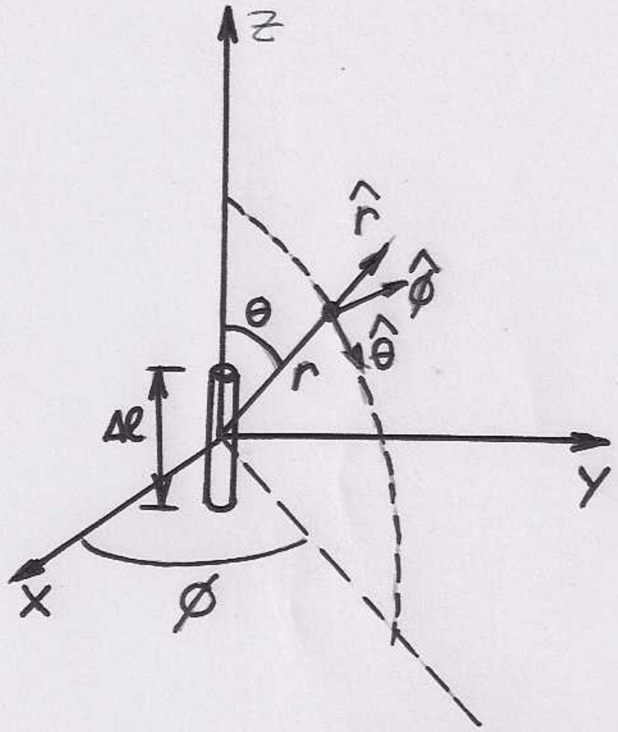
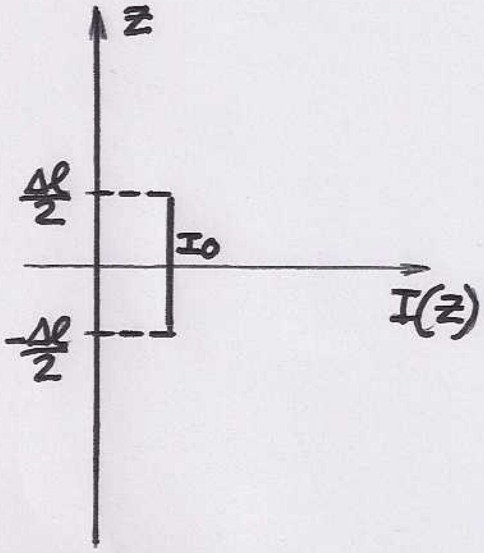
ELEMENTO DE CORRIENTE





DISTRIBUCION DE CORRIENTE.

$$\nabla^2 \vec{A} + \omega^2 \mu \epsilon \vec{A} = -\mu \vec{J}$$ EC. DIF. PARA UN DADO $\vec{J}$

$$\vec{A} = \mu \int_{V'} \frac{\vec{J} e^{-j\beta R}}{4\pi R} dv'$$

$$\vec{J} = I_0 \delta(x')\delta(y') \cdot \hat{z}$$

$$\vec{A} = \mu \iiint \frac{I_0 \delta(x')\delta(y') e^{-j\beta R} \cdot \hat{z}}{4\pi R} dx'dy'dz'$$

$$\vec{A} = \mu \int_{-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} \frac{I_0 e^{-j\beta R}}{4\pi R} \cdot dz' \hat{z}$$

COMO $\Delta l \ll \lambda$ y $\Delta l \ll R$

$$\vec{A} = \mu \frac{I_0 e^{-j\beta R}}{4\pi R} \cdot \Delta l \cdot \hat{z} = \mu \frac{I_0 e^{-j\beta r} \Delta l . \hat{z}}{4\pi r}$$

$\vec{A} = A_z \hat{z}$.

HAY QUE PASAR $\vec{A}$ COORD. CARTESIANAS A ESFERICAS.

