# Delineating Efficient Portfolios

## Exam Focus

This is an optional reading that addresses fundamental concepts regarding portfolio return and volatility. It is important to be familiar with the calculations of expected return and volatility for a two-asset portfolio and understand the importance of correlation in portfolio diversification. It is also important to understand the shape of the portfolio possibilities curve and what is meant by the minimum variance portfolio. Additionally, you should know what the efficient frontier looks like and how short sales and riskless borrowing affect it. We have included Concept Checkers at the end of this reading for additional practice with these concepts.

## Expected Return and Volatility of a Two-asset Portfolio

The expected return on a portfolio is a weighted average of the expected returns on the individual assets that are included in the portfolio. For example, for a two-asset portfolio:

$$E(R_P) = w_1E(R_1) + w_2E(R_2)$$

where:
$E(R_P)$ = expected return on Portfolio P
$w_i$ = proportion (weight) of the portfolio allocated to Asset i
$E(R_i)$ = expected return on Asset i

The weights ($w_1$ and $w_2$) must sum to 100% for a two-asset portfolio.

The variance of a two-asset portfolio equals:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}_{1,2}$$

where:
$\sigma_P^2$ = variance of the returns for Portfolio P
$\sigma_1^2$ = variance of the returns for Asset 1
$\sigma_2^2$ = variance of the returns for Asset 2
$w_i$ = proportion (weight) of the portfolio allocated to Asset i
$\text{Cov}_{1,2}$ = covariance between the returns of the two assets

The covariance, $Cov_{1,2}$, measures the strength of the relationship between the returns earned on assets 1 and 2. The covariance is unbounded (ranges from negative infinity to positive infinity); therefore, it is not a very useful measure of the strength of the relationship between two asset's returns. Instead, we often scale the covariance by the standard deviations of the two assets to derive the correlation coefficient, $\rho_{1,2}$:

$$\rho_{1,2} = \frac{Cov_{1,2}}{\sigma_1 \sigma_2}$$

From the previous equation, notice that the covariance equals $\rho_{1,2}\sigma_1\sigma_2$. Therefore, the variance of the two-asset portfolio can also be written as:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

The portfolio standard deviation or portfolio volatility is the positive square root of the portfolio variance.

$$\sigma_P = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2\right]^{1/2}$$

**Example: Expected return and volatility for a two-asset portfolio**

Using the information in the following figure, **calculate** the expected return and standard deviation of the two-asset portfolio.

**Characteristics for a Two-Stock Portfolio**

| | *Caffeine Plus* | *Sparklin'* |
|---|---|---|
| Amount invested | $40,000 | $60,000 |
| Expected return | 11% | 25% |
| Standard deviation | 15% | 20% |
| Correlation | 0.30 | |

**Answer:**

First, determine the weight of each stock relative to the entire portfolio. Since the investments are $40,000 and $60,000, we know the total value of the portfolio is $100,000:

$w_c$ = investment/portfolio value = $40,000 / $100,000 = 0.40

$w_s$ = investment/portfolio value = $60,000 / $100,000 = 0.60

Next, we determine the expected return on the portfolio:

$$E(R_p) = w_c E(R_c) + w_s E(R_s)$$

$$E(R_p) = (0.40)(0.11) + (0.60)(0.25) = 0.1940 = 19.40\%$$

Then, we calculate the variance of the portfolio:

$$\begin{aligned}\sigma_p^2 &= w_c^2\sigma_c^2 + w_s^2\sigma_s^2 + 2w_c w_s \rho_{cs}\sigma_c\sigma_s \\ &= (0.40)^2(0.15)^2 + (0.60)^2(0.20)^2 + 2(0.40)(0.60)(0.30)(0.15)(0.20) \\ &= 0.02232\end{aligned}$$

And, finally, the standard deviation of the portfolio:

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.02232} = 0.1494 = 14.94\%$$

## The Portfolio Possibilities Curve

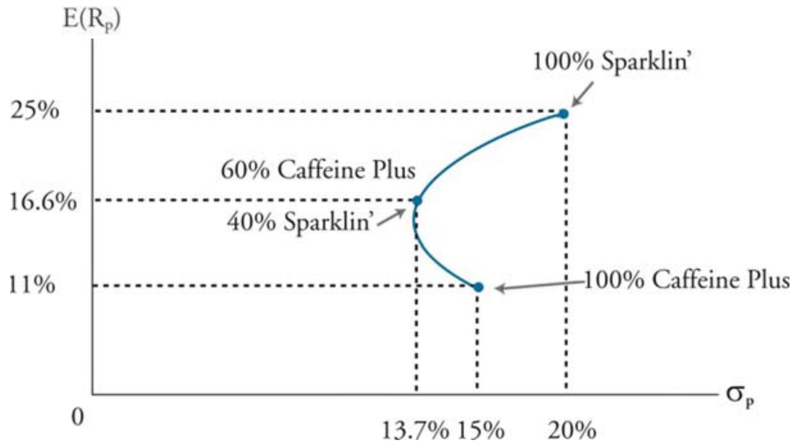
In the Caffeine Plus and Sparklin' example, we calculated the expected return and volatility of one possible combination: 40% in Caffeine Plus and 60% in Sparklin'. However, an infinite number of combinations of the two stocks are possible. We can plot these combinations on a graph with expected return on the *y*-axis and standard deviation on the *x*-axis, commonly referred to as plotting in risk/return "space." The graph of the possible portfolio combinations is referred to as the **portfolio possibilities curve**. Figure 1 shows some of these combinations.

