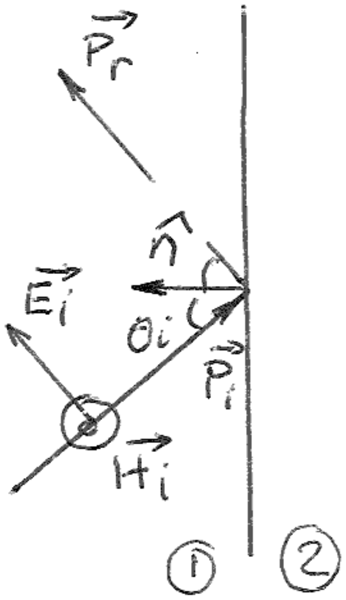
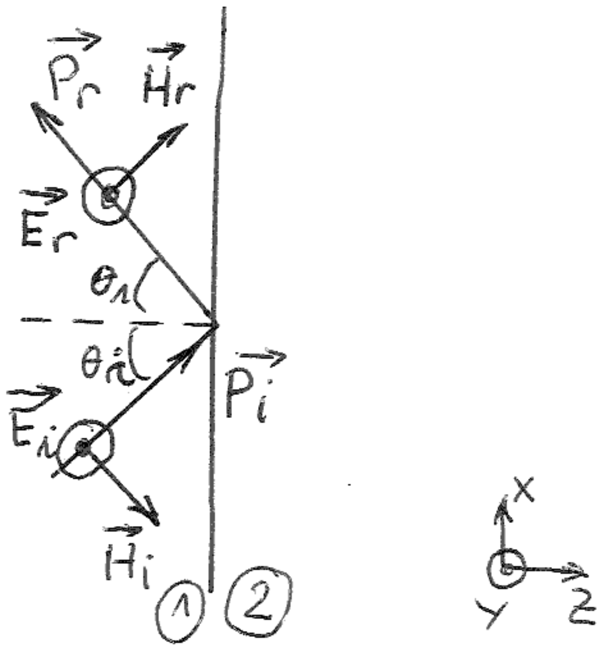
INCIDENCIA OBLICUA

