Mechanism Design for ZK-Rollup Prover Markets

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Abstract

In ZK-Rollups, provers spend significant computational resources to generate validity proofs. Their costs should be compensated properly, so a sustainable prover market can form over time. Existing transaction fee mechanisms (TFMs) such as EIP-1559, however, do not work in this setting, as EIP-1559 only generates negligible revenue because of burning, while provers often create or purchase specialized hardware in hopes of creating long-term revenue from proving, somewhat reminiscent of proof-of-work miners in the case of chains like Bitcoin. In this paper, we explore the design of transaction fee mechanisms for prover markets. The desiderata for such mechanisms include efficiency (social welfare is maximized), incentive compatibility (it is rational to bid honestly), collusion resistance (no profitable collusion among provers exists), and off-chain agreement proofness (no profitable collusion between users and provers exists). To demonstrate the difficulties of our new setting, we put forward several simple strawman mechanisms, and show they suffer from notable deficiencies.

1 Introduction

ZK-Rollup systems rely on provers to generate zero-knowledge proofs (ZKPs) to finalize ZK-Rollup blocks. Despite recent efficiency improvement [24, 8, 6], generating ZKPs for ZK-Rollups remains a demanding job that requires significant computational resources [12]. Currently, ZK-Rollup systems typically run their own centralized provers [56]. However, previous works on similar centralized approaches find they are susceptible to censorship [51] and may increase costs for users, while allowing free entry for service providers can increase competitiveness and lower costs [19]. An appealing alternative is to build an open and permissionless marketplace where provers compete to generate ZKPs at low costs. In a prover market, user fees should at the very least cover the cost of provers and, ideally, also provide some sustainable profit margins. This, together with a series of design goals, calls for a properly designed transaction fee mechanism for prover markets.

Prover markets differ from classical fee markets (e.g., that in Bitcoin and Ethereum) in three ways. Firstly, a prover market is a two-sided market where users demand proof capacity, and provers

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supply it. In a prover market, users have heterogeneous valuations of proof capacity, and provers may have heterogeneous cost and capacity (e.g., some provers may invest in specialized hardware that has low proving costs). This is in contrast to block space markets where validators' cost to produce blocks is nominal. Secondly, compensating provers for their work would be essential, whereas fee markets do not aim to compensate validators (e.g., EIP-1559 [36] burns most revenue; instead, validators are compensated by block rewards, etc.) Thirdly, fee markets are usually not concerned with selecting validators (e.g., in Ethereum, validators are selected by the consensus protocol), but prover markets need to select provers from a group of candidates when there is a supply surplus. Ideally, this process will select the provers with the lowest costs, but provers need not report their true costs.

In this paper, we explore the design space of prover markets. A prover market mechanism \mathcal{M} generally proceeds in rounds. In each round, a user submits transaction TX with a per-constraint fee f; a prover submits a bid specifying its capacity and cost to generate proof within the round. \mathcal{M} chooses a subset of user transactions and appoints a set of provers to generate proofs for chosen transactions. Ideally, if \mathcal{M} magically learns the true valuation from users and true costs and capacity from provers, it can solve for an optimal outcome that maximizes social welfare (e.g., by picking transactions with high values until the prover's capacity or cost constraints are met.) The challenge, though, is that both users and provers may be strategic. A user may bid lower than her true valuation, and a prover may report a higher than the true cost. Moreover, a subset of users and provers may collude so that users can "get in" with a fee lower than honest users, creating an unfairness amongst users.

Desirable properties. As with TFM for fee markets (e.g., [36]), the main goal of TFM for prover markets is to disincentive undesirable strategic behaviors that can reduce efficiency or cause unfairness, summarized as follows.

- Efficiency: An efficient prover market should maximize total social welfare, which means that the transactions with the highest values are proven up to the constraints of the prover's constraints (when demand exceeds supply) and provers with the lowest prover costs are selected (when supply exceeds demand.)
- Incentive-compatibility: There is a (Bayesian) incentive-compatible strategy for each user and each prover, i.e., it is rational for users and provers to act honestly. This is similar to UIC and MIC in [36, 15, 10], with the necessary modifications for our case. This implies eliminating a number of practical attacks by individual parties, such as Sybil attacks where a user (or a prover) submits fake bids under fake identities. Note that by the definition of Nash equilibrium, collusion between strategic players is not considered, so we explicitly capture the threats by collusion in the next point.
- Collusion resistance for provers: Provers cannot collude to be better off. Prover collusion is a new threat that is not considered in TFMs, as TFMs typically consider only one miner or discuss multiple miners in the context of MPC-assisted mechanisms [41, 53].
- Off-chain agreement (OCA) proofness: A coalition of users and provers cannot deviate from the protocol in a profitable way. This is analogous to OCA-proofness defined in [36], but in prover markets, OCA may involve multiple provers (whereas in TFMs typically only one validator is involved.)

While the desiderata overlap with existing definitions, the fact that prover markets involve multiple provers requires new definitions and new techniques to meet these requirements.

1.1 Paper Organization

The rest of the paper is organized as follows. In the rest of this section we present a strawman design, which motivates $Proo\varphi$, with an overview description of $Proo\varphi$ that follows. Section 2 presents related work. Section 3 presents a model of ZK-Rollups and the role of prover markets. Section 4 provides a preliminary analysis of $Proo\varphi$. Finally, Section 5 concludes.

1.2 Strawman Design Options

At a high level, the designs we propose involve a double auction among transaction-submitting users and proof producers. Specifically, a user bids (TX, f) where TX is the transaction to be proven, and f is the fee that the user is willing to pay for each constraint in TX ; we use $|\mathsf{TX}|$ to denote the number of constraints in TX , and we assume constraints are equivalent to each other in terms of its proving cost. Each prover bids (s,p), meaning that she can generate a proof of up to s constraints by the end of the round if she is paid at least p for each constraint. \mathcal{M} receives bids from users and provers simultaneously and decides four things:

- 1. the transactions batch;
- 2. the fee charged from each transaction;
- 3. the set of provers selected;
- 4. the payment to each prover.

Strawman mechanism #1. A natural attempt is to start with the simple TFM in Ethereum before EIP-1559 where users engage in first-price auctions (i.e., they pay what they bid.) In order to solicit prover costs, we can consider a truthful auction among provers, such as the Vickrey auction [50]. Combining the two, our first attempt is described as follows:

First attempt

• Given user bids $\mathcal{B}_U = \{(\mathsf{TX}_i, f_i)\}$ and prover bids $\mathcal{B}_P = \{(s_i, p_i)\}$, the mechanism selects a set of transactions \mathcal{T} and a set of provers \mathcal{P}

$$\mathcal{T} = \arg\max \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \tag{1}$$

s.t.
$$\sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \le \sum_{j \in \mathcal{P}} s_j \tag{2}$$

$$\wedge \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \ge \sum_{j \in \mathcal{P}} s_j \cdot \mathsf{VCG}(j, \mathcal{T}, \mathcal{B}_U, \mathcal{B}_P) \tag{3}$$

That is, \mathcal{T} is the set of transactions that maximizes total payment subject to 1) selected provers have enough capacity to prove transactions in \mathcal{T} , and 2) total fee paid by transactions in \mathcal{T} is enough to cover the total payment to provers (determined by VCG

payment rule.)

