old)}} = \frac{V_{GS} - V_T + \mathcal{E}_{sat(old)} L}{V_{GS} - V_T + \mathcal{E}_{sat(new)} L} = \frac{V_{GS} + 4}{V_{GS} + 2}$$



Notice that increase v_{sat} help the I_{Dsat} at high V_G while increasing μ_{eff} help I_{Dsat} at low V_G . As we normal desire to have high on current at high V_G , larger v_{sat} is more desirable.

9.18 (a)

$$I_D = \mu WC_{\alpha} (V_G - V_T - V(y)) \frac{dV(y)}{dy}$$

$$\text{integrate: } I_D y = \mu WC_{\alpha} \left[(V_G - V_T) V(y) - \frac{V^2(y)}{2} \right]$$

$$\therefore V(y) = (V_G - V_T) - \sqrt{(V_G - V_T)^2 - \frac{2I_D y}{\mu WC_{\alpha}}}$$

$$\mathcal{E}(y) = - \frac{dV(y)}{dy} = \frac{I_D}{\sqrt{\mu WC_{\alpha} (V_G - V_T)^2 - 2I_D \mu WC_{\alpha} y}}$$

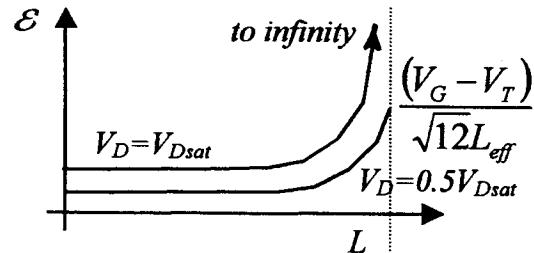
note : to have the expression with terminal voltage only, put $I_D = \mu C_{\alpha} \frac{W}{L} \left[(V_G - V_T) V_D - \frac{V_D^2}{2} \right]$

(b)

$$\text{at } V_D = 0.5V_{D_{sat}}, \quad I_D = \frac{\mu W C_\alpha}{8L_{eff}} (V_G - V_T)^2$$

$$\therefore \mathcal{E}(y) = \frac{(V_G - V_T)}{\sqrt{4L_{eff}(4L_{eff} - y)}}$$

also use the result in part (a) we obtain :



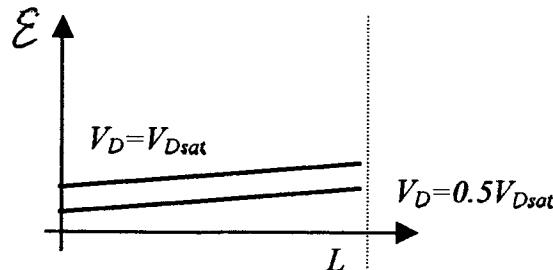
(c)

$$I_D = \mu W C_\alpha (V_G - V_T - V(y)) \frac{dV(y)}{dy} - \frac{I_D}{\mathcal{E}_{sat}} \frac{dV(y)}{dy}$$

$$\text{integrate: } I_D y = \left[\mu W C_\alpha (V_G - V_T) - \frac{I_D}{\mathcal{E}_{sat}} \right] V(y) - \mu W C_\alpha \frac{V^2(y)}{2}$$

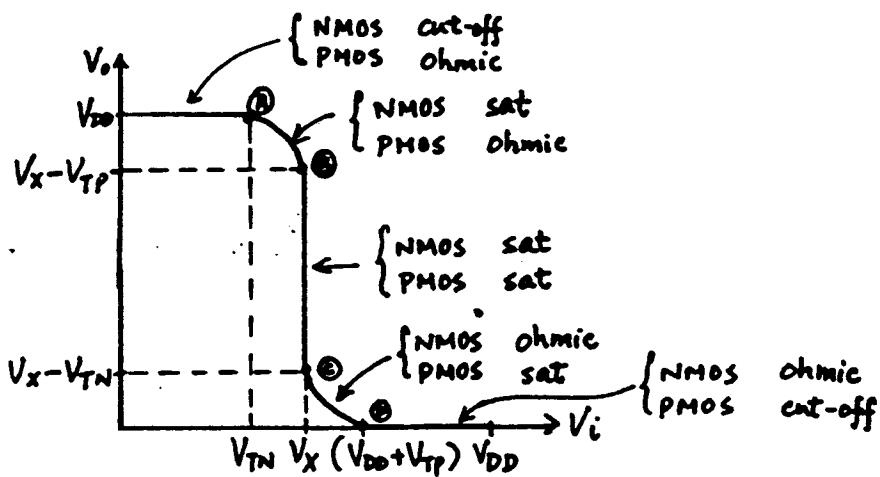
$$\therefore V(y) = (V_G - V_T) - \frac{I_D}{\mu W C_\alpha \mathcal{E}_{sat}} - \sqrt{\left((V_G - V_T) - \frac{I_D}{\mu W C_\alpha \mathcal{E}_{sat}} \right)^2 - \frac{2I_D y}{\mu W C_\alpha}}$$

$$(y) = -\frac{dV(y)}{dy} = I_D \left[\frac{I_D^2}{\mathcal{E}_{sat}^2} - \frac{2I_D \mu W C_\alpha (V_G - V_T)^2}{\mathcal{E}_{sat}} + [\mu W C_\alpha (V_G - V_T)]^2 - 2I_D \mu W C_\alpha y \right]^{-\frac{1}{2}}$$



9.19

(a)



(b) To find V_x , let's take a look at the point C.

Since NMOS is in ohmic region,

$$I_{DN} = k_N [(V_x - V_{TN})(V_x - V_{TN}) - \frac{(V_x - V_{TN})^2}{2}] \text{ from eq. (9.1.5)}$$

Since PMOS is saturated,

$$I_{DP} = \frac{k_P}{2}(V_{DD} - V_x + V_{TP})^2 \text{ from eq. (9.1.6)}$$

But $I_{DN} = I_{DP}$.

$$\therefore 40[(V_x - 1)^2 - \frac{(V_x - 1)^2}{2}] = \frac{35}{2}(5 - V_x - 1)^2$$

$$40(V_x - 1)^2 = 35(4 - V_x)^2 \Rightarrow V_x = 2.45 \text{ V}$$

Thus,

	$V_i (\text{V})$	$V_o (\text{V})$
A	1	5
B	2.45	3.45
C	2.45	1.45
D	4	0

9.20(a) For $V_{DS} \leq V_{GS} - V_T$,

$$I_D = K \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \text{ from eq. (9.1.5)}$$

For $V_{DS} \geq V_{GS} - V_T$,

$$I_D = \frac{K}{2} (V_{GS} - V_T)^2 \text{ from eq. (9.1.6).}$$

$$K = 40 \times 10^{-6} \text{ A/V}^2 \text{ and } V_T = 2 \text{ V}$$

With $V_{GS} = 0$ and 2 V , $I_D = 0$.With $V_{GS} = 4 \text{ V}$:

$$\text{for } V_{DS} \leq 2 \text{ V}, \quad I_D = 40 \times 10^{-6} \left[2 V_{DS} - \frac{V_{DS}^2}{2} \right] \quad (\text{A})$$

$$\text{for } V_{DS} \geq 2 \text{ V}, \quad I_D = \frac{40 \times 10^{-6}}{2} \times 2^2 = 80 \mu\text{A}$$

