Efficiently Building and Characterizing Electromagnetic Models of Multi-Qubit Superconducting Circuits

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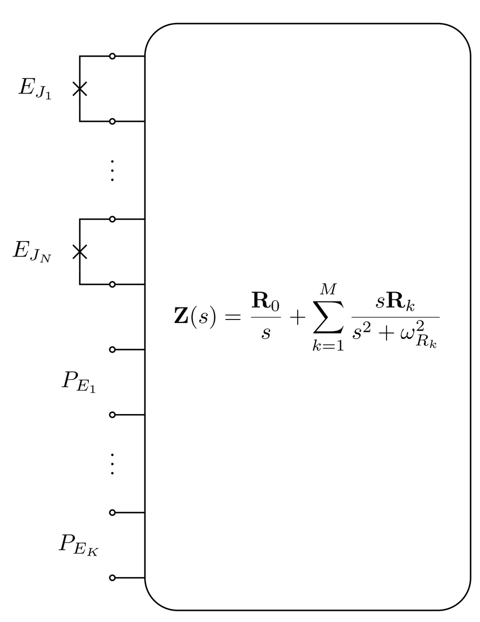
Abstract

In an attempt to better leverage superconducting quantum computers, scaling efforts have become the central concern. These efforts have been further exacerbated by the increased complexity of these circuits. The added complexity can introduce parasitic couplings and resonances, which may hinder the overall performance and scalability of these devices. We explore a method of modeling and characterization based on multiport impedance functions that correspond to multi-qubit circuits. By combining vector fitting techniques with a novel method for interconnecting rational impedance functions, we are able to efficiently construct Hamiltonians for multi-qubit circuits using electromagnetic simulations. Our methods can also be applied to circuits that contain both lumped and distributed element components. The constructed Hamiltonians account for all the interactions within a circuit that are described by the impedance function. We then present characterization methods that allow us to estimate effective qubit coupling rates, state-dependent dispersive shifts of resonant modes, and qubit relaxation times.

Lossless Reciprocal Impedance Functions

An arbitrary lossless reciprocal impedance function obtained using vector fitting [1] can be synthesized by the canonical Cauer representation [2,3]:



$$\mathbf{Z}(s) = \frac{\mathbf{R}_0}{s} + \sum_{k=1}^{M} \frac{s\mathbf{r}_k^T\mathbf{r}_k}{s^2+\omega_{R_k}^2}$$

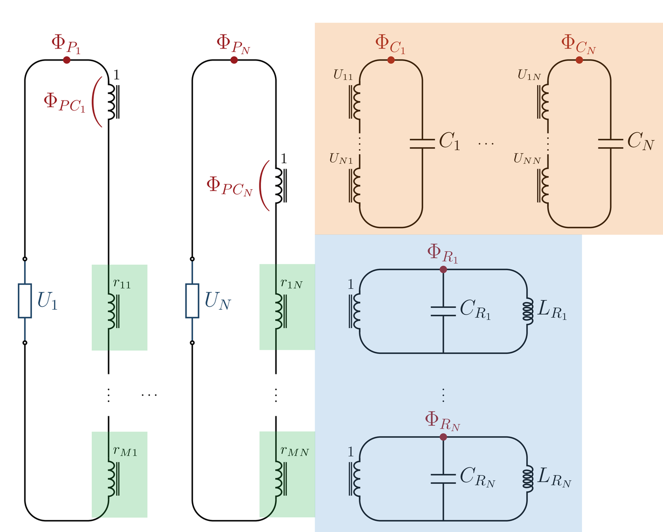
$$s = \sigma + j\omega$$

$$\mathbf{C_0} = \mathrm{diag}(C_1, \ldots, C_N)$$

$$\mathbf{R}_0 = \mathbf{U}\mathbf{C}_0^{-1}\mathbf{U}^T$$

$$\mathbf{r}_k = (r_{k1}, \ldots, r_{kN})$$

$$\mathbf{R} = \begin{pmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_M \end{pmatrix}$$



