MAD-HTLC: Because HTLC is Crazy-Cheap to Attack

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Abstract—Smart Contracts and transactions allow users to implement elaborate constructions on cryptocurrency blockchains like Bitcoin and Ethereum. Many of these constructions, including operational payment channels and atomic swaps, use a building block called Hashed Time-Locked Contract (HTLC).

In this work, we distill from HTLC a specification (HTLC-Spec), and present an implementation called Mutual-Assured-Destruction Hashed Time-Locked Contract (MAD-HTLC). MAD-HTLC employs a novel approach of utilizing the existing blockchain operators, called miners, as part of the design. If a user misbehaves, MAD-HTLC incentivizes the miners to confiscate all her funds. We prove MAD-HTLC's security using the UC framework and game-theoretic analysis. We demonstrate MAD-HTLC's efficacy and analyze its overhead by instantiating it on Bitcoin's and Ethereum's operational blockchains.

Notably, current miner software makes only little effort to optimize revenue, since the advantage is relatively small. However, as the demand grows and other revenue components shrink, miners are more motivated to fully optimize their fund intake. By patching the standard Bitcoin client, we demonstrate such optimization is easy to implement, making the miners natural enforcers of MAD-HTLC.

Finally, we extend previous results regarding HTLC vulnerability to bribery attacks. An attacker can incentivize miners to prefer her transactions by offering high transaction fees. We demonstrate this attack can be easily implemented by patching the Bitcoin client, and use game-theoretic tools to qualitatively tighten the known cost bound of such bribery attacks in presence of rational miners. We identify bribe opportunities occurring on the Bitcoin and Ethereum main networks where a few dollars bribe could yield tens of thousands of dollars in reward (e.g., \$2 for over \$25K).

I. INTRODUCTION

Blockchain-based cryptocurrencies like Bitcoin [1] and Ethereum [2] are monetary systems with a market cap of \$400B [3]. They enable simple *transactions* of internal tokens and implementation of more elaborate *smart contracts*. The transactions create the smart contracts and interact with them. Entities called *miners* create data structures called *blocks* that contain transactions. They publish and order the blocks to form a blockchain, thus *confirming* the included transactions and achieving system progress. The system state is obtained by parsing the transactions according to the block order. Blockchain security relies on *incentives*, rewarding miners with tokens for carrying out their tasks.

A prominent smart-contract design pattern is the *Hashed Time-Locked Contract* (HTLC), set up for two participants, Alice, \mathcal{A} , and Bob, \mathcal{B} (§II). It asserts that \mathcal{A} gets tokens for presenting a hash preimage of a specific value before a certain timeout, otherwise \mathcal{B} gets them. A variety of more

elaborate smart-contract designs rely on *HTLC* as a building block. These include high-frequency payment channels [4]–[10], atomic swaps [11]–[15], contingent payments [16]–[20], and vaults [21]–[24]. We identify the specification required by the variety of contracts using *HTLC* and call it *HTLC-Spec*.

Unfortunately, HTLC is vulnerable to incentive manipulation attacks [25]–[27]. Winzer et al. [28] showed that \mathcal{B} can bribe miners using designated smart contracts to ignore \mathcal{A} 's transactions until the timeout elapses. Similarly, Harris and Zohar [29] show that \mathcal{B} can delay \mathcal{A} 's transaction confirmation by overloading the system with his own transactions. Both of these allow \mathcal{B} to obtain the HTLC tokens while depriving \mathcal{A} of them, even if \mathcal{A} published the preimage.

In this work, we provide a secure implementation of *HTLC-Spec*, and further analyze *HTLC*'s susceptibility to bribery.

We begin by describing the model (§III) for an underlying blockchain mechanism like that of Bitcoin or Ethereum. The system's state is a set of contracts; each contract comprises a token amount and a predicate; transactions redeem contract tokens by providing inputs that satisfy their predicates. Users publish transactions initiating new contracts, assigning them with the redeemed tokens while also offering some as fees. In each round one miner adds a block with a transaction to the chain and receives its fee.

We proceed to present MAD-HTLC, our HTLC-Spec implementation (§IV). MAD-HTLC relies on the fact that miners can participate in a smart contract execution, and thus their interests should be taken into account. MAD-HTLC utilizes miners as enforcers of its correct execution, allowing and incentivizing them to seize its contract tokens in case of any bribery attempt. That, in turn, incentivizes \mathcal{A} and \mathcal{B} to refrain from such attempts and to interact with MAD-HTLC as intended. To the best of our knowledge, this is the first work to utilize miner incentives in this way.

In addition to the preimage specified by HTLC-Spec, which we denote pre_a , MAD-HTLC uses a second preimage, pre_b , known only to \mathcal{B} . MAD-HTLC comprises a main deposit contract (MH-Dep) and an auxiliary collateral contract (MH-Col), which work as follows. MH-Dep has three so-called $redeem\ paths$. First, it allows \mathcal{A} to redeem it with a transaction including the predefined preimage, pre_a . Alternatively, it allows \mathcal{B} to redeem it after a timeout with a transaction including the other preimage, pre_b . This is essentially the specification, but MH-Dep provides another option, allowing any user, and specifically any miner, to redeem it herself with a transaction including both pre_a and pre_b .

Now, if both \mathcal{A} and \mathcal{B} try to redeem MH-Dep, then their transactions must reveal preimages pre_a and pre_b , respectively. Any miner can then simply take these preimages and issue her own transaction that uses the third redeem path to seize the tokens for herself. Specifically, if \mathcal{A} tries to redeem MH-Dep, then \mathcal{B} is assured that she cannot do so – if she tries to redeem the tokens then the miners would get them instead. Assuming \mathcal{B} is benign, i.e., rational but prefers to act honestly for the same reward, then this construction is sufficient to satisfy HTLC-Spec. But we can do better.

If \mathcal{B} is spiteful, then he will prefer to reduce \mathcal{A} 's reward if it does not affect his. When \mathcal{A} knows pre_a and tries to redeem MH-Dep, \mathcal{B} cannot redeem it as well, but he can publish a redeeming transaction nonetheless allowing the miners to collect the tokens instead of \mathcal{A} .

We strengthen MAD-HTLC such that \mathcal{B} is strictly incentivized to refrain from such deviant behavior with the auxiliary contract MH-Col. It can be redeemed only after the same timeout as MH-Dep, either by a transaction of \mathcal{B} , or by any miner that provides both pre_a and pre_b . Now, if \mathcal{A} knows pre_a then she can redeem MH-Dep and \mathcal{B} can redeem MH-Col. If instead \mathcal{B} contends with \mathcal{A} for MH-Dep, both still lose the MH-Dep for the miners; but now both pre_a and pre_b are revealed, allowing miners to seize the MH-Col tokens as well. \mathcal{B} is therefore strictly incentivized not to contend, allowing \mathcal{A} to receive the MH-Dep tokens as required.

This means the *MAD-HTLC* construction is secure against the known incentive manipulation attacks [25]–[29] – \mathcal{B} cannot incentivize miners to exclude \mathcal{A} 's transaction and getting his confirmed instead.

MAD-HTLC utilizes the mutual assured destruction [30], [31] principle: If a party misbehaves then all parties lose everything. Although penalizing the well-behaved party as well, this mechanism design [32] technique ensures rational players act as intended.

To prove the security of MAD-HTLC (§V), we first bound the possible leakage and interactions of system entities using the UC framework [33]. These interactions include the setup, initiation and redeeming of MAD-HTLC. Then, we formalize MAD-HTLC as a game played by \mathcal{A} , \mathcal{B} and the miners, where the action space comprises the aforementioned possible interactions. We model all parties as rational non-myopic players, and show the prescribed behavior is *incentive-compatible* [34].

We prove the efficacy of *MAD-HTLC* by implementing it both in the less expressive Bitcoin Script [35] and in the richer Ethereum Solidity [36] smart-contract languages (§VI). We deploy it on Bitcoin's and Ethereum's main networks, and show it bears negligible overhead (e.g., 2.2e-6 BTC) compared to the secured amount (e.g., 2.6 BTC). Specifically for payment-channels [4]–[10], this negligible overhead is only incurred in the abnormal case of a dispute.

MAD-HTLC relies on miners non-myopically optimizing their transaction choices, often referred to as *Miner Extractable Value (MEV)* [37]–[39]. While such optimizations are common in the Ethereum network, as of today, Bitcoin's default cryptocurrency client only offers basic optimization.

Changes in miners' revenue structure will make better optimizations more important. To demonstrate miners can easily enhance transaction choice optimization once they choose to do so, we patch the standard Bitcoin client [40] to create *Bitcoin-MEV infrastructure*, allowing for easy additions of elaborate logic. In particular, we implement the logic enabling miners to benefit from enforcing the correct execution of *MAD-HTLC*.

We then revisit the security of the prevalent HTLC implementation and refine previous results [28] regarding its vulnerability to bribing attacks (§VII). We show that HTLC is vulnerable even in blockchains with limited Script-like languages, and bribe can be made using the built-in transaction fee mechanism. We analyze miners' behavior as a game for the HTLC timeout duration. Each suffix of the game can be analyzed as a subgame, and all players have perfect knowledge of the system state. \mathcal{B} can take advantage of this setting to incentivize miners to withhold \mathcal{A} 's transaction until the timeout, making this the single $subgame\ perfect\ equilibrium\ [41]$. So, in presence of rational non-myopic miners the required bribe cost is independent of the timeout duration. This matches the lower bound, and qualitatively tightens the exponential-intimeout upper bound, both presented by Winzer et al. [28].

In our Bitcoin-compatible attack variation, miners only have to be non-myopic for the attack to succeed, a simple optimization we implement by patching the standard Bitcoin client with merely 150 lines of code. We identify several potential bribe opportunities on the Bitcoin and Ethereum main networks, including examples of a few dollars bribe would have yielded tens of thousands of dollars in payout (e.g., payment channel where bribe of \$2 could have yielded payout of \$25K).

We conclude by discussing future directions (\S VIII), including attacks and mitigations in a weaker model where \mathcal{A} or \mathcal{B} have mining capabilities, and using MAD-HTLC to reduce latency in systems using HTLC-Spec.

In summary, we make the following contributions:

- We formalize the specification *HTLC-Spec* of the prevalent *HTLC* contract;
- present MAD-HTLC that satisfies HTLC-Spec utilizing miners as participants;
- prove MAD-HTLC is secure and incentive compatible;
- implement, deploy, and evaluate MAD-HTLC on the Bitcoin and Ethereum main networks;
- patch the prevalent Bitcoin Client to create Bitcoin-MEV infrastructure, and to specifically support enforcing correct MAD-HTLC execution;
- prove HTLC is vulnerable to bribery attacks in limited smart-contract environments; and
- qualitatively tighten the bound of Winzer et al. [28] and implement the required rational miner behavior.

Open Source and Responsible Disclosure: We completed a responsible-disclosure process with prominent blockchain development groups. We intend to open source our code, subject to security concerns of the community.

II. RELATED WORK

We are not aware of prior work utilizing miners' incentives to use them as enforcers of correct smart contract execution. We review previous work on bribing attacks in blockchains (§II-A), detail exhibited and postulated mining behavior with respect to transaction selection (§II-B), and present systems and applications using *HTLC-Spec* (§II-C).

A. Bribery Attacks

Winzer et al. [28] present attacks that delay confirmation of specific transactions until a given timeout elapses. Their attacks apply to HTLC where \mathcal{B} delays the confirmation of \mathcal{A} 's redeeming transaction until he can redeem it himself. Their presented attack requires predicates available only in rich smart contract languages like Ethereum's Solidity [36], [42] and Libra's Move [43], [44], but not Bitcoin's Script [35]. Specifically, the attack requires setting a *bribe contract* that monitors what blocks are created and rewards participants accordingly.

In contrast, our attack variation works with Bitcoin's Script as well, as we demonstrate by implementation. It therefore applies a wider range of systems [45]–[47].

Winzer et al. [28] present two results regarding the attack costs. First, they show that \mathcal{B} 's attack cost for making miner's collaboration with the attack a Nash-equilibrium grows linearly with the *size* (i.e., the relative mining capabilities) of the smallest miner. However, all miners not cooperating with the attack is also a Nash equilibrium. Therefore, they analyze \mathcal{B} 's cost for making the attack a dominant strategy, i.e., to incentivize a to support the attack irrespective of the other miners' strategies. This bound grows linearly with relative miner sizes, and exponentially with the *HTLC* timeout.

Our analysis improves this latter bound by taking account the miners all know the system state and each others' incentives. This insight allows us to use the *subgame perfect equilibrium* [31], [34], [41], [48]–[54] solution concept, a refinement of Nash-equilibrium suitable for games of dynamic nature. We consider the game played by non-myopic rational participants aware of the game dynamics, and show that a linear-in-miner-size cost (as in [28]) suffices for the existence of a *unique* subgame perfect equilibrium.