- Each transaction $i \in \mathcal{T}$ is charged $|\mathsf{TX}_i| \cdot f_i$.
- Prover payment $VCG(i, \mathcal{T}, \mathcal{B}_U, \mathcal{B}_P)$ is decided by the payment rule in the VCG auction.
- The excessive fees are burnt.

The following greedy algorithm can be used to solve the optimization problem: We sort prover bids with cost so that $p_1 > p_2 \cdots$, and sort user bids using payments so that $f_1 > f_2 \cdots > f_N$. Then, consider each transaction from i = 1 to N: assign TX_i to the lowest cost prover who has spare capacity; stop if all provers are fully utilized or the total fees cannot cover the payment.

Example 1 (Concrete example). For a concrete example, suppose there are 1 transaction with f = 1,000, 5 transactions with f = 500, and 10 transactions with f = 150 (assuming all transactions have a normalized number of constraints, i.e., |TX| = 1); on the prover side, suppose there are two provers with $(s_1 = 10, p_1 = 100)$ and $(s_2 = 10, p_2 = 500)$. Note that according to the VCG auction rules, prover 1 will be assigned for the first 10 transactions then prover 2 will be assigned. The payment for prover 1 is exactly the cost of prover 2, i.e., 500 (intuitively, VCG pays the lowest-cost prover the second-lowest-cost).

Running through the above algorithm, the top-7 transactions (one with f=1,000, five with f=500, and one with f=150 respectively) will be allocated to prover 1. Prover 2 is not selected at all. The total fee paid is 3,650, and VCG payment to prover 1 is $500 \times 7 = 3,500$. Note that adding the 8th transaction (with f=150) will cause a deficiency, as the mechanism must pay $500 \times 8 = 4,000$ to prover 1, but the selected transactions pay a total of 3,800.

However, a serious problem with this scheme is that the price-setting prover (the prover 2) can be incentivized to cheat. We illustrate the problem using the following example.

Example 2 (First attempt is prone to off-chain agreements). Consider the second transaction (with f = 500). The sender of it can collude with Prover 2 as follows: it asks prover 2 to bid 400 (as opposed to her true cost 500), and the sender would bid 200 instead.

After this collusion takes place, 1 transaction with bid 1,000, 4 transactions with bid 500, 1 transaction with bid 200, and 3 transactions with bid 150 will be included in the batch. Therefore, the sender of the second transaction and prover 2 will have a utility gain equal to 500 - 200 = 300.

This collusion means that the first attempt is susceptible to off-chain agreements (OCA). Moreover, since Prover 2 is not allocated in either case, such collusion incurs no cost and is risk-free.

We observe that this severe OCA problem is partly caused by the dependence between the payment to provers and the bids of the other provers. Next, we resort to auction schemes that do not have such dependency for a fix. These auctions are said to have separable payment rules [15].

Strawman mechanism #2. First-price auction has separable payment rules when fixing the total allocation, as allocated bidders pay what they bid, regardless of others' bids. As a second attempt, we change the auction among provers from VCG to a generalized first-price auction (GFP). We summarize this as our second attempt as follows:

Second attempt

• Given user bids $\mathcal{B}_U = \{(\mathsf{TX}_i, f_i)\}$ and prover bids $\mathcal{B}_P = \{(s_j, p_j)\}$, the mechanism selects a set of transactions \mathcal{T} and a set of provers \mathcal{P}

$$\mathcal{T} = \arg\max \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \tag{4}$$

s.t.
$$\sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \le \sum_{j \in \mathcal{P}} s_j \tag{5}$$

$$\wedge \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \ge \sum_{j \in \mathcal{P}} s_j \cdot \mathsf{GFP}(j, \mathcal{T}, \mathcal{B}_U, \mathcal{B}_P) \tag{6}$$

That is, \mathcal{T} is the set of transactions that maximizes total payment subject to the prover's capacity and cost constraints.

- Each transaction $i \in \mathcal{T}$ is charged $|\mathsf{TX}_i| \cdot f_i$.
- The payment of prover j is $GFP(j, \mathcal{T}, \mathcal{B}_U, \mathcal{B}_P) = s_i \cdot p_j$.
- The excessive fees are burnt.

This design avoids the specific OCA problem of Example 2, however, it has a different OCA problem summarized as follows.

Example 3. The settings of the transactions and the provers are the same as Example 1. There are 1 user who bids 1,000, 5 users who bid 500, and 10 users who bid 150. There are two provers, denoted as prover 1 and prover 2. Prover 1 has a capacity of 10 with a unit cost of 100, and Prover 2 has a capacity of 10 with a unit cost of 500.

We assume that the provers know the valuations of each other. When all provers bid individually, prover 1 will bid $500 - \epsilon$ where ϵ is an arbitrarily small positive value, and prover 2 will bid 500. Then 1 transaction with bid 1,000, 5 transactions with bid 500, and 1 transaction with bid 150 will be included in \mathcal{T} .

However, one user who bids 150 and is not included in the batch can collude with prover 1. Prover 1 will compensate it so that the user can bid 500 instead of 150. This coalition will have the user transaction included in the batch, and the sum of utility within the coalition will increase by 50.

This example of OCA is not as severe as Example 2, because prover 1 in the coalition needs to trust the user in the coalition really bids 150 originally, or otherwise some user who originally bids 500 can trick the prover into compensating it. Yet in Example 2, the prover in the OCA does not have any risk. The existence of OCA concludes that a separable first-price auction prevents some OCA problems but does not eliminate them.

1.3 Our Design: Prooφ

The previous attempts suffer from two OCA problems. We make the following observations that lead to our design of a prover market mechanism we call $Proo\phi$.

Observation 1: Associate prover revenue with total user fee. The design attempts so far suffer from OCA problems, partly because they do not maximize the social welfare of the provers

and the users. In both attempts, a part of the fee collected from users is burnt and not distributed to the provers. Intuitively, given that the allocation is fixed, one may attempt to mitigate this issue by transferring all user payments to provers, i.e., by eliminating burning. However, we emphasize that this does not prevent OCA where users and provers collude to manipulate the allocation.

Observation 2: Prover payment with uniform price. In the first attempt, we adopted the DSIC VCG auction's payment rule; in the second attempt, we adopted the payment rule of a generalized first-price auction. We observe that another separable payment rule can be paying each prover a uniform price per constraint. We pay each selected prover the average unit fee collected from users, where the average unit fee is the weighted average of the bid of selected transactions weighted over the size of transactions. In this payment rule the payment to each prover is only dependent on their bids, and there is no burning.

Observation 3: Adding a capacity parameter can partially mitigate OCA. In previous attempts, OCA is possible because proof capacity is not exhausted, so utilizing the spare capacity yields extra profit as long as the user pays a fee higher than the cost via OCA. The third observation is we can artificially constraint the proof capacity. We do so by adding a capacity parameter C that serves as the upper bound of the batch. Namely, the mechanism can only select transactions with a total of C constraints. Intuitively, setting capacity parameter property can mitigate OCA, as provers can only add transactions through OCA if they "kick out" a transaction, which may lower the net profit. We derive conditions of the capacity parameter under which the protocol is OCA-proof in Section 4.