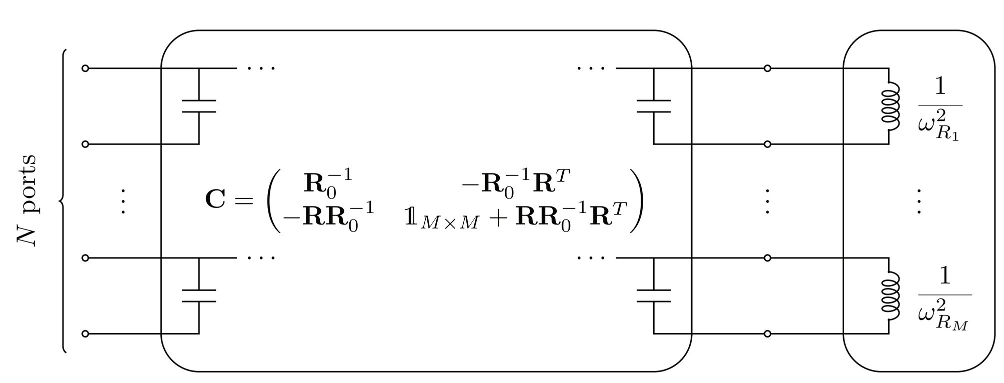
The Hamiltonian of the above above circuit [3] suggests that there is an alternative synthesis of the impedance function that has a cascade structure:

$$\mathcal{H} = \frac{1}{2}\mathbf{Q}^T\mathbf{C}^{-1}\mathbf{Q} + \frac{1}{2}\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} + \sum_{i=1}^{N} U_i(\Phi_{P_i})$$

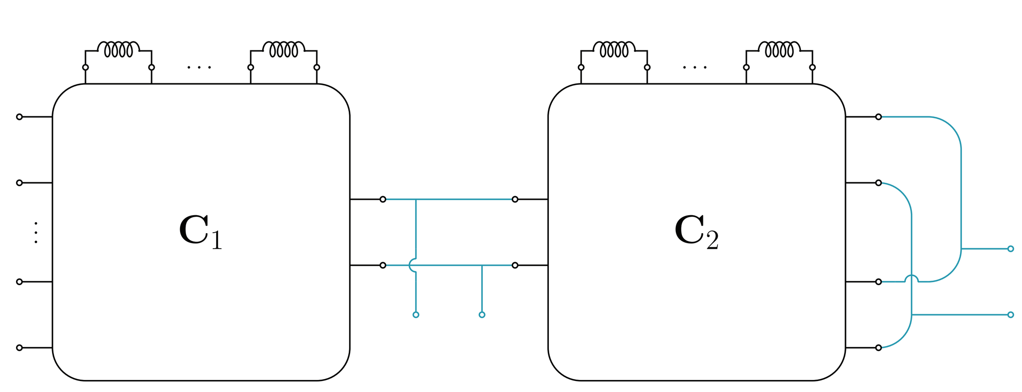
$$\mathbf{C} = \begin{pmatrix} \mathbf{R}_0^{-1} & -\mathbf{R}_0^{-1}\mathbf{R}^T \\ -\mathbf{R}\mathbf{R}_0^{-1} & \mathbf{C}_R + \mathbf{R}\mathbf{R}_0^{-1}\mathbf{R}^T \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{0}_{N\times N} & \mathbf{0}_{N\times M} \\ \mathbf{0}_{M\times N} & \mathbf{M}_R \end{pmatrix}$$

$$\mathbf{M}_R = \mathrm{diag}(L_{R_1}^{-1}, \ldots, L_{R_M}^{-1})$$



The cascade synthesis allows multiple rational impedances to be interconnected such that the final network also has the same cascade structure.