EL VECTOR DE POYNTING INCIDE CON UN ANGULO θ_i CUALQUIERA



POLARIZACIÓN PARALELA



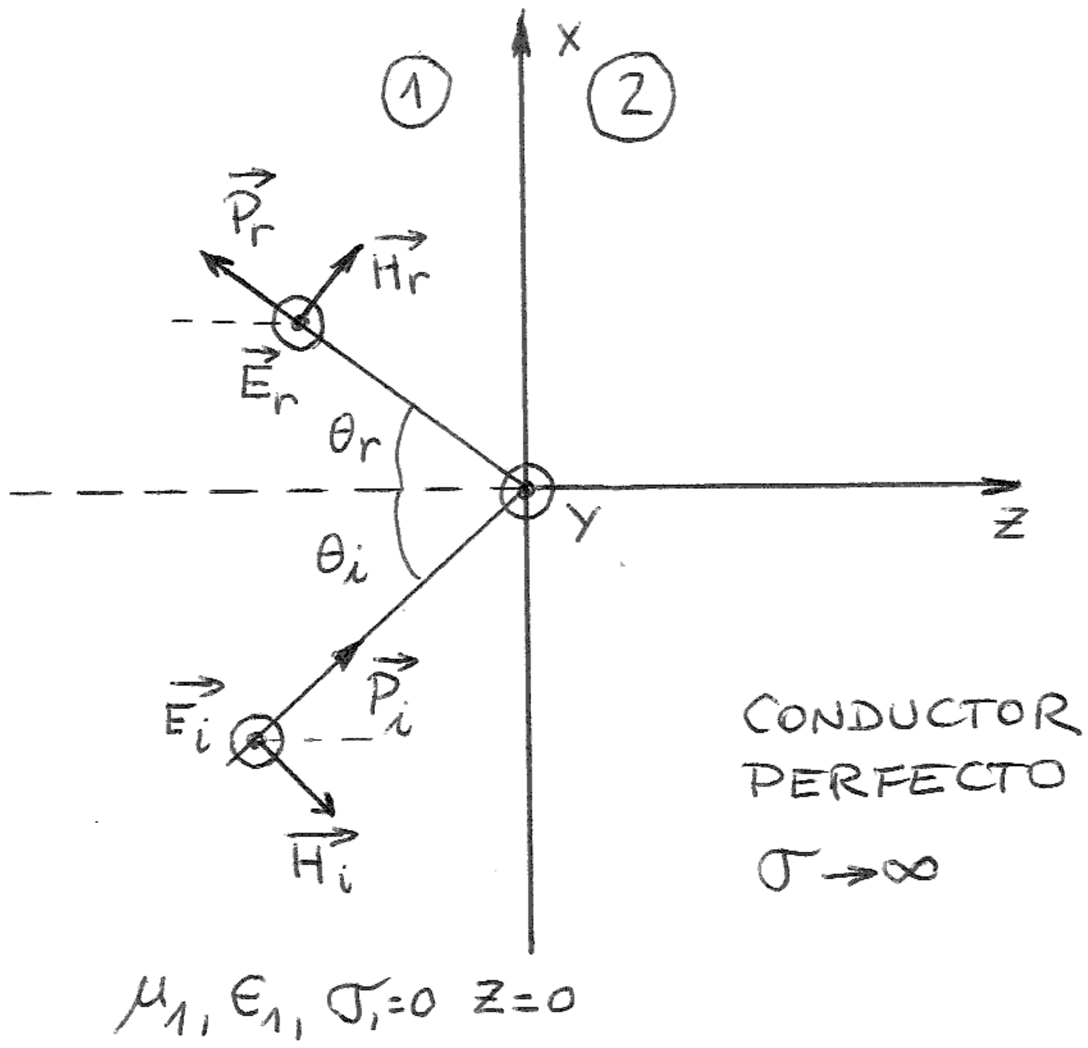
POLARIZACIÓN PERPENDICULAR

EL PLANO DE INCIDENCIA ESTÁ DEFINIDO POR $\vec{P}_i$ Y EL VERSOR NORMAL A LA INTERFAZ $\hat{n}$.

SE VAN A ESTUDIAR MEDIO 1: DIEL. PERFECTO Y MEDIO 2: CONDUCTOR. LUEGO EL CASO DE QUE AMBOS MEDIOS SEAN DIEL. PERFECTOS.