1.4 Overview of Proop

In $Proo\phi$, a capacity parameter C is set and the mechanism runs a generalized first-price auction among user transactions such that total constraints do not exceed C. For provers, the mechanism identifies the provers whose costs do not exceed the average unit fee of the included user transactions, and we denote this set of provers as \mathcal{P} . The total capacity of the prover in \mathcal{P} is expected to exceed C, and then the mechanism runs a lottery to select provers in \mathcal{P} to prove the user transactions.

Prooφ description (Short Version)

- A batch capacity parameter C is set and fixed.
- Given user bids $\mathcal{B}_U = \{(\mathsf{TX}_i, f_i)\}$ and prover bids $\mathcal{B}_P = \{(s_j, p_j)\}$, the mechanism selects a set of transactions \mathcal{T} and a set of provers \mathcal{P} . Let $\bar{f} := \sum_{i \in \mathcal{T}} f_i \cdot |\mathsf{TX}_i| / \sum_{i \in \mathcal{T}} |\mathsf{TX}_i|$ denote the average unit user fee.

$$\mathcal{T} = \arg\max \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \tag{7}$$

$$\mathcal{T} = \arg \max \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \tag{7}$$
s.t.
$$\sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \le C \tag{8}$$

$$\wedge \ \bar{f} \ge p_j, \forall j \in \mathcal{P} \tag{9}$$

That is, \mathcal{T} is the set of transactions that maximizes total payment and the total number

of constraints in \mathcal{T} does not exceed the capacity parameter C; \mathcal{P} is the set of provers such that the cost of each prover in \mathcal{P} is not greater than the average unit fee paid by the transactions in \mathcal{T} . We make the assumption that $\sum_{j\in\mathcal{P}} s_j \geq C$ always holds.

To determine \mathcal{T} , the mechanism greedily adds the highest-bid transactions until no other transactions can be added given the total constraints of the transactions in \mathcal{T} does not exceed C.

- Then the provers in \mathcal{P} will be selected with the same uniform probability. Let $C' := \sum_{i \in \mathcal{T}} |\mathsf{TX}_i|$ be the actual number of constraints in the batch. If a prover j is selected it is allocated capacity $\min\{C', s_j\}$. The mechanism updates $\mathcal{P} := \mathcal{P} \{j\}$ and $C' := C' s_j$, and this process is repeated if C' > 0.
- Each transaction $i \in \mathcal{T}$ is charged $|\mathsf{TX}_i| \cdot f_i$.
- The payment of each prover j is $q_j \cdot \bar{f}$, where q_j is the number of constraints that the prover is allocated.

In Section 4, we provide a preliminary analysis of the properties of this proposed mechanism under the assumption that the set of chosen provers \mathcal{P} , which is a function of user bids and prover bids, has a total capacity of at least C, i.e., $\sum_{j\in\mathcal{P}} s_j \geq C$.

1.5 Limitations and Open Problems

We now discuss some practical considerations and the limitations of the proposed mechanisms. We note that our proposed mechanism alone is not a complete solution for ZK-Rollup prover markets, as it relies on assumptions that require system-level solutions to realize properly.

- Trusted orchestration surface. The proposed prover market mechanism is meant to be executed by a trusted third party. The trusted third party will collect bids from users and provers, select and charge the set of transactions, and allocate and pay the set of provers. In practice, a decentralized sequencer like Espresso Sequencer [13] can act as such a trusted party. Alternatively, a blockchain can be used to eliminate third-party trust, but this assumes that that concerns around transaction latency and throughput have been adequately addressed.
- Sybils. Prooφ has problems in handling Sybil attacks. Some provers can gain profit if they create Sybil provers (see Examples 9 and 10). This problem can be mitigated with system-level measures such as staking or a registration process that verifies the prover's identity. Sybil-resistant Decentralized Identities (DIDs) can be used to facilitate on-chain identity verification. "Proof of work" can also be employed: in order to register a prover with capacity s, a prover needs to demonstrate (perhaps interactively) that she can generate a proof for s constraints within a set deadline. A potential problem, though, is that provers can rent computing power during registration. One mitigation is to randomly ask a prover to regenerate proofs of capacity throughout their tenure as a registered prover. These measures can also be combined.
- Collusion. If all provers collude, they can monopolize the market and charge a fee higher than the market-clearing price. Nurturing a market with a large number of provers can increase the barrier to form a total collusion. However, monopoly is a bigger concern in prover markets than fee markets, as it is very likely there will be fewer provers than validators

given the hardware needs for efficient proving (e.g., Ethereum, for instance, has about 1 million validators.)

Another mitigation is to introduce a *default prover* run by the community. The default prover would have a reasonable price and proof generation capacity on its own, and will also participate in the auction. For instance, ZK-Rollups might want to collectively sponsor such a default prover. Having a default prover effectively sets a reserve price in the prover auction.

• Capacity. The *Proo*φ mechanism has a fixed capacity parameter. In practice, the capacity should be adjusted as the demand and supply of the prover market change. More specifically, if there are a large number of user transactions with high bids that cannot be selected because of the limited capacity parameter, and if the total proof generation capacity of the provers is also high, then the capacity parameter should increase and vice versa.

Yet the job of adjusting the capacity parameter is challenging since the capacity parameter needs to conform with the bids of the users and provers in the market. If the capacity parameter is too high, the mechanism may not be able to select provers with enough proof generation capacity to prove. On the other hand, if the capacity parameter is too low there will be inefficiency in the market. We leave the work of coming up with a robust capacity parameter adjustment scheme as an open question.

- Partial collusion. *Proo*φ is vulnerable to partial prover collusion as demonstrated in Examples 11 and 12. Partial prover collusion, in contrast to full prover collusion, is a collusion performed by a few provers which increases their joint utility. Partial collusion needs to be prevented since it is not fair for the other provers who bid honestly in the prover market.
- Supporting flexible transaction ordering policies. As presented, *Proo*φ implies an ordering of transactions by fees. Supporting flexible ordering policies, such as first-come-first-served-based (FCFS) ordering [25, 22, 21], or leaving ordering to sequencers, may be desirable. *Proo*φ can support FCFS ordering with a slight modification: the mechanism is given an ordered list of user transactions and an additional constraint that the chosen transactions must be a prefix of the input (so the given ordering is respected.) The problem, however, is that if users pay different amounts in the same batch, users paying high fees essentially subsidize those paying a lower fee, potentially incentivizing untruthful bidding. Second, the additional constraints may make the matching infeasible. An alternative design is to charge users a uniform base fee, decided by the mechanism to cover the cost. However, since we cannot burn the base fee, this may give rise to OCA problems. We leave addressing these problems as future work.

In an alternative model where the sequencer can arbitrarily order transactions to extract MEV, the provers' cost can be covered by the sequencer. A possible design is to run auctions between sequencers and provers. Scroll's PSS design [40, 49] falls in this category, but their mechanism chooses provers based on their stakes, instead of their proof cost. Our goal is to choose provers with lower cost.

• Full incentive compatibility. In TFMs, we have mechanisms such as the TFM in EIP-1559 that possess good properties like incentive compatibility for users and miners, OCA-proofness, etc. In prover markets, however, it is challenging to design a mechanism with all the desired properties. We raise the open question of how to design a mechanism to forestall collusion, Sybil attacks, and/or OCA with mechanism design.