Other work [25]–[27] analyzes bribing attacks on the consensus mechanism of cryptocurrency blockchains. Unlike this work, bribes in these papers compete with the total block reward (not just a single transaction's fee) and lead miners to violate predefined behavior. These attacks are therefore much more costly and more risky than the bribery we consider, where a miner merely prioritizes transactions for confirmation.

A recent and parallel work [55] also suggests using Bitcoin's fee mechanism to attack *HTLC*. It assumes miners below a certain hash-rate threshold are myopic (sub-optimal) while those above it are non-myopic; it presents safe timeout values given Bitcoin's current hash-rate distribution. In this work, we assume all miners are non-myopic and prove that in this model the attack costs are independent of the timeout. We also

present *MAD-HTLC*, which is secure against these attacks with both myopic and non-myopic miners.

B. Transaction-Selection Optimization

MAD-HTLC incentivizes rational entities to act in a desired way. It relies on the premise that all involved parties are rational, and specifically, that they monitor the blockchain state and issue transactions accordingly.

Indeed, previous work [28], [37], [56]–[61] shows this premise is prominent, and that system users and miners engage in carefully-planned transaction placing, manipulating their publication times and offered fees to achieve their goals. Other work [62]–[68] asserts the profitability of such actions is expected to rise as the underlying systems mature, enabling constructions such as *MAD-HTLC*, which rely on these optimizations.

C. HTLC-Spec usage

A variety of smart contracts [16]–[24] critically rely on *HTLC-Spec*. To the best of our knowledge, all utilize *HTLC*, making them vulnerable once miners optimize their transaction choices. We review some prominent examples.

a) Off-chain state channels: A widely-studied smart contract construction [5]–[10], [12], [69]–[71] with implementations on various blockchains [4], [72]–[76] is that of an off-chain channel between two parties, A and B.

The channel has a *state* that changes as \mathcal{A} and \mathcal{B} interact, e.g., pay one another by direct communication. In the simplest case, the state is represented by a *settlement transaction* that \mathcal{B} can place on the blockchain. The settlement transaction terminates the channel by placing its final state back in the blockchain. The transaction initiates an HTLC with a hash digest of \mathcal{B} 's choice. \mathcal{B} can redeem the contract after the timeout or, alternatively, \mathcal{A} can redeem it before the timeout if \mathcal{B} had shared the preimage with her.

When \mathcal{A} and \mathcal{B} interact and update the channel state, \mathcal{B} revokes the previous settlement transaction by sending his preimage to \mathcal{A} . This guarantees that if \mathcal{B} places a revoked settlement transaction on the blockchain, \mathcal{A} can redeem the tokens within the timeout. Alternatively, if \mathcal{A} becomes unresponsive, \mathcal{B} can place the transaction on the blockchain and redeem the tokens after the timeout elapses.

Note that this scheme assumes synchronous access to the blockchain – \mathcal{A} should monitor the blockchain, identify revoked-state transactions, and issue her own transaction before the revocation timeout elapses. To remove this burden, services called *Watchtowers* [77]–[79] offer to replace \mathcal{A} in monitoring the blockchain and issuing transactions when needed. However, these also require the same synchronous access to the blockchain, and the placement of transactions is still at the hands of bribable miners. *MAD-HTLC* can be viewed as turning the miners themselves into watchtowers – watchtowers that directly confirm the transactions, without a bribable middleman.

b) Atomic swaps: These contracts enable token exchange over multiple blockchain systems [11]–[15], [80], where a set of parties transact their assets in an atomic manner, i.e., either all transactions occur, or none.

Consider two users, \mathcal{A} and \mathcal{B} , that want to have an atomic swap over two blockchains. \mathcal{A} picks a preimage and creates an HTLC on the first blockchain with timeout T_1 . Then, \mathcal{B} creates an HTLC requiring the same preimage (\mathcal{B} knows only its hash digest) and a timeout $T_2 < T_1$ on the second blockchain. \mathcal{A} publishes a transaction redeeming the HTLC on the second blockchain, revealing the preimage and claiming the tokens. \mathcal{B} learns the preimage from \mathcal{A} 's published transaction, and publishes a transaction of his own on the first blockchain. If \mathcal{A} does not publish her transaction before T_2 elapses, then the swap is canceled.

III. MODEL

We start by describing the system participants and how they form a chain of blocks that contain transactions (§III-A). Next, we explain how the transactions are parsed to define the system state (§III-B). Finally, we detail the required contract specification *HTLC-Spec* (§III-C).

A. Blockchain, Transactions and Miners

We assume an existing blockchain-based cryptocurrency system, facilitating *transactions* of internal system *tokens* among a set of *entities*. All entities have access to a digital signature scheme [81] with a security parameter μ . Additionally, they have access to a hash function $H: \{0,1\}^* \to \{0,1\}^{\mu}$, mapping inputs of arbitrary length to outputs of length μ . We assume the value of μ is sufficiently large such that the standard cryptographic assumptions hold: the digital signature scheme is existentially unforgeable under chosen message attacks (*EU-CMA*) [81], [82], and that H upholds preimage resistance [83], [84].

The blockchain serves as an append-only ledger storing the system state. It is implemented as a linked list of elements called *blocks*. A subset of the entities are called *miners*, who aside from transacting tokens also extend the blockchain by creating new blocks. We refer to non-mining entities as *users*.

There is a constant set of n miners. Each miner is associated a number representing its relative block-creation rate, or mining power. Denote the mining power of miner i by λ_i , where $\sum_{i=1}^n \lambda_i = 1$. Denote the minimal mining power by $\lambda_{\min} = \min_i \lambda_i$. As in previous work [28], [64], [85], [86], these rates are common knowledge, as in practice miners can monitor the blockchain and infer them [87].

Block creation is a discrete-time, memoryless stochastic process. At each time step exactly one miner creates a block. As in previous work [4], [9]–[11], [64], we disregard miners deliberately [85], [88], [89] or unintentionally [90]–[92] causing transient inconsistencies (called *forks* in the literature).

Blocks are indexed by their location in the blockchain. We denote the first block by b_1 and the j'th block by b_j .

Transactions update the system state. An entity creates a transaction locally, and can *publish* it to the other entities.

Transaction publication is instantaneous, and for simplicity we abstract this process by considering published transactions to be part of a publicly-shared data structure called the *mempool*. As in previous work [9], [10], [64], all entities have synchronous access to the mempool and the blockchain.

Unpublished and mempool transactions are *unconfirmed*, and are yet to take effect. Miners can include unconfirmed transactions of their choice when creating a block, thus *confirming* them and executing the stated token reassignment.

The system limits the number of included transactions per block, and to simplify presentation we consider this limit to be one transaction per block.

The system progresses in steps. Each step j begins with system entities publishing transactions to the mempool. Then, a single miner is selected at random proportionally to her mining power, i.e., miner i is selected with probability λ_i . The selected miner creates block b_j , either empty or containing a single transaction, and adds it to the blockchain. This confirms the transaction, reassigning its tokens and awarding that miner with its fee. The system then progresses to the next step.

B. System State

The system state is a set of token and *predicate* pairs called *contracts*. Transactions *reassign* tokens from one contract to another. We say that a transaction *redeems* a contract if it reassigns its tokens to one or more new *initiated* contracts.

To redeem a contract, a transaction must supply input values such that the contract predicate evaluated over them is true. Transactions that result in negative predicate value are *invalid*, and cannot be included in a block. We simply disregard such transactions.

We say that an entity *owns* tokens if she is the only entity able to redeem their contract, i.e., the only entity that can provide input data in a transaction that results in positive evaluation of the contract's predicate.

Transactions reassign tokens as follows. Each transaction lists one or more input contracts that it redeems, each with its respective provided values. Each transaction also lists one or more output contracts that it initiates. A transaction is only valid if the aggregate amount in the output contracts is not larger than the amount in its redeemed input contracts. The difference between the two amounts is the transaction's *fee*. The fee is thus set by the entity that creates the transaction.

The system state is derived by parsing the transactions in the blockchain by their order. Each transaction reassigns tokens, thus updating the contract set. Transaction fees are reassigned to a contract supplied by the confirming miner.

Two transactions *conflict* if they redeem the same contract. Both of them might be valid, but only one can be placed in the blockchain. Once one of them is confirmed, a block containing the other is invalid. We disregard such invalid blocks, and assume miners only produce valid ones.

There is always at least one unconfirmed valid transaction in the mempool [64], [66], [68], [93], [94], and the highest offered fee by any mempool transaction is f, referred to as the *base* fee. Miners act rationally to maximize their received

fees (see §II-B). Users are also rational, and offer the minimal sufficient fee for having their transactions confirmed.

Predicates have access to three primitives of interest:

- vSig (sig; pk): validate that a digital signature sig provided by the transaction (on the transaction, excluding sig) matches a public key pk specified in the contract.
- vPreImg (pre; dig): validate that a preimage pre provided by the transaction matches a hash digest dig specified in the contract, i.e., that H (pre) = dig.
- vTime (T): validate that the transaction trying to redeem
 the contract is in a block at least T blocks after the
 transaction initiating it.

A predicate can include arbitrary logic composing those primitives. In predicates that offer multiple redeem options via *or* conditions, we refer to each option as a *redeem path*.

We note that once a transaction is published, its content becomes available to all entities. We say that an entity *knows* data if it is available to it.

C. HTLC-Spec Specification

We formalize as HTLC-Spec the following contract specification, used in variety of blockchain-based systems and algorithms [4], [7]–[14], [17]–[19]. HTLC-Spec is specified for two users, $\mathcal A$ and $\mathcal B$. It is parameterized by a hash digest and a timeout, and contains a certain deposit amount, v^{dep} . $\mathcal A$ gets the deposit if she publishes a matching preimage before the timeout elapses, otherwise $\mathcal B$ does.

In a blockchain setting, \mathcal{A} and \mathcal{B} redeem the deposit with a transaction that offers a fee. We assume the contract token amount v^{dep} is larger than the base fee f, otherwise the contract is not applicable.

The redeeming transaction by \mathcal{A} or \mathcal{B} (according to the scenario) should require a fee negligibly larger than the base fee f. Specifically, the fee amount is independent of v^{dep} .

To construct HTLC-Spec, \mathcal{A} and \mathcal{B} choose the included hash digest, the timeout, and the token amount, v^{dep} . Then either of them issues a transaction that generates the contract with v^{dep} tokens and the parameterized predicate. Either \mathcal{A} or \mathcal{B} initially knows the preimage, depending on the scenario.

For simplicity, we assume that \mathcal{A} either knows the preimage when the transaction initiating HTLC-Spec is confirmed on the blockchain, or she never does.

IV. MAD-HTLC DESIGN

We present MAD-HTLC, an implementation of HTLC-Spec. MAD-HTLC comprises two sub contracts 1 — MH-Dep, the core implementation of the HTLC-Spec functionality, and MH-Col, an auxiliary contract for collateral, used to disincentivize spiteful behavior by \mathcal{B} .

MAD-HTLC includes additional variables and parameters along those of HTLC-Spec, facilitating its realization. It includes two preimages, pre_a and pre_b ; the former corresponds to the preimage of HTLC-Spec; the latter is an addition in

MAD-HTLC, chosen by \mathcal{B} , used in the various redeem paths. It also includes the *HTLC-Spec* deposit token amount v^{dep} , but also utilizes v^{col} collateral tokens.

Essentially, MH-Dep lets either \mathcal{A} redeem v^{dep} with preimage pre_a , or \mathcal{B} after the timeout with preimage pre_b , or any party with both preimages pre_a and pre_b . MH-Col has v^{col} redeemable only after the timeout, either by \mathcal{B} , or by any party with both preimages pre_a and pre_b .

We present protocol $\Pi_{\text{mad-htlc}}$ for setup, initiation and redeeming of a *MAD-HTLC* (§IV-A), and detail the specifics of *MH-Dep* (§IV-B) and *MH-Col* (§IV-C).

A. Protocol $\Pi_{mad-htle}$

Recall that HTLC-Spec is used in several scenarios differing in which party chooses the preimage, when that chosen preimage is shared, and who initiates the contract on the blockchain (§II-C). However, in all scenarios, once the contract is initiated, $\mathcal A$ can redeem $v^{\rm dep}$ by publishing the preimage before the timeout elapses, and $\mathcal B$ can redeem them only after.

So, there are several variants for any protocol that implements HTLC-Spec, and we focus on the variant where \mathcal{B} picks the first preimage pre_a , potentially shares it with \mathcal{A} , either \mathcal{A} or \mathcal{B} can initiate the contract on chain, and either can redeem it using the various redeem paths. This corresponds to the *off-chain payment channels* scenario (§II-C).