INCIDENCIA OBLICUA EN CONDUCTOR PERF

POLARIZACION PERPENDICULAR

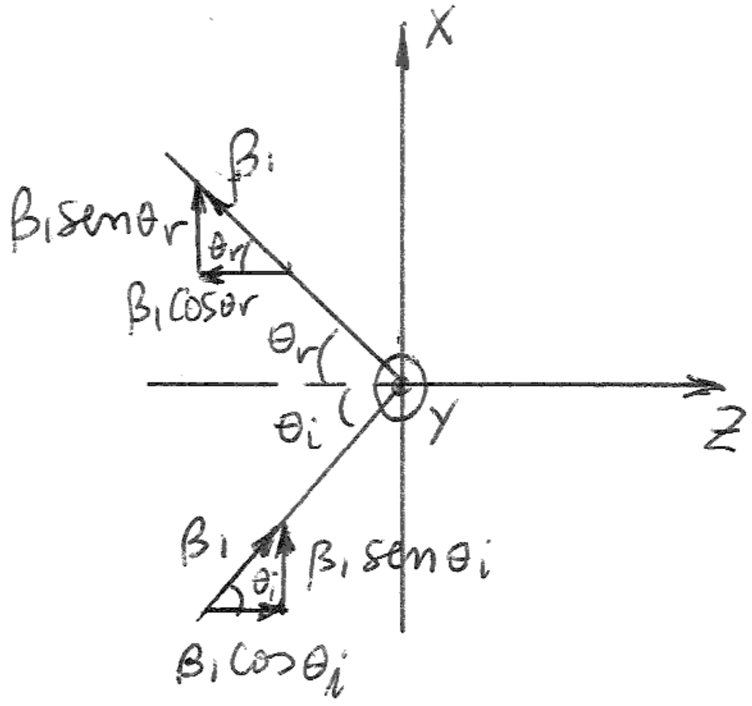


LOS VECTORES DE POYNTING SON:

$$\vec{P}_i = P_i \overbrace{\left(\hat{x} \operatorname{sen}\theta_i + \hat{z} \cos\theta_i\right)}^{\hat{a}_i}$$

$$\vec{P}_r = P_r \underbrace{\left(\hat{x} \operatorname{sen}\theta_r - \hat{z} \cos\theta_r\right)}_{\hat{a}_r}$$

$$\vec{P}_t = 0$$



RECORDANDO QUE EL CAMPO SE PUEDE ESCRIBIR COMO:

$$\vec{E_i} = \hat{y} E_{i_1} e^{-j\vec{k_i}\cdot\vec{r}}$$

ENTONCES

$$\vec{E_i} = \hat{y} E_{i_1} e^{-j\beta_1 \hat{a}_i \cdot \vec{r}} = \hat{y} E_{i_1} e^{-j\beta_1 (x\, sen\theta_i + z\cos\theta_i)}$$

$$\vec{H_i} = \frac{E_{i_1}}{Z_1} \cdot \left(-\hat{x}\cos\theta_i + \hat{z}\, sen\theta_i\right) \cdot e^{-j\beta_1 (x\, sen\theta_i + z\cos\theta_i)}$$

EL CAMPO MAGNÉTICO SE OBTIENE DE

$$\vec{H_i} = \frac{\nabla \times \vec{E_i}}{-j\omega\mu_1} = \frac{\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_Y & 0 \end{vmatrix}}{-j\omega\mu_1} = \frac{\hat{x}\left(-\partial E_Y/\partial z\right) + \hat{z}\left(\partial E_Y/\partial x\right)}{-j\omega\mu_1}$$

$$\vec{H_i} = \frac{1}{-j\omega\mu_1}\left[+j\beta_1 \cos\theta_i E_{i_1} e^{-j\beta_1 (x\, sen\theta_i + z\cos\theta_i)}\, \hat{x} + (-j\beta_1)\, sen\theta_i E_{i_1} e^{-j\beta_1 (x\, sen\theta_i + z\cos\theta_i)}\, \hat{z} \right]$$

$$\vec{H_i} = \frac{E_{i1}}{Z_1} e^{-j\beta_1 (x \operatorname{sen}\theta_i + z \cos\theta_i)} \cdot \left[-\hat{x}\cos\theta_i + \hat{z}\operatorname{sen}\theta_i\right]$$

EL CAMPO REFLEJADO

$$\vec{E_r} = \hat{y} E_{r_1} e^{-j\beta_1 (x \operatorname{sen}\theta_r - z\cos\theta_r)}$$

$$\vec{H_r} = \frac{\nabla \times \vec{E_r}}{-j\omega\mu_1} = \frac{\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}}{-j\omega\mu_1} = \frac{\hat{x}\left(\frac{-\partial E_y}{\partial z}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x}\right)}{-j\omega\mu_1}$$

$$\vec{H_r} = \frac{1}{-j\omega\mu_1}\Big[-j\beta_1 \cos\theta_r E_{r_1} e^{-j\beta_1 (x\operatorname{sen}\theta_r - z\cos\theta_r)}\hat{x} + (-j\beta_1)\operatorname{sen}\theta_r E_{r_1} e^{-j\beta_1(x\operatorname{sen}\theta_r - z\cos\theta_r)}\hat{z}\Big]$$

$$\vec{H_r} = \frac{E_{r1}}{Z_1}\left[\hat{x}\cos\theta_r + \hat{z}\operatorname{sen}\theta_r\right] e^{-j\beta_1 (x\operatorname{sen}\theta_r - z\cos\theta_r)}$$