2 Related Work

2.1 Existing Fee Mechanisms for ZK Proofs

=Nil;. =Nil; is a market for ZK proofs [29] where users can purchase proofs for pre-built circuits. The proof market keeps an order book for each circuit with buy orders from users and sell orders from provers. The price of generating a proof is discovered with the order book mechanism.

Mina. Mina [44] is a layer 1 blockchain where the correctness of state transitions is proven by ZK-SNARK. Block producers are responsible for submitting proof for transactions in the block [27]. They can generate the proof themselves or buy proof from SNARK workers (provers) in a proof market (e.g. nil). The TFM on the user side is a first-price auction, like pre-EIP1559 Ethereum. Fees are collected by block producers, but the TFM does not explicitly take proof cost into consideration.

Aztec. Aztec Network [35] proposed a way for different provers to generate proofs in parallel. The proofs are then aggregated and submitted to layer 1. There is a recent call for proposals for decentralized prover coordination from Aztec [35]. The winning proposal is Sidecar [34, 42]. In this proposal, the sequencer is responsible for delivering the proof in a predefined amount of time or gets slashed. The sequencer can either generate the proof themselves or outsource the proving process to other entities (e.g. through a proof market). In other words, the sequencers also play the role of prover in the protocol, although the sequencer and the prover may be different entities in practice.

Taiko [45, 48, 47]. In Taiko, provers are selected based on their proof capacity C (total number of blocks they can prove in parallel), expected price per gas R and stake A. The expected price per gas R is similar to the prover's cost p in our protocol. The weight W of a prover is $W = \frac{A}{R^2}$. The top 32 provers with the highest scores are considered active. Each block is assigned to a random active prover, with a probability proportional W. If the chosen prover fails to deliver the proof in the predefined proving time window (3 times the average proof delay), a certain percentage of their total stake will be confiscated. The transaction fee paid by the users is determined by a mechanism like EIP-1559. The base fee is paid to the Taiko DAO. The tip is used to cover the L1 transaction fee and the prover fee. The rest of the tip is the profit of the sequencer.

Scroll. Scroll proposes to separate the sequencer and the prover through PSS (prover-sequencer-separation) [40], a similar way as MEV-Boost. A PoS protocol runs among the provers and one prover is selected for each slot. The sequencers are in charge of building blocks that extract maximum value from transactions. The payment to the prover is decided by the first-price auction among the sequencers. We notice that this mechanism does not take the size of the block into consideration. In Ethereum, proposing a larger block may incur a similar cost as a smaller block. However, in the ZK-Rollup, the prover may also need to know the block's size to determine which bid to take. The fee paid by the user is composed of the L1 fee and the L2 fee. The L1 fee per gas is supplied by an oracle, while the L2 fee per gas is determined by the first price auction of user [49].

Starknet. In the current protocol, the transaction fee in Starknet is determined based on the estimated cost of publication on layer 1 [16]. The transaction fee is posted and transactions are included in a first-come, first-serve manner. There is a recent effort to decentralize the Starknet [43], where the sequencer is selected by running Tendermint, a PoS consensus protocol. For the prover's network, the potential solutions are discussed [46], but there has not been a concrete protocol yet.

Protocol	Payment of Users	Prover Selection	Payment to Provers
Ρrοοφ	first price auction	posted price and lottery	average user fee × assigned capacity
Nil	order book	order book	order book
Mina	first price auction	same as sequencer	out of scope of protocol
Aztec (Sidecar)	N.A.	same as sequencer	out of scope of protocol
Taiko	EIP-1559-like mechanism	weighted random selection	specified by prover
Scroll	first price auction	PoS	first price auction by sequencer
Starknet	posted price	N.A.	N.A.
Gevulot	posted price	randomly selected	posted price

Table 1: Comparison of prover market proposals. N.A. = Information Not Available.

Gevulot. Gevulot is a layer 1 blockchain with a decentralized prover network [20]. To join the prover network, a node needs to deposit a stake and complete designated proof of workload tasks. The proof generation jobs are randomly allocated to the prover nodes using the verifiable random function (VRF). The pre-paid compute fee for a transaction is calculated by the network, based on the customized parameter specified by the user [17]. Once the transaction is proven, the user may get a rebate. The longer the time taken to generate the proof, the higher the rebate. The rebate serves as an incentive for the prover to submit the proof as quickly as possible, since the prover will be awarded with compute fee - rebate.

Comparison between protocols. We compare the protocols mentioned above across three dimensions: how provers are selected (prover selection), how the fee paid to the prover is decided (payment to provers) and how to decide the amount of money paid by the user (payment of users). The result is summarized in Table 1.

2.2 Transaction Fee Mechanisms for Layer 1 Blockchains

The line of research on transaction fee mechanisms (TFMs) for layer 1 blockchains veers close to ours, yet with important differences stemming from the market structure for ZK-Rollups: Most literature on L1 TFMs analyze a myopic setting where a single miner is responsible for allocating transactions to a size-limited block and can do so however it wants without incurring any cost for processing transactions [23, 54, 4, 15, 10, 41, 9, 14]. Notably, Roughgarden [36] considers miners' marginal costs, which are assumed to be fixed and public, meaning that all transactions incur the same marginal cost for all miners. In contrast, in our case capacity is bounded by the amount provers are willing to supply, with marginal costs being heterogeneous and private. These considerations imply that, for mechanisms to be efficient in our setting, they must elicit competitive proving costs from provers while ensuring that user demand is met without incurring losses to provers. Another difference is that while miners can choose which transactions to allocate, most ZK-Rollup designs rely on entities called sequencers for this role, thereby limiting the ability of provers to misbehave.

TFM desiderata. Lavi, Sattath, and Zohar [23] were the first to cast L1 TFMs as auctions, while noting that the blockchain setting introduces new challenges: the auctioneer (the miner) cannot be trusted, unlike in "traditional" auction theory (Akbarpour and Li [1] later considered the related notion of "credible" auctions; see Roughgarden [36] for a comparison). Thus, they note that TFMs should be incentive compatible both for users (i.e., they should find it most profitable to specify a fee that corresponds to their private valuation of having their transaction processed) and

miners (i.e., a miner should not profit from allocating "fake" miner-created transactions or from ignoring "honest" user-created transactions that should be allocated). Roughgarden [36] notes that miners can collude with users "off-chain", and advanced what has since become the canonical TFM desiderata: TFMs should be UIC, MIC, and OCA-proof (off-chain agreement proof).

TFM impossibilities. Chung and Shi [10] show it is impossible to design TFMs that are UIC, collusion-resistant, and produce more than 0 miner revenue. Gafni and Yaish [15] find that relaxing the UIC requirement in a Bayesian manner allows "simple" TFM designs such as Bitcoin's pay-as-bid first-price auction to produce high revenue. A study of collusion-resistance in TFMs is performed by Gafni and Yaish [14], who show that the SCP collusion notion considered by Chung and Shi [10] is stricter than the OCA of Roughgarden [36], and proceed to show a 0 revenue impossibility for deterministic UIC, MIC and OCA-proof TFMs, and for general classes of randomized mechanisms; they furthermore bound the efficiency of any randomized mechanism. Chung, Roughgarden, and Shi [9] prove a 0 revenue result for deterministic and randomized UIC, MIC, and OCA-proof TFMs with bounded inclusion rules that are weakly symmetric, a property that means the TFM cannot make use of auxiliary information besides the bid, such as the bidder's identity.