Protocol $\Pi_{\text{mad-htlc}}$ (Protocol 1) progresses in phases, and is parameterized by the timeout T and the token amounts v^{dep} and v^{col} . First, in the setup phase, $\mathcal B$ randomly draws (denoted by $\stackrel{\mathcal R}{\leftarrow}$) the two preimages pre_a and pre_b . He then derives their respective hash digests $dig_a \leftarrow H(pre_a)$ and $dig_b \leftarrow H(pre_b)$, shares dig_a and dig_b with $\mathcal A$. Upon $\mathcal A$'s confirmation $\mathcal B$ creates a transaction tx_{init} that initiates a MAD-HTLC with parameters T, dig_a , dig_b , v^{dep} , v^{col} and shares tx_{init} with $\mathcal A$.

In the following initiation phase, \mathcal{B} can share pre_a with \mathcal{A} . Additionally, either \mathcal{A} or \mathcal{B} can publish tx_{init} to the mempool, allowing miners to confirm it and initiate the MAD-HTLC.

In the final redeeming phase, once the MAD-HTLC is initiated, \mathcal{A} and \mathcal{B} can redeem v^{dep} and v^{col} from MH-Dep and MH-Col, respectively. Specifically, \mathcal{A} redeems v^{dep} only if she received pre_a from \mathcal{B} , and otherwise \mathcal{B} redeems v^{dep} . Either way, \mathcal{B} redeems v^{col} .

B. MH-Dep

The MH-Dep contract is initiated with v^{dep} tokens. Its predicate is parameterized with \mathcal{A} 's and \mathcal{B} 's public keys, pk_a and pk_b , respectively; a hash digest of the predefined preimage $dig_a = H(pre_a)$ such that any entity other than \mathcal{A} and \mathcal{B} does not know pre_a , and \mathcal{A} or \mathcal{B} know pre_a according to on the specific use case; another hash digest dig_b such that $H(pre_b) = dig_b$, where only \mathcal{B} knows pre_b ; and a timeout T. The contract has three redeem paths, denoted by $dep-\mathcal{A}$, $dep-\mathcal{B}$ and $dep-\mathcal{M}$, and presented in Predicate 1. Table I shows the possible redeeming entities of MH-Dep.

In the dep-A path (line 1), A can redeem MH-Dep by creating a transaction including pre_a and sig_a , a signature created using her secret key sk_a . Such a transaction can

¹Separating *MAD-HTLC* into two sub contracts is for Bitcoin compatibility; these can be consolidated to a single contract in blockchains supporting, richer smart-contract languages, see §VI.

Protocol $\Pi_{\mathrm{mad-htlc}}$ run by $\mathcal A$ and $\mathcal B$ details the setup, initiation and redeeming of a MAD-HTLC in the scenario where $\mathcal B$ picking pre_a . It is parameterized by timeout T, and token amounts v^{dep} and v^{col} .

setup

 \mathcal{B} draws $pre_a \overset{R}{\leftarrow} \{0,1\}^{\mu}$, $pre_b \overset{R}{\leftarrow} \{0,1\}^{\mu}$ and sets $dig_a \leftarrow H\left(pre_a\right)$, $dig_b \leftarrow H\left(pre_b\right)$. Then \mathcal{B} sends dig_a and dig_b to \mathcal{A} for confirmation. Afterwards \mathcal{B} , compiles a transaction tr_{init} that initiates a MAD-HTLC (both MH-Dep and MH-Col) with dig_a , dig_b , T, v^{dep} , v^{col} as parameters and shares it with \mathcal{A} . tr_{init} is not published yet.

initiation

 ${\cal B}$ can send pre_a to ${\cal A}$. If so, ${\cal A}$ expects to receive pre such that $dig_a=H\left(pre\right)$, and ignores other values.

Either A or B publish tx_{init} to the mempool, and it is eventually included in a block b_j , initiating MAD-HTLC.

redeeming

If $\mathcal A$ had received pre_a , she creates and publishes tx_a^{dep} , a transaction redeeming MH-Dep using the $dep\mathcal A$ redeem path.

 \mathcal{B} waits for the creation of block b_{j+T-1} . If by then \mathcal{A} did not publish tx_a^{dep} then \mathcal{B} publishes $tx_a^{\text{dep+col}}$, redeeming both MH-Dep and MH-Col through dep- \mathcal{B} and col- \mathcal{B} redeem paths, respectively. If \mathcal{A} did publish tx_a^{dep} then \mathcal{B} publishes tx_b^{col} , redeeming only MH-Col using the col- \mathcal{B} redeem path.

Protocol 1: $\Pi_{mad-htlc}$

Predicate 1: MH-Dep

Table I: Possible redeeming entity of MH-Dep.

	pre _b published	pre _b not published
pre _a published	Any entity	\mathcal{A}
pre _a not published	\mathcal{B}	_

Table II: Possible redeeming entity of MH-Col.

	pre _b published	pre _b not published
pre _a published	Any entity	\mathcal{B}
prea not published	\mathcal{B}	\mathcal{B}

be included even in the next block b_{j+1} . This path is only available to A, since only she ever knows sk_a .

In the $dep-\mathcal{B}$ path (line 2), \mathcal{B} can redeem MH-Dep by creating a transaction including pre_b and sig_b , a signature created using his secret key sk_b . Such a transaction can be included in a block at least T blocks after MH-Dep's initiation, that is, not earlier than block b_{j+T} . This path is only available to \mathcal{B} , since only he ever knows sk_b .

In the $dep-\mathcal{M}$ path (line 3), any entity can redeem MH-Dep by creating a transaction including both pre_a and pre_b . A transaction taking this redeem path does not require a digital signature, and can be included even in the next block b_{j+1} . This path is therefore available to any entity, and specifically to any miner, that knows both pre_a and pre_b .

C. MH-Col

The MH-Col contract is initiated with $v^{\rm col}$ tokens. Its predicate is parameterized with \mathcal{B} 's public key pk_b ; the hash digest of the predefined secret $dig_a = H\left(pre_a\right)$ such that any entity other than \mathcal{A} and \mathcal{B} does not know pre_a , and \mathcal{A} and \mathcal{B} know pre_a based on the specific use case; the hash digest dig_b such that $H\left(pre_b\right) = dig_b$, where only \mathcal{B} knows pre_b ; and a timeout T. It has two redeem paths, denoted by col- \mathcal{B}

Predicate 2: MH-Col

```
\begin{array}{ll} \text{Parameters: } pk_b, T, dig_a, dig_b \\ \text{MH-Col } (pre_1, pre_2, sig) \coloneqq \\ \text{1} \quad \text{VTime } (T) \land \\ \text{2} \quad \left[ \text{VSig } (sig; pk_b) \lor & // \textit{col-B} \\ \text{3} \quad \left( \text{VPreImg } (pre_1; dig_a) \land \text{VPreImg } (pre_2; dig_b) \right) \right] \quad // \textit{col-M} \end{array}
```

and $col-\mathcal{M}$, and presented in Predicate 2. Table II shows the possible redeeming entities of MH-Col.

Both paths are constrained by the timeout T, meaning a redeeming transaction can only be included in a block at least T blocks after the MH-Col initiation (line 1).

In the col- \mathcal{B} path (line 2), \mathcal{B} can redeem MH-Col by creating a transaction including sig_b , a signature created using his secret key sk_b . Only \mathcal{B} can redeem MH-Col using this path as he is the only one able to produce such a signature. This path allows \mathcal{B} to claim the collateral tokens in case either he or \mathcal{A} , but not both, publish a transaction redeeming MH-Dep.

The $col-\mathcal{M}$ path (line 3) allows any entity to redeem MH-Col by creating a transaction including both pre_a and pre_b , not requiring any digital signature. This path allows miners to claim the MH-Col tokens in case \mathcal{B} tries contesting \mathcal{A} on redeeming MH-Dep, thus disincentivizing his attempt.

V. MAD-HTLC SECURITY ANALYSIS

To prove the security of *MAD-HTLC* we first show what actions the participants can take to interact with it (§V-A). We prove with the UC framework [33] the security of the setup, initiation and redeeming of a *MAD-HTLC*. This analysis yields a set of conditions on which entity can redeem tokens from *MAD-HTLC*.

Then, we move to analyze how the entities should act to maximize their gains. We formalize the redeeming of an initiated MAD-HTLC as a game played by \mathcal{A} , \mathcal{B} and the miners ($\S V\text{-}B$), and show that they are all incentivized to act as intended ($\S V\text{-}C$).

A. Setup, Initiation and Redeeming Transactions Security

Our first goal is to prove the setup and initiation of *MAD-HTLC* are secure and to show which valid transactions each participant can generate based on the mempool and blockchain state. We present an overview of the security claims and proofs, and bring the details in the extended report [95].

Like prior work [4], [9], [28], [31], [64], [81], [84], [96]–[107], we assume the blockchain and predicate security holds, including the digital signature scheme and the hash function.

We make the following observation: Transaction invalidity due to vTime is temporal; this predicate becomes true once sufficiently many blocks are created. In contrast, two valid transactions can conflict, so only one of them can be confirmed. We neglect both invalidity reasons and show which valid transactions can be created; clearly, any transaction that is invalid under this relaxation is also invalid without it. Additionally, we consider only transactions relevant to our protocol, ignoring unrelated transactions.

We formalize parties' ability to redeem the contract under this relaxation using the rPred() function (Eq. 1): Denote $path \in \{dep-\mathcal{A}, dep-\mathcal{B}, dep-\mathcal{M}, col-\mathcal{B}, col-\mathcal{M}\}; \mathcal{P}$ the

redeeming party; $h_a=1$ if the redeeming party has a suitable preimage for dig_a , and 0 otherwise; and $h_b=1$ if the redeeming party has a suitable preimage for dig_b , and 0 otherwise. Then the relaxed contract predicate is expressed by the function $rPred(path, \mathcal{P}, h_a, h_b)$. We note redeeming transactions are published in the mempool, hence publish any included preimages.

$$rPred (path, \mathcal{P}, h_a, h_b) = \begin{cases} (\mathcal{P} = \mathcal{A}) \wedge h_a & path = \text{dep-}\mathcal{A} \\ (\mathcal{P} = \mathcal{B}) \wedge h_b & path = \text{dep-}\mathcal{B} \end{cases}$$

$$\begin{cases} h_a \wedge h_b & path \in \{\text{dep-}\mathcal{M}, \text{col-}\mathcal{M}\} \\ \mathcal{P} = \mathcal{B} & path = \text{col-}\mathcal{B} \end{cases}$$

$$(1)$$

We move to consider the setup, initiation, and redeeming of a single (relaxed) contract with respect to the mempool and the blockchain. We focus on a mempool and blockchain projection (mbp) functionality of a relaxed MAD-HTLC, and we model it as a single ideal functionality, \mathcal{G}_{mbp} . This functionality captures the parameter setup of a single contract by A and B, its initiation, and redeeming transaction validity due to the vPreImg and vSig predicates, disregarding conflicts and timeouts. To facilitate the vPreImg predicate and its underlying preimage-resistant hash function H, we model the latter as a global random oracle ideal functionality \mathcal{H} [33], [108], [109]. We abstract away digital signatures by considering authenticated channels among parties and functionalities. We consider an adversary that learns messages sent to \mathcal{G}_{mbp} and that messages were sent between parties but not their content. This modeling is similar to previous work [8], [9], [84], [98].

We then define the $(\mathcal{H}, \mathcal{G}_{mbp})$ -hybrid world [33] (hereinafter, simply the hybrid world), where the \mathcal{H} and \mathcal{G}_{mbp} ideal functionalities reside. In this hybrid world we then define the relaxed MAD-HTLC (rmh) protocol Π_{rmh} that is similar to $\Pi_{mad\text{-htlc}}$ (Protocol 1), but (1) it is defined with \mathcal{H} and \mathcal{G}_{mbp} ; (2) it considers system entities other than \mathcal{A} and \mathcal{B} , and specifically miners, represented as a third party \mathcal{M} ; and (3) it disregards timeouts and transaction conflicts.

The transition from Π_{rmh} to $\Pi_{mad-htlc}$ is straightforward, and we bring Π_{rmh} in the extended report [95].

Then, our goal is to prove the following lemma, detailing the possible valid transactions the entities can create and publish.

Lemma 1. Let there be a contract setup and initiated as described by Π_{rmh} , let pub_a and pub_b be indicators whether the preimages pre_a and pre_b were published in \mathcal{G}_{mbp} , respectively, and let shared indicate if \mathcal{B} shared pre_a with \mathcal{A} . So, initially $pub_a \leftarrow 0$, $pub_b \leftarrow 0$ and shared $\leftarrow 0$. Then, parties \mathcal{A} , \mathcal{B} and \mathcal{M} can only create and publish the following valid redeeming transactions:

• \mathcal{B} can publish a valid redeeming transaction using the dep- \mathcal{B} , dep- \mathcal{M} , col- \mathcal{B} or col- \mathcal{M} redeem paths. Doing so with either dep- \mathcal{M} or col- \mathcal{M} sets pub_a \leftarrow 1, and with either dep- \mathcal{B} , dep- \mathcal{M} , or col- \mathcal{M} sets pub_b \leftarrow 1. In addition to transaction creation and publication, \mathcal{B} can share pre_a with \mathcal{A} (and by doing so sets shared \leftarrow 1).