Hamiltonians of Transmon Networks

Using the rational impedance function corresponding to a transmon network, we can build the following circuit Hamiltonian:

$$\begin{aligned}\hat{H} = &\sum_{i=1}^{N}\left(\omega_{J_i}\hat{b}_i^\dagger\hat{b} + \frac{\beta_{J_i}}{2}\hat{b}_i^\dagger\hat{b}_i^\dagger\hat{b}_i\hat{b}_i + \sum_{j>i}^{j=N} g_{ij}(\hat{b}_i^\dagger\hat{b}_j + \hat{b}_i\hat{b}_j^\dagger - \hat{b}_i^\dagger\hat{b}_j^\dagger - \hat{b}_i\hat{b}_j)\right)\\ &+ \sum_{k=1}^{M}\left(\omega_{R_k}\hat{a}_k^\dagger\hat{a}_k + \frac{\alpha_{R_k}}{2}\hat{a}_k^\dagger\hat{a}_k^\dagger\hat{a}_k\hat{a}_k + \sum_{\ell>k}^{\ell=M} g_{R_k,R_\ell}(\hat{a}_k^\dagger\hat{a}_\ell + \hat{a}_k\hat{a}_\ell^\dagger - \hat{a}_k^\dagger\hat{a}_\ell^\dagger - \hat{a}_k\hat{a}_\ell)\right)\\ &+ \sum_{i=1}^{N}\sum_{k=1}^{M} g_{i,R_k}(\hat{b}_i^\dagger\hat{a}_k + \hat{b}_i\hat{a}_k^\dagger - \hat{b}_i^\dagger\hat{a}_k^\dagger - \hat{b}_i\hat{a}_k)\end{aligned}$$

We can use the following transformation to approximately block-diagonalize the above Hamiltonian:

$$\hat{U} = e^{\hat{S}} = \exp\left(\sum_{i=1}^{N}\sum_{k=1}^{M}\left[\frac{g_{i,R_k}}{\Delta_{i,R_k}}(\hat{b}_i^\dagger\hat{a}_k - \hat{b}_i\hat{a}_k^\dagger) - \frac{g_{i,R_k}}{\Sigma_{i,R_k}}(\hat{b}_i^\dagger\hat{a}_k^\dagger - \hat{b}_i\hat{a}_k)\right]\right)$$

Applying the transformation yield an effective transmon Hamiltonian:

$$\hat{H}_{\mathrm{eff}} \approx \sum_{i=1}^{N}\left(\tilde{\omega}_{J_i}\hat{b}_i^\dagger\hat{b} + \frac{\tilde{\beta}_{J_i}}{2}\hat{b}_i^\dagger\hat{b}_i^\dagger\hat{b}_i\hat{b}_i + \sum_{j>i}^{j=N}\tilde{g}_{ij}(\hat{b}_i^\dagger\hat{b}_j + \hat{b}_i\hat{b}_j^\dagger - \hat{b}_i^\dagger\hat{b}_j^\dagger - \hat{b}_i\hat{b}_j)\right) + \hat{H}_{\mathrm{eff}}^{R} + \hat{H}_{\mathrm{eff}}^{DS} + \hat{H}_{\mathrm{eff}}^{CK}$$

$$\tilde{g}_{ij} = g_{ij} + \frac{1}{2}\sum_{k=1}^{M} g_{i,R_k}g_{j,R_k}\left(\frac{1}{\Delta_{i,R_k}} + \frac{1}{\Delta_{j,R_k}} - \frac{1}{\Sigma_{i,R_k}} - \frac{1}{\Sigma_{j,R_k}}\right)$$

$$\hat{H}_{\mathrm{eff}}^{DS} = \sum_{i=1}^{N}\sum_{k=1}^{M} 2g_{i,R_k}^2(\beta_{J_i} + \alpha_{R_k})\left(\frac{1}{\Delta_{i,R_k}^2} + \frac{1}{\Sigma_{i,R_k}^2}\right)\hat{b}_i^\dagger\hat{b}_i\hat{a}_k^\dagger\hat{a}_k$$

$$\hat{H}_{\mathrm{eff}}^{CK} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j>i}^{j=N}\sum_{k=1}^{M}\left(\frac{g_{i,R_k}g_{j,R_k}}{\Delta_{i,R_k}\Delta_{j,R_k}}\right)^2(\beta_{J_i} + \beta_{J_j} + 4\alpha_{R_k})\hat{b}_i^\dagger\hat{b}_i\hat{b}_j^\dagger\hat{b}_j$$