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \operatorname{sen}\theta\cos\phi & \operatorname{sen}\theta\operatorname{sen}\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\operatorname{sen}\phi & -\operatorname{sen}\theta \\ -\operatorname{sen}\phi & \cos\phi & 0 \end{bmatrix} \cdot \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} A_z\cos\theta \\ -A_z\operatorname{sen}\theta \\ 0 \end{bmatrix} = \begin{bmatrix} \mu \dfrac{I_0 e^{-j\beta r}}{4\pi r}\Delta l\cos\theta \\ -\mu \dfrac{I_0 e^{-j\beta r}}{4\pi r}\Delta l\operatorname{sen}\theta \\ 0 \end{bmatrix}$$

COMO $\vec{B} = \nabla \times \vec{A}$ y $\vec{B} = \mu \vec{H}$

$$\vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$$

$$\cancel{\nabla \times \vec{A} = \frac{\hat{r}}{r\operatorname{sen}\theta}\left[\frac{\partial}{\partial\theta}(\underbrace{A_\phi}_{=0}\operatorname{sen}\theta) - \underbrace{\frac{\partial A_\theta}{\partial\phi}}_{=0}\right]} + \cancel{\frac{\hat{\theta}}{r}\left[\frac{1}{\operatorname{sen}\theta}\underbrace{\frac{\partial A_r}{\partial\phi}}_{=0} - \frac{\partial}{\partial r}(r\underbrace{A_\phi}_{=0})\right]}$$

$$+\frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$A = A(r\theta) \Rightarrow \dfrac{\partial A}{\partial\phi} = 0$

$A_\phi = 0$

$$\nabla \times \vec{A} = \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

1º término: $\dfrac{\hat{\phi}}{r}\dfrac{\partial}{\partial r}\left(\cancel{r}\left(\dfrac{-\mu I_0 e^{-j\beta r}}{4\pi \cancel{r}}\Delta l\operatorname{sen}\theta\right)\right)$

$$\frac{j\beta I_0 \mu e^{-j\beta r}\Delta l\operatorname{sen}\theta}{4\pi r}\cdot\hat{\phi}$$

2^o termino: $-\frac{\hat{\phi}}{r}\cdot\frac{\partial A_r}{\partial \theta} = -\frac{\hat{\phi}}{r}\cdot\frac{\partial}{\partial \theta}\left(\mu \frac{I_o e^{-j\beta r}}{4\pi r}\Delta l \cos\theta\right)$

$$\hat{\phi}\,\frac{\mu I_o e^{-j\beta r}\Delta l \,sen\theta}{4\pi r^2}$$

$$\vec{H} = j\beta \frac{I_o \cancel{\mu} e^{-j\beta r}}{\cancel{\mu}\, 4\pi r}\Delta l\, sen\theta + \frac{\cancel{\mu}}{\cancel{\mu}}\frac{I_o e^{-j\beta r}}{4\pi r^2}\Delta l\, sen\theta \quad \hat{\phi}$$

$$\boxed{\vec{H} = \frac{I_o e^{-j\beta r}\Delta l\, sen\theta}{4\pi r}\left(j\beta + \frac{1}{r}\right)\hat{\phi}}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{E} = \frac{\nabla \times \vec{H}}{j\omega\epsilon}$$

$$\nabla \times \vec{H} = \frac{\hat{r}}{r\,sen\theta}\left[\frac{\partial}{\partial\theta}\left(H_\phi\, sen\theta\right) - \cancel{\left(\frac{\partial H_\theta}{\partial \phi}\right)}\right]$$

$$+\frac{\hat{\theta}}{r}\left[\cancel{\frac{1}{sen\theta}\frac{\partial H_r}{\partial\phi}} - \frac{\partial}{\partial r}\left(r H_\phi\right)\right]$$

$$+\frac{\hat{\phi}}{r}\cdot\left[\cancel{\frac{\partial}{\partial r}\left(r H_\theta\right)} - \cancel{\frac{\partial H_r}{\partial\theta}}\right]$$

$$\nabla \times \vec{H} = \frac{\hat{r}}{r\,sen\theta}\cdot\left(\frac{\partial}{\partial\theta}\left(H_\phi\, sen\theta\right)\right) - \frac{\hat{\theta}}{r}\cdot\frac{\partial}{\partial r}\left(r H_\phi\right)$$

$$\nabla \times \vec{H} = \frac{\hat{r}}{r\,sen\theta}\cdot\frac{\partial}{\partial\theta}\left(\frac{I_o e^{-j\beta r}\Delta l\, sen^2\theta}{4\pi r}\left(j\beta + \frac{1}{r}\right)\right)$$

$$-\frac{\hat{\theta}}{r}\frac{\partial}{\partial r}\left(\frac{\cancel{r} I_o e^{-j\beta r}\Delta l\, sen\theta}{4\pi \cancel{r}}\cdot\left(j\beta + \frac{1}{r}\right)\right)$$