Additional works. For more details on TFMs, we refer the reader to Roughgarden [36] and Gafni and Yaish [14]; both give an overview of recent developments in the literature.

2.3 Double Auctions

Our setting is inherently that of a two-sided market: on the demand side, users submit transactions that require proving, while on the supply side, provers provide proving capacity. Double auctions are commonly used to coordinate trade in such markets, i.e., to choose which agents get to trade in the market and at what prices. In particular, while there is some overlap between the "traditional" setting explored by auction theory literature and the blockchain setting, we note that the latter introduces new challenges: provers may collude, either amongst themselves or with users.

Double auction desiderata. Although the Vickrey auction has good properties in the one-sided setting (in particular, it is incentive compatible for agents), adapting it to the two-sided setting may require the auctioneer to subsidize trades [50], thereby running the risk of incurring a deficit; that is, the mechanism is not budget balanced. Mechanisms that finance subsidies by other means, such as charging entrance fees from agents, may result in agents incurring losses and thus are not individually rational for agents. Ideally, double auction mechanisms should be incentive compatible, budget balanced, and individually rational, while maintaining efficiency, i.e. the requirement that items should be given to those who value them most. Unfortunately, Myerson and Satterthwaite [28] show that it is impossible to design mechanisms that satisfy these desiderata. Subsequent works explore relaxed versions of this desiderata under various settings [30]. In particular, McAfee [26] present a mechanism that is incentive compatible, individually rational, budget-balanced and asymptotically efficient (i.e., its efficiency approaches the optimal one when the number of agents is increased) for unit-demand buyers and unit-supply sellers (i.e., each buyer wishes to purchase a single item, and each seller offers a single item for sale).

Budget balance. Although the mechanism of McAfee [26] is budget-balanced, it may retain some of the buyers' payments and not transfer them in full to sellers, a property referred to as being *weakly* budget balanced. One may consider *strongly* budget balanced mechanisms, where sellers receive

all payments. For example, Colini-Baldeschi et al. [11] present posted-price mechanisms that are incentive compatible, individually rational, strongly budget-balanced, and which achieve a constant approximation of the social welfare in the unit supply and demand case. Notably, Segal-Halevi, Hassidim, and Aumann [39] advance an asymptotically efficient mechanism satisfying the same set of properties, and which is moreover *prior-free*, i.e., requires no assumption on the distribution of agent valuations. However, the mechanism relies in part on a random selection of sellers, thus its efficiency guarantees hold only in expectation. The two approximation approaches are combined by Babaioff et al. [3], who present a simple prior-free mechanism that guarantees both an ex-ante constant approximation and an ex-post realization-dependent asymptotically optimal efficiency.

Bulow-Klemperer. In the one-sided setting, Bulow and Klemperer [5] show that while Myerson's optimal-revenue auction requires some prior knowledge of the distribution of bidder valuations, an auctioneer without this knowledge can obtain equivalent revenue from the second price auction by recruiting a single additional bidder. Similar results for the two-sided case are given by Babaioff, Goldner, and Gonczarowski [2] for their Buyer Trade Reduction (BTR) mechanism, which iteratively matches the highest-bid buyer to the lowest-cost seller until the prospective buyer's bid is lower than the seller's cost; then, this buyer's bid is offered as the trading price to all matches. If at least one matched pair declines to trade at this price, the last pair is "reduced" from the matching and does not trade, with the bid of that pair's buyer used as the trading price to all remaining pairs. The analysis of Babaioff, Goldner, and Gonczarowski [2] partly assumes that the sellers' distribution is first-order stochastically dominated by the buyers', leading Cai et al. [7] to consider general distributions that do not satisfy this assumption and prove bounds on the amount of additional agents that allow the trade reduction mechanism to match the performance of the optimal one.

Sybil-proofness. Yokoo, Sakurai, and Matsubara [55] explore the possibility of both buyers and sellers submitting "false-name" bids, equivalent to our notion of sybil-proofness (i.e., a single agent, regardless of whether it is a buyer or a seller, can pose as multiple "fake" buyers and sellers).

Additional works. For a broad review of the literature on auctions, we refer the reader to the survey conducted by Parsons, Rodriguez-Aguilar, and Klein [31], which covers a variety of auction formats, including double auctions. Furthermore, Segal-Halevi, Hassidim, and Aumann [39] provide a comparison of several notable double auction mechanisms, and Cai et al. [7] go over recent approximation and Bulow-Klemperer-style results.

3 Defining Prover Markets

3.1 ZK-Rollup Model

We first introduce the workflow of transaction processing in a ZK-Rollup based on the design of zkSync Era [52]. The high-level idea of ZK-Rollup is to outsource transaction execution to untrusted off-chain parties, with an on-chain smart contract storing transactions (for data availability) and verifying the execution results using SNARKs. In a ZK-Rollup system, there are three types of participants: users who create transactions, a (distributed) sequencer who orders transactions and builds L2 blocks, and provers who generate validity proofs for state changes caused by ordered transactions. The L1 smart contract verifies validity proofs and updates the state root, finalizing the execution of user transactions.

- Submission. A user submits her transaction TX to the sequencer, along with a bid f_i as the input to the transaction fee mechanism. We assume that the number of constraints implied by TX in the underlying SNAKR system is known, denoted |TX|.
- **Batching.** The sequencer groups user transactions into *L2 blocks* and stores them on L1 for data availability. As a performance optimization, the sequencer further groups consecutive blocks into *batches* to amortize the proof cost.
- **Proving.** For each batch of transactions, the provers generate and submit a SNARK to the L1 smart contract, finalizing the execution results. In existing ZK-Rollups, it is currently typical that proofs are generated by in-house provers (usually centralized) [33, 38, 52].

3.2 Prover Market

The goal of a prover market is to replace the in-house provers with an open market where provers compete to provide low-cost services to Rollups and users. A prover market is a two-sided market where users demand proofs (for their transactions), and providers supply proof generation services.

User and prover model. In our model, there are multiple provers and users. We assume that each user has a private willingness-to-pay known to them before they send transactions. We assume each prover has a fixed proof capacity s_i known to themselves—prover i can generate a proof for up to s_i constraints before some predefined deadline. We also assume that the cost to generate a proof is *linear* in the number of constraints to be proven, and each prover has a per-constraint cost p_i . The cost incurred by provers is covered by the transaction fees collected from users.

Note that all prices and costs mentioned hereafter are per-constraint, unless otherwise specified. If a transaction has c constraints, and the per-constraint price for that transaction is p, then the user who submits that transaction needs to pay $c \cdot p$.

The user demand is expressed as the total number of constraints D(p) demanded by users given a per-constraint price p. The user demand is non-increasing in p, since when the price increases there will be fewer transactions willing to pay such prices.

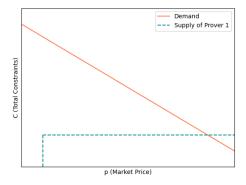
The prover supply is expressed as the total number of constraints S(p) supplied by provers given a per-constraint price p. The prover supply is non-decreasing in p, since when the price increases there will be more provers with cost below that price.