With $V_{GS} = 6 \text{ V}$:

$$\text{for } V_{DS} \leq 4 \text{ V}, \quad I_D = 40 \times 10^{-6} \left[4 V_{DS} - \frac{V_{DS}^2}{2} \right] \quad (\text{A})$$

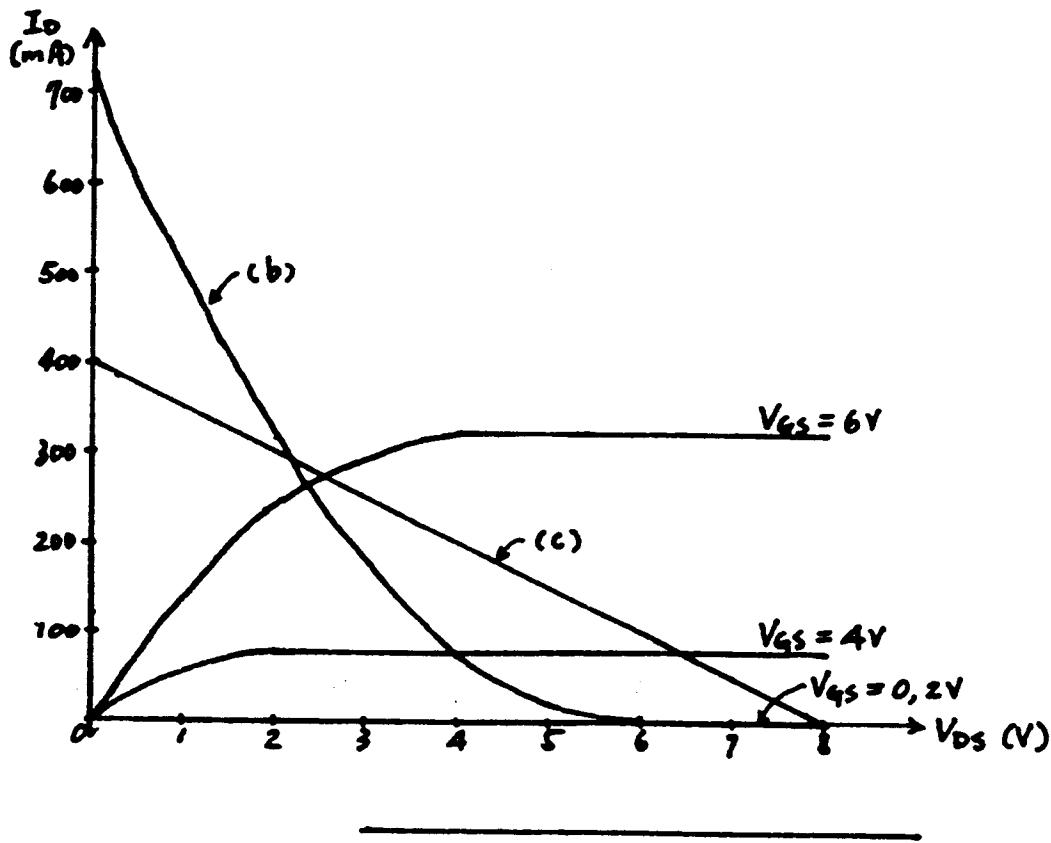
$$\text{for } V_{DS} \geq 4 \text{ V}, \quad I_D = \frac{40 \times 10^{-6}}{2} \times 4^2 = 320 \mu\text{A}$$

(b) The load transistor is saturated when it is turned on because V_{DS} is always greater than $V_{GS} - V_T$.

$$\begin{aligned} \therefore I_D &= \frac{K}{2} (V_{GS} - V_T)^2 = \frac{K}{2} (V_{DD} - V_o - V_T)^2 \\ &= 20 \times 10^{-6} (6 - V_o)^2 \quad (\text{A}) \end{aligned}$$

$$(\text{c}) \quad I_D = \frac{(V_{DD} - V_o)}{20k\alpha} = \frac{(8 - V_o)}{20 \times 10^3} \quad (\text{A})$$

The plot is on the next page.

9.21

- (a) Assume that the enhancement-mode transistor is not saturated, while the depletion-mode transistor is saturated. This should be verified latter.

Using Eq. (9.1.5),

$$\begin{aligned} I_{DS} &= k_E [(V_i - V_{TE}) V_o - \frac{V_o^2}{2}] \\ &= 50(4V_o - 0.5V_o^2) = 200V_o - 25V_o^2 \mu A \end{aligned}$$

Using Eq. (9.1.6),

$$I_{DD} = \frac{k_D}{2} (0 - V_{TD})^2 = 45 \mu A$$

Since $I_{DS} = I_{DD}$, $200V_o - 25V_o^2 = 45$

$$V_o = 0.232 \text{ } \cancel{.77}$$

Now, we can check the above assumption, and can easily see that the assumption is correct.

$$\therefore V_o = 0.232 \text{ V}$$

$$(b) V_{TD} = V_{TD}(0) + \gamma (\sqrt{2|A_p|} + |V_{SB}| - \sqrt{2|A_p|}) \\ = -3 + 0.4 (\sqrt{0.6+5} - \sqrt{0.6}) = -2.36 \text{ V}$$

(c) For the first try, let's use $V_o = 0.232 \text{ V}$

$$\text{Then, } V_{TD} = -3 + 0.4 (\sqrt{0.6+0.232} - \sqrt{0.6}) \\ = -2.945 \text{ V}$$

$$\therefore I_{DD} = \frac{k_p}{2} (0 - V_{TD})^2 = 43.36 \mu\text{A}$$

Since $I_{DE} = I_{DD}$, $200V_o - 25V_o^2 = 43.36$

$$V_o = 0.223 \text{ V}$$

Now, let's iterate it.

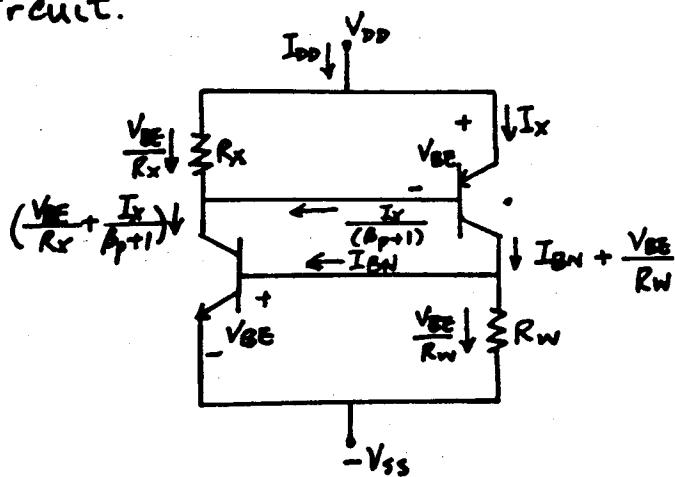
$$V_{TD} = -3 + 0.4 (\sqrt{0.6+0.223} - \sqrt{0.6}) = -2.947 \text{ V}$$

$$I_{DD} = 43.42 \mu\text{A}$$

$$\therefore V_o = 0.223 \text{ V}$$

9.22

In a latched condition, we have the following equivalent circuit.