Qubit Decay Rates

Using the cascade representation of impedance functions, we can use the equations of motion for the circuit to estimate the qubit decay rates to the external environment. The admittance parameter of the lossy network can also be used to estimate these decay rates.

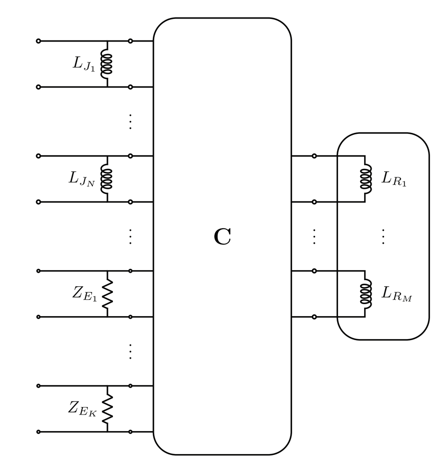
$$\ddot{\mathbf{\Phi}} + \mathbf{C}^{-1}\mathbf{Z}^{-1}\dot{\mathbf{\Phi}} + \mathbf{C}^{-1}\mathbf{M}\mathbf{\Phi} = 0$$

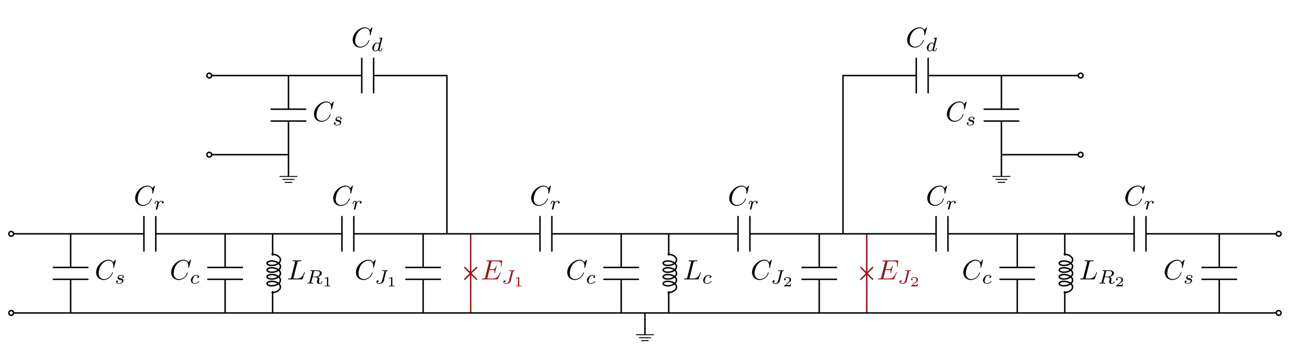
$$\begin{pmatrix} \dot{\mathbf{\Phi}} \\ \dot{\mathbf{V}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbb{1} \\ -\mathbf{C}^{-1}\mathbf{M} & -\mathbf{C}^{-1}\mathbf{Z}^{-1} \end{pmatrix}\begin{pmatrix} \mathbf{\Phi} \\ \mathbf{V} \end{pmatrix}$$