**Figure 1: Portfolio Returns for Various Weights of Two Assets**

| $w_{\text{Caffeine Plus}}$<br>$w_{\text{Sparklin'}}$ | 100%<br>0% | 80%<br>20% | 60%<br>40% | 40%<br>60% | 20%<br>80% | 0%<br>100% |
|---|---|---|---|---|---|---|
| $\hat{R}_p$ | 11.00% | 13.80% | 16.60% | 19.40% | 22.20% | 25.00% |
| $\sigma_p$ | 15.00% | 13.74% | 13.72% | 14.94% | 17.10% | 20.00% |

The plot in Figure 2 represents all possible expected return and standard deviation combinations attainable by investing in varying amounts of Caffeine Plus and Sparklin'.

**Figure 2: Expected Return and Standard Deviation Combinations**



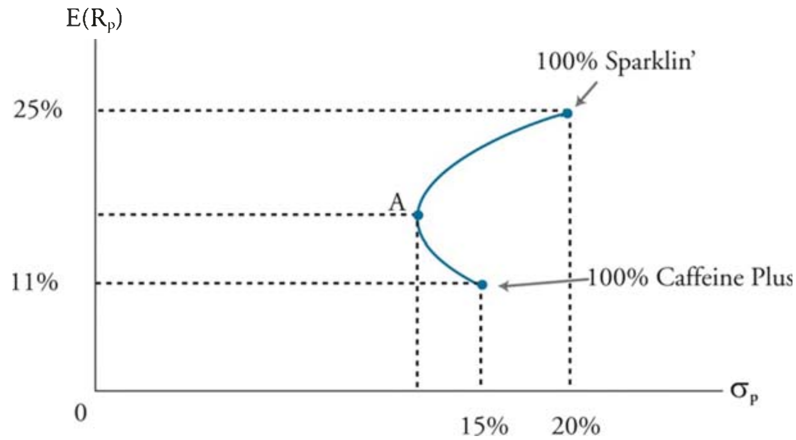

There are several things to notice about Figure 2:

- If 100% of the portfolio is allocated to Caffeine Plus, the portfolio will have the expected return and standard deviation of Caffeine Plus (i.e., Caffeine Plus is the portfolio), and the investment return and risk combination is at the lower end of the curve.
- As the investment in Caffeine Plus is decreased and the investment in Sparklin' is increased, the investment moves up the curve to the point where the portfolio's expected return is 16.6% with a standard deviation of 13.72% (labeled 60% Caffeine Plus/40% Sparklin').
- Finally, if 100% of the portfolio is allocated to Sparklin', the portfolio will have the expected return and standard deviation of Sparklin', and the investment return and risk combination is at the upper end of the curve (e.g., higher risk and higher expected return).

## Minimum Variance Portfolio

The **minimum variance portfolio** is the portfolio with the smallest variance among all possible portfolios on a portfolio possibilities curve. The minimum variance portfolio consisting of Caffeine Plus and Sparklin' contains approximately 70% Caffeine Plus and 30% Sparklin' and has an expected return of 15.3% and a standard deviation of 13.6%. On the portfolio possibilities curve, the minimum variance portfolio represents the left-most point on the curve. Figure 3 illustrates the minimum variance portfolio for Caffeine Plus and Sparklin' (point A).

**Figure 3: Minimum Variance Portfolio**




## Correlation and Portfolio Diversification

### Perfect Positive Correlation

In the case where two assets have perfect positive correlation (i.e., $\rho = 1$), the portfolio standard deviation reduces to the simple weighted average of the individual standard deviations indicating no diversification. This is shown mathematically as:

$$\sigma_p = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \times 1 \times \sigma_1\sigma_2\right]^{1/2} = w_1\sigma_1 + w_2\sigma_2$$

Since expected portfolio return is a linear combination of the individual asset returns, and risk is a linear combination of the individual asset volatilities, the portfolio possibilities curve for two perfectly correlated assets is a straight line. This line is given as:

$$E(R_P) = \left(E(R_2) - \frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2}\sigma_2\right) + \left(\frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2}\right)\sigma_P$$

No diversification is achieved if the correlation between assets equals +1. As the correlation between two assets *decreases*, however, the benefits of diversification *increase*. As the correlation decreases, there is less tendency for stock returns to move together. The separate movements of each stock serve to reduce the volatility of a portfolio to a level that is less than the weighted sum of its individual components (e.g., less than $w_1\sigma_1 + w_2\sigma_2$).