$$\nabla \times \vec{H} = \frac{\hat{r}}{r\cancel{sen\theta}} \cdot \frac{I_o e^{-j\beta r} \Delta l}{4\pi r}\left(j\beta + \frac{1}{r}\right) 2\cancel{sen\theta} \cdot cos\theta$$

$$- \frac{\hat{\theta}}{r} \frac{\partial}{\partial r}\left(\frac{I_o e^{-j\beta r} \Delta l \, sen\theta}{4\pi} \cdot \left(j\beta + \frac{1}{r}\right)\right)$$

$$\nabla \times \vec{H} = \hat{r} \frac{I_o e^{-j\beta r} \Delta l}{4\pi r^2}\left(j\beta + \frac{1}{r}\right) 2 cos\theta$$

$$- \frac{\hat{\theta}}{r} \frac{\partial}{\partial r} \frac{I_o \Delta l \, sen\theta}{4\pi}\left(j\beta(-j\beta) e^{-j\beta r} + \frac{(-j\beta) e^{-j\beta r}}{r} + e^{-j\beta r}\left(-\frac{1}{r^2}\right)\right)$$

$$\nabla \times \vec{H} = \hat{r} \frac{I_o e^{-j\beta r} \Delta l}{4\pi r^2}\left(j\beta + \frac{1}{r}\right) 2 cos\theta$$

$$- \hat{\theta} \frac{I_o \Delta l \, sen\theta}{4\pi r} \cdot \left(\beta^2 e^{-j\beta r} - \frac{j\beta e^{-j\beta r}}{r} - \frac{e^{-j\beta r}}{r^2}\right)$$

$$\nabla \times \vec{H} = \hat{r} \frac{I_o e^{-j\beta r} \Delta l}{4\pi r^2}\left(j\beta + \frac{1}{r}\right) 2 cos\theta$$

$$+ \hat{\theta} \cdot \frac{I_o \Delta l \, sen\theta \, e^{-j\beta r}}{4\pi r} \cdot \left(-\beta^2 + \frac{j\beta}{r} + \frac{1}{r^2}\right)$$

$$\vec{E} = \frac{\nabla \times \vec{H}}{j\omega\epsilon}$$

$$\vec{E} = \frac{\hat{r}.\, I_0 e^{-j\beta r} \Delta l}{j\omega\epsilon\, 4\pi r^2} \cdot \left(j\beta + \frac{1}{r} \right) 2\cos\theta$$

$$+ \frac{\hat{\theta}\, I_0 e^{-j\beta r} \Delta l}{j\omega\epsilon\, 4\pi r} \cdot \left(-\beta^2 + \frac{j\beta}{r} + \frac{1}{r^2} \right) \operatorname{sen}\theta$$

$$\frac{\beta}{\omega\epsilon} = \frac{\cancel{\omega}}{c} \frac{1}{\cancel{\omega}\epsilon} = \frac{1}{\frac{\epsilon}{\sqrt{\mu\epsilon}}} = \frac{1}{\sqrt{\frac{\epsilon}{\mu}}} = \sqrt{\frac{\mu}{\epsilon}} = Z_{00}$$

$$\boxed{\begin{aligned} \vec{E} = {} & \hat{r}\, \frac{I_0 e^{-j\beta r} \Delta l . 2\cos\theta}{4\pi} \cdot \left(\frac{Z_{00}}{r^2} + \frac{1}{j\omega\epsilon r^3} \right) \\ & + \hat{\theta}\, \frac{I_0 e^{-j\beta r} \Delta l \operatorname{sen}\theta}{4\pi} \left(\frac{j\omega\mu}{r} + \frac{Z_{00}}{r^2} + \frac{1}{j\omega\epsilon r^3} \right) \end{aligned}}$$

$$\frac{-\beta^2}{j\omega\epsilon} = -\left(\frac{\omega}{c}\right)^2 \cdot \frac{1}{j\omega\epsilon} = \frac{-\omega^{\cancel{2}}}{\frac{1}{\mu\cancel{\epsilon}} \cancel{j\omega\epsilon}} = j\omega\mu$$

A GRANDES DISTANCIAS DE LA FUENTE

$$(E) \quad \frac{j\omega\mu}{r} \gg \frac{Z_{00}}{r^2} \Rightarrow \frac{j\omega\mu}{r} \gg \sqrt{\frac{\mu}{\epsilon}} \frac{1}{r^2} \Rightarrow \boxed{\beta r \gg 1}$$

$$(H) \quad j\beta \gg \frac{1}{r} \Rightarrow j\beta r \gg 1 \qquad \boxed{\beta r \gg 1}$$