Ideal functionality \mathcal{F}_{rmh} in the ideal world represents the setup, initiation and redeeming transaction publication of the contract for session id sid. It interacts with parties $\mathcal{A}, \mathcal{B}, \mathcal{M}$, and simulator Sim. It internally stores indicators $setup_a^{\rm mh}$, $setup_b^{\rm mh}$, $shared^{\rm mh}$, $published^{\rm mh}$, $init^{\rm mh}$, $pub_1^{\rm mh}$ and $pub_2^{\rm mh}$, all with initial value of 0.

- Upon receiving (setup-B, sid) from B when setup_b^{mh} = 0, set setup_b^{mh} ← 1 and leak (setup-B, sid) to Sim.
- Upon receiving (setup-A, sid) from $\mathcal A$ when $setup_b^{\rm mh}=1 \land setup_a^{\rm mh}=0$, set $setup_a^{\rm mh}\leftarrow 1$ and leak (setup-A, sid) to Sim.
- Upon receiving (share, sid) from \mathcal{B} when $setup_a^{\rm mh}=1 \land shared^{\rm mh}=0$, set $shared^{\rm mh}\leftarrow 1$, and leak (share, sid) to Sim.
- Upon receiving (publish, sid) from either \mathcal{A} or \mathcal{B} when $setup_a^{mh} = 1 \land published^{mh} = 0$, set $published^{mh} \leftarrow 1$, and leak (publish, sid) to Sim.
- Upon receiving (init, sid) from \mathcal{M} when published^{mh} = $1 \wedge init^{mh} = 0$, set $init^{mh} \leftarrow 1$, and leak (init, sid) to Sim.
- Upon receiving (redeem, sid, path) from any party \mathcal{P} such that $path \in \{dep-\mathcal{A}, dep-\mathcal{B}, dep-\mathcal{M}, col-\mathcal{B}, col-\mathcal{M}\}$ when $init^{\mathrm{mh}} = 1$, set $pub_1^{\mathrm{mh}} \leftarrow pub_1^{\mathrm{mh}} \lor ((\mathcal{P} = \mathcal{A}) \land shared^{\mathrm{mh}} \land (path = dep-\mathcal{A})) \lor ((\mathcal{P} = \mathcal{B}) \land path \in \{dep-\mathcal{A}, dep-\mathcal{M}, col-\mathcal{M}\})$ and $pub_2^{\mathrm{mh}} \leftarrow pub_2^{\mathrm{mh}} \lor ((\mathcal{P} = \mathcal{B}) \land path \in \{dep-\mathcal{B}, dep-\mathcal{M}, col-\mathcal{M}\})$, denote $res^{\mathrm{mh}} \leftarrow rPred (path, \mathcal{P}, pub_1^{\mathrm{mh}}, pub_2^{\mathrm{mh}})$, leak (redeem, sid, path, \mathcal{P}) to Sim, and return res^{mh} to \mathcal{P} .
- Upon receiving (update, sid, i) for $i \in \{0,1\}$ from Sim through the influence port, set $pub_i^{\rm mh} \leftarrow 1$.

Functionality 1: \mathcal{F}_{rmh} in the ideal world.

- If $\operatorname{pub}_a \vee \operatorname{shared} = 1$ then \mathcal{A} can publish a valid redeeming transaction using the dep- \mathcal{A} redeem path (and by doing so she sets $\operatorname{pub}_a \leftarrow 1$). If $(\operatorname{pub}_a \vee \operatorname{shared}) \wedge \operatorname{pub}_b = 1$, then \mathcal{A} can publish a valid redeeming transaction with either the dep- \mathcal{M} or col- \mathcal{M} redeem paths (and by doing so sets $\operatorname{pub}_a \leftarrow 1$ and $\operatorname{pub}_b \leftarrow 1$).
- If pub_a ∧ pub_b = 1 then M can publish a valid redeeming transaction with either dep-M or col-M.

To prove Lemma 1 we consider an ideal world, where we define a *relaxed MAD-HTLC* ideal functionality \mathcal{F}_{rmh} (Functionality 1) that implements the setup, initiation and redeeming of a relaxed *MAD-HTLC* contract.

 $\mathcal{F}_{\mathrm{rmh}}$ maintains indicators $\mathit{setup_a^{\mathrm{mh}}}$, $\mathit{setup_b^{\mathrm{mh}}}$, $\mathit{shared^{\mathrm{mh}}}$, $\mathit{published^{\mathrm{mh}}}$, $\mathit{init^{\mathrm{mh}}}$, and $\mathit{pub_1^{\mathrm{mh}}}$ and $\mathit{pub_2^{\mathrm{mh}}}$, corresponding to execution of $\mathit{MAD-HTLC}$ (Protocol 1): $\mathit{setup_a^{\mathrm{mh}}}$ and $\mathit{setup_b^{\mathrm{mh}}}$ correspond to to \mathcal{A} and \mathcal{B} completing their setup; $\mathit{shared^{\mathrm{mh}}}$ is set if \mathcal{B} shared $\mathit{pre_a}$ with \mathcal{B} ; $\mathit{published^{\mathrm{mh}}}$ and $\mathit{init^{\mathrm{mh}}}$ indicate if the execution reached the initiation and redeeming phases, respectively; and $\mathit{pub_1^{\mathrm{mh}}}$ and $\mathit{pub_2^{\mathrm{mh}}}$ are set if $\mathit{pre_a}$ and $\mathit{pre_b}$ are published with a transaction.

 \mathcal{F}_{rmh} leaks messages to Sim and receives a special update instruction that sets either pub_1^{mh} or pub_2^{mh} . Looking ahead, this allows the simulator to notify \mathcal{F}_{rmh} of a publication by a corrupted party.

The construction of \mathcal{F}_{rmh} and the definition of rPred (Eq. 1) imply that the properties described by Lemma 1 trivially hold in the ideal world.

We then prove Π_{rmh} UC-realizes \mathcal{F}_{rmh} , i.e., for any PPT adversary Adv, there exists a PPT simulator Sim such that for any PPT environment \mathcal{Z} , the execution of Π_{rmh} in the hybrid world with Adv is computationally indistinguishable from the execution of \mathcal{F}_{rmh} in the ideal world with Sim.

We prove the aforementioned by showing how to construct such a Sim for any Adv, and the derived indistinguishability towards \mathcal{Z} . Sim internally manages two preimages on its own, which are indistinguishable from to those chosen by \mathcal{B} in the hybrid world: for an honest \mathcal{B} , Sim draws these two preimages from the same distribution as in the real world; for a corrupted \mathcal{B} , Sim learns the chosen preimages throughout the execution. Additionally, Sim internally-simulates \mathcal{H} and \mathcal{G}_{mbp} , and interacts with \mathcal{F}_{rmh} through leakage and influence ports.

The existence of these simulators shows Lemma 1 applies to the hybrid world as well, meaning it details the possible valid redeeming transactions of a relaxed MAD-HTLC.

Recall the relaxed version considers only the vPreImg and vSig predicates while disregarding vTime and transactions conflicts, which we now move to consider.

B. MAD-HTLC Game

The MAD-HTLC construction within the blockchain system gives rise to a game: the participants are \mathcal{A} , \mathcal{B} and the system miners; their utilities are their tokens; and the action space is detailed by Lemma 1 while considering the timeout constraints and transaction conflicts.

Note that Lemma 1 considers party \mathcal{M} representing any system miner, while the upcoming analysis considers all the miners and their individual rewards.

The MAD-HTLC game begins when the MH-Dep and MH-Col contracts are initiated in some block b_j . The game, which we denote by Γ^{MH} , comprises T rounds, representing the creation of blocks $b_{j+1},...,b_{j+T}$. Each round begins with $\mathcal A$ and $\mathcal B$ publishing redeeming transactions, followed by a miner creating a block including a transaction of her choice.

 \mathcal{A} and \mathcal{B} 's strategies are their choices of published transactions – which transactions to publish, when, and with what fee. Miner strategies are the choices of which transaction to include in a block if they are chosen to create one.

To accommodate for the stochastic nature of the game [110] we consider entity utilities as the expected number of tokens they own at game conclusion, i.e., after the creation of T blocks. \mathcal{A} and \mathcal{B} 's utilities depend on the inclusion of their transactions and their offered fees, and miner utilities on their transaction inclusion choices.

We present the game details ($\S V-B1$) and the suitable solution concept ($\S V-B2$).

1) Game Details: The game progresses in rounds, where each round comprises two steps. First, A and B alternately publish transactions, until neither wishes to publish any more.

Note that all published transactions of the current and previous rounds are in the mempool. Since miners prefer higher fees, for the analysis we ignore any transaction tx if there is another transaction tx such that both were created by the same entity, both redeem the same contracts, and tx pays a higher fee than tx or arrives before tx.

Tokens are discrete, hence there is a finite number of fees A and B may offer, meaning the publication step is finite.

Then, a single miner is picked at random proportionally to her mining power and gets to create a block including a transaction of her choice, receiving its transaction fees. She can also create a new transaction and include it in her block.

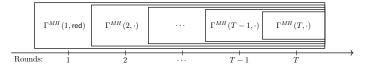


Figure 1: Γ^{MH} subgames.

a) Subgames: The dynamic and turn-altering nature of the game allows us to define subgames, representing suffixes of Γ^{MH} . For any $k \in [1,T]$ we refer to the game starting just before round k as the k'th subgame (Fig. 1).

Note that as miners create blocks and confirm transactions, the system state, including the state of *MAD-HTLC*, changes. Specifically, if the *MH-Dep* is already redeemed, future blocks do not allow inclusion of conflicting transactions that redeem the *MH-Dep* as well.

Hence, when considering MAD-HTLC states we distinguish whether MH-Dep is redeemable or irredeemable, which we denote by red and irred, respectively. We also note that MH-Col cannot be redeemed until the very last T'th subgame.

Consequently, each subgame $k \in [1,T]$ is defined by the number of remaining blocks to be created k, and the MH-Dep state $s \in \{\text{red}, \text{irred}\}$. We denote such a subgame by $\Gamma^{MH}(k,s)$.

We use \cdot to denote sets of subgames, e.g., $\Gamma^{MH}(\cdot, \text{red})$ denotes the set of subgames where the contract state s is red.

We refer to $\Gamma^{MH}(T,\cdot)$ as the *final* subgames, as once played, the full game Γ^{MH} is complete. We refer to all other subgames as *non-final*.

The game begins when there are T blocks to be created, \mathcal{A} and \mathcal{B} did not publish any transactions, and the MH-Dep is redeemable. Thus, the initial, complete game is $\Gamma^{MH}(1, \text{red})$.

Once the first round of a non-final subgame is complete, the system transitions to the *subsequent* subgame.

b) Actions: A and B's actions are the publication of transactions in any $\Gamma^{MH}(\cdot,\cdot)$ subgame.

 ${\cal A}$ can only redeem MH-Dep and only if she has pre_a (shared = 1), hence has a single transaction of interest $k_a^{\rm dep}$, offering fee of $f_a^{\rm dep}$ tokens. Note $t_a^{\rm dep}$ has to outbid unrelated transactions and thus has to offer a fee $f_a^{\rm dep} > f$, however, cannot offer more tokens than the redeemed ones, so $f_a^{\rm dep} < v^{\rm dep}$. This transaction utilizes the $dep\text{-}{\cal A}$ redeem path of MH-Dep, hence publishing it also publishes pre_a .

 \mathcal{B} can redeem $MH\text{-}Dep,\ MH\text{-}Col$ or both. We thus consider three transactions of interest: $tx_b^{\text{dep}},\ \text{redeeming}\ MH\text{-}Dep$ while offering fee $f_b^{\text{dep}};\ tx_b^{\text{col}},\ \text{redeeming}\ MH\text{-}Col$ while offering fee $f_b^{\text{col}};\ \text{and}\ tx_b^{\text{dep+col}},\ \text{redeeming}\ \text{both}\ MH\text{-}Dep\ \text{and}\ MH\text{-}Col$ while offering fee $f_b^{\text{dep+col}}$. To redeem $MH\text{-}Dep\ \mathcal{B}$ uses the $dep\text{-}\mathcal{B}$ redeem path, hence publishing transactions tx_b^{dep} or $tx_b^{\text{dep+col}}$ also publishes pre_b . Redeeming MH-Col is by the $col\text{-}\mathcal{B}$ redeem path. Similarly to \mathcal{A} 's fee considerations, \mathcal{B} 's transactions have to outbid unrelated transactions, and cannot offer more tokens than they redeem, so $f < f_b^{\text{dep}} < v^{\text{dep}},\ f < f_b^{\text{col}} < v^{\text{col}}$ and $f < f_b^{\text{dep+col}} < v^{\text{dep}} + v^{\text{col}}$.