APLICANDO LAS CONDICIONES DE CONTORNO

$z = 0 \qquad E_{tang\,1} = 0$

$$E_i(x\,z)\Big|_{z=0} + E_r(x\,z)\Big|_{z=0} = 0$$

$$E_{i1}\, e^{-j\beta_1 (x\operatorname{sen}\theta_i + \underbrace{\cancel{z\cos\theta_i}}_{=0})} + E_{r1}\, e^{-j\beta_1 (x\operatorname{sen}\overbrace{\theta_r}^{=\theta_i} + \underbrace{\cancel{z\cos\theta_r}}_{=0})} = 0$$

$$(E_{i1} + E_{r1})\, e^{-j\beta_1 x \operatorname{sen}\theta_i} = 0 \Rightarrow \boxed{E_{r1} = -E_{i1}}$$

EL CAMPO REFLEJADO SERÁ:

$$\vec{E}_r = -\hat{y}\, E_{i1}\, e^{-j\beta_1 (x\,\text{sen}\theta_i - z\cos\theta_i)}$$

$$\vec{H}_r = \frac{-E_{i1}}{Z_1}\left[\hat{x}\cos\theta_i + \hat{z}\,\text{sen}\theta_i\right] e^{-j\beta_1 (x\,\text{sen}\theta_i - z\cos\theta_i)}$$

EL CAMPO TOTAL EN EL MEDIO 1 SERÁ:

$$\vec{E}_1 = \hat{y}\, E_{i1}\, e^{-j\beta_1 (x\,\text{sen}\theta_i + z\cos\theta_i)} - \hat{y}\, E_{i1}\, e^{-j\beta_1 (x\,\text{sen}\theta_i - z\cos\theta_i)}$$

$$= \hat{y}\, E_{i1}\, e^{-j\beta_1 x\,\text{sen}\theta_i} \cdot \underbrace{\left[-e^{j\beta_1 z\cos\theta_i} + e^{-j\beta_1 z\cos\theta_i}\right]}_{-2j\,\text{sen}(\beta_1 z\cos\theta_i)}$$

$$\vec{E}_1 = -2j\,\text{sen}(\beta_1 z\cos\theta_i)\, E_{i1}\, e^{-j\beta_1 x\,\text{sen}\theta_i}\, \hat{y}$$

ANALOGAMENTE

$$\vec{H}_1 = -2\frac{E_{i1}}{Z_1}\left[\hat{x}\cos\theta_i\cos(\beta_1 z\cos\theta_i) + \hat{z}\, j\,\text{sen}\theta_i\,\text{sen}(\beta_1 z\cos\theta_i)\right] \cdot e^{-j\beta_1 x\,\text{sen}\theta_i}$$

LA DENSIDAD DE POT. PROMEDIO TEMPORAL.

$$\langle\vec{P}_1\rangle = \frac{1}{2}\,\text{Re}\left[\vec{E}_1 \times \vec{H}_1^*\right]$$