$\boxed{\beta r \gg 1}$ CONDICION DE CAMPO LEJANO

A GRANDES DISTANCIAS DE LA FUENTE $(\beta r \gg 1)$

$$\vec{H} = \frac{I_o e^{-j\beta r} \Delta l \,\text{sen}\theta}{4\pi r}\left(j\beta + \frac{1}{r}\right)\hat{\phi} \cong \frac{I_o e^{-j\beta r} \Delta l \,\text{sen}\theta \, j\beta}{4\pi r}\hat{\phi}$$

$$\vec{E} \cong \frac{I_o e^{-j\beta r} \Delta l \,\text{sen}\theta}{4\pi} \cdot \frac{j\omega\mu}{r}\hat{\theta}$$

CAMPOS DE RADIACIÓN

$$\vec{H} = H_\phi \hat{\phi} \qquad \vec{E} = E_\theta \hat{\theta}$$

E Y H ESTAN EN FASE

$$Z = \frac{E}{H} = \frac{\omega\mu}{\beta} = \frac{\cancel{\omega}\mu}{\cancel{\omega}\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\boxed{Z = Z_{oo} = 120\pi\,\Omega}$$

IMPEDANCIA INTRÍNSECA DEL VACÍO.

$$\langle\vec{P}\rangle = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)$$

$$\langle\vec{P}\rangle = \frac{1}{2}\mathrm{Re}\left(E_\theta\hat{\theta}\times H_\phi^*\hat{\phi}\right)$$

$$\langle\vec{P}\rangle = \frac{1}{2}\mathrm{Re}\left(E_\theta H_\phi^*\right)\underbrace{\left(\hat{\theta}\times\hat{\phi}\right)}_{\hat{r}}$$

$$\langle\vec{P}\rangle = \frac{1}{2}\mathrm{Re}\left(\frac{I_o e^{-j\beta r}\Delta l\,\text{sen}\theta}{4\pi}\frac{j\omega\mu}{r}\cdot\left(\frac{I_o e^{-j\beta r}\Delta l\,\text{sen}\theta\, j\beta}{4\pi r}\right)^*\right)\hat{r}$$

$$\langle\vec{P}\rangle = \frac{1}{2}\mathrm{Re}\left(\frac{I_o^2\Delta l^2\text{sen}^2\theta\, j\omega\mu(-j\beta)}{(4\pi r)^2}\right)\hat{r}$$

$$\langle\vec{P}\rangle = \frac{1}{2}\frac{I_o^2\Delta l^2\text{sen}^2\theta}{(4\pi r)^2}\cdot\omega\mu\beta\hat{r} = \frac{1}{2}\frac{I_o^2\Delta l^2\text{sen}^2\theta}{(4\pi r)^2}\cdot\beta^2 Z_{oo}\hat{r}$$

$$\omega\mu\beta = \omega\mu\,\omega\sqrt{\mu\epsilon} = \underbrace{\omega^2\mu\epsilon}_{\beta^2}\sqrt{\mu\epsilon}\,\frac{1}{\epsilon} = \beta^2\sqrt{\frac{\mu}{\epsilon}} = \beta^2 Z_{oo}$$

$$\boxed{\langle\vec{P}\rangle = \frac{I_o^2\Delta l^2\text{sen}^2\theta\,\beta^2 Z_{oo}\hat{r}}{32 r^2\pi^2}}$$

A PEQUEÑAS DISTANCIAS. $(\beta r \ll 1)$

$$\vec{H} = \frac{I_o e^{-j\beta r} \Delta l \, \text{sen}\theta}{4\pi r}\left(j\beta + \frac{1}{r}\right)\hat{\phi}$$

$$\vec{H} = \frac{I_o e^{-j\beta r} \Delta l \, \text{sen}\theta}{4\pi r}\frac{1}{r}\hat{\phi} = \frac{I_o e^{-j\beta r} \Delta l \, \text{sen}\theta}{4\pi r^2}\cdot\hat{\phi}$$