### Perfect Negative Correlation

The greatest diversification is achieved in the case where two assets have perfect negative correlation (i.e., $\rho = -1$). In this case, the portfolio standard deviation reduces to two linear equations, which are:

$$\sigma_p = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \times -1 \times \sigma_1\sigma_2\right]^{1/2} = w_1\sigma_1 - w_2\sigma_2 \text{ or } -w_1\sigma_1 + w_2\sigma_2$$

When two assets have perfect negative correlation, it is possible to construct a portfolio with zero volatility by setting the standard deviation equal to zero and solving for the portfolio weights. The portfolio with zero volatility has portfolio weights of:

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

$$w_2 = 1 - w_1$$

Given that the standard deviation reduces to two linear equations, the portfolio possibilities curve for two assets with perfect negative correlation will be two line segments.

### Zero Correlation

When the correlation between two assets is zero, the covariance term in the portfolio standard deviation expression is eliminated, and the resulting expression is:

$$\sigma_p = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \times 0 \times \sigma_1\sigma_2\right]^{1/2} = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2\right]^{1/2}$$

In this case, the standard deviation expression reduces to a non-linear equation, and the portfolio possibilities curve will be non-linear.

Assuming that the standard deviations of the individual assets are greater than zero, it is impossible to construct a portfolio with zero volatility. The weights of the minimum variance portfolio can be solved as previously discussed. The weights are calculated as:

$$w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$w_2 = 1 - w_1$$

### Moderate Positive Correlation

Most equities are positively correlated (i.e., $0 < \rho < 1$). If we assume that two assets are moderately correlated (e.g., $\rho = 0.5$), then the portfolio standard deviation reduces to:

$$\sigma_p = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \times \frac{1}{2} \times \sigma_1\sigma_2\right]^{1/2} = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_1w_2\sigma_1\sigma_2\right]^{1/2}$$

Similar to the case of zero correlation, assets with moderate correlation have non-linear portfolio possibilities curves.

### An Example of Correlation and Portfolio Diversification

To illustrate the effects of correlation on diversification, consider the expected return and standard deviation data derived for domestic stocks, DS, and domestic bonds, DB as shown in Figure 4.

**Figure 4: Diversification Example**

| | *Expected Return* | *Standard Deviation* |
|---|---|---|
| Domestic Stocks (DS) | 0.20 | 0.30 |
| Domestic Bonds (DB) | 0.10 | 0.15 |

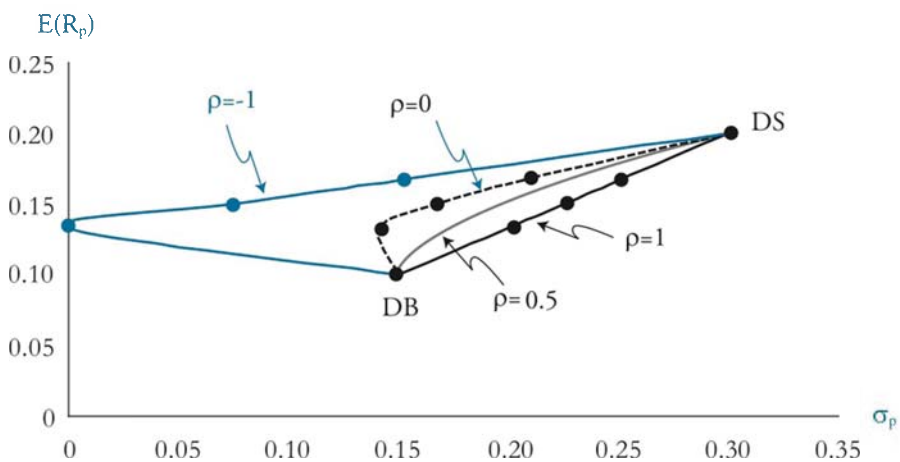
Figure 5 shows the expected return and standard deviation combinations for various portfolio percentage allocations to domestic stocks and domestic bonds for each of the correlations +1, 0.5, 0, and –1.

**Figure 5: Expected Return/Standard Deviation Combinations for Various Allocations**

| | | | | | $\sigma_P$ | |
|---|---|---|---|---|---|---|
| *DS % Allocation* | *DB % Allocation* | $E(R_p)$ | $\rho = 1$ | $\rho = 0.5$ | $\rho = 0$ | $\rho = -1$ |
| 100.00 | 0.00 | 0.200 | 0.300 | 0.300 | 0.300 | 0.300 |
| 66.67 | 33.33 | 0.167 | 0.250 | 0.229 | 0.206 | 0.150 |
| 50.00 | 50.00 | 0.150 | 0.225 | 0.198 | 0.168 | 0.075 |
| 33.33 | 66.67 | 0.133 | 0.200 | 0.173 | 0.141 | 0.000 |
| 0.00 | 100.00 | 0.100 | 0.150 | 0.150 | 0.150 | 0.150 |

Figure 6 shows the plot of the expected returns and standard deviations for each of the four correlations.