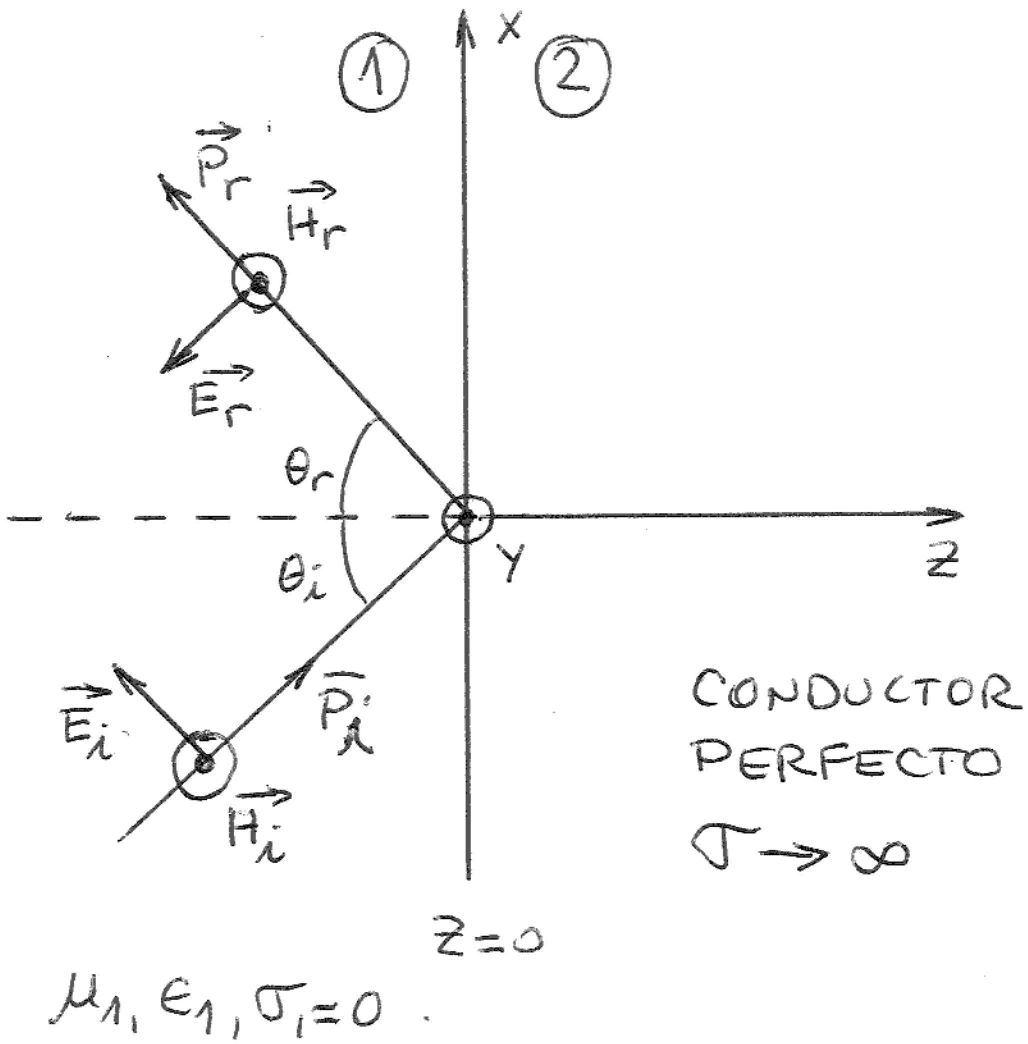
VERIFICAR RESULTADO

$$\boxed{\langle\vec{P}_1\rangle = \hat{x}\,\frac{2E_{i1}^2}{Z_1}\cdot\text{sen}^2(\beta_1 z\cos\theta_i)\,\text{sen}\,\theta_i}$$

SE OBTIENE QUE LA ONDA SE PROPAGA SOLAMENTE EN $\hat{x}$ Y EN $-\hat{z}$ SE TIENE UNA ONDA ESTACIONARIA PURA.

LA ONDA SE PROPAGA PARALELA A LA SUPERFICIE DEL CONDUCTOR, CONSTITUYE UNA "ONDA GUIADA" POR LA SUPERFICIE DEL CONDUCTOR.

