Given equation

$y = mx + b$

data points $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (3, 6)$

$m = -1$

$b = 1$

$\alpha = 0.1$

No of iterations $= 4$

No of data points $(n) = 2$

Predicted values $\hat{y}_i = mx_i + b$

Mean Squared Error $= \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$

$$\frac{\partial J}{\partial m} = \frac{2}{n} \sum_{i=1}^{n} x_i (\hat{y}_i - \hat{y}_i)$$

$$\frac{\partial J}{\partial b} = \frac{2}{n} \sum_{i=1}^{n} (\hat{y}_i - \hat{y}_i)$$

So

$$m_{new} = m_{old} - \alpha \cdot \frac{\partial J}{\partial m}$$

$$b_{new} = b_{old} - \alpha \cdot \frac{\partial J}{\partial m}$$

Iteration 1:

$m = -1, b = 1$

Predictions ($y = mx + b$):

So:

$\hat{y}_1 = -1(1) + 1 = 0$

$\hat{y}_2 = -1(3) + 1 = -2$

Errors ($e_i = \hat{y}_i - y_i$):

$e_1 = 0 - 3 = -3$

$e_2 = -2 - 6 = -8$

Gradients:

$$\frac{\partial J}{\partial m} = -\frac{2}{2}[(3)(1) + (8)(3)]$$

$$\frac{\partial J}{\partial m} = -\frac{2}{2}[3 + 24]$$

$$\frac{\partial J}{\partial m} = -1(27)$$

$$= -27$$

$$\frac{\partial J}{\partial m} = -\frac{2}{2}(3+8)$$

$$= -1(11)$$

$$= 11$$

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Updates

$$m_{new} = m_{old} - \alpha \frac{\partial J}{\partial m}$$

Since $\frac{\partial J}{\partial m} = -27$

$$m_{new} = -1 - (0.1)(-27)$$

$$m_{new} = -1 + 2.7$$
$$= 1.7$$

$$b_{new} = b_{old} - \alpha \frac{\partial J}{\partial b}$$

~~Hy~~ Since $\frac{\partial J}{\partial b} = -1$

$$b_{new} = 1 - 0.1(-11)$$
$$= 1 + 1.1$$
$$= 2.1$$

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Iteration 2

$\hat{y}_1 = m \cdot x_1 + b$, $\hat{y}_2 = m \cdot x_2 + b$.

predictions: $m = 1.7$, $b = 2.1$.

$\hat{y}_1 = (1.7) \times (1) + 2.1 = 3.8$

$\hat{y}_2 = 1.7 \times 3 + 2.1 = 7.2$

Errors: $e_1 = \hat{y}_1 - y_1 = 3.8 - 3 = 0.8$

$e_2 = y_2 - y_2 = 7.2 - 6 = 1.2$

Gradients: $\dfrac{\partial L}{\partial m} = \dfrac{2}{2}\left[x_1(\hat{y}_1 - y_1) + x_2(\hat{y}_2 - y_2)\right]$

$\Rightarrow \dfrac{2}{2}\left[(1) \times (0.8) + 3(1.2)\right]$

$= 0.8 + 3.6 = \underline{\underline{4.4}}$

$\dfrac{\partial L}{\partial b} = \dfrac{2}{2}\left[(\hat{y}_1 - y_1) + (\hat{y}_2 - y_2)\right]$

$= \dfrac{2}{2}\left[0.8 + 1.2\right] = \underline{\underline{2}}$

update m and b.

$m = m - \alpha \cdot \dfrac{\partial L}{\partial m} = 1.7 - 0.1(4.4) = 1.26$

$b = b - \alpha \cdot \dfrac{\partial L}{\partial b} = 2.1 - 0.1(2) = 1.9$

$m_{new} = \underline{\underline{1.26}}$ $\qquad b_{new} = \underline{\underline{1.9}}$

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Iteration 3 .