**Figure 6: Effects of Correlation on Portfolio Risk**




As indicated in Figure 6, the lower the correlation between the returns of the stocks in the portfolio, the greater the diversification benefits. If the correlation equals +1 (the solid black line), the minimum-variance frontier is a straight line between the two points (DB and DS), and there is no benefit to diversification. If the correlation equals –1 (the solid blue line), the minimum-variance frontier is two straight-line segments, and there exists a portfolio combination of stocks and bonds with a standard deviation of *zero* (the allocation of 66.67% to domestic bonds and 33.33% to domestic stocks).

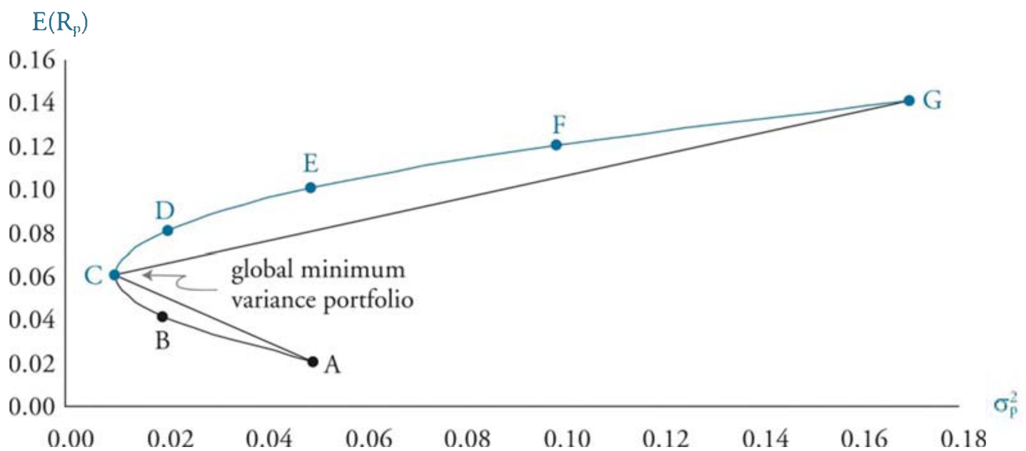
## The Shape of the Portfolio Possibilities Curve

*Professor's Note: In this section, we are not considering the special cases where the portfolio possibilities curve is a straight line (i.e., $\rho = 1$) or two line segments (i.e., $\rho = -1$). In all other cases, the portfolio possibilities curve is a curve similar to Figure 7.*

Looking at Figure 7, the shape of the portfolio possibilities curve is best described in two pieces.

- The piece of the portfolio possibilities curve that lies above the minimum variance portfolio (from point C through point G) is concave.
- The piece of the portfolio possibilities curve that lies below the minimum variance portfolio (from point A through point C) is convex.

**Figure 7: Shape of the Portfolio Possibilities Curve**




*Professor's Note: A concave function is one where the function lies above a straight-line segment connecting any two points on the function. A convex function lies below a straight-line segment connecting any two points on the function.*

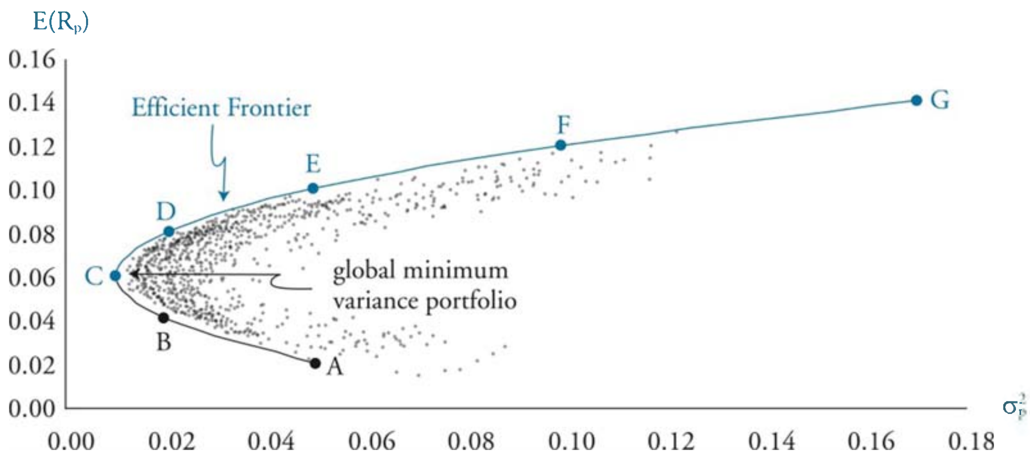
*In Figure 7, the function is above the line segment from C to G. Therefore, the portion of the portfolio possibilities curve from C to G is concave. The function is below the line segment from A to C. Therefore, the portion of the portfolio possibilities curve from A to C is convex.*