For npn transistor,

$$I_{BN} = \frac{1}{B_N} \left(\frac{V_{BE}}{R_x} + \frac{I_x}{B_p+1} \right)$$

For pnp transistor,

$$\frac{I_x}{(\beta_p + 1)} = \frac{1}{\beta_p} \left(I_{BN} + \frac{V_{BE}}{R_W} \right)$$

$$I_x = \frac{\beta_p + 1}{\beta_p} \left[\frac{1}{\beta_n} \left(\frac{V_{BE}}{R_X} + \frac{I_x}{\beta_p + 1} \right) + \frac{V_{BE}}{R_W} \right]$$

$$I_x \left(1 - \frac{1}{\beta_p \beta_n} \right) = \frac{\beta_p + 1}{\beta_p \beta_n} \frac{V_{BE}}{R_X} + \frac{\beta_p + 1}{\beta_p} \frac{V_{BE}}{R_W}$$

$$I_x = \frac{\beta_p + 1}{\beta_p \beta_n - 1} \frac{V_{BE}}{R_X} + \frac{\beta_n (\beta_p + 1)}{\beta_n \beta_p - 1} \frac{V_{BE}}{R_W}$$

Thus,

$$\begin{aligned} I_{DD} &= \frac{V_{BE}}{R_X} + I_x \\ &= \frac{1}{\beta_n \beta_p - 1} \left[\frac{(\beta_n \beta_p - 1)V_{BE}}{R_X} + \frac{(\beta_p + 1)V_{BE}}{R_X} + \frac{\beta_n (\beta_p + 1)V_{BE}}{R_W} \right] \\ &= \frac{\left(\frac{V_{BE}}{R_X} \right) \beta_p (\beta_n + 1) + \left(\frac{V_{BE}}{R_W} \right) \beta_n (\beta_p + 1)}{\beta_n \beta_p - 1} \end{aligned}$$

9.2.4

(a) Poisson's Equation (Eq. (4.1.10))

$$\frac{d^2\phi}{dx^2} = -\frac{P}{\epsilon_{si}} = -\frac{q}{\epsilon_{si}} (P - n + Nd - Na), \quad \phi = \frac{1}{q}(E_f - E_i)$$

$$P = n_i \exp\left(-\frac{q\phi}{kT}\right), \quad n = n_i \exp\left(\frac{q\phi}{kT}\right) \quad (\text{Eq. (8.3.3)})$$

$$P = -q(n_i \exp\left(-\frac{q\phi}{kT}\right) - n_i \exp\left(\frac{q\phi}{kT}\right) + Nd - Na)$$

$$U = \left(\frac{q\phi}{kT}\right) \text{ Normalized Potential.}$$

$$U_B = \left(\frac{q\phi_0}{kT}\right) \text{ Normalized Substrate Potential.}$$

$$\begin{aligned} P &= -q(n_i \exp(-U_B) - n_i \exp(U_B) + Nd - Na) \\ P &= -q(-2n_i \sinh(U_B) + Nd - Na) \end{aligned} \quad \left. \begin{array}{l} U = U_B \\ \text{in substrate} \end{array} \right\}$$

$$\text{In the substrate: } P = 0 \text{ and } Nd - Na = n - P$$

$$= n_i(e^{U_B} - e^{-U_B})$$

$$\therefore Nd - Na = 2n_i \sinh(U_B)$$

(b) Poisson's Equation (Eq. (4.1.10))

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}} (P - n + Nd - Na)$$

$$U = \frac{q\phi}{kT} \text{ Normalized Potential}$$

$$\phi = \frac{KTU}{q}$$

$$\left(\frac{KT}{q}\right) \frac{d^2U}{dx^2} = -\frac{q}{\epsilon_{si}} (P - n + Nd - Na) \quad \begin{array}{l} P - n = -2n_i \sinh(u) \\ Nd - Na = 2n_i \sinh(U_B) \end{array}$$

$$\frac{d^2U}{dx^2} = -\frac{q^2}{KT\epsilon_{si}} (-2n_i \sinh(u) + 2n_i \sinh^2(U_B))$$

$$\frac{d^2U}{dx^2} = \underbrace{\left(\frac{2q^2 n_i}{\epsilon_s k T}\right)}_{Y_{LDi}} (\sinh(u) - \sinh(U_B))$$

$$\therefore \frac{d^2U}{dx^2} = \frac{1}{L_{Di}^2} (\sinh(u) - \sinh(U_B)), \quad L_{Di} \equiv \left(\frac{\epsilon_s k T}{2q^2 n_i}\right)^{1/2}$$

$$(c) L_{Di} = \left(\frac{e_s k T}{2q^2 n_i} \right)^{1/2}, \quad e_s = (11.7)(8.854 \times 10^{-14} \text{ F cm}^{-1})$$

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$T = 300 \text{ } ^\circ\text{K}$$

$$q = 1.602 \times 10^{-19} \text{ C}$$

$$n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$$

$$L_{Di} = \left[\frac{(11.7)(8.854 \times 10^{-14} \text{ F cm}^{-1})(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ } ^\circ\text{K})}{2(1.602 \times 10^{-19} \text{ C})^2(1.45 \times 10^{10} \text{ cm}^{-3})} \right]^{1/2}$$

$$= 2.4 \times 10^{-3} \text{ cm} = 24 \mu\text{m}$$

9.25

(a) From Problem 10.5,

$$\frac{d^2u}{dx^2} = \frac{1}{L_{Di}^2} [\sinh(u) - \sinh(u_B)], \quad L_{Di} = \left(\frac{e_s k T}{2q^2 n_i} \right)^{1/2} \quad u = \frac{q\phi}{kT}$$

$$\text{Integration Factor: } 2 \left(\frac{du}{dx} \right) - \frac{du}{dx} = \left(\frac{q}{kT} \right) \varepsilon \quad \left(-\frac{d\phi}{dx} = +\varepsilon \right)$$

$$(2 \frac{du}{dx}) \frac{d^2u}{dx^2} = (2 \frac{du}{dx}) \frac{1}{L_{Di}^2} [\sinh(u) - \sinh(u_B)]$$

$$\frac{d}{dx} \left(\frac{du}{dx} \right)^2 = \frac{2}{L_{Di}^2} [\sinh(u) - \sinh(u_B)] \frac{du}{dx}$$

Integration from the substrate towards the surface yields:

$$\int_{u_B}^0 d \left(\frac{du}{dx} \right)^2 = \frac{2}{L_{Di}^2} \int_u^{u_B} [\sinh(u) - \sinh(u_B)] du, \quad u_B = \text{substrate normalized potential}$$

$$\left(\frac{q}{kT} \right)^2 \int_{\varepsilon}^0 d(\varepsilon)^2 = \frac{2}{L_{Di}^2} [(\cosh(u_B) - \cosh(u)) - (u_B - u) \sinh(u_B)]$$

$$\varepsilon^2 = \frac{2}{L_{Di}^2} \left(\frac{kT}{q} \right)^2 [(\cosh(u_B) - \cosh(u)) - (u_B - u) \sinh(u_B)]$$

$$\therefore \varepsilon(x) = \pm \sqrt{2} \left(\frac{kT}{q L_{Di}} \right) [(\cosh(u_B) - \cosh(u(x))) - (u_B - u(x)) \sinh(u_B)]^{1/2}$$