A miner's action is the choice of a transaction to include if she is chosen to create a block. First, she can include a transaction unrelated to MAD-HTLC in any $\Gamma^{MH}(\cdot,\cdot)$ subgame.

She can also include any of the following transactions, assuming they were previously published by \mathcal{A} or \mathcal{B} , and as a function of the contract state: tx_a^{dep} if MH-Dep is redeemable, that is, in any $\Gamma^{MH}(\cdot, \text{red})$; tx_b^{col} if the timeout has elapsed, that is, in any $\Gamma^{MH}(T, \cdot)$; and tx_b^{dep} or $tx_b^{\text{dep+col}}$ if the timeout has elapsed and MH-Dep is redeemable, that is, in $\Gamma^{MH}(T, \text{red})$.

Conditioned on knowing pre_a and pre_b through published transactions, a miner can also create and include the following transactions, redeeming the contracts herself:

- Transaction tx_m^{dep} redeeming MH-Dep, using the dep-M redeem path, and getting the v^{dep} tokens of MH-Dep as reward. This action is only available if the miner knows both pre_a and pre_b, and if MH-Dep is redeemable, that is, in any Γ^{MH}(·, red) subgame where tx_a^{dep} and either of tx_b^{dep} or tx_b^{dep+col} were published.
 Transaction tx_m^{col} redeeming MH-Col, using the col-M
- Transaction $tx_m^{\rm col}$ redeeming MH-Col, using the $col\text{-}\mathcal{M}$ redeem path, and getting the $v^{\rm col}$ tokens of MH-Col as reward. This action is only available if the miner knows both pre_a and pre_b , and the timeout has elapsed, that is, in any $\Gamma^{MH}(T, \text{red})$ subgame where $tx_a^{\rm dep}$ and either of $tx_b^{\rm dep}$ or $tx_b^{\rm dep+col}$ were published.
- Transaction $tx_m^{\text{dep+col}}$ redeeming both *MH-Dep* and *MH-Col*, using the dep- \mathcal{M} and col- \mathcal{M} redeem paths, and getting the $v^{\text{dep}}+v^{\text{col}}$ tokens of *MH-Dep* and *MH-Col* as reward. This action is only available if the miner knows both pre_a and pre_b , the *MH-Dep* is redeemable, and the timeout has elapsed, that is, in subgame $\Gamma^{MH}(T, \text{red})$ where tx_a^{dep} and either of tx_b^{dep} or $tx_b^{\text{dep+col}}$ were published.

We disregard actions that are trivially dominated [52], such as \mathcal{A} and \mathcal{B} sharing their secret keys or publishing the relevant preimages not via a transaction; a miner including a transaction of another entity that redeems either of the contracts using the two preimages instead of redeeming it herself; and a miner creating an empty block instead of including an unrelated transaction.

- c) Strategy: A strategy σ is a mapping from each subgame to a respective feasible action, stating that an entity takes that action in the subgame. We call the strategy vector of all entities in a game a strategy profile, denoted by $\bar{\sigma}$.
- d) Utility: Recall an entity's utility is her expected accumulated token amount at game conclusion. We define the utility of an entity in a subgame as the expected token amount she accumulates within the subgame until its conclusion. We denote the utility of entity i when all entities follow $\bar{\sigma}$ in subgame $\Gamma^{MH}(k,s)$ by u_i $(\bar{\sigma},\Gamma^{MH}(k,s))$.
- 2) Solution Concept: Note that block-creation rates, entity utilities and their rationality are all common knowledge, and that when choosing an action an entity is aware of the current system state. That means any subgame $\Gamma^{MH}(k,s)$ is of perfect information [111], [112]. We are thus interested in strategy profiles that are subgame perfect equilibria [41], [48]–[54].

A strategy profile $\bar{\sigma}$ is a subgame perfect equilibrium in $\Gamma^{MH}(k,s)$ if, for any subgame, no entity can increase her utility by deviating to a different strategy, where it knows how

the other players would react based on their perfect knowledge. This implies that for each subgame, the actions stated by $\bar{\sigma}$ are a Nash equilibrium.

We say that a prescribed strategy profile is *incentive compatible* [34] if it is a subgame perfect equilibrium, and the utility of each player is not lower than her utility in any other subgame perfect equilibrium. So an entity cannot deviate to increase her utility, and there are no other more favorable equilibria.

Our analysis utilizes the common technique of *backward induction* [54], [113]–[115], suitable for perfect-information finite games. Intuitively, to determine her best action, a player analyzes the game outcome for each possible action, repeating the process recursively for each possible game suffix.

C. MAD-HTLC Incentive Compatibility

We now show the MAD-HTLC prescribed behavior (Protocol 1) is incentive compatible and implements HTLC-Spec.

We first analyze A's and B's utilities when both follow the prescribed strategy, starting with the scenario where A knows the preimage pre_a (i.e., when shared = 1).

Lemma 2. In Γ^{MH} (1, red), if A knows pre_a and A and B both follow the prescribed strategies, then miners' best-response strategy leads to A redeeming MH-Dep for $v^{dep} - f_a^{dep}$ tokens, and B redeeming MH-Col for $v^{col} - f_b^{col}$ tokens.

Proof. The prescribed strategy states that \mathcal{A} publishes tx_a^{dep} during the first T-1 rounds, and that \mathcal{B} publishes tx_b^{col} in round T.

Note that \mathcal{B} does not publish tx_b^{dep} and $tx_b^{\text{dep+col}}$, hence miners do not know pre_b . The transactions tx_a^{dep} and tx_b^{col} offer f_a^{dep} and f_b^{col} fees, respectively, both greater than the base fee f.

The induced subgames therefore enable miners to include $tx_a^{\rm dep}$ in one of the first T-1 blocks, and including $tx_b^{\rm col}$ in the last one. Using backward induction shows the subgame perfect equilibrium is to include $tx_a^{\rm dep}$ in its published round, and $tx_b^{\rm col}$ in the last.

So both tx_a^{dep} and tx_b^{col} are included in blocks, and $\mathcal A$ and $\mathcal B$ get $v^{\text{dep}}-f_a^{\text{dep}}$ and $v^{\text{col}}-f_b^{\text{col}}$ tokens, respectively.

We now consider \mathcal{A} that does not know pre_a (i.e., when shared = 0).

Lemma 3. In $\Gamma^{MH}(1, red)$, if A does not know pre_a and A and B both follow the prescribed strategies, then miners' best-response strategy leads to B redeeming both MH-Dep and MH-Col for $v^{dep} + v^{col} - f_b^{dep+col}$ tokens, and A gets none.

Proof. As A does not know pre_a she does not publish any transaction, hence redeems no contract and receives no tokens.

By the prescribed strategy \mathcal{B} publishes $tx_b^{\text{dep+col}}$, offering fee $f_b^{\text{dep+col}} > f$ and revealing pre_b . However, pre_a is not published, so miners cannot redeem MH-Dep and MH-Col themselves. Therefore, miners maximize their utility by including $tx_b^{\text{dep+col}}$ in the last round.

cluding $tx_b^{\text{dep+col}}$ in the last round.

That means $tx_b^{\text{dep+col}}$ is included in a block, and $\mathcal A$ and $\mathcal B$ get 0 and $v^{\text{dep}} + v^{\text{col}} - f_b^{\text{dep+col}}$ tokens, respectively.

We now present three lemmas, considering potential deviations from the prescribed strategy, and showing that any such deviation is strictly dominated. We provide the gist of the proofs, with the details deferred to Appendix A.

We first show that if A and B contend then the miners do not take their transactions in the last round.

Lemma 4. In the last round of the game, i.e. subgame $\Gamma^{MH}(T,\cdot)$, if tx_a^{dep} and either tx_b^{dep} or $tx_b^{dep+col}$ are published then miners' best-response strategy is not to include any of \mathcal{A} 's or \mathcal{B} 's transactions in this round.

This holds because in the described scenario any miner can simply redeem all the tokens herself. Then we show \mathcal{A} 's cannot deviate to increase her utility.

Lemma 5. In $\Gamma^{MH}(1, red)$, A cannot increase her utility by deviating from the prescribed strategy.

This holds as publishing at the last round or not publishing at all results with A not getting any tokens. Similarly, we claim B does not gain from deviating.

Lemma 6. In $\Gamma^{MH}(1, red)$, \mathcal{B} cannot increase his utility by deviating from the prescribed strategy.

Intuitively, if \mathcal{B} publishes when \mathcal{A} also does then \mathcal{B} loses all the tokens, whilst refraining from doing so earns him the collateral.

Following directly from Lemma 5 and Lemma 6), we obtain:

Corollary 1. The prescribed strategy of MAD-HTLC is a unique subgame perfect equilibrium, and as such, incentive compatible.

We are now ready to prove our main theorem:

Theorem 1. MAD-HTLC satisfies HTLC-Spec with rational PPT participants.

Proof. Lemma 1 shows the possible redeeming transactions for PPT participants, disregarding invalidity due to timeouts and transaction conflicts. Consequently, the game description considering the timeouts and conflicts (§V-B) captures the possible redeeming transactions of PPT participants.

The game analysis (Corollary 1) shows the prescribed strategy (Protocol 1) is incentive compatible, and Lemma 2 and Lemma 3 show the prescribed strategy matches HTLC-Spec. Note that matching HTLC-Spec, Protocol 1 states the redeeming transaction fee should be negligibly larger than f, and is independent of v^{dep} .

Myopic Miners: MAD-HTLC's design deters \mathcal{B} from bribe attempts as he knows rational non-myopic miners will seize his funds if he acts dishonestly.

However, even in the presence of unsophisticated, myopic miners, MAD-HTLC still satisfies HTLC-Spec. The common transaction selection logic [116]–[119] as of today has miners myopically optimize for the next block. Since \mathcal{B} 's transaction can only be confirmed in the last round, these miners will simply include \mathcal{A} 's transaction, achieving the desired outcome.

Table III: Bitcoin contract and redeeming transaction sizes.

Contract	Size [bytes]	Redeem path	Redeeming tx [bytes]
UTLC	99	htlc-A	291
HILC	HTLC 99	htlc-B	259
		dep-A	323
MH-Dep	129	dep-B	322
-	dep-M	282	
MH-Col	88	col-B	248
	00	col-M	241

Only miners that are sophisticated enough to be non-myopic but not sophisticated enough to take advantage of the dep- \mathcal{M} path would cooperate with the attack. But even in the presence of such miners, it is sufficient for one miner (or user) to take advantage of the dep- \mathcal{M} path during the T rounds in order to thwart the attack.

VI. MAD-HTLC IMPLEMENTATION

We demonstrate the efficacy of *MAD-HTLC* by evaluating it in Bitcoin and Ethereum. We discuss the deployment of *MAD-HTLC* and its overhead (§VI-A), and our implementation of a framework for implementing MEV infrastructure [37]–[39] on Bitcoin (§VI-B), used to facilitate *MAD-HTLC* guarantees.

A. Contract Implementation, Overhead and Deployment

We implement *MH-Dep* and *MH-Col* in Bitcoin's Script [35] and Ethereum's Solidity [36], [42] smart contract languages. We also implement a version of the standard *HTLC* for reference. We bring the code in Appendix B.

We briefly discuss these implementations, show their transaction-fee overhead is negligible compared to the secured amounts, and present main network deployments.

Bitcoin implementation: Bitcoin's transaction fees are determined by the transaction sizes. Our contracts use P2SH [120] (non SegWit [121]) addresses, so the initiating transactions contain only the hashes of the scripts, and each contract initiation within a transaction requires 28 bytes. The redeeming transactions provide the full predicate script along with its inputs. Table III presents the script and redeeming transaction sizes of HTLC, MH-Dep and MH-Col.

A transaction redeeming *MH-Dep* is about 50 bytes larger than one redeeming *HTLC*. At the current Bitcoin common fees [122] and exchange rate [123] implies an additional cost of a mere \$0.02. Including the auxiliary *MH-Col* implies an additional cost of about \$0.10.

Further size-reduction optimizations such as using SegWit transactions and merging multiple transactions can also be made, but are outside the scope of this work.

Ethereum implementation: Compared to Bitcoin's Script, Solidity [36], [42] is a richer smart contract language, allowing MAD-HTLC to be expressed as a single contract consolidating MH-Dep and MH-Col.

On the Ethereum platform transactions pay fees according to their so-called *gas* usage, an inner form of currency describing the cost of each operation the transaction performs. We compare the initiation and redeeming costs of *HTLC* and *MAD-HTLC*. Note that *MAD-HTLC* contains about twice the

Table IV: Ethereum gas for contract initiation and redeeming.