$y_1 = m_1 x_1 + b$, $y_2 = m_2 x_2 + b$

Predictions : $M = 1.26$, $b = 1.9$

$\hat{y}_1 = (1.26)(1) + (1.9) = \underline{\underline{3.16}}$

$\hat{y}_2 = (1.26)(3) + (1.9) = \underline{\underline{5.68}}$

Errors : $e_1 = \hat{y}_1 - y_1 = 3.16 - 3 = \underline{\underline{0.16}}$

$e_2 = \hat{y}_2 - y_2 = 5.68 - 6 = \underline{\underline{-0.32}}$

Gradients : $\frac{\partial L}{\partial M} = \frac{2}{2}\left[x_1(\hat{y}_1 - y_1) + x_2(\hat{y}_2 - y_2)\right]$

$= \left[1(0.16) + 3(-0.32)\right]$

$= \underline{\underline{-0.8}}$

$\frac{\partial L}{\partial b} = \frac{2}{2}\left[(\hat{y}_1 - y_1) + (\hat{y}_2 - y_2)\right]$

$= (0.16) + (-0.32)$

$= \underline{\underline{-0.16}}$

Updated M and b

$M = m - \alpha \cdot \frac{\partial L}{\partial m} = 1.26 - (0.1)(-0.8)$

$= 1.26 + 0.08$

$= \underline{1.34}$

$$b = b - \alpha \cdot \frac{\partial L}{\partial b} = 1.9 - (0.1)(-0.16)$$

$$= 1.9 + 0.016$$

$$= \underline{\underline{1.916}}$$

$$M_{new} = \boxed{1.34}$$

$$b_{new} = \boxed{1.916}$$

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Gradient Descent - Iteration 4

Given Information

- Current values: $m = 1.34$, $b = 1.916$
- Learning rate: $\alpha = 0.1$
- Data points: (1,3) and (3,6)
- Number of points: $n = 2$

Formulas:

Linear Equation: $\hat{y} = mx + b$

Gradient Formulas:

$dJ/dm = -(2/n) \times \Sigma(y_i - \hat{y}_i) \times x_i$

$dJ/db = -(2/n) \times \Sigma(y_i - \hat{y}_i)$

$m_{new} = m_{old} - \alpha \times (dJ/dm)$

$b_{new} = b_{old} - \alpha \times (dJ/db)$

ITERATION 4: STEP BY STEP SOLUTION

Step 1: Calculate Predicted values

For point (1,3): $\hat{y}_1 = 1.34 \times 1 + 1.916$

$\hat{y}_1 = 3.256$

For point (3,6): $\hat{y}_2 = 1.34 \times 3 + 1.916$

$= 5.936$

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Step 2: Calculate Errors $(y_i - \hat{y}_i)$

Error for point 1: $3 - 3.256 = -0.256$

Error for point 2: $6 - 5.936 = 0.064$

Step 3: Compute Gradients

Gradient with respect to m:

$dJ/dm = -(2/2) \times [(0.256) \times 1 + (0.064) \times 3]$

$dJ/dm = -1 \times [-0.256 + 0.192] = 0.064$

Gradient with respect to b:

$dJ/db = -(2/2) \times [(y_1 - \hat{y}_1) + (y_2 - \hat{y}_2)]$

$dJ/db = -1 \times [(-0.256) + (0.064)]$

$dJ/db = -1 \times (0.192) = 0.192$

Step 4: Update parameters

Update m:

$m\_new = 1.34 - 0.1 \times 0.064 = 1.3336$

Update b:

$b\_new = 1.916 - 0.1 \times 0.192 = 1.8968$

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Final results for Iteration 4

New m: 1.336 , New b: 1.8968

Absolute errors: $|-0.256| + |0.064| = 0.32$

Parameter changes: $\Delta m = 0.0064$, $\Delta b = 0.019$[illegible]

Errors continue to decrease, and parameter changes are getting small indicating were approaching the optimal solution

Iteration	m	b	T. Absolute Error	Δm	Δb
0	1.0	1.0	-	~~2.7~~	-
1	1.7	2.1	11.0	2.7	1.1
2	1.26	1.9	2.0	0.44	0.2
3	1.34	1.916	0.48	0.08	0.016
4	1.3336	1.898	0.32	0.0064	0.0[illegible]