If $u_B < u(x)$ then + sign

If $u_B > u(x)$ then - sign

$$(\varepsilon(x) = -\frac{kT}{q} \frac{du}{dx})$$

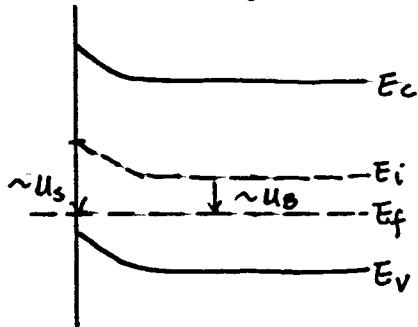
$$(b) \text{ Define: } F_s = +\sqrt{2} \left[(U_B - U_s) \sinh(U_B) - (\cosh(U_B) - \cosh(U_s)) \right]^{1/2}$$

U_s = Surface Normalized Potential

$$\therefore E(x) = \pm \sqrt{2} \left(\frac{kT}{qL_{di}} \right) F_s(U_s, U_B)$$

(c) For P-type silicon:

accumulation

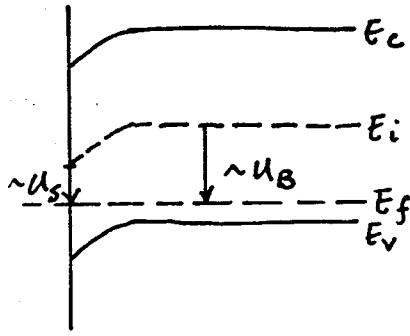


$$u = \frac{\frac{1}{q}(E_f - E_i)}{KT} = \frac{(E_f - E_i)}{KT}$$

U_s, U_B negative but $|U_s| > |U_B|$

$\therefore (U_B - U_s)$ is positive $(U_B - U_s) > 0$

depletion/inversion



U_B negative

U_s negative or positive

$|U_B| > |U_s|$ if U_s negative

$\therefore U_B - U_s$

$\therefore (U_B - U_s) < 0$

9.26

$$Q_s = \pm \left(\frac{\epsilon_s kT}{qL_{di}} \right) F(U_s, U_B)$$

$$N_A = 10^{15} \text{ cm}^{-3}, T = 300 \text{ K}, L_{di} = 24 \mu\text{m}$$

$$Q_s = -(1.12 \times 10^{-11} \text{ C/cm}^2) F(U, U_B)$$

(- sign, \because n-channel device)

$$P \approx N_A = n_i \exp(-U_B)$$

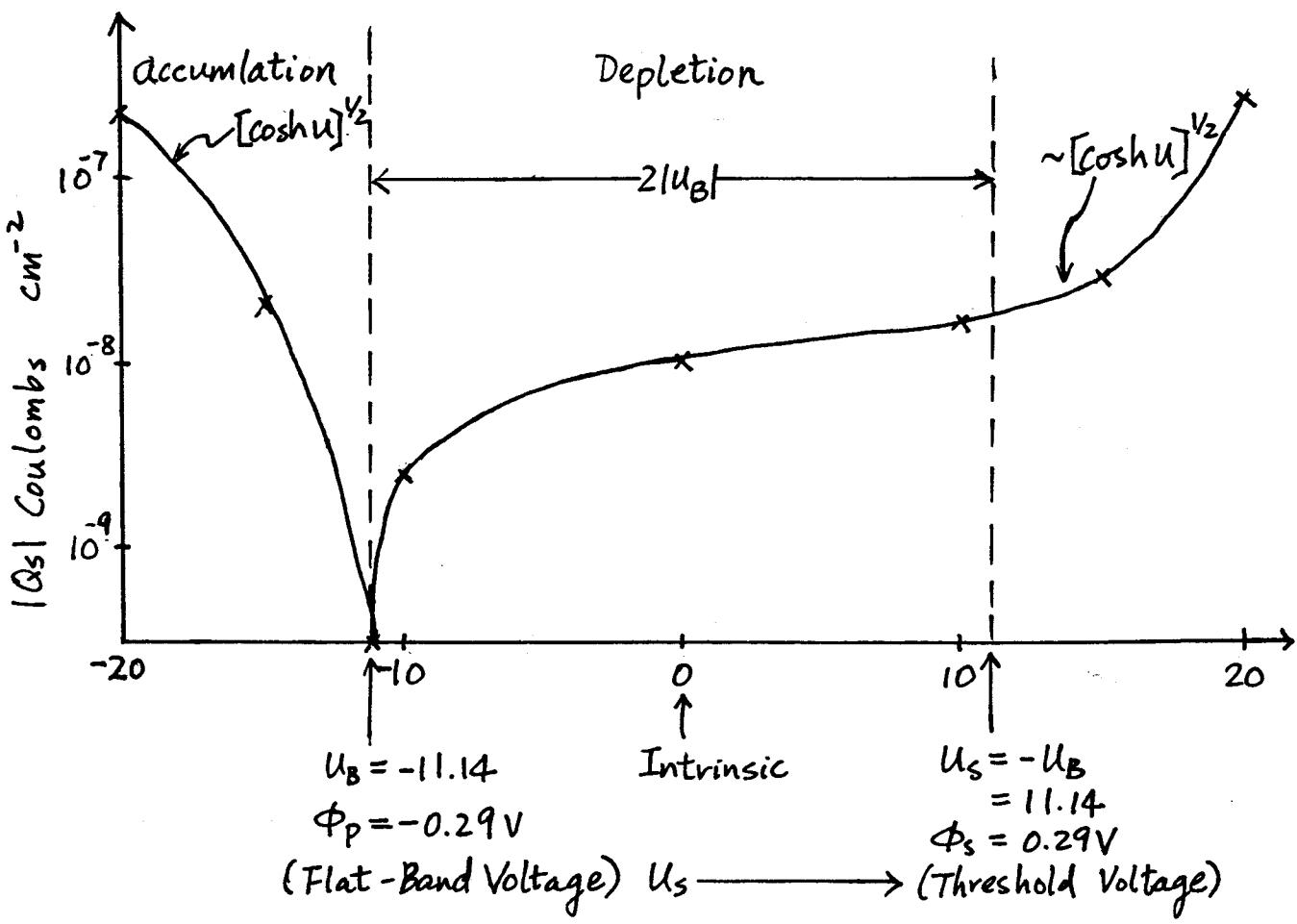
$$U_B = \ln\left(\frac{n_i}{N_A}\right) = \ln\left(\frac{1.45 \times 10^{10} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}}\right) = -11.14$$

$$|Q_s| = (1.12 \times 10^{-11} \text{ C/cm}^2) \sqrt{2} [(-11.14 - U_s) \sinh(-11.14) - (\cosh(-11.14) - \cosh(U_s))]^{1/2}$$

Values [calculator]

U_s	0	-10	10	15	20
$ Q_s $	1.08×10^{-8}	2.69×10^{-9}	1.52×10^{-8}	2.63×10^{-8}	2.47×10^{-7}