$$\vec{E} = \frac{\hat{r}\, I_o e^{-j\beta r} \Delta l \, 2\cos\theta}{4\pi}\cdot\frac{1}{j\omega\epsilon r^3} + \frac{\hat{\theta}\, I_o e^{-j\beta r} \Delta l \, \text{sen}\theta}{4\pi}\,\frac{1}{j\omega\epsilon r^3}$$

$$\vec{E} = -j\hat{r}\,\frac{I_o e^{-j\beta r} \Delta l \, 2\cos\theta}{4\pi\omega\epsilon r^3} - j\hat{\theta}\,\frac{I_o e^{-j\beta r} \Delta l \, \text{sen}\theta}{4\pi\omega\epsilon r^3}$$

$$\langle\vec{P}\rangle = \frac{1}{2}\text{Re}\left(\vec{E}\times\vec{H}^*\right) = 0.$$

EN CAMPO CERCANO.
LA ENERGÍA ACTIVA ES CERO.

$$\frac{1}{2}\text{Im}\left(\vec{E}\times\vec{H}^*\right) = \frac{1}{2}\,\frac{I_o^2 \Delta l^2\, 2\cos\theta\,(-j)\,\text{sen}\theta}{4\pi\omega\epsilon r^3 \; 4\pi r^2}\cdot\overbrace{\left(\hat{r}\times\hat{\phi}\right)}^{-\hat{\theta}}$$

$$+\frac{1}{2}\,\frac{I_o^2 \Delta l^2\, \text{sen}^2\theta\,(-j)}{4\pi\omega\epsilon r^3 \; 4\pi r^2}\cdot\underbrace{\left(\hat{\theta}\times\hat{\phi}\right)}_{\hat{r}}$$

HABRA ENERGÍA REACTIVA

E Y H ESTAN EN CUADRATURA

AQUI LA ENERGÍA ES ALMACENADA COMO EN UN DISPOSITIVO REACTIVO.

POTENCIA TOTAL RADIADA

CAMPOS DE RADIACION O LEJANO.

$$W = \oint_S \langle \vec{P} \rangle \cdot \vec{ds} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} P.\ r^2 \operatorname{sen}\theta\, d\theta\, d\phi \qquad \hat{r} \parallel \hat{ds}$$

$$W = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{I_0^2 \Delta l^2 \operatorname{sen}^2\theta\, \beta^2 Z_{00}}{32 \cancel{r^2} \pi^2} \cancel{r^2} \operatorname{sen}\theta\, d\theta\, d\phi$$

$$W = \frac{2\pi\ I_0^2 \Delta l^2 \beta^2 Z_{00}}{32\pi^2} \cdot \int_{\theta=0}^{\pi} \operatorname{sen}^3\theta\, d\theta$$

$$\int_0^{\pi} \operatorname{sen}^3\theta\, d\theta = \left(-\cos\theta + \frac{\cos^3 x}{3}\right)\Bigg|_0^{\pi} = 1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) = 2 - \frac{2}{3} = \frac{4}{3}$$

$$W = \frac{\cancel{2}\pi\ I_0^2 \Delta l^2 \beta^2 Z_{00}}{\cancel{32}\ \pi^2} \cdot \frac{\cancel{4}}{3} \qquad \cancel{16}\ 4$$

$$\beta^2 = \left(\frac{\omega}{c}\right)^2 = \frac{(2\pi f)^2}{\left(\frac{1}{\sqrt{\mu\varepsilon}}\right)^2}$$

$$\beta^2 = \left(\frac{\omega}{c}\right)^2 = \left(\frac{2\pi f}{c}\right)^2 = \left(\frac{2\pi \cancel{c}}{\cancel{c}\,\lambda}\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

$$W = \frac{\pi I_0^2 \Delta l^2 \cancel{4\pi^2} Z_{00}}{3 \cdot \cancel{4} \cdot \lambda^2 \cancel{\pi^2}} = \frac{\pi I_0^2 \Delta l^2\ 120\pi\,\Omega}{3\lambda^2}$$

$$\boxed{W = I_0^2\, 40\pi^2 \left(\frac{\Delta l}{\lambda}\right)^2 (\Omega)} \qquad \frac{I_0}{\sqrt{2}} = I_{ef}$$