Example 4. Here is an example of a prover market. If there is one prover with limited proof capacity in the market, the supply of the prover can be drawn as Fig. 1. If there are two provers in the market, the total supply of the provers can be presented as Fig. 2.

Market clearance price. With an ideal mechanism, the price of a prover market should be set based on *true* user values and proper costs to maximize the number of transactions to be proven. This is called *market clearance price* [32].

With the user demand function D(p) and the prover supply function S(p), we define the market clearance price as follows. Intuitively, the price where supply meets demand is the market clearance price.

Definition 1 (Market Clearance Price [32]). Suppose we have complete information of all user demand and prover supply, and therefore have the demand curve D(p) and the supply curve S(p).



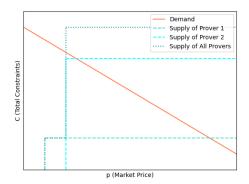


Figure 1: Prover market with one prover

Figure 2: Prover market with two provers

The market clearance price p^* should satisfy

$$p^* = \underset{p}{\operatorname{arg\,min}} [D(p) \le S(p)]. \tag{10}$$

Social welfare and efficiency cost. In a prover market, we define the economics concept *social welfare* [32] as follows.

Definition 2 (Prover Market Social Welfare). The social welfare within a prover market is defined as the sum of the utility of all participants in the market. If a user transaction is not allocated or if a prover is not allocated to prove, their utility is zero.

In a prover market, if each user and each prover are given the market clearance price as the posted price, the social welfare will be maximized [32]. However, the prover market could be less efficient for a given market mechanism, represented by the reduced social welfare compared with the optimal social welfare. We define *efficiency cost* of a prover market mechanism as the difference of the social welfare compared with the optimal social welfare.

Definition 3 (Efficiency Cost of a Prover Market Mechanism [32]). The efficiency cost of a prover market mechanism is the difference between the social welfare in the equilibrium of the mechanism and the maximum social welfare.

In a real prover market, the price for users or for provers is possibly not the market clearance price. We define the price difference of a prover market as the ratio of the price difference of the average user price compared with the market clearance price.

Definition 4 (Price Difference of a Prover Market). Let p' denote the average per-constraint price for the users. The efficiency loss L of the prover market is defined as

$$L = (p' - p^*)/p^*. \tag{11}$$

Desired prover market dynamics. On a high level, the prover market dynamics should satisfy two properties. Firstly, the competition within the prover market should be fair for each market

participant. This indicates that the prover market mechanism should be resistant to attacks and behaving honestly is the optimal strategy. Secondly, the prover market should be efficient in the sense that the average price of the users of the market should be the market clearance price.

3.3 Threat Model

We describe the possible attacks that may happen in the context of ZK-Rollups. Attacks may happen if the mechanism for the prover market is not well-designed, and the attacks may harm the fairness of competition or negatively impact the market efficiency. We consider that the sequencer and the layer-1 that the ZK-Rollup is settling on are trusted. The attacks on the market may occur in the following forms.

Misreporting bids. If any user or prover can benefit from misreporting their bid, we consider such action as an attack on the prover market. For example, if in a trivial mechanism all user transactions are included in each batch uniformly randomly, all users will bid 0 fee-per-constraint which is not truthful.

Sybil attacks. Sybil attacks can occur in the form of a user submitting fake bids as fake users, a user submitting fake bids as fake provers, a prover submitting fake bids as fake users, and a prover submitting fake bids as a fake prover.

Prover collusion. We refer to prover collusion as several provers can communicate with each other and bid strategically to increase their joint utility. If such prover collusion harms the utility of other honest parties, we consider it an attack. Below we give an example of an attack that stems from prover collusion.

Example 5. Suppose the user demand is D(p) = 100 - p, and there are two provers with costs 5 and 10 respectively, and each prover has 45 proof generation capacity. If they bid honestly, their joint revenue will be 225. If they collude to bid 25 proof generation capacity, their joint revenue will be 1350. However, in this case, the users whose valuations are within the range of (25, 45) cannot be included in the batch.

There are cases where collusion among a subset of provers can harm user utility.

Example 6. Suppose the user demand is D(p) = 100 - p, and there are three provers with costs 5, 10, and 20, respectively. We denote them as prover 1, 2, and 3. Each prover has 40 proof generation capacity. The prover market mechanism is running a second-price Vickrey auction among the prover. If all provers bid honestly, then prover 1 and prover 2 will win, and the joint utility of prover 1 and prover 3 be 600. If prover with 1 and prover 3 collude, where prover 3 bids 25 instead, then their joint utility will be 800. This will result in prover 2 getting less.

Off-Chain agreements. Instead of signaling their bid in the prover market, one or several provers and a set of users can communicate with each other directly, agreeing to submit untruthful bids to increase their joint utility. We refer to this as *off-chain agreements*.

3.4 Prover Market Desiderata

Recall that our high-level goal is that the market should be fair in the sense that no participant may gain an advantage by deviating from its honest strategy. From this high-level goal, we

discuss the desiderata of a good prover market mechanism. Among them, we focus on incentive-compatibility for users (UIC), incentive-compatibility for provers (PIC), and off-chain agreement proofness (OCA-Proofness). Some of the definitions here are standard in the works on transaction fee mechanism (TFM) [37, 36, 15]. UIC is a property frequently analyzed in Layer 1 fee market mechanisms, and PIC is similar to miner incentive compatibility at large. Yet our definition of OCA-Proofness is different from the definitions of OCA-Proofness in TFM works. In TFMs, there will only be one miner that can collude with users; in prover markets, however, more than one prover may form a coalition with users. The detailed desiderata are defined as follows.

Incentive-compatibility for users (UIC). It is desired that users will bid truthfully. A prover market mechanism is *incentive-compatible for users* only if it satisfies the notion of DSIC [37] (Definition 5) or BIC [15] (Definition 6) for users. Both of the definitions capture the idea that each user bidding honestly will maximize its utility.

Definition 5 (Dominant Strategy Incentive-Compatibility (DSIC) [1]). A mechanism is dominant strategy incentive compatible if it is always best for each participant to bid its true valuation.

Definition 6 (Bayesian Incentive-Compatibility (BIC) [15]). A mechanism is Bayesian incentive-compatible if each participant bidding honestly is a Bayesian-Nash equilibrium (BNE).

We give an example of a mechanism that is BIC but is not DSIC.

Example 7. Bitcoin's TFM is not DSIC, but is BIC [15].

Beyond DSIC and BIC, a prover market is UIC only if each user cannot increase its utility by introducing fake bids either as a fake user or as a fake prover. Combining these considerations, we formally define UIC in Definition 7.

Definition 7 (Incentive-Compatibility for Users (UIC) [15, 37]). A prover market mechanism is incentive-compatible for users if the mechanism is DSIC or BIC for users, and if each user cannot increase its utility by creating any fake bids as fake users or as fake provers. Furthermore, if the mechanism is UIC and is DSIC for users, we call it DSIC-UIC; if the mechanism is UIC and is BIC but not DSIC for users, we call it BIC-UIC.

Incentive-compatibility for provers (PIC). Similar to the definition of UIC, we define PIC in Definition 8.