U_s	-15	-20
$ Q_s $	1.9×10^{-8}	2.47×10^{-7}



Chapter 10

10.1

$$\begin{aligned}
 I_D(\Delta L) &= I_{Dsat} \left(\frac{V_G - V_T + \mathcal{E}_{sat} L}{V_G - V_T + \mathcal{E}_{sat}(L - \Delta L)} \right) \\
 &= I_{Dsat} \left(1 - \frac{\Delta L}{L + \frac{V_G - V_T}{\mathcal{E}_{sat}} + L} \right)^{-1} \\
 &\approx I_{Dsat} \left(1 + \frac{\Delta L}{L + \frac{V_G - V_T}{\mathcal{E}_{sat}}} \right) \quad \text{by Taylor expansion of } \left(\frac{1}{1-x} \right) \text{ for small } x
 \end{aligned}$$

then

$$\frac{dI_D}{dV_D} = \frac{I_{Dsat}}{L + \frac{V_G - V_T}{\mathcal{E}_{sat}}} \frac{d\Delta L}{dV_D} \quad \text{where} \quad \Delta L = l \ln \left(\frac{\frac{V_D - V_{Dsat}}{l} + \mathcal{E}_m}{\mathcal{E}_{sat}} \right)$$

$$\text{For } \frac{V_D - V_{Dsat}}{l} \gg \mathcal{E}_{sat} \text{ (in saturation), } \mathcal{E}_m \approx \frac{V_D - V_{Dsat}}{l}$$

Therefore

$$\begin{aligned}
 \frac{d\Delta L}{dV_D} &= \frac{l \mathcal{E}_{sat} \left(\frac{1}{\mathcal{E}_{sat} l} + \frac{1}{\mathcal{E}_{sat}} \frac{d}{dV_D} \frac{1}{\mathcal{E}_m} \right)}{\frac{V_D - V_{Dsat}}{l} + \mathcal{E}_m} = \frac{1}{\mathcal{E}_m} \\
 \Rightarrow R_{out} &= \left(\frac{dI_D}{dV_D} \right)^{-1} = \frac{I_{Dsat}}{I_{Dsat}} \left(L + \frac{V_G - V_T}{\mathcal{E}_{sat}} \right)
 \end{aligned}$$

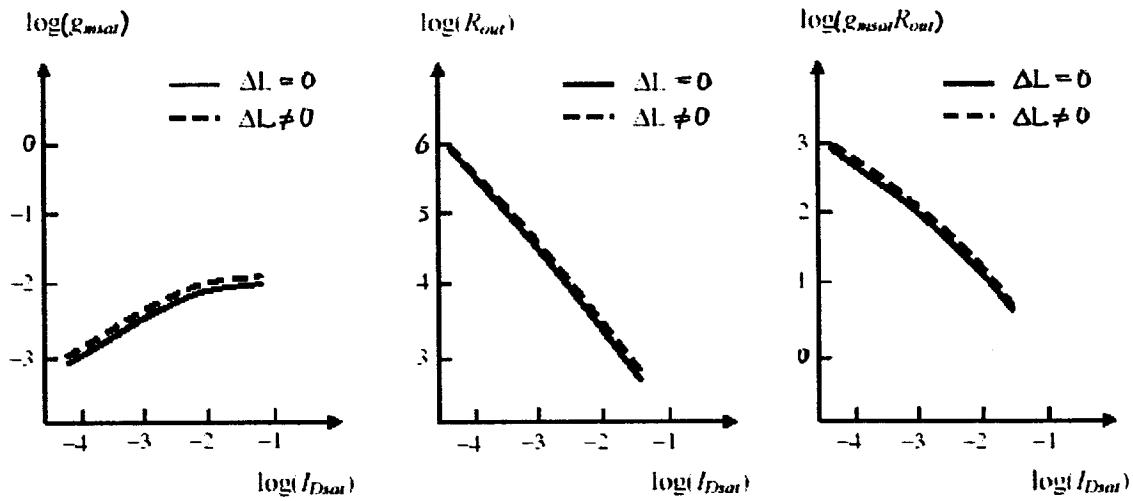
$$10.2 \quad R_{out} = \frac{dV_D}{dI_{Dsat}} = \left(L + \frac{V_G - V_T}{\mathcal{E}_{sat}} \right) \left(\frac{\mathcal{E}_m}{I_{Dsat}} \right)$$

$$g_{msat} = \frac{dI_{Dsat}}{dV_G} = Wv_{sat}C_{ox} \left\{ 1 - \frac{(\mathcal{E}_{sat}L')^2 + L'(V_G - V_T)^2 \left[\frac{2v_{sat}}{\mu_0} \frac{v}{\mathcal{E}_0} \left(\frac{\mathcal{E}_{eff}}{\mathcal{E}_0} \right)^{v-1} \frac{1}{6t_{ox}} \right]}{(V_G - V_T + \mathcal{E}_{sat}L')^2} \right\}$$

where $L' = L - \Delta L$, $\mathcal{E}_{sat} = \frac{2v_{sat}}{\mu_{eff}}$, $\mu_{eff} = \frac{\mu_0}{1 + \left(\frac{\mathcal{E}_{eff}}{\mathcal{E}_0} \right)^v}$, $\mathcal{E}_{eff} = \frac{V_G - V_T}{6t_{ox}} + \frac{V_T + V_a}{3t_{ox}}$

$$\Delta L = l \ln \left(\frac{V_D - V_{Dsat} + \mathcal{E}_m}{\mathcal{E}_{sat}} \right), \quad V_{Dsat} = \frac{(V_G - V_T)\mathcal{E}_{sat}L'}{(V_G - V_T) + \mathcal{E}_{sat}L'}, \quad \mathcal{E}_m \approx \frac{V_D - V_{Dsat}}{l}, \quad l \approx 0.22t_{ox}^{1/2}x_j^{1/2}$$

(Take some particular values $V_D = 5V$, $V_T = 0.7V$, $V_a = 0.5V$, $v_{sat} = 10^7 \text{ cm/s}$, $\mu_0 = 670 \text{ cm}^2/\text{V-s}$, $v = 1.6$, $\mathcal{E}_0 = 0.67 \text{ MV/cm}$)



10.3

$$\left(\frac{I_{sub}}{I_D} \right)_{BD} = \frac{A}{B} (V_{BD} - V_{Dsat}) \exp\left(-\frac{Bl}{V_{BD} - V_{Dsat}} \right)$$

where $\left(\frac{I_{sub}}{I_D} \right)_{BD} = 0.05$ and $l = 0.22t_{ox}^{1/3}x_j^{1/2} = 0.152 \mu m$

solve $V_{BD} - V_{Dsat}$ with iteration, we have $V_{BD} - V_{Dsat} = 5.3 V$

Therefore: $V_{BD} = 5.3 + \frac{(V_G - V_T) \mathcal{E}_{sat} L}{V_G - V_T + \mathcal{E}_{sat} L}$

Similarly for p-MOSFET, $V_{BD} - V_{Dsat} = 9.3 V$ and $V_{BD} = 9.3 + \frac{(V_G - V_T) \mathcal{E}_{sat} L}{V_G - V_T + \mathcal{E}_{sat} L}$