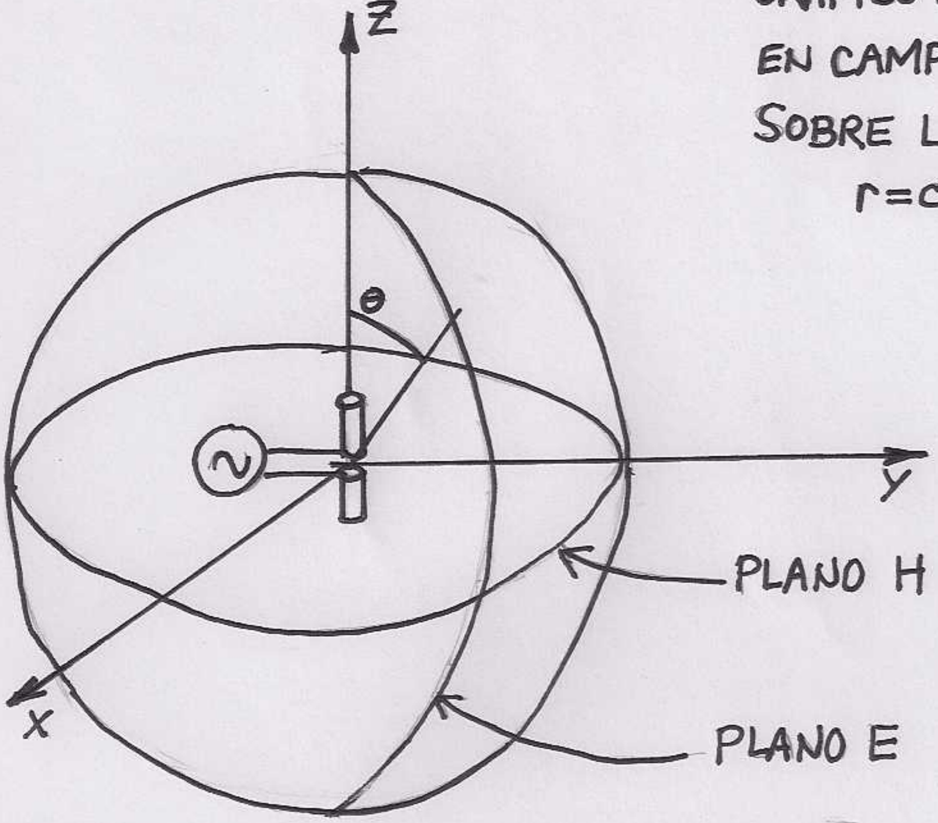
$$W = 2 I_{ef}^2\, 40\pi^2 \left(\frac{\Delta l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{\Delta l}{\lambda}\right)^2 \cdot I_{ef}^2 = R_{rad} \cdot I_{ef}^2$$

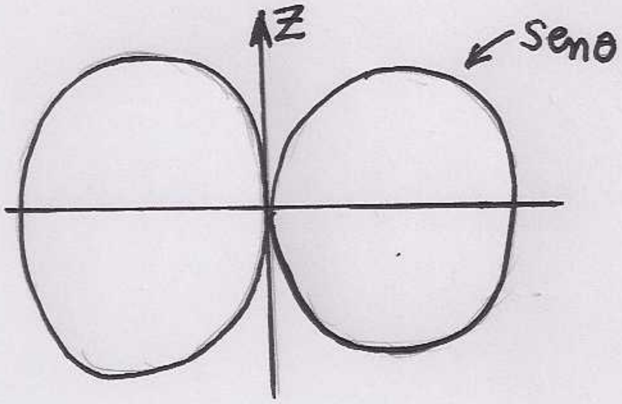
$$\boxed{R_{rad} = 80\pi^2 \left(\frac{\Delta l}{\lambda}\right)^2}$$

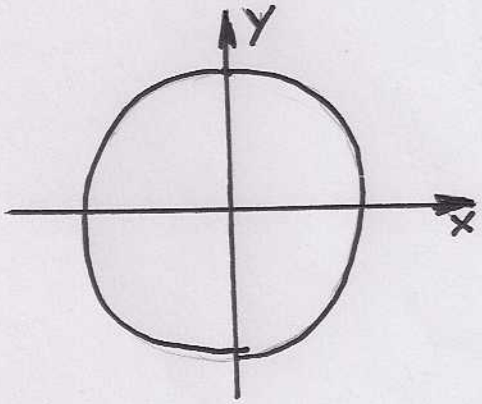
RESISTENCIA DE RADIACION DEL ELEMENTO DE CORRIENTE.