Contract	Initiation [gas]	Redeem path	Redeeming [gas]
HTLC 362,000		htlc-A	34,734
HILC	362,000	htlc-B	32,798
	600,000	dep-A	58,035
MAD-HTLC		dep-B	58,885
		dep-M	59,043
		col-B	41,175
		col-M	44,887

code of *HTLC*, and as expected, its operations are more gasconsuming. We bring the details in Table IV.

We stress these numbers regard the most basic, straightforward implementation, and that Ethereum and Solidity enable further optimizations – for example, deploying a more elaborate *library contract* once [124], and simpler *contract instances* that use the library, achieving significantly reduced amortized costs. More importantly, the additional fee costs are independent of (e.g., \$0.2 [125]), and can be negligible compared to, the secured amounts (e.g., \$6.2K [125]).

Recall that for off-chain channels this overhead is incurred only in the abnormal unilateral channel closure.

Main network deployment: We deployed *MAD-HTLC* on both blockchains (Appendix C details the transaction IDs).

For Bitcoin, we deployed three *MH-Dep* instances on the main network and redeemed them using its three redeem paths. We also deployed two *MH-Col* instances and redeemed using its two redeem paths.

For Ethereum, we deployed a consolidated MAD-HTLC, and posted transactions redeeming the v^{dep} through both $dep\text{-}\mathcal{A}$ and $dep\text{-}\mathcal{B}$. These transactions offered relatively low fees, so were not included in a block by any miner, and only revealed pre_a and pre_b . At this point there were no other transactions trying to redeem the MAD-HTLC, although users and miners monitoring the blockchain could have created a transaction redeeming the v^{dep} using the $dep\text{-}\mathcal{M}$ with the revealed pre_a and pre_b . We deduce this optimization currently does not take place on the Ethereum main network.

Then, we published a transaction of our own using $dep-\mathcal{M}$, revealing (again) pre_a and pre_b , offering a relatively-high fee. Nevertheless, our transaction was slightly out-bid by another transaction, which also used $dep-\mathcal{M}$, and took the deposit. It was likely published by a front-running bot [37], [56], [60], presenting yet another example of entities monitoring the blockchain looking for MEV opportunities [37], [56], [60], as required for MAD-HTLC security.

B. Bitcoin-MEV Infrastructure

By default, cryptocurrency clients [116]–[119] only perform myopic transaction-inclusion optimizations, trying to generate a single maximal-fee block each time. As recently shown [37], [56], [60] (including in our deployment above), miners and other entities perform more sophisticated optimizations on the Ethereum network.

In contrast, we are not aware of similar optimizations taking place on the Bitcoin network. Specifically, Bitcoin Core, which is used by roughly 97% of current Bitcoin nodes [40], maintains a local *mempool* data structure that only contains unconfirmed transactions whose timeouts (if any) have elapsed.

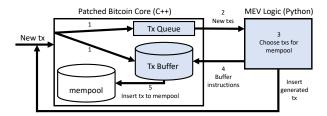


Figure 2: Bitcoin-MEV, new components shaded.

This implementation prevents miners from optimizing based on transaction pending on a timeout. However, this limitation is not a consensus rule, but an implementation choice. Taking more elaborate considerations into account when choosing transactions is not a violation of legitimate miner behavior.

As noted (§II-B), optimizing transaction revenue is becoming more important for miners over time. To demonstrate the ease of achieving broader optimizations, including non-myopic considerations, we implemented *Bitcoin-MEV*, an infrastructure allowing to easily incorporate any logic over Bitcoin Core received transactions.

Bitcoin-MEV's main design goal is to enable users to deploy their own optimization algorithms. It comprises a patched (140 LoC) C++ Bitcoin Core node with additional RPCs, and a Python script (Fig. 2, new components shaded), working as follows.

When the node receives a new transaction, instead of directly placing it in its mempool, it pushes the transaction to a designated *new transaction queue*. The Python script monitors this queue with a dedicated RPC, fetches new transactions and parses them. Then, based on the implemented optimization algorithm, it can instruct the node how to handle the transaction – insert it to the mempool, discard it, or keep it for future use. The Python script can also generate new transactions and send them to the node.

We implemented and locally tested a Python script (350 LoC) for enforcing MAD-HTLC by taking advantage of the opportunities it provides miners to increase their revenue. We screen received transactions, tease out \mathcal{A} and \mathcal{B} 's preimages, and create a transaction redeeming the MAD-HTLC contracts using the extracted preimages.

VII. HTLC

The prevalent implementation of *HTLC-Spec* is a direct translation of the specification to a single contract called *HTLC* (§VII-A).

It relies on the premise that miners benevolently enforce the desired execution, namely include \mathcal{A} 's transaction in a block before the timeout elapses. However, this assumption contradicts the core principle of cryptocurrency permissionless systems — miners operate to make profit [57], [58], [62]–[67], and include transactions that benefit their personal gains [37], [56], [59], [61]. Specifically, \mathcal{B} can incentivize miners with a bribe [26]–[28] to exclude \mathcal{A} 's transaction until the timeout elapses, and then redeem the HTLC himself.

Predicate 3: HTLC

```
\begin{array}{ll} \text{Parameters: } pk_a, pk_b, T, dig_a \\ HTLC (pre, sig) \coloneqq & & & & & & & \\ \text{In } (\mathsf{vPrelmg} \ (pre; dig_a) \land \mathsf{vSig} \ (sig; pk_a)) \lor & & & & & & \\ \text{In } (\mathsf{vSig} \ (sig; pk_b) \land \mathsf{vTime} \ (T)) & & & & & & \\ \end{array}
```

We analyze the security of HTLC by formalizing the game played by the entities ($\S VII-B$), and showing how cheap \mathcal{B} 's required bribe is ($\S VII-C$). We show miner fee optimization is easy by implementing a bribery-accepting (i.e., rational and non-myopic) miner ($\S VII-D$), and conclude by estimating the actual attack cost using numbers from operational contracts ($\S VII-E$).

A. Construction

 ${\cal A}$ and ${\cal B}$ execute HTLC-Spec by having an HTLC contracted with $v^{\rm dep}$ tokens placed in some block b_j . The HTLC's predicate is parameterized with ${\cal A}$'s and ${\cal B}$'s public keys, pk_a and pk_b , respectively; a hash digest of the predefined secret $dig_a = H \ (pre_a)$ such that any entity other than ${\cal A}$ and ${\cal B}$ does not know $pre_a \ ({\cal A} \ {\rm or} \ {\cal B} \ {\rm know} \ pre_a$ based on the specific use case); and a timeout T.

HTLC has two redeem paths, denoted htlc- \mathcal{A} and htlc- \mathcal{B} , and presented in Predicate 3. In htlc- \mathcal{A} (line 1), \mathcal{A} can redeem with a transaction including pre_a and sig_a , a signature with her secret key sk_a . In htlc- \mathcal{B} (line 2), \mathcal{B} can redeem with a transaction including sig_b , a signature with his secret key sk_b . This transaction can only be included in a block at least T blocks after HTLC's initiation, that is, block b_{j+T} .

As only $\mathcal A$ and $\mathcal B$ know their respective secret keys, other entities cannot redeem the contract.

The intended way \mathcal{A} and \mathcal{B} should interact with HTLC is as follows. If \mathcal{A} knows the predefined preimage pre_a , she publishes a transaction tx_a^h offering a fee $f_a^h > f$ that redeems the HTLC. She publishes this transaction right after the creation of block b_j , that is, before the creation of block b_{j+1} . If \mathcal{A} does not know the predefined preimage pre_a she does not publish any transactions.

 ${\mathcal B}$ observes the published transactions in the mempool, watching for $tx_a^{\rm h}$. If by block b_{j+T-1} ${\mathcal A}$ did not publish $tx_a^{\rm h}$ then ${\mathcal B}$ publishes $tx_b^{\rm h}$ with a fee $f_b^{\rm h}>f$, redeeming the HTLC. If ${\mathcal A}$ did publish $tx_a^{\rm h}$ by block b_{j+T-1} then ${\mathcal B}$ does not publish any transactions.

B. HTLC Game

HTLC operation gives rise to a game, denoted by Γ^H , played among \mathcal{A} , \mathcal{B} and the miners. It is similar to that of the MAD-HTLC game (\S V-B), so we present the differences.

a) Subgames: The game state is simply the number of blocks (k) created so far and state of the HTLC, which can be either redeemable (red) or irredeemable (irred), so denoted $\Gamma^H(k, \text{red/irred})$.

The game begins when one block (initiating the HTLC) was created, \mathcal{A} and \mathcal{B} did not publish any transactions, and the HTLC is redeemable. Thus, the initial, complete game is $\Gamma^H(1, \text{red})$.

b) Actions: \mathcal{A} can redeem the HTLC with a transaction $tx_a^{\rm h}$, offering $f_a^{\rm h}$ tokens as fee. Note $tx_a^{\rm h}$ has to outbid unrelated transactions and thus has to offer a fee $f_a^{\rm h} > f$, however, cannot offer more tokens than the redeemed ones, so $f_a^{\rm h} < v^{\rm dep}$. \mathcal{A} redeems HTLC using the htlc- \mathcal{A} redeem path, so $tx_a^{\rm h}$ can be confirmed in any round.

 \mathcal{B} can redeem HTLC with a transaction $tx_b^{\rm h}$, offering $f_b^{\rm h}$ tokens as fee. Similarly, $f_b^{\rm h}$ is bounded such that $f < f_b^{\rm h} < v^{\rm dep}$. \mathcal{B} redeems HTLC using the htlc- \mathcal{B} redeem path, so $tx_b^{\rm h}$ can only be confirmed in the last round.

Any miner can include the following transactions: an unrelated transaction in any $\Gamma^H(\cdot,\cdot)$ subgame; tx_a^h in any $\Gamma^H(\cdot,\mathsf{red})$ subgame; and tx_b^h in the $\Gamma^H(T,\mathsf{red})$ subgame.

C. Bribe Attack Analysis

We now show the HTLC prescribed strategy (§VII-A) is not incentive compatible. Specifically, we show that if \mathcal{A} commits to the prescribed strategy, then \mathcal{B} strictly gains by publishing a conflicting transaction, outbidding \mathcal{A} 's fee, thus incentivizing miners to exclude \mathcal{A} 's transaction and include his instead.

Let \mathcal{A} publish $tx_a^{\rm h}$ with fee $f_a^{\rm h}$ in the first round, and \mathcal{B} publish a transaction $tx_b^{\rm h}$ with fee $f_b^{\rm h} > \frac{f_a^{\rm h} - f}{\lambda_{\rm min}} + f$. Focusing on miner actions, we show through a series of lemmas they are incentivized to include $tx_b^{\rm h}$ and to exclude $tx_a^{\rm h}$, resulting with lower utility for \mathcal{A} , higher utility for \mathcal{B} , and a violation of the HTLC-Spec.

First, we show miner utilities for subgames where the HTLC is irredeemable. Denote by $\bar{\sigma}$ the best response strategy of all miners in this setting.

Lemma 7. For any $k \in [1,T]$, the utility of miner i in subgame $\Gamma^H(k, \text{irred})$ is $u_i(\bar{\sigma}, \Gamma^H(k, \text{irred})) = \lambda_i(T-k+1)f$.

Proof. Since HTLC is irredeemable, the only available action for miners is to include an unrelated transaction, yielding a reward of f.

Consider any $\Gamma^H(k, \mathsf{irred})$ subgame. There are T-k+1 remaining blocks to be created, and miner i creates any of them with probability λ_i . This scenario can be viewed as a series of T-k+1 Bernoulli trials with success probability λ_i . The number of successes is therefore Binomially distributed, and the expected number of blocks miner i creates is λ_i (T-k+1). The reward for each block is f, so miner i's utility is u_i $(\bar{\sigma}, \Gamma^H(k, \mathsf{irred})) = \lambda_i (T-k+1) f$.

We now consider miner utilities for $\Gamma^H(\cdot, \text{red})$ subgames, where the HTLC is redeemable. We begin with the final subgame $\Gamma^H(T, \text{red})$, creating block B_{j+T} .

Lemma 8. Choosing to include tx_b^h is a unique subgame perfect equilibrium in $\Gamma^H(T, \textit{red})$, and miner i's utility when doing so is $u_i\left(\bar{\sigma}, \Gamma^H(T, \textit{red})\right) = \lambda_i f_b^h$.

Proof. In the $\Gamma^H(T, \text{red})$ subgame, the miner that creates the block has three transactions to pick from: an unrelated transaction for the base fee f, tx_a^h for f_a^h , or tx_b^h for f_b^h .

As $f_b^{\rm h}>\frac{f_a^{\rm h}-f}{\lambda_{\rm min}}+f$, $0<\lambda_{\rm min}<1$ and $f_a^{\rm h}>f$, it follows that $f_b^{\rm h}>f_a^{\rm h}$ and $f_b^{\rm h}>f$. That means including $tx_b^{\rm h}$ yields strictly greater reward than all other actions, thus being a unique subgame perfect equilibrium in this subgame.