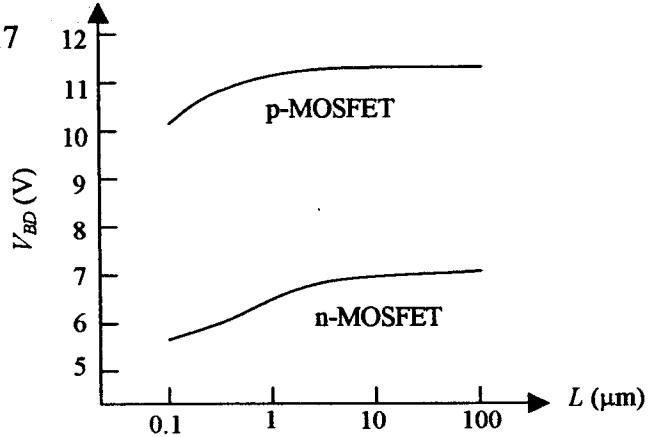
Calculating \mathcal{E}_{sat} using equation 9.2.4, 9.2.7

with values given in table 9.3. We have

$$\mathcal{E}_{sat} (\text{n-MOS}) = 3.3 \times 10^4 V$$

$$\mathcal{E}_{sat} (\text{p-MOS}) = 7.6 \times 10^4 V$$

The resulting plot is shown in the figure.



10.4 (a) $l = 0.22t_{ox}^{1/3}x_j^{1/2} = 0.152 \mu m$

At $\mathcal{E}_m = 2 \times 10^5 V/cm$, $I_{sub}/I_D = 7.4 \times 10^{-4}$, using

$$\left(\frac{I_{sub}}{I_D} \right)_{BD} = \frac{A}{B} (V_{BD} - V_{Dsat}) \exp\left(-\frac{Bl}{V_{BD} - V_{Dsat}} \right)$$

$$(b) \quad V_{D_{sat}} = \frac{(V_G - V_T) \mathcal{E}_{sat} L}{V_G - V_T + \mathcal{E}_{sat} L} = 1.46V \quad \text{at } V_G = 3V,$$

Therefore $V_{DD(max)} = V_{D_{sat}} + \mathcal{E}_m I = 4.35V$

10.5 for $\mathcal{E}_{sat}L \gg V_G - V_T$, $V_{D_{sat}} \approx V_G - V_T$, and $I_{D_{sat}} = \mu_0 C_{ox} W (V_G - V_T)^2 / (2L)$, then

$$I_{sub} \approx \frac{A_i \mu_0 C_{ox} W}{B_i 2L} (V_G - V_T)^2 \exp\left(-\frac{IB_i}{V_D - (V_G - V_T)}\right)$$

$$\text{setting } \frac{dI_{sub}}{dV_G} = 0 \Rightarrow V_G = V_T + \frac{4V_D + IB_i}{4} \left[1 - \sqrt{1 - \frac{16V_D^2}{(4V_D + IB_i)^2}} \right] \approx V_T + \frac{2V_D^2}{4V_D + IB_i}$$

$$\text{and peak } I_{sub} \text{ is given by } I_{sub(peak)} \approx \frac{A_i \mu_0 C_{ox} W}{B_i 2L} \left(\frac{2V_D^2}{4V_D + IB_i} \right)^2 \exp\left(-\frac{IB_i(4V_D + IB_i)}{2V_D^2 + IB_i V_D}\right)$$

10.6 V_{Dsat} is needed to calculate \mathcal{E}_m . V_{Dsat} should be calculated at the V_G which gives the worst hot-carrier conditions, i.e. at peak I_{sub} . From problem 10.5, this V_G is

$$V_G \approx V_T + \frac{2V_D^2}{4V_D + lB_i}, \text{ and therefore } V_{Dsat} \approx V_G - V_T \approx \frac{2V_D^2}{4V_D + lB_i}$$

$$\mathcal{E}_m \approx \frac{V_D - V_{Dsat}}{l} \leq 2 \times 10^5 \text{ V/cm} \Rightarrow V_D \leq l \mathcal{E}_m + V_{Dsat} \approx l \mathcal{E}_m + \frac{2V_D^2}{4V_D + lB_i}$$

solving for V_D , we have

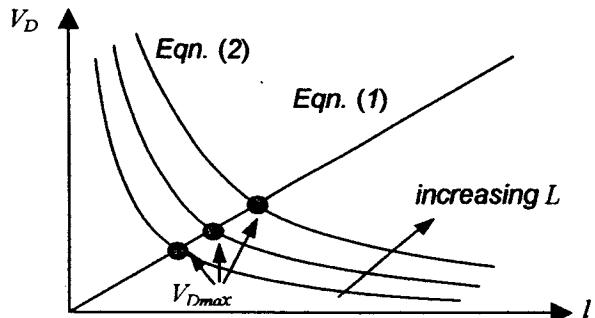
$$V_D \leq \frac{1}{6} \left(-9 \times 10^5 l + \sqrt{81 \times 10^{10} l^2 + 408 \times 10^{10} l^2} \right) = 2.19 \times 10^5 l \quad \dots \dots \dots (1)$$

$$\text{for } \Delta V_T \text{ consideration, } \Delta V_T = 2V_D e^{-\frac{L}{l}} \leq 0.2 \text{ V} \Rightarrow V_D \leq 0.1 e^{-\frac{L}{l}} \quad \dots \dots \dots (2)$$

The allowed V_D 's have to satisfy both equation (1) and (2) and the maximum values are illustrated in the figure. Equation (1) and (2), we have

$$2.19 \times 10^5 l (\mu\text{m}) = 0.1 e^{-\frac{L}{l}} \dots (3)$$

solving by iteration, we obtain



L=	2 μm		1.2 μm		0.6 μm		0.3 μm	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
$l (\mu\text{m})$ (from (3))	0.4	0.36	0.25	0.24	0.12	0.14	0.043	0.081
V_D (V) (from (1))	9.5	8.6	5.9	5.7	2.9	3.3	1.0	1.9

10.7

(a) & (b)

$$\text{From } + \int_0^{X_{ox}} E_o dx = 20V, E_o = \frac{20V}{30 \times 10^{-7} \text{ cm}} \\ = 6.7 \times 10^6 \text{ V/cm}$$

Since the voltage across the oxide is maintained at 20V, the areas under the electric field curves are same for the both conditions.