Another important aspect regarding the shape of the portfolio possibilities curve is that the curve must lie to the left of a line segment connecting any two points on the curve. From the discussion of portfolio diversification and correlation, combinations of two assets with perfect positive correlation result in a straight line. Combinations of assets with lower correlation will always lie to the left of that line.

## The Efficient Frontier

Plotting all risky assets and potential combinations of risky assets will result in a graph similar to Figure 8.

**Figure 8: Efficient Frontier**




Notice that the graph includes some portfolios that no rational investor would select. All portfolios lying on the inside of the curve are inefficient. Additionally, some portfolios offer higher returns with identical risk. For example, portfolios A and E have identical risk; however, Portfolio E has a much higher expected return, and a similar contrast exists for Portfolio D versus Portfolio B. All rational investors would prefer Portfolio D over Portfolio B, and Portfolio E over Portfolio A.

Portfolios such as D and E are called **efficient portfolios**, which are portfolios that have:

- Minimum risk of all portfolios with the same expected return.
- Maximum expected return for all portfolios with the same risk.

The **efficient frontier** is a plot of the expected return and risk combinations of all efficient portfolios, all of which lie along the upper-left portion of the possible portfolios (from Point C to Point G in Figure 8).

### Short Sales and the Efficient Frontier

When short sales are allowed, the shape of the efficient frontier changes. To examine how it changes, consider again the Caffeine Plus and Sparklin' example.

Referring back to the example, Caffeine Plus has an expected return of 11% and a standard deviation of 15%, and Sparklin' has an expected return of 25% and a standard deviation of 20%. The correlation between Caffeine Plus and Sparklin' is 0.30. Although neither stock has a negative return, it may make sense to short sell one of the stocks. In this case, Sparklin' has a higher expected return, so shorting Caffeine Plus and investing in Sparklin' would expand the efficient frontier. Figure 9 highlights the portfolio return and volatility for combinations of Sparklin' and Caffeine Plus including short sales.

**Figure 9: Portfolio Returns for Various Weights of Two Assets (w/ Short Sales)**

| $w_{\text{Caffeine Plus}}$ | 100% | 80% | 60% | 40% | 20% | 0% | –20% | –40% | – 60% | –80% | – 100% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_{\text{Sparklin}^{®}}$ | 0% | 20% | 40% | 60% | 80% | 100% | 120% | 140% | 160% | 180% | 200% |
| $\hat{R}_p$ | 11.00% | 13.80% | 16.60% | 19.40% | 22.20% | 25.00% | 27.80% | 30.60% | 33.40% | 36.20% | 39.00% |
| $\sigma_p$ | 15.00% | 13.74% | 13.72% | 14.94% | 17.10% | 20.00% | 23.28% | 26.82% | 30.53% | 34.36% | 38.28% |

When allowing for short sales, the efficient frontier expands up and to the right. By shorting, it is possible to create higher return and higher volatility portfolio combinations that would not be possible otherwise. Theoretically, with no limitations on shorting, it would be possible to construct a portfolio with infinite return.

*Professor's Note: Up to this point, we have discussed risky assets. Now, we add the risk-free asset to the set of asset choices and examine the effect it has on investment choices.*

## Combining the Risk-Free Rate with the Efficient Frontier

So far, our portfolios have consisted of risky assets only. However, in reality, investors usually allocate their wealth across both risky and risk-free assets. The following discussion illustrates the effects of the inclusion of the risk-free asset. A risk-free asset is a security that has a return known ahead of time, so the variance of the return is zero.

Consider the task of creating portfolios comprising the risk-free asset, F, and a risky portfolio, P. Assume that Portfolio P lies on the efficient frontier of risky assets. Various combinations (weightings) of Portfolio P and the risk-free asset can be created. By adding the risk-free asset to the investment mix, a very important property emerges: *The shape of the efficient frontier changes from a curve to a line.*

Recall that the expected return for a portfolio of two assets equals the weighted average of the asset expected returns. Therefore, the expected return on Investment C that combines the risk-free asset and risky Portfolio P equals:

$$E(R_C) = w_F R_F + w_P E(R_P)$$

where:
$w_F$ = percentage allocated to the risk-free asset
$w_P$ = percentage allocated to Portfolio P

Also, recall that the variance of the portfolio of two assets (F and P) equals:

$$\sigma_C^2 = w_F^2\sigma_F^2 + w_P^2\sigma_P^2 + 2w_F w_P \text{Cov}_{FP}$$

where:

$\sigma_C^2$ = variance for Investment C
$\sigma_F^2$ = variance for the risk-free asset
$\sigma_P^2$ = variance for Portfolio P
$\text{Cov}_{FP}$ = covariance between F and P

Observe that since we know that the variance and the standard deviation of the risk-free asset both equal zero, and that the covariance of the risk-free asset with any risky asset also equals zero, the equations for the variance and standard deviation for Investment C simplify to:

$$\sigma_C^2 = w_P^2\sigma_P^2$$

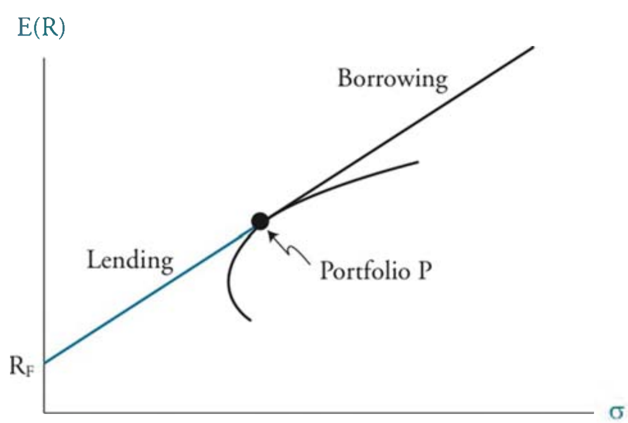
$$\sigma_C = w_P\sigma_P$$

Because the expected return and portfolio standard deviation of the combination of a risk-free asset and risky portfolio are both linear, the efficient frontier reduces to a linear equation. That is, by including the risk-free asset, we have caused the efficient frontier to become a straight line. The equation for the efficient frontier becomes the **capital market line** (**CML**).

$$E(R_C) = R_F + \left(\frac{E(R_P) - R_F}{\sigma_P}\right)\sigma_C$$

Figure 10 illustrates the combination of the risk-free asset with the risky portfolio.

**Figure 10: Efficient Frontier including the Risk-Free Asset**




When the risk-free asset is combined with the risky Portfolio P, the efficient frontier becomes a line with:

- The intercept equal to the risk-free rate, and
- The slope equal to the reward-to-risk ratio for the risky portfolio.

Note that the capital market line is tangent to the efficient frontier. The point of tangency, Portfolio P, is known as the market portfolio. This portfolio contains all available risky assets in proportion to their total market values.

If all investors agree on the efficient frontier (i.e., they have homogeneous expectations regarding the risks and returns for all risky assets), they will hold a combination of the market portfolio and the risk-free asset. Risk-averse investors will create lower risk portfolios by lending (i.e., investing in the risk-free asset). More risk-tolerant investors will increase portfolio return by borrowing at the risk-free rate. This result is known as the separation theorem.

## Concept Checkers

1. Assume the following information for stocks A and B.
   - Expected return on Stock A = 18%.
   - Expected return on Stock B = 23%.
   - Correlation between returns of Stock A and Stock B = 0.10.
   - Standard deviation of returns on Stock A = 40%.
   - Standard deviation of returns on Stock B = 50%.

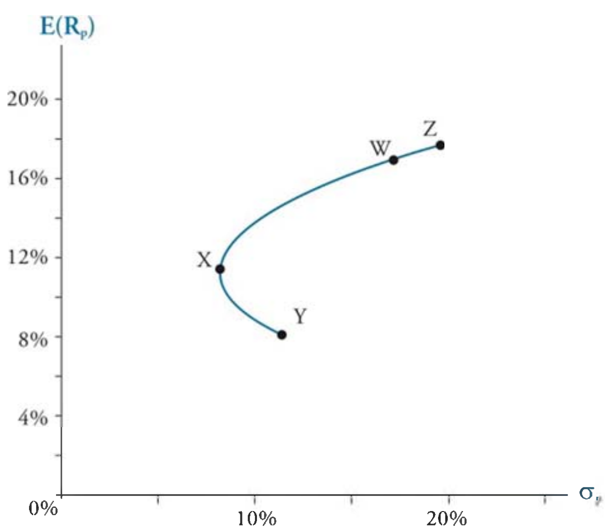
   The expected return and standard deviation of an equally weighted portfolio of stocks A and B are closest to:

| | Expected return (%) | Standard deviation (%) |
|---|---|---|
| A. | 20.5 | 33.54 |
| B. | 20.5 | 11.22 |
| C. | 33.5 | 11.22 |
| D. | 33.5 | 33.54 |

**Use the following data to answer Questions 2 and 3.**

Assume the expected return on stocks is 18% (represented by *Z* in the figure), and the expected return on bonds is 8% (represented by point *Y* on the graph).