DIAGRAMA DE RADIACION

GRAFICO DE $E(\theta,\phi)$
EN CAMPO LEJANO
SOBRE LA ESFERA
$r=cte$

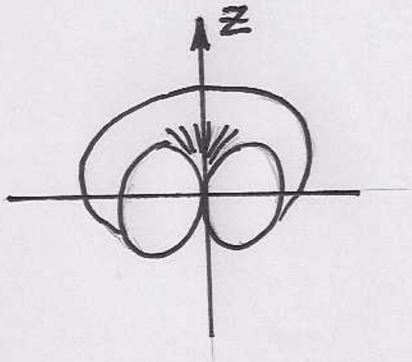






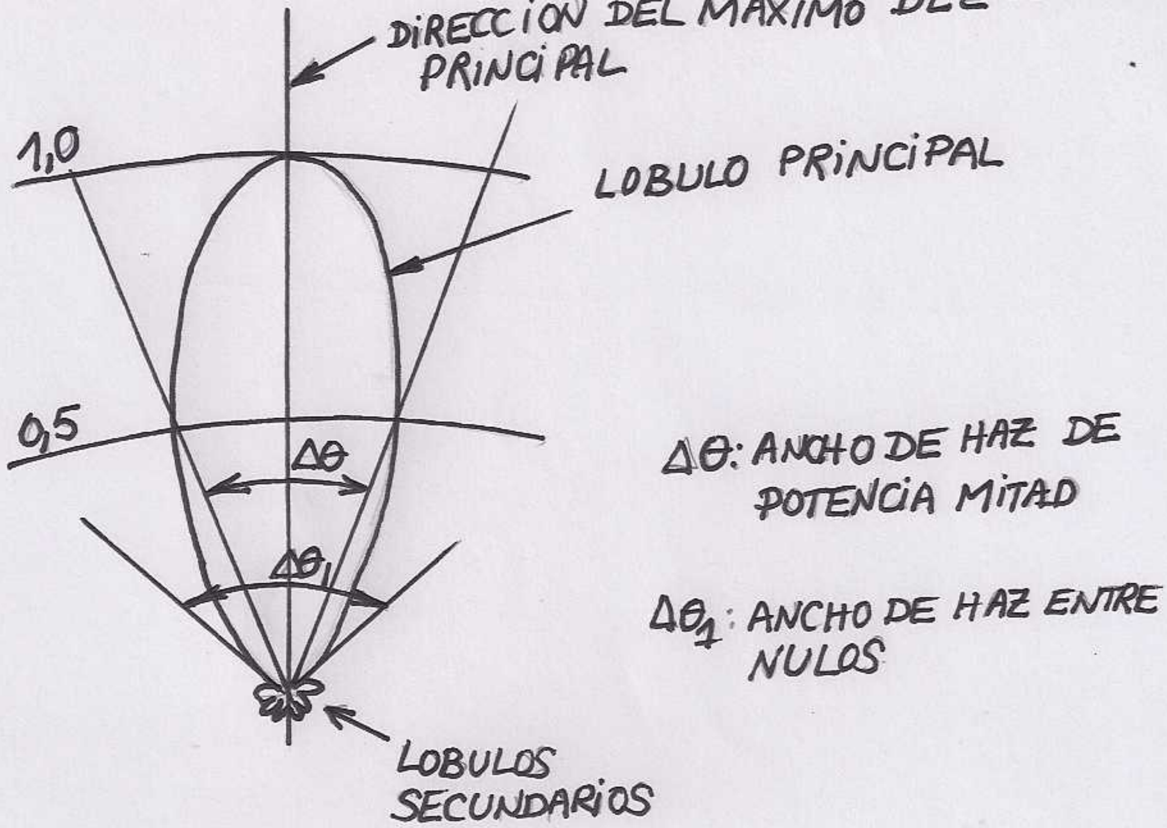
PLANO E

PLANO H



E y H SE LLAMAN
PLANOS PRINCIPALES

DIAGRAMA EN POTENCIA



$\Delta\theta$: ANCHO DE HAZ DE POTENCIA MITAD

$\Delta\theta_1$: ANCHO DE HAZ ENTRE NULOS

$$E = f(\theta) \cdot cte_1 \qquad \text{(CAMPO)}$$

$$W = f^2(\theta)\, cte_2 = F(\theta)\, cte_2 \qquad \text{(POTENCIA)}$$

IMPORTANTE

EL DIAGRAMA EN CAMPO Y POTENCIA EN DB ES IGUAL