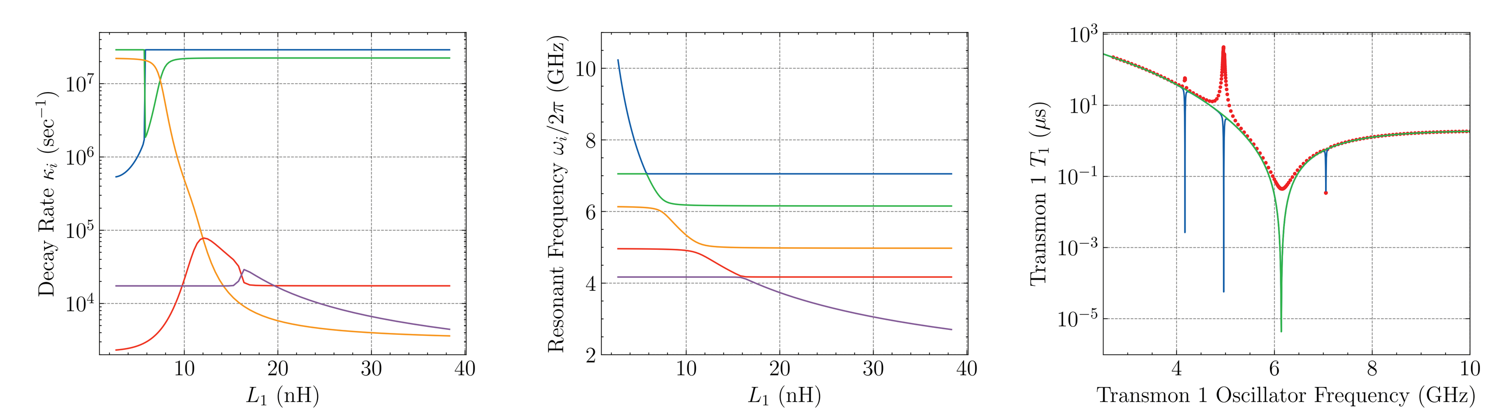
$$\downarrow$$

$$s_i = -\frac{\kappa_i}{2} \pm i\omega_i$$

$$\kappa_{J_i} \approx \tilde{C}_{J_i}^{-1}\,\mathrm{Re}(\mathbf{Y}_{ii}(\omega_{J_i}))$$