POLARIZACION PARALELA



LOS VECTORES DE POYNTING SON IGUALES AL CASO DE POL. PERP.

$$\vec{P}_i = P_i \left(\underbrace{\hat{x}\,\mathrm{sen}\,\theta_i + \hat{z}\cos\theta_i}_{\hat{a}_i} \right)$$

$$\vec{P}_r = P_r \left(\underbrace{\hat{x}\,\mathrm{sen}\,\theta_r - \hat{z}\cos\theta_r}_{\hat{a}_r} \right)$$

$$\vec{P}_t = 0$$

LA FASE DE LOS CAMPOS $\vec{E_i}$ Y $\vec{E_r}$ SON IGUALES AL CASO DE POL. PERP.

$$\vec{E_i} = E_{i1}(\hat{x}\cos\theta_i - \hat{z}\,\mathrm{sen}\,\theta_i)\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i + z\cos\theta_i)}$$

$$\vec{H_i} = \hat{y}\,\frac{E_{i1}}{Z_1}\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i + z\cos\theta_i)}$$

LOS CAMPOS REFLEJADOS SERÁN

$$\vec{E_r} = E_{r1}(-\hat{x}\cos\theta_i - \hat{z}\,\mathrm{sen}\,\theta_i)\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i - z\cos\theta_i)}$$

$$\vec{H_r} = \hat{y}\,\frac{E_{r1}}{Z_1}\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i - z\cos\theta_i)}$$

AQUÍ SE USO $\theta_r = \theta_i$

APLICANDO LAS CONDICIONES DE CONTORNO:

$z=0 \qquad E_{tang_1} = 0$

$$E_{i_x}(xz)\Big|_{z=0} + E_{r_x}(xz)\Big|_{z=0} = 0$$

$$E_{i1}\,\hat{x}\cos\theta_i \cdot e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} + E_{r1}(-\hat{x}\cos\theta_i)\,e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} = 0$$

$$E_{i1} - E_{r1} = 0 \quad \Rightarrow \quad \boxed{E_{r1} = E_{i1}}$$

EL CAMPO TOTAL EN EL MEDIO 1 :

$$\vec{E_1} = \vec{E_i} + \vec{E_r}$$

$$= E_{i1}(\hat{x}\cos\theta_i - \hat{z}\,\mathrm{sen}\,\theta_i)\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i + z\cos\theta_i)}$$

$$- E_{i1}(\hat{x}\cos\theta_i + \hat{z}\,\mathrm{sen}\,\theta_i)\,e^{-j\beta_1(x\,\mathrm{sen}\,\theta_i - z\cos\theta_i)}$$

$$\vec{E}_1 = E_{i1} e^{-j\beta_1 x \,\mathrm{sen}\,\theta_i} \Big[(\hat{x}\cos\theta_i - \hat{z}\,\mathrm{sen}\,\theta_i) e^{-j\beta_1 z \cos\theta_i}$$

$$- (\hat{x}\cos\theta_i + \hat{z}\,\mathrm{sen}\,\theta_i) e^{+j\beta_1 z\cos\theta_i} \Big]$$

$$\vec{E}_1 = E_{i1} e^{-j\beta_1 x \,\mathrm{sen}\,\theta_i} \cdot \Big[-\hat{x}\cos\theta_i \, 2j\,\mathrm{sen}(\beta_1 z\cos\theta_i) \cdot$$

$$- \hat{z}\,\mathrm{sen}\,\theta_i \, 2\cos(\beta_1 z \cos\theta_i) \Big]$$

ADEMÁS:

$$\vec{H}_1 = \hat{y}\, 2\frac{E_{i1}}{Z_1} e^{-j\beta_1 x\,\mathrm{sen}\,\theta_i} \cdot \cos(\beta_1 z\cos\theta_i)$$

$$\langle P \rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*) = \hat{x}\,\frac{2E_{i1}^2}{Z_1}\cos^2(\beta_1 z\cos\theta_i)\,\mathrm{sen}\,\theta_i$$

SE OBTIENE UNA ONDA QUE SE PROPAGA EN $\hat{x}$ Y LA ONDA ESTACIONARIA ESTÁ EN $\hat{z}$