**Portfolio Possibilities Curve: Stocks and Bonds**




2. The graph shows the portfolio possibilities curve for stocks and bonds. The point on the graph that most likely represents a 90% allocation in stocks and a 10% allocation in bonds is Portfolio:
   A. W.
   B. X.
   C. Y.
   D. Z.

3. The efficient frontier consists of the portfolios between and including:
   A. X and W.
   B. Y and Z.
   C. X and Z.
   D. Y and X.

4. Which of the following best describes the shape of the portfolio possibilities curve?
   A. The curve is strictly convex.
   B. The curve is strictly concave.
   C. The curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.
   D. The curve is convex above the minimum variance portfolio and concave below the minimum variance portfolio.

5. When short sales are possible (i.e., there are no short sale restrictions), the efficient frontier is:
   A. a straight line between the risk-free asset and the market portfolio.
   B. two line segments, which indicate a negative relationship between short and long positions.
   C. expanded to include portfolios with higher return and lower volatility.
   D. expanded to include portfolios with higher return and higher volatility.

## Concept Checker Answers

1. A $E(R_P) = w_A E(R_A) + w_B E(R_B) = (0.50)(0.18) + (0.50)(0.23) = 0.205 = 20.5\%$

$$\sigma_p = \left[w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{AB}\sigma_A\sigma_B\right]^{1/2}$$

$$\sigma_p = \begin{bmatrix}(0.5)^2(0.4)^2 + (0.5)^2(0.5)^2 \\ +2(0.5)(0.5)(0.1)(0.4)(0.5)\end{bmatrix}^{1/2} = 0.3354 = 33.54\%$$

2. A Since the return to *W* is the nearest to *Z* (stocks), it is logical to assume that point *W* represents an allocation of 90% stocks/10% bonds. The return for *W* is lower than *Z*, but it also represents a reduction in risk.

3. C The *efficient frontier* consists of portfolios that have the maximum expected return for any given level of risk (standard deviation or variance). The efficient frontier starts at the global minimum-variance portfolio and continues above it. Any portfolio below the efficient frontier is dominated by a portfolio on the efficient frontier. This is because efficient portfolios have higher expected returns for the same level of risk.

4. C The portfolio possibilities curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.

5. D When short sales are allowed, the efficient frontier expands up and to the right (i.e., higher return and higher volatility portfolio combinations become feasible). When considering two stocks, by shorting the stock with lower expected return and using the proceeds to increase the investment in the other stock, it is possible to increase portfolio return. This increased return comes at a cost of higher volatility, though.

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# The Standard Capital Asset Pricing Model

Topic 10

## Exam Focus

This topic discusses the capital market line (CML) and the capital asset pricing model (CAPM). The CAPM requires many assumptions, such as the existence of a risk-free asset and that all investors have the same type of utility function and expectations. The existence of a risk-free asset means the efficient frontier becomes a straight line, which allows for the use of simple expressions to analyze price risk. It is important to have a firm grasp on the CAPM calculation methodology.

## The Capital Asset Pricing Model (CAPM)

### LO 10.2: Describe the assumptions underlying the CAPM.

The **capital asset pricing model** (CAPM), derived by Sharpe, Lintner, and Mossin, is one of the most celebrated models in all of finance. The model describes the relationship we should expect to see between risk and return for individual assets. Specifically, the CAPM provides a way to calculate an asset's expected return (or "required" return) based on its level of systematic (or market-related) risk, as measured by the asset's beta.

### CAPM Assumptions

In the derivation of any economic or scientific model, simplifying assumptions regarding the market, which the model represents, must be made. The CAPM has a number of underlying assumptions:

1. Investors face no transaction costs when trading assets. This assumption simplifies the computation of returns. If transaction costs were considered, returns would be a function of transaction costs, which would then have to be estimated.
2. Assets are infinitely divisible. It is possible to hold fractional shares.
3. There are no taxes; therefore, investors are indifferent between capital gains and income or dividends.
4. Investors are price takers whose individual buy and sell decisions have no effect on asset prices. The market for assets is perfectly competitive.
5. Investors' utility functions are based solely on expected portfolio return and risk. This assumption provides a framework for how investors make investment decisions.
6. Unlimited short-selling is allowed. Investors can sell an unlimited number of shares of an asset short.

7. Investors can borrow and lend unlimited amounts at the risk-free rate.
8. Investors are only concerned about returns and risk over a single period, and the single period is the same for all investors.
9. All investors have the same forecasts of expected returns, variances, and covariances. This is known as homogeneous expectations.
10. All assets are marketable, including human capital.