To find E_x ,

$$20V = E_x (0.9 \times 30 \times 10^{-7}) + 7 \times 10^6 \times 0.1 \\ \therefore E_x = 6.63 \times 10^6 \text{ V/cm}$$

Hence, the change of E -field at the trapping sites is

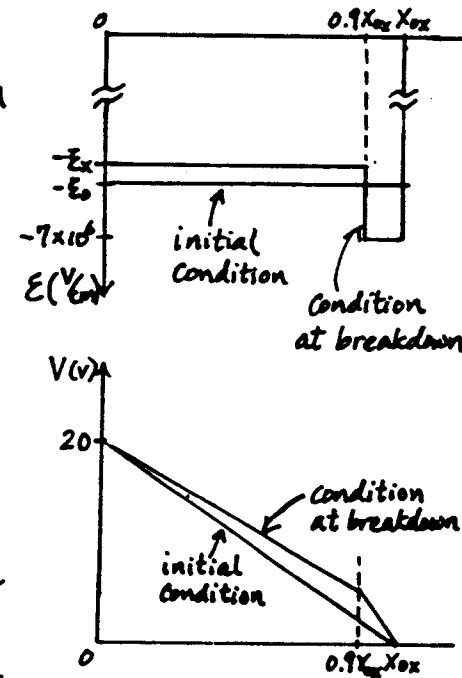
$$\Delta E = 7 \times 10^6 - 6.63 \times 10^6 = 0.37 \times 10^6 \text{ V/cm}$$

From Gauss' law, $\epsilon_{ox} \Delta E = Q_{ot}$

$$\therefore Q_{ot} = 3.9 \times 8.854 \times 10^{-14} \text{ F/cm} \times 0.37 \times 10^6 \text{ V/cm} \\ = 1.28 \times 10^{-7} \text{ C/cm}^2$$

The time required to trap this quantity of charge is

$$t = \frac{Q_{ot}}{J_{ot}} = \frac{1.28 \times 10^{-7}}{5 \times 10^{-8}} = 2.56 \text{ sec}$$

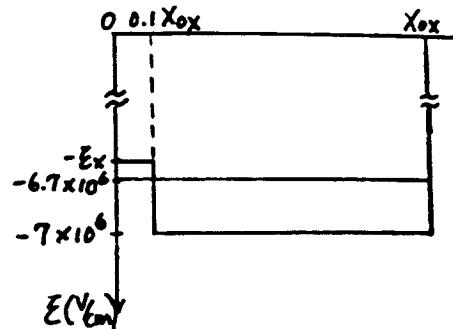


(c)

$$20V = E_x (0.1 \times 30 \times 10^{-7}) \\ + 7 \times 10^6 \times 0.9 \times 30 \times 10^{-7} \\ \therefore E_x = 3.67 \times 10^6 \text{ V/cm}$$

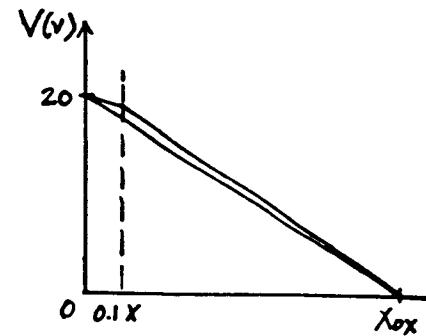
$$\text{Thus, } \Delta E = 7 \times 10^6 - 3.67 \times 10^6 \\ = 3.33 \times 10^6 \text{ V/cm}$$

$$Q_{ot} = \epsilon_{ox} \Delta E = 1.15 \times 10^{-6} \text{ C/cm}^2$$



\therefore The time required to trap this quantity of charge is

$$t = \frac{1.15 \times 10^{-6}}{5 \times 10^{-8}} = 23.0 \text{ sec.}$$



10.8

(a) From the continuity of normal component of \vec{D} ,

$$\epsilon_{ox} E_{ox} = \epsilon_{si} E'$$

(Note that ϵ_{ox} is constant across the oxide.)

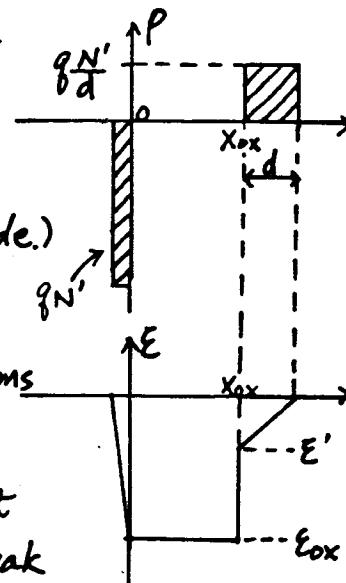
$$\therefore E' = \frac{\epsilon_{ox}}{\epsilon_{si}} \epsilon_{ox}$$

But $E' = \frac{q}{\epsilon_{si}} N'$, where N' is the electrons

per unit area on a MOSFET gate.

Hence, the number of electrons per unit area that can cause an oxide to break down is

$$N' = \frac{\epsilon_{ox}}{q} \epsilon_{crit.}$$



Therefore, the number of electrons that can cause an oxide to break down is a function only of the gate area and the oxide permittivity.

$$(b) N = \frac{A \epsilon_{ox}}{q} \epsilon_{crit.}$$

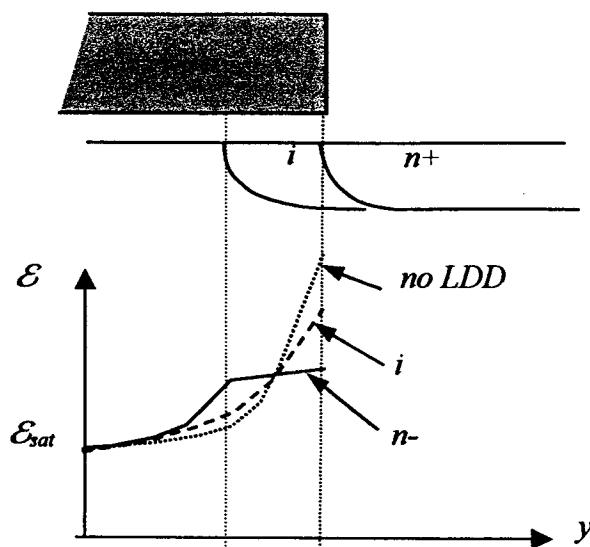
$$= \frac{6 \times 10^{-4} \times 30 \times 10^{-4} \times 3.9 \times 8.854 \times 10^{-14} \times 7 \times 10^6}{1.6 \times 10^{-19}}$$

$$= 2.72 \times 10^7 \text{ electrons}$$

$$(c) t = \frac{qN}{I_{av}} = \frac{1.6 \times 10^{-19} \times 2.72 \times 10^7}{10^{-12}}$$

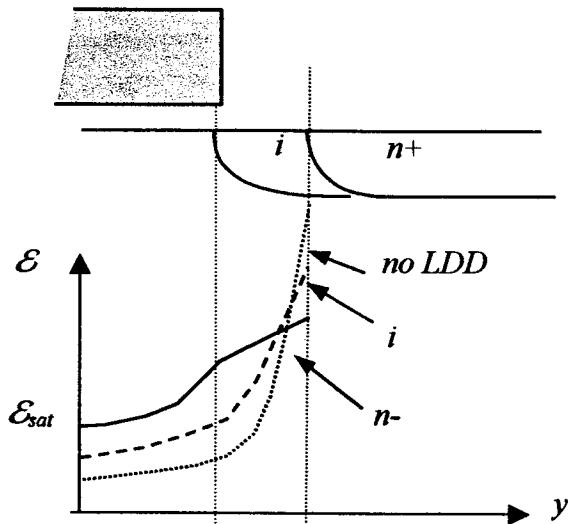
$$= 4.35 \text{ sec.}$$

10.9



Note: areas under the curves have to be the same

10.10



More voltage will be dropped in the "LDD" region and may force the device to fall out of the velocity saturation region.