Miner i creates the block with probability λ_i , and so her expected profit, i.e. utility, is $u_i(\bar{\sigma}, \Gamma^H(T, \text{red})) = \lambda_i f_b^h$. \square

We now move on to consider any earlier $(k \in [1, T-1])$ subgame (Blocks B_{j+1} to B_{j+T-1}) where the HTLC is redeemable.

Lemma 9. For any $k \in [1, T-1]$, the unique subgame perfect equilibrium is that every miner includes an unrelated transaction in $\Gamma^H(k, \text{red})$, and miner i's utility when doing so is $u_i(\bar{\sigma}, \Gamma^H(k, \text{red})) = \lambda_i((T-k)f + f_b^h)$.

To prove this lemma we show that for any $k \in [1, T-1]$, including \mathcal{A} 's transaction in subgame $\Gamma^H(k, \text{red})$ results with lower overall utility at game conclusion – intuitively, it redeems the contract, so in the last subgame miners cannot include \mathcal{B} 's transaction. The proof is by induction on k, and we bring it in full in Appendix D.

We conclude with the main theorem regarding *HTLC* susceptibility to bribing attacks:

Theorem 2. Alice's prescribed behavior of HTLC allows \mathcal{B} to bribe miners at a cost of $\frac{f_{\alpha}^{h}-f}{\lambda_{min}}+f$.

Proof. The proof follows directly from Lemma 8 and Lemma 9, both showing that if \mathcal{A} naively follows the prescribed strategy then subgame perfect equilibrium of the initial subgame is for all miners to place unrelated transactions until round T and then place \mathcal{B} 's transaction.

Note that by Theorem 2, the bribing cost required to attack HTLC is independent in T, meaning that simply increasing the timeout does contribute to HTLC's security.

Of course once \mathcal{A} sees an attack is taking place she can respond by increasing her fee. In turn, this could lead to \mathcal{B} increasing his fee as well, and so forth. Instead of focusing on these bribe and counter-bribe dynamics, we conclude by showing that \mathcal{A} can preemptively prevent the attack, or assure winning with a counter-bribe, by paying a high fee dependent on v^{dep} ,. We note that such a high fee is in violation of the HTLC-Spec.

Corollary 2. B cannot bribe the miners in this manner if A's tx_a^h offers at least $f_a^h > \lambda_{min} (v^{dep} - f) + f$.

Proof. In order to achieve the attack, \mathcal{B} ought to make placing unrelated transactions until T and placing his transaction at T a subgame perfect equilibrium. As shown (Theorem 2), the threshold to incentivize the smallest miner is $f_b^h > \frac{f_a^h - f}{\lambda_{\min}} + f$. Recall the fee f_b^h of the bribing transaction tx_b^h is upper bounded by the HTLC tokens v^{dep} . Therefore, to achieve the attack it must hold that $v^{\text{dep}} > \frac{f_a^h - f}{\lambda_{\min}} + f$. By choosing $f_a^h > \lambda_{\min} \left(v^{\text{dep}} - f\right) + f$, \mathcal{A} can prevent \mathcal{B} from paying a fee adhering to the bounds.

Myopic Miners: This bribery attack variant relies on all miners being rational, hence considering their utility at game conclusion instead of myopically optimizing for the next block. If a portion of the miners are myopic and any of them gets to create a block during the first T-1 rounds, that miner would include \mathcal{A} 's transaction and \mathcal{B} 's bribery attempt would have failed.

In such scenarios the attack succeeds only with a certain probability – only if a myopic miner does not create a block in the first T-1 rounds. The success probability therefore decreases exponentially in T. Hence, to incentivize miners to support the attack, $\mathcal B$ has to increase his offered bribe exponentially in T.

The analysis relies on assumptions on the mining power distribution, and is outside the scope of this work. Notably, for the simpler case when all other miners are myopic, miner i is incentivized to support the attack only when it is her dominant strategy, matching the upper bound of Winzer et al. [28].

D. Non-Myopic Bribery-Accepting Miner Implementation

Aside from the Bitcoin-MEV infrastructure (§VI-B), we also implemented a simpler Bitcoin Core patch supporting the mentioned bribe attack on *HTLC*.

When the patched client receives transactions with an unexpired timeout (*waiting* transactions) it stores them in a data structure instead of discarding them. When creating a new block, the client first checks if any of the timeouts have elapsed, and if so, moves the relevant transactions to the mempool. When receiving conflicting transactions, instead of accepting the first and discarding the second, it accepts the transaction that offers a higher fee. In case of a conflict with a waiting transaction, it chooses based on the condition described in Theorem 2.

The simplicity of this patch (150 LoC, no external modules) demonstrates that miners can trivially achieve non-myopic transaction selection optimization.

E. Real-World Numbers

We conclude this section by presenting three examples of *HTLC* being used in running systems, and show the substantial costs to make them resistant against bribery attacks.

Table V presents for each example the *HTLC* tokens v^{dep} , the base fee f, and the ratio of required tokens for bribery resistance (Theorem 2) and the base fee $\frac{\lambda_{\min}(v^{\text{dep}}-f)+f}{f}$. To estimate the base fee we conservatively take the actual paid fee, which is an upper bound. We conservatively estimate $\lambda_{\min} = 0.01$ [87]; miners with lower mining power are less likely due to economy-of-scale [126].

The first example is of a Bitcoin Lightning channel [127], [128], where the required fee to secure the contract against a bribery is 1.34e4 times the actual fee. Plugging in \$10K as the average Bitcoin price at the time [123], we get that an attack requires about a \$2 bribe for a payoff of over \$25K. Note this is just an arbitrary example, and there are plenty of such low-fee, high-capacity channels, in all a few dollars bribe is sufficient to yield tens of thousands of dollars as reward [127].

Table V: HTLC bribe resistance cost examples.

Name	$v^{ m dep}$	f	$\frac{\lambda_{\min}\left(v^{\text{dep}}-f\right)+f}{}$
			f
Lightning channel (BTC) [127], [128]	2.684	2.22e-6	1.34e4
Litecoin atomic swap (LTC) [129]	1.337	3.14e-4	435.7
Liquality [130] atomic swap (ETH) [125]	12	0.0004	301
Liquality [130] atomic swap (BTC) [131]	0.278	5.76e-6	483.63

The second example is of a Litecoin atomic swap [129], requiring 436 times higher fee to be secured against bribes. The last two examples are the two sides of a BTC-ETH atomic swap conducted by Liquality [130], requiring more than 300X and 480X fees to be secure, respectively.

VIII. FUTURE DIRECTIONS

We briefly present two future research directions. First, we discuss attacks and mitigations in a weaker model, where either \mathcal{A} or \mathcal{B} have significant mining power (§VIII-1). Then, we discuss how using MAD-HTLC can reduce latency in systems utilizing HTLC-Spec (§VIII-2)

1) Mining \mathcal{A} or \mathcal{B} : As in previous work [26], [28], the security analysis of MAD-HTLC assumes that \mathcal{A} and \mathcal{B} have no mining capabilities and do not collude with any miner. Indeed, acquiring mining capabilities (or forming collusion agreements) requires a significant investment, substantially higher than necessary for a simple bribe. Removing this assumption extends the game space considerably, and brings in timing and probability considerations that are outside the scope of this work. Nevertheless, we briefly present the issue and a potential low-overhead modification that disincentivizes such attacks.

 ${\cal A}$ with mining capabilities that knows pre_a can stall until the timeout elapses and ${\cal B}$ publishes pre_b , and then redeem both MH-Dep (using either dep- ${\cal A}$ or dep- ${\cal M}$) and MH-Col (using col- ${\cal M}$). This requires ${\cal A}$ to create the block right after the timeout elapses, otherwise another miner would include ${\cal B}$'s transactions. The potential profit is the $v^{\rm col}$ tokens, whose number is in the order of a transaction fee.

 \mathcal{B} with mining capabilities can redeem MH-Dep (using dep- \mathcal{M}) if he knows pre_a . This requires \mathcal{B} to create the first block after the MAD-HTLC initiation, otherwise another miner would include \mathcal{A} 's transaction. The potential damage for this case is similar to the HTLC bribery (Winzer et al. [28] and $\S VII$ -C), and note that any miner will be able to redeem MH-Col once the timeout elapses.

Both variants require the miner to reveal pre_a and pre_b by creating a block at a specific height, meaning they only succeed with some probability. As such, their profitability depends on the relative mining size of the miner, the deposit and collateral amounts, and the transaction fees.

Nevertheless, these are vulnerabilities of MAD-HTLC, and we propose the following countermeasure: Instead of having a single MH-Dep and a single MH-Col, have multiple of each, all with the same dig_a and dig_b , but each with a different timeout T, and split $v^{\rm dep}$ and $v^{\rm col}$ among them.

As one of the timeouts elapse, if the miner attacks then she loses her advantage, as once she exposes pre_a and pre_b , any miner can compete for the remaining contracts. Therefore, this mechanism diminishes the attack profitability.

This adjustment's overhead is only due to the fees for creating and redeeming more contracts. However, those can be small, independent of the secured amount.

2) Latency Reduction: Systems utilizing HTLC-Spec must set the timeout parameter T, facing a trade-off. Too short timeouts result in a security risk – \mathcal{B} might get the tokens unjustly because \mathcal{A} 's transaction was not yet confirmed. Too long timeouts imply an opportunity cost due to the unavailability of the locked coins, and increase susceptibility to various attacks [12], [132], [133].

MAD-HTLC can allow for significantly reduced timeouts compared to HTLC, since instead of waiting for confirmation, it now suffices to consider transaction publication. The analysis depends on mempool and congestion properties that are outside the scope of this work.

IX. CONCLUSION

We introduce a novel approach of utilizing miner's rationality to secure smart contracts, and use it to design MAD-HTLC, a contract implementing HTLC-Spec. We show using the UC framework and with game-theoretic analysis that MAD-HTLC is secure. We also show the prevalent HTLC is vulnerable to cheap bribery attacks in a wider variety of systems, and qualitatively tighten the known cost bound in presence of rational miners. We demonstrate the efficacy of our approach by implementing and executing MAD-HTLC on Bitcoin and Ethereum. We also demonstrate the practicality of implementing a rational miner by patching the standard Bitcoin client.

Both the attack against *HTLC* and the secure alternative *MAD-HTLC* have direct impact on a variety of contracts using the *HTLC-Spec* design pattern. As miners' incentives to act rationally increase, those systems will become vulnerable and can directly adopt *MAD-HTLC* as a plug-in alternative.

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APPENDIX A

MAD-HTLC INCENTIVE COMPATIBILITY LEMMA PROOFS

This section presents the proofs for Lemma 4, Lemma 5 and Lemma 6. We recall the lemmas for readability.

Lemma (4). In the last round of the game, i.e. subgame $\Gamma^{MH}(T,\cdot)$, if tx_a^{dep} and either tx_b^{dep} or $tx_b^{dep+col}$ are published then miners' best-response strategy is not to include any of A's or B's transactions in this round.

Proof. Since A and B published their transactions, both pre_a and pre, are available to all miners. Therefore, any miner can create a transaction redeeming MH-Dep and MH-Col herself.

If MH-Dep is irredeemable $(\Gamma^{MH}(T, irred))$, then miners can create $tx_m^{\rm col}$ and redeem MH-Col themselves in round T, getting $v^{\rm col}$ tokens as reward. Alternatively, if $tx_b^{\rm col}$ is published they can include it in a block, getting a fee of f_b^{col} tokens. As $f_b^{\text{col}} < v^{\text{col}}$, including tx_b^{col} is strictly dominated by including tx_m^{col} . In this case miners can also not include tx_a^{dep} as the MH-Dep is irredeemable.

If MH-Dep is redeemable $(\Gamma^{MH}(T, red))$, miners can also create $tx_m^{\text{dep+col}}$, include it in a block, and get $v^{\text{dep}} + v^{\text{col}}$ in reward. Alternatively, they can include either tx_a^{dep} , tx_b^{dep} , tx_b^{col} or $tx_b^{\text{dep+col}}$ (whichever was published). However, any of these offers fees lower than $v^{\text{dep}} + v^{\text{col}}$, making them strictlydominated by including $tx_m^{\text{dep+col}}$

Either way, including any of A's or B's transactions results with a strictly lower reward, hence miners avoid doing so. \Box

Lemma (5). In Γ^{MH} (1, red), A cannot increase her utility by deviating from the prescribed strategy.

Proof. First, if A does not know pre_a , she can take no action, hence trivially complies with the prescribed strategy.

If A does know pre_a , then her possible deviations are not publishing tx_a^{dep} at all, or publishing it only in the last round T.

Not publishing tx_a^{dep} at all is strictly dominated — she gets no tokens; if she instead abides by the prescribed strategy then she cannot get a lower revenue but can get more, e.g., if \mathcal{B} also follows the prescribed strategy (Lemma 2).