## The Capital Market Line (CML)

### LO 10.3: Interpret the capital market line.

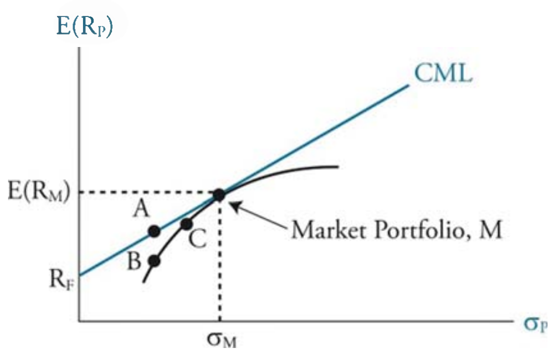
In the presence of riskless lending and borrowing, the efficient frontier transforms from a curve to a line tangent to the previous curve. Investors will choose to invest in some combination of their tangency portfolio and the risk-free asset. Assuming investors have identical expectations regarding expected returns, standard deviations, and correlations of all assets, there will be only one tangency line, which is referred to as the **capital market line** (CML).

Under the assumptions of the CML, all investors agree on the exact composition of the optimal risky portfolio. This universally agreed upon optimal risky portfolio is called the **market portfolio**, M, and it is defined as the portfolio of all marketable assets weighted in proportion to their relative market values. For instance, if the market value of Asset X is $1 billion, and the market value of all traded assets is $100 billion, then the weight allocated to Asset X in the market portfolio equals 1%.

The key conclusion of the CML can be summarized as follows: *All investors will make optimal investment decisions by allocating between the risk-free asset and the market portfolio.*

Figure 1 provides a graph of the CML.

**Figure 1: The Capital Market Line**



The equation for the CML is:

$$E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_P$$

The slope of the CML is often called the *market price of risk* and equals the reward-to-risk ratio (or Sharpe ratio) for the market portfolio. This is calculated as:

$$\frac{E(R_M) - R_F}{\sigma_M}$$

*Professor's Note: We will examine the calculation of risk-adjusted return measures, such as the Sharpe ratio, in Topic 11.*

The CML is useful for computing the expected return for an efficient (diversified) portfolio; however, it cannot compute the expected return for inefficient portfolios or individual securities. The CAPM must be used to compute the expected return for any inefficient portfolio or individual security.

## Beta

### LO 10.5: Interpret beta and calculate the beta of a single asset or portfolio.

The sensitivity of an asset's return to the market return is referred to as the asset's **beta**. Beta is a standardized measure of the covariance of the asset's return with the market return. Beta can be calculated as follows:

$$\beta_i = \frac{\text{covariance of Asset i's return with the market return}}{\text{variance of the market return}} = \frac{Cov_{i,M}}{\sigma_M^2}$$

We can use the definition of the correlation between the returns on Asset *i* with the returns on the market to get the covariance equation:

$$\rho_{i,M} = \frac{Cov_{i,M}}{\sigma_i \sigma_M}$$

$$Cov_{i,M} = \rho_{i,M}\sigma_i\sigma_M$$

Therefore, by substituting for $Cov_{i,M}$ in the equation for $\beta_i$, we can also calculate beta as:

$$\beta_i = \frac{\rho_{i,M}\sigma_i\sigma_M}{\sigma_M^2} = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$$

**Example: Calculating an asset's beta**

The standard deviation of the market return is estimated as 20%.

1. If Asset A's standard deviation is 30% and its correlation of returns with the market index is 0.8, what is Asset A's beta?

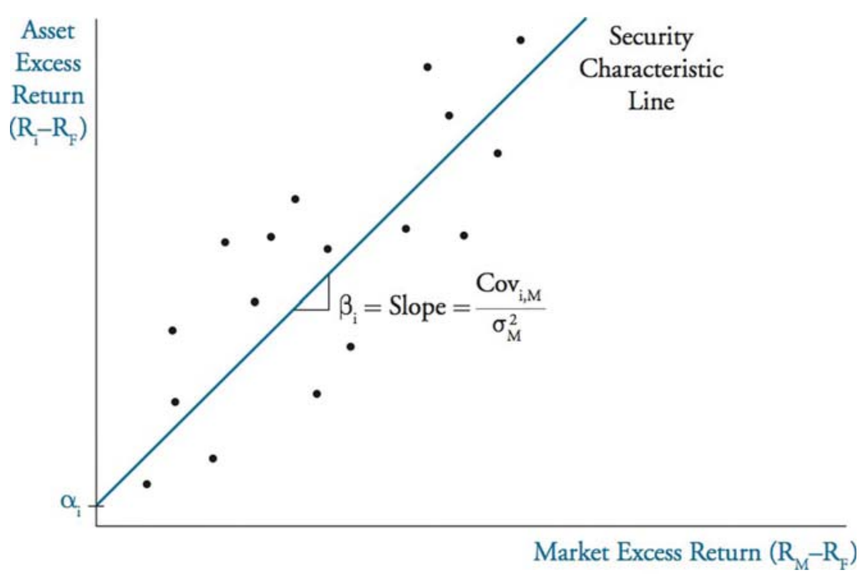
   Using the