Definition 8 (Incentive-Compatibility for Provers (PIC)). A prover market mechanism is incentive-compatible for users if the mechanism is DSIC or BIC for provers, and if each prover cannot increase its utility by creating any fake bids as fake users or as fake provers. Furthermore, if the mechanism is PIC and is DSIC for provers, we call it DSIC-PIC; if the mechanism is PIC and is BIC but not DSIC for provers, we call it BIC-PIC.

Prover collusion resistance. Finally, a set of provers can increase their joint utility by colluding with each other in an off-chain agreement. We say a mechanism is *prover collusion-resistant* (Prover CR) if any collision of a specific number of provers cannot increase their joint utility (Definition 9)

Definition 9 (Prover Collusion Resistant (Prover CR)). We say a mechanism is c_P -Prover-Collusion-Resistant (c_P -Prover-CR) if a coalition of up to c_P provers cannot increase their joint utility by forming the coalition.

4 Analyzing Prooφ

4.1 Mechanism Description

In this section, we introduce our design of a prover market mechanism, named $Proo\varphi$. On a high level, $Proo\varphi$ sets a batch capacity parameter C and runs a pay-as-bid greedy auction (PABGA) among user transactions and selects user transactions to maximize the fee paid by selected users. For provers, the mechanism includes all provers whose costs do not exceed the average unit transaction fee in a lottery pool. Then the provers in the lottery pool are selected at random and paid the unit price which is the average unit fee of the selected transactions. In more detail, we describe this mechanism as follows.

$Proo\phi$ description

- Parameters: The global parameter is the batch capacity parameter C, and the proof generation window T.
- Collect Bids: Recall that the mechanism collects user bids $\mathcal{B}_U = \{(\mathsf{TX}_i, f_i)\}_{i=1}^n$, and collects prover bids $\mathcal{B}_P = \{(s_j, p_j)\}_{i=1}^N$.
- Allocation: Let $\bar{f} := \sum_{i \in \mathcal{T}} f_i \cdot |\mathsf{TX}_i| / \sum_{i \in \mathcal{T}} |\mathsf{TX}_i|$. The mechanism selects a set of transactions \mathcal{T} and a set of provers \mathcal{P} .

$$\mathcal{T} = \arg\max \sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \cdot f_i \tag{12}$$

s.t.
$$\sum_{i \in \mathcal{T}} |\mathsf{TX}_i| \le C \tag{13}$$

$$\wedge \ \bar{f} \ge p_j, \forall j \in \mathcal{P} \tag{14}$$

That is, \mathcal{T} is the set of transactions that maximizes total payment and the total number of constraints in \mathcal{T} does not exceed the capacity parameter C; \mathcal{P} is the set of provers such that the cost of each prover in \mathcal{P} is not greater than the average unit fee paid by the transactions in \mathcal{T} . We make the assumption that $\sum_{j\in\mathcal{P}} s_j \geq C$ always holds.

To determine \mathcal{T} , the mechanism greedily adds the highest-bid transactions until the total constraints in \mathcal{T} reaches C.

Then the provers in \mathcal{P} will be selected with the same uniform probability. Let $C' := \sum_{i \in \mathcal{T}} |\mathsf{TX}_i|$ be the actual number of constraints in the batch. If a prover j is selected it is allocated capacity $\min\{C', s_j\}$. The mechanism updates $\mathcal{P} := \mathcal{P} - \{j\}$ and $C' := C' - s_j$, and this process is repeated if C' > 0.

- **Payment**: Each transaction $i \in \mathcal{T}$ is charged $|\mathsf{TX}_i| \cdot f_i$. The payment of each prover j is $q_j = q_j \cdot \bar{f}$, where q_j is the number of constraints that the prover is allocated.
- Staking and Slashing: Each prover needs to stake S before submitting bids. This stake will be slashed if the provers fail to complete the allocated proof generation task within the fixed time window T.

The following is an example of the execution of $Proo\varphi$.

Example 8. Suppose the batch capacity C is set to 20. There are 3 transactions $\mathsf{TX}_1, \mathsf{TX}_2, \mathsf{TX}_3$ of size 10, and their bids are respectively $f_1 = 5, f_2 = 3, f_3 = 1$. Suppose there are 3 provers P_1, P_2, P_3 in the prover market. The bids of provers are $(s_1, p_1) = (10, 1), (s_2, p_2) = (10, 2), (s_3, p_3) = (20, 3)$.

Because C=20, TX_1 and TX_2 will be included in the batch, and the total fee collected is 80. P_1 , P_2 and P_3 will be selected included in the lottery set \mathcal{P} . Then the mechanism uniformly randomly selects a prover in \mathcal{P} , and we assume that P_1 is selected. P_1 is allocated $q_1 = \min\{C, s_1\} = 10$. Then it updates $C := C - q_1 = 10$, and $\mathcal{P} = \{2,3\}$. Since there is the remaining capacity that is not allocated, the mechanism uniformly randomly selects another prover in the updated \mathcal{P} , and we suppose the selected prover is P_3 . P_3 is allocated $q_3 = \min\{C, s_3\} = 10$. Then it updated $C := C - q_3 = 0$. Since C is 0 it means all user transactions are fully allocated, and the mechanism is finished for this batch.

4.2 Analysis

In this section, we provide a preliminary analysis of $Proo\phi$ under the assumption that $\sum_{j\in\mathcal{P}} s_j \geq C$, i.e., the available capacity of provers in the lottery pool is greater than the batch capacity parameter.¹

Assumptions. We describe our analysis assumptions below.

- The capacity parameter C satisfies $\sum_{j\in\mathcal{P}} s_j \geq C$, i.e., the total proof generation capacity of the provers in \mathcal{P} is greater than the capacity parameter C. Note that this is an assumption of user bids, prover bids, and the capacity parameter.
- All transactions have the same number of constraints.
- All parties are myopic, i.e., they are only concerned with their utility within one batch of transactions.
- Each user has exactly one transaction and does not know the information of the other pending transactions unless it colludes with other users.
- If a user transaction is not included in a batch, its utility is zero; if a prover is not selected to prove anything, its utility is zero.

Prover Incentive Compatibility. Considering the prover side, recall that a prover market is PIC if provers cannot perform profitable Sybil attacks, and the mechanism is DSIC or BIC for the provers. However, we show that the mechanism is not PIC because of the existence of Sybil attacks. The provers with large proof generation capacity can generate Sybils to increase their chance of winning the prover-side allocation (Example 9).

Example 9. Suppose the capacity is C = 1 and the average unit transaction fee is f = 1. Also suppose there are two provers with proof generation costs $p_1, p_2 = 0.5, 0.5$ and proof generation capacity $s_1, s_2 = 1, 1$. If both provers submit bids honestly, the probability of both provers getting allocated to generate proof is 1/2. However, p_1 can create 99 Sybil bids, which will increase its chance of generating proof to 99%.

¹We are currently working on the proofs of some properties. Please see https://arxiv.org/abs/2404.06495 for the up-to-date analysis.

Sybil attack problems may still exist even if a prover does not have the capacity of the entire batch (Example 10).