The inclusion of tx_a^{dep} in the last block depends on what transactions \mathcal{B} publishes throughout the game (Lemma 4). That is, if \mathcal{B} published either tx_b^{dep} or $tx_b^{\text{dep+col}}$ then miners' best-response is not to include tx_a^{dep} , and \mathcal{A} gets no tokens. Otherwise, miners' best response is to include the transaction that offers the highest fee, which can be either tx_a^{dep} or another, resulting with A receiving $v^{\text{dep}} - f_a^{\text{dep}}$ and 0 tokens, respectively.

So, A cannot gain, and in several scenarios strictly lose, by deviating from her prescribed strategy.

Lemma (6). In Γ^{MH} (1, red), \mathcal{B} cannot increase his utility by deviating from the prescribed strategy.

Proof. Consider all of \mathcal{B} 's possible actions. His potential maximal utility is from having $tx_b^{\text{dep+col}}$ included, which he obtains by following the prescribed strategy in the scenario where A does not know pre_a (Lemma 3). So, he has no incentive to deviate in this case.

Now, consider the case where A knows pre_a , hence accord-

ing to Lemma 5 publishes tx_a^{dep} in the first T-1 rounds. \mathcal{B} can publish tx_b^{dep} , $tx_b^{\text{dep+col}}$ and tx_b^{col} throughout the game. If he publishes tx_b^{dep} or $tx_b^{\text{dep+col}}$ in any round then none of his transactions are included (Lemma 4) and he gets no reward. However, if he only publishes tx_h^{col} then by Lemma 2 he receives $v^{\text{col}} - f_b^{\text{col}} > 0$ tokens.

Not publishing tx_b^{col} at all results with the minimal utility of 0, and an earlier publication still leads miners to include both tx_b^{col} and tx_a^{dep} (cf. 6), obtaining the same utility as of the prescribed behavior.

APPENDIX B

MAD-HTLC BITCOIN AND ETHEREUM IMPLEMENTATIONS

Fig. 3 shows the Bitcoin Script implementation of MH-Dep, MH-Col and HTLC. It also presents the required input data for each redeem path.

Script is stack-based, and to evaluate input data and a contract the latter is concatenated to the former, and then executed: constants are pushed into the stack, instructions operate on the stack. For a successful evaluation the stack must hold exactly one element with value 1 after all operations are executed.

a) MH-Dep: The script expects either two or three data elements. It hashes the first two and checks if they match dig_a and dig_b .

If the first matches dig_a but the second does not match dig_b (dep-A), then the script verifies the existence of a third data element, and that it is a signature created with A's secret key.

If the first does not match dig_a but the second matches dig_b (dep-B), then the script verifies the existence of a third data element, and that it is a signature created with \mathcal{B} 's secret key. It also verifies the timeout has elapsed.

If both the first and the second data elements match dig and dig_b (dep- \mathcal{M}), respectively, then the script expects no third data element and evaluates successfully.

MH-Dep	MH-Col	HTLC
OP_HASH160 dig_a OP_EQUAL OP_EQUAL OP_SWAP OP_HASH160 dig_b OP_EQUAL OP_IF OP_IF OP_IF OP_LSE T OP_CHECKSEQUENCEVERIFY OP_DROP pk_b OP_CHECKSIG OP_ENDIF OP_ELSE OP_CHECKSIG OP_ENDIF OP_ELSE OP_VERIFY pk_a OP_CHECKSIG OP_ENDIF	T OP_CHECKSEQUENCEVERIFY OP_DROP OP_HASH160 dig_a OP_EQUAL OP_IF OP_HASH160 dig_b OP_EQUAL OP_ELSE pk_b OP_CHECKSIG OP_ENDIF	OP_HASH160 dig_a OP_EQUAL OP_IF
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

Figure 3: MH-Dep, MH-Col and HTLC Bitcoin Script implementations.

Table VI: Bitcoin main-net experiment transaction IDs.

Description	Transaction ID
Initiate MILDen	d032175260145055860296cbca8f7462
Initiate MH-Dep	4f30334ddf948d5da12f0c7414d80cc0
MII Don noth 1	33c957bb2f75e797d240a38504ce49a3
MH-Dep path 1	aeaaceb72f8577096b4f2ff23f5b3a1e
MII Don noth 2	cd090c90afaacc0e2648834fe96f6177
MH-Dep path 2	ec2f967b7e50245537afdaf0d5a80263
MH-Dep path 3	505c7f1f3862b7f5c6b78f72cce5e37a
MH-Dep patil 3	655b946fbdc7d03526055f7ea206781a
Initiate MH-Col	ea830dba56000b3486cf1c5122fedcf8
	8169ab596536fd406b4f989e7761c1b4
MIL Coll mode 1	4c06ebff8de6bb56242c75849767a633
MH-Col path 1	9e40a0442f815a2487fd9d6237c51b9f
MH-Col path 2	68270b94ca80281e31e193dac6779d3a
MH-Cor paul 2	22d2799fe2afff8cef66c0ec6b420c88

b) MH-Col: The script expects exactly two data elements. It begins by verifying timeout has elapsed, and then hashes the first element and checks if it matches dig_a .

If not $(col-\mathcal{B})$, the script then verifies the second data is a signature created with \mathcal{B} 's secret key. Otherwise $(col-\mathcal{M})$, the script hashes the second data element and verifies it matches dig_h .

We bring the Ethereum Solidity implementation of *MAD-HTLC* and *HTLC* in the extended report [95].

APPENDIX C

MAD-HTLC BITCOIN AND ETHEREUM DEPLOYMENT

Tables VI and VII show the transaction IDs in our Bitcoin and Ethereum deployments (§VI-A), respectively. Their details can be viewed with online block explorers.

APPENDIX D HTLC Bribe Attack Analysis Proof

We recall Lemma 9 and prove it.

Lemma (9). For any $k \in [1, T-1]$, the unique subgame perfect equilibrium is that every miner includes an unrelated transaction in $\Gamma^H(k, \text{red})$, and miner i's utility when doing so is $u_i(\bar{\sigma}, \Gamma^H(k, \text{red})) = \lambda_i((T-k)f + f_h^h)$.

Table VII: Ethereum main-net experiment transaction IDs.

Description	Transaction ID
Initiation	f10be5e53b9ad8a6f10d7e9b9bfbd63a b8737c50274885182a67e7adc3fa59c2
dep-A	36e349b4fdc5385ef57a88d077837223 b3a26b0e6afc75f90bbaf2860d9295fd
dep-B	84aa626d659b63e0554f8de1a3d6e204 41d8d778b7e1e79d0a36ded325afedb4
dep-M (ours)	ebdb267e8b612d59910bc2348a95eec8 388e62dbd6d64458c982f0cdacea67d9
dep-M (other)	74e87bba99ccd7a0bd794b793f108674 5b462390df01594ce057a430c122635a

Proof. Note that in $\Gamma^H(k, \text{red})$ there are two actions available, either include an unrelated transaction and receive f reward, or include tx_a^h and receive f_a^h reward.

Consider any miner i. Denote by λ_u^k the accumulated block-creation rates of miners, excluding miner i, that choose to include an unrelated transaction in $\Gamma^H(k, \text{red})$. Therefore, the accumulated probabilities of miners that choose to include tx_a^h , excluding miner i, is $1 - \lambda_u^k - \lambda_i$.

If miner i chooses to include an unrelated transaction then either of the following occurs. First, with probability λ_i miner i gets to create a block, includes an unrelated transaction and receives a reward of f. The subsequent subgame is $\Gamma^H(k+1,\text{red})$. Alternatively, with probability λ_u^k another miner that includes an unrelated transaction gets to create a block, miner i gets no reward and the subsequent subgame is $\Gamma^H(k+1,\text{red})$. Finally, with probability $1-\lambda_u^k-\lambda_i$ another miner that includes tx_a^h gets to create a block, miner i gets no reward and the subsequent subgame is $\Gamma^H(k+1,\text{irred})$.

Therefore, miner *i*'s utility when including an unrelated transaction in these subgames is

$$\begin{aligned} u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k, \text{red}\right)\right) &= \\ \lambda_{i} \cdot \left(f + u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k + 1, \text{red}\right)\right)\right) + \\ \lambda_{u}^{k} \cdot u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k + 1, \text{red}\right)\right) + \\ \left(1 - \lambda_{i} - \lambda_{u}^{k}\right) \cdot u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k + 1, \text{irred}\right)\right) \,. \end{aligned} \tag{2}$$

Similarly, if miner i chooses to include tx_a^h than either of the

following occurs. First, with probability λ_i miner i gets to create a block, includes $tx_a^{\rm h}$ and receives a reward of $tx_a^{\rm h}$. The subsequent subgame is $\Gamma^H(k+1,{\rm irred})$. Alternatively, with probability λ_u^k another miner that includes an unrelated transaction gets to create a block, miner i gets no reward and the subsequent subgame is $\Gamma^H(k+1,{\rm red})$. Finally, with probability $1-\lambda_u^k-\lambda_i$ another miner that includes $tx_a^{\rm h}$ gets to create a block, miner i gets no reward and the subsequent subgame is $\Gamma^H(k+1,{\rm irred})$.

Therefore, miner i's utility when including tx_a^h in these subgames is

$$\begin{aligned} u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k, \mathsf{red}\right)\right) &= \\ \lambda_{i} \cdot \left(f_{a}^{\mathsf{h}} + u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k+1, \mathsf{irred}\right)\right)\right) + \\ \lambda_{u}^{k} \cdot u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k+1, \mathsf{red}\right)\right) + \\ \left(1 - \lambda_{i} - \lambda_{u}^{k}\right) \cdot u_{i}\left(\bar{\sigma}, \Gamma^{H}\left(k+1, \mathsf{irred}\right)\right) \; . \end{aligned} \tag{3}$$

To prove the lemma we need to show that for any $k \in [1, T-1]$ the utility from including an unrelated transaction (Eq. 2) exceeds that of including tx_a^h (Eq. 3). This reduces to showing that

$$f + u_i \left(\bar{\sigma}, \Gamma^H \left(k + 1, \text{red} \right) \right) > f_a^h + u_i \left(\bar{\sigma}, \Gamma^H \left(k + 1, \text{irred} \right) \right) , \tag{4}$$

which we do inductively.

a) Base: First, consider k=T-1. Using Lemma 8 and Lemma 7 we get the condition presented in Eq. 4 is $f+\lambda_i f_b^h > f_a^h + \lambda_i f$, or alternatively,

$$f_b^{\mathsf{h}} > \frac{f_a^{\mathsf{h}} - f}{\lambda_i} + f \ . \tag{5}$$

Since $\lambda_{\min} \leq \lambda_i$ and $f_b^h > \frac{f_a^h - f}{\lambda_{\min}} + f$, the condition (Eq. 5) holds, meaning that in any subgame perfect equilibrium miner i is strictly better by including an unrelated transaction in subgame $\Gamma^H(T-1, \text{red})$.

Therefore, all miners choose to include unrelated transactions in such subgames, meaning $\lambda_u^j=1-\lambda_i$ and $1-\lambda_i-\lambda_u^j=0$. Therefore, miner i's utility (Eq. 2) is $u_i\left(\bar{\sigma},\Gamma^H\left(k,\mathrm{red}\right)\right)=\lambda_i\left(f+f_b^\mathrm{h}\right)$.

- b) Assumption: Consider any $k \in [1, T-2]$ and assume that the claim holds for k+1. That is, the unique subgame perfect equilibrium in subsequent games $\Gamma^H(k+1, \text{red})$ is for all miners to include an unrelated transaction, and the utility of miner i when doing so is $u_i\left(\bar{\sigma}, \Gamma^H(k+1, \text{red})\right) = \lambda_i\left((T-k) + f_b^h\right)$.
- c) Step: Using the inductive assumption and Lemma 7 the condition of Eq. 4 translates to $f + \lambda_i \left((k+1) f + f_b^h \right) > f_a^h + \lambda_i \left((k+1) f \right)$, or alternatively,

$$f_b^{\rm h} > \frac{f_a^{\rm h} - f}{\lambda_i} + f \ . \tag{6}$$

Again, since $\lambda_{\min} \leq \lambda_i$ and $f_b^h > \frac{f_a^h - f}{\lambda_{\min}} + f$, the condition (Eq. 6) holds, meaning that in the subgame perfect equilibrium miner i's strict best response is to include an unrelated transaction in subgame $\Gamma^H(k, \text{red})$.

Since all miners include unrelated transactions, we get $\lambda_u^j = 1 - \lambda_i$ and $1 - \lambda_i - \lambda_u^j = 0$. Therefore, miner i's utility (Eq. 2) is $u_i \left(\bar{\sigma}, \Gamma^H \left(k, \mathsf{red} \right) \right) = \lambda_i \left((T - k) \, f + f_b^h \right)$.