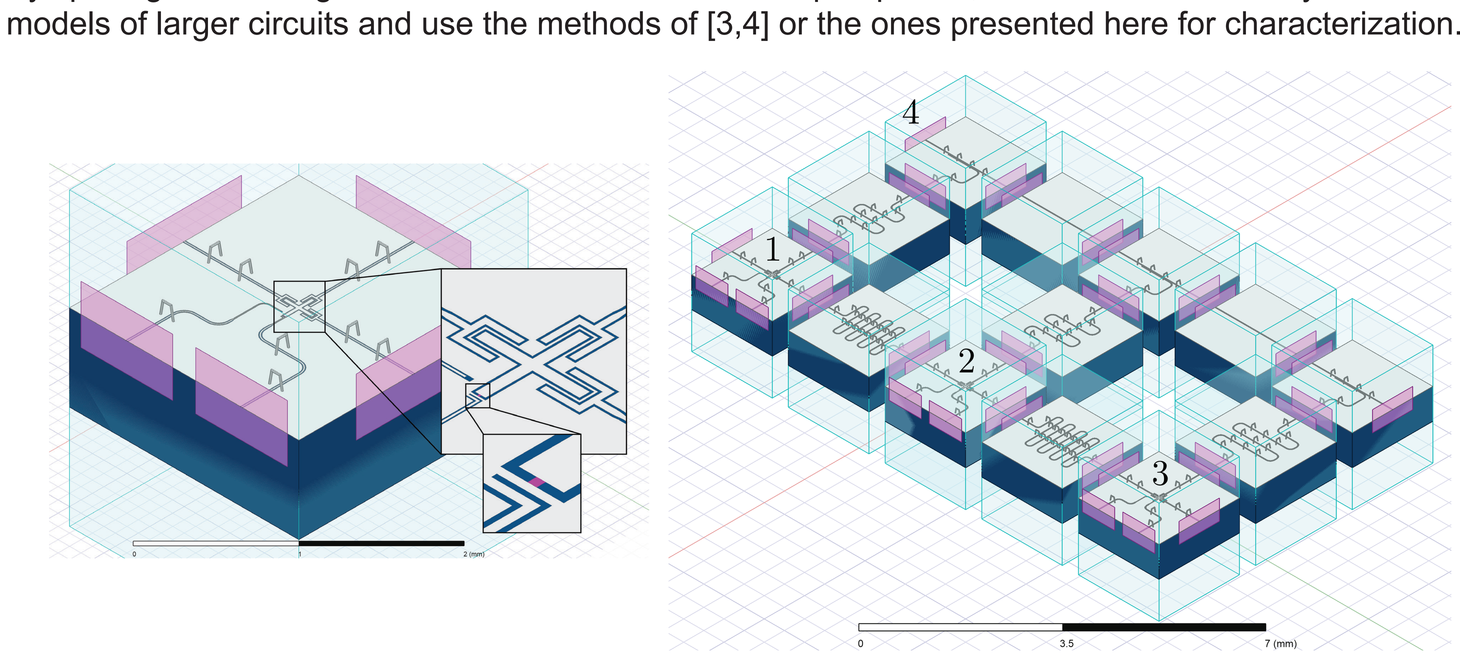


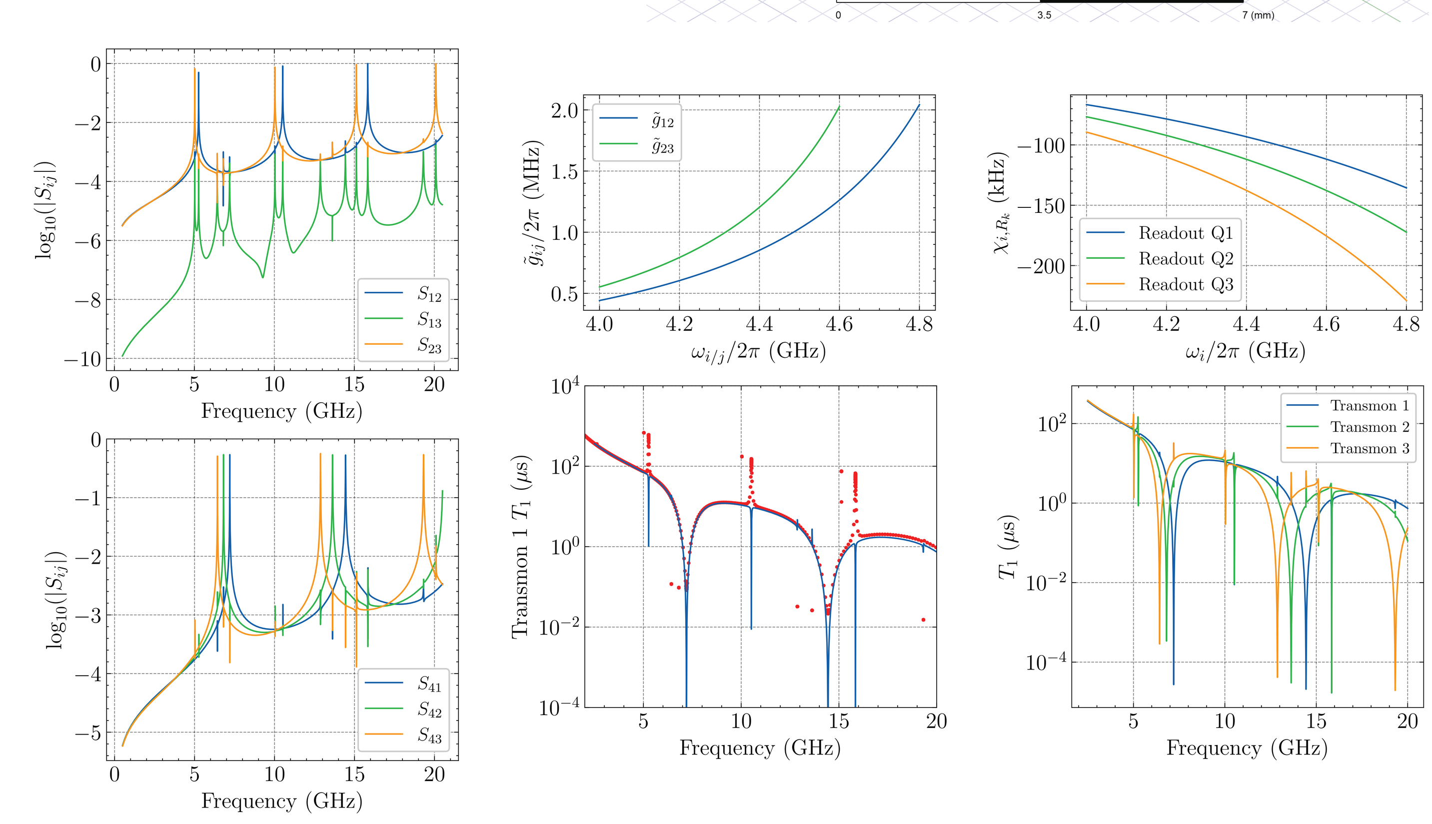


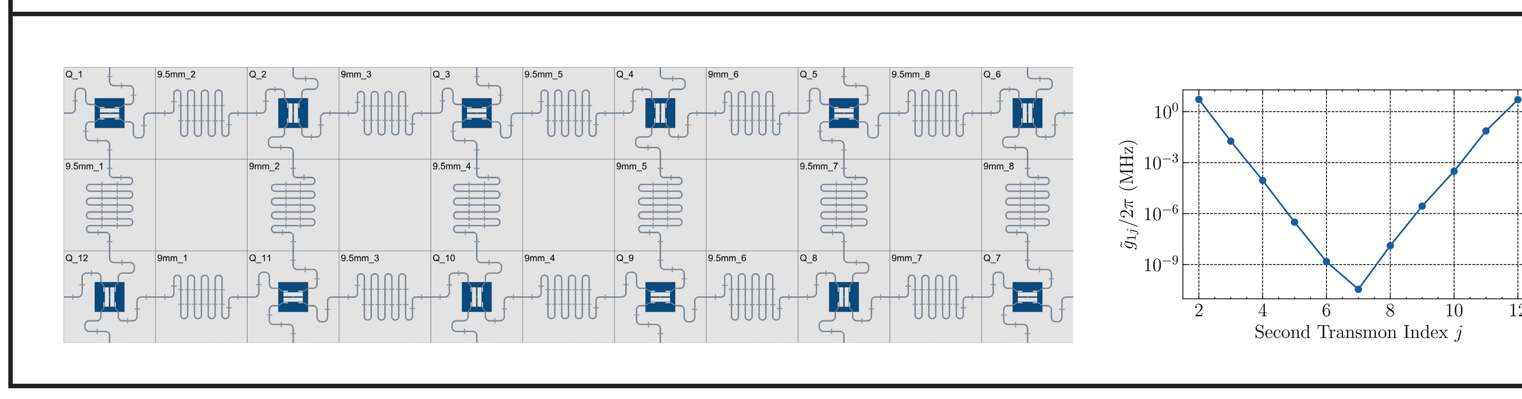


Brick Building Approach

By splitting electromagnetic models into smaller and simpler pieces, we can more efficiently build models of larger circuits and use the methods of [3,4] or the ones presented here for characterization.







[1] B. Gustavsen and A. Semlyen, IEEE Transactions on Power Delivery 14, 1052 (1999)
[2] R. Newcomb, Linear multiport synthesis (McGraw-Hill Book Company, New York, 1966)
[3] F. Solgun, D. P. DiVincenzo, and J. M. Gambetta, IEEE Transactions on Microwave Theory and Techniques 67, 928 (2019)
[4] L. Labarca, O. Benhayoune-Khadraoui, A. Blais, and A. Parra-Rodriguez, arXiv:2312.08354