Example 10. Suppose the stake of a prover is S with proof generation capacity 2 transactions, and suppose the size of \mathcal{P} is N_0 . Also, suppose that the batch capacity parameter is the size of one transaction. Then the prover has probability $\frac{1}{N_0-1}$ to be selected if it bids its capacity honestly. If the prover creates a Sybil, its probability of being punished is $\frac{1}{N_0(N_0-1)}$. Therefore, if

$$\left(\frac{2}{N_0 - 1} - \frac{1}{N_0(N_0 - 1)}\right) \cdot U \ge \frac{2S}{N_0(N_0 - 1)},$$

i.e., $U(2N_0-1) \ge 2S$, then the user has enough incentive to create Sybils. Here U is the reward of proving one transaction for the user.

Still, if the batch capacity and the sum of the proof generation capacity of provers in the set \mathcal{P} are identical to each other, i.e., $C = \sum_{j \in \mathcal{P}} s_j$, then it is incentive compatible for each prover to bid honestly.

One can mitigate the Sybil attacks with system-level designs such as adding a prover registration process, or with a Proof-of-Stake random selection process where the probability of each prover being selected in the lottery pool is proportional to its stake.

Prover Collusion. Recall that a mechanism is Prover CR if no profitable coalition among provers is possible. This mechanism is not Prover CR even if each prover cannot bid its proof generation capacity s greater than its authentic proof generation capacity s_0 and cannot create Sybils, which is demonstrated in the following example.

Example 11. Suppose the capacity is C = 1 and the average unit transaction fee is f = 1. There are 3 provers with proof generation cost $p_1, p_2, p_3 = 0.5, 0.5, 2$ and proof generation capacity $s_1, s_2, s_3 = 1, 1, 1$. If all provers submit their bids truthfully, then the joint expected utility of prover 2 and prover 3 will be 1/4. However, if prover 2 and prover 3 collude and bid $p'_2, p'_3 = 0.5, 1$ then the joint expected utility of prover 2 and prover 3 will be 1/3.

We further assume that each prover's capacity is less than the batch capacity, and the stake of each prover is sufficiently large. Under these assumptions, the mechanism is still not Prover CR.

Example 12. Suppose the capacity is C=1 and the average unit transaction fee is f=2. Suppose there are N_0 provers in \mathcal{P} , and that each prover in \mathcal{P} has proof generation capacity 1. Also suppose there are two provers with proof generation cost $p_1, p_2 = 1, 2.2$. Note that the first prover is in \mathcal{P} and the second prover is not in \mathcal{P} . If all provers submit their bids truthfully, the joint expected utility of prover 1 and prover 2 will be $\frac{1}{N_0-1}$. However, if prover 1 and prover 2 collude and bid $p_1', p_2' = 0.5, 1$ then the joint expected utility of prover 1 and prover 2 will be at least

$$1 \cdot \left(\frac{2}{N_0} - \frac{1}{N_0(N_0 + 1)}\right) + 0.8 \cdot \frac{1}{N_0(N_0 + 1)} > \frac{1}{N_0 - 1}.$$

Efficiency. The efficiency of $Proo\phi$ depends on the value of the capacity parameter C. Recall that the efficiency cost of a mechanism is defined as the difference of social welfare from the maximum market social welfare at market clearance price. In the following, let p^* denote the market clearance price defined in Definition 1.

In the cases where $C < \sum_{j \in \mathcal{P}} s_j$, there will be market efficiency cost in $Proo\phi$, as demonstrated in the following example.

Example 13. Suppose the capacity is C = 1 and there are 10 transactions with bid f = 1 and 1 prover with cost 0 and proof generation capacity 10. Then the market clearance price is $p^* = 0$ and the optimal market social welfare is 10. However, since C = 1 only 1 transaction can be included so the market social welfare of this mechanism is 1. Therefore, the market efficiency cost will be 10 - 1 = 9.

4.3 Discussions

Remark 1 (Can we adaptively adjust batch parameter within each block?). One may propose an alternative in-time capacity parameter adjustment scheme, where C is adjusted to the total constraint demanded by users at market clearance price in each run of the mechanism. However, this in-time adjustment will lead to OCA. As an example, there are 3 transactions in the batch with fee $f_1, f_2, f_3 = 1, 2, 3$. There are 2 provers with capacity $s_1, s_2 = 1, 2$ and cost $p_1, p_2 = 0.5, 2$. The market clearance price is $\bar{f} = 2$, and the capacity is adjusted to be C = 2. However, prover 1 can reach OCA with transaction 1, such that transaction 1 is also included in the batch.

Alternatively, C may also be adjusted such that the total number of transactions is maximized and $\sum_{j\in\mathcal{P}} s_j \cdot \bar{f} < \sum_{i\in\mathcal{I}} f_i \cdot |\mathsf{TX}_i|$. However, this will lead to another OCA problem. As an example, there are 3 transactions in the batch with fee $f_1, f_2, f_3 = 1, 2, 3$. There are 2 provers with capacity $s_1, s_2 = 1, 2$ and cost $p_1, p_2 = 0.5, 2$. Then if all parties bid honestly, the capacity should be C = 3, and all transactions are proven. However, prover 1 and transaction 1 can reach an OCA, where transaction 1 will not participate in the bidding. As a consequence, prover 1 will gain 2.5 instead of 2.

Remark 2 (Why can't we have user-side posted price with burning like EIP-1559?). The main problem is with OCA-proofness. In $Proo\phi$, it is not possible to burn all the fees collected from users, since the cost of provers to generate proofs is not negligible. Therefore, if there is one prover who has dominant proof generation power, it is able to reach off-chain agreements when the demand is high.

Consider the example where there are 10 transactions with true fee-per-constraint 2 and 10 transactions with fee-per-constraint 10, and the posted price is 5. The cost of the prover is 1 and the capacity of the prover is 10. Then the prover can reach off-chain agreements with the users whose true fee-per-constraint is 2 and include them in the block. This is not fair for the users whose transactions have fee-per-constraint 10.

Remark 3 (Why can't we have user-side second price auctions?). If the users run a second-price auction to determine which transactions get allocated in the ZK-Rollup blocks, then a prover may submit a fake user bid to increase the average payment from users and therefore gain more revenue.

Consider the example where we run a second-price auction for the users in the lottery-based mechanism. We have 3 users with fee-per-constraint $f_1 = 1$, $f_2 = 5$, $f_3 = 9$ and one prover with cost-per-constraint p = 3 and a large proof generation capacity. The batch capacity is 2 transactions. Then the prover can generate a fake transaction with fee-per-constraint f' = 4, and can therefore increase its revenue.

5 Conclusions

This paper presents $Proo\varphi$, a ZK-Rollup transaction fee mechanism aiming to meet the unique demands of the ZK-Rollup prover market. The mechanism incorporates a capacity parameter and adopts a payment rule to provers to prevent off-chain agreements while ensuring incentive compatibility. We provide a preliminary analysis of $Proo\varphi$ w.r.t. incentive compatibility, OCA-proofness, and efficiency.

We acknowledge the limitations of $Proo\varphi$. $Proo\varphi$ is vulnerable to Sybil attacks and partial collusion attacks, and it is difficult to dynamically adjust the capacity parameter in $Proo\varphi$. These limitations and challenges may provide insights for further research into prover market mechanisms, including exploring anti-collusion mechanisms, anti-Sybil attack mechanisms, and dynamic capacity adjustments to accommodate different prover market conditions.

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