10.11 First solve the boundary condition in the LDD region.

Assume at the edge of the n- region, the \mathcal{E} field is \mathcal{E}_m , and the voltage is V_{n-}

$$\mathcal{E}_m \approx \frac{V_{n-} - V_{Dsat}}{l}$$

Also, the \mathcal{E} field in the n- region is constant and equal to \mathcal{E}_m , so we have

$$V(y) = V_D - \mathcal{E}_m(L-y)$$

where y start from the edge of the n- region, and L is the length of the n- region

At the edge of the n- region, the above equation becomes $V_{n-} = V_D - \mathcal{E}_m L$

We can solve for \mathcal{E}_m and V_{n-} for the boundary condition, where

$$\mathcal{E}_m = \frac{V_D - V_{Dsat}}{L+l} \quad \text{and} \quad V_{n-} = V_D - \frac{V_D - V_{Dsat}}{L+l} L$$

Therefore, using: $\frac{d^2V}{dy^2} = \frac{V(y) - V_{n-}}{l^2} - q \frac{N_D(y)}{\epsilon_{Si}} = \frac{d\mathcal{E}_y}{dy} = 0$

We have: $N_D(y) = \frac{\epsilon_{Si}}{ql^2} (V_D - V_{n-} - \mathcal{E}_m L + \mathcal{E}_m y) = \frac{\epsilon_{Si}}{ql^2} \mathcal{E}_m y = \frac{\epsilon_{Si}}{ql^2} \frac{V_D - V_{Dsat}}{L+l} y$

10.12 From the method given in Chapter 9, $V_{Dsat}=0.98V$ and $I=0.22t_{ox}^{1/3}x_j^{1/2}=0.124\mu m$.

$$\text{Therefore } \mathcal{E}_m \approx \frac{V_D - V_{Dsat}}{l} = 1.63 \times 10^5 \text{ V/cm}$$

$$\text{Also } I_{Dsat} = WC_{ox}v_{sat}(V_G - V_T - V_{Dsat}) = 9.1 \text{ mA}$$

$$\text{Therefore } I_{sub} = \frac{A}{B}(V_D - V_{Dsat})I_{Dsat}e^{\left(-\frac{B_l}{\mathcal{E}_m}\right)} = 6.92 \times 10^{-5} A$$

$$\text{With } 0.1\mu m \text{ LDD with optimal profile, } \mathcal{E}_m \approx \frac{V_D - V_{Dsat}}{l + 0.1\mu m} = 9.03 \times 10^4 \text{ V/cm}$$

(or \mathcal{E}_m is reduced by $7.27 \times 10^4 \text{ V/cm}$)

Effective V_D now becomes $V_D - \mathcal{E}_m L_n = 2.097V$

$$\text{So } I_{sub} = \frac{A}{B}(V_{D(eff)} - V_{Dsat})I_{Dsat}e^{\left(-\frac{B_l}{\mathcal{E}_m}\right)} = 8.56 \times 10^{-9} A$$

10.13 $R_{LDD} = 1000\Omega \times 0.1/50 = 2\Omega$

Only the source side resistance R_S will affect the drain current. Assume the impact of R_S to the body effect is very small and V_T is invariant, we then need to solve the following equation for I_{Dsat} in the presence of source resistance

$$I_{Dsat} \approx WC_{ox}v_{sat} \left[(V_{GS} - I_{Dsat}R_S - V_T) - \frac{(V_{GS} - I_{Dsat}R_S - V_T) \mathcal{E}_{sat}L}{(V_{GS} - I_{Dsat}R_S - V_T) + \mathcal{E}_{sat}L} \right]$$

$$\text{or } (WC_{ox}v_{sat}R_S^2 + R_S)I_{Dsat}^2 - WC_{ox}v_{sat}R_S(3(V_{GS} - V_T) + \mathcal{E}_{sat}L)I_{Dsat} + WC_{ox}v_{sat}(V_{GS} - V_T)^2 = 0$$

giving $I_{Dsat} = 9.035 \text{ mA}$

Compared with the original value of 9.13mA (from problem 10.12), I_{Dsat} is reduced by 1.04%.

10.14 Initial charging current as calculated in the example in section 10.5 is 0.9pA. Following similar procedure as the example with the V_T measured from the control gate to be 1.7V, the required Q_{fg} is given by

$$\frac{Q_{fg}}{\varepsilon_{ox}} d_1 = 1V \Rightarrow Q_{fg} = \frac{\varepsilon_{ox}}{d_1} = 3.45 \times 10^{-7} \text{ C/cm}^2$$

Total charge to be injected = $Q_{fg} \times 0.5\mu\text{m} \times 1\mu\text{m} = 1.75 \times 10^{-9}\text{C}$

Effective floating gate voltage at the end of charging = 5V

Following similar procedure in the example,

$$V_{Dsat} = 2.05\text{V}, I_{Dsat} = 0.745\text{mA}, \mathcal{E}_m = 2.38 \times 10^5 \text{V/cm}$$

Therefore $I_G \approx CI_{Dsat} e^{-\frac{q}{\mathcal{E}_m \lambda}} \approx 1 \times 10^{-3} \times 0.745\text{mA} \times \exp\left(\frac{-3.32}{2.38 \times 10^5 \times 73 \times 10^{-8}}\right) \approx 3.66\text{fA}$

$$\text{Mean charging current} = 0.5 \times (3.66\text{fA} + 0.231\text{fA}) = 1.95\text{fA}$$

$$\text{Charging time} = 1.75\text{C} / 1.95\text{fA} = 0.9 \text{ second}$$

10.15 Effective floating gate voltage under the condition in problem 10.14 is given by:

$$V_{FG} = 6 - \frac{Q_{fg}}{\varepsilon_{ox}} d_1$$

We obtains:

$$\begin{aligned} V_{Dsat} &\approx 6 - \frac{Q_{fg}}{\varepsilon_{ox}} d_1 - 0.25 = 5.75 - \frac{Q_{fg}}{\varepsilon_{ox}} d_1 \\ I_{Dsat} &\approx \mu C_{ox} \frac{W}{2L} \left(5.75 - \frac{Q_{fg}}{\varepsilon_{ox}} d_1 \right)^2 \\ &\approx \frac{1}{l} \left(V_D - 5.75 + \frac{Q_{fg}}{\varepsilon_{ox}} d_1 \right) = \frac{1}{l} \left(0.25 + \frac{Q_{fg}}{\varepsilon_{ox}} d_1 \right) \end{aligned}$$

Therefore

$$I_G \approx CI_{Dsat} e^{-\frac{Q_f}{\mu C_\alpha \lambda}} \approx 1 \times 10^{-3} \times \mu C_\alpha \frac{W}{2L} \left(5.75 - \frac{Q_f}{\varepsilon_\alpha} d_1 \right)^2 \times \exp \left(\frac{-3.32l}{\left(0.25 + \frac{Q_f}{\varepsilon_\alpha} d_1 \right) \times 73 \times 10^{-8}} \right)$$

Maximum I_G occurs at $V_{FG} \approx V_D$ and lead to

$$\frac{Q_f}{\varepsilon_\alpha} d_1 = 1 \Rightarrow Q_f = 0.345 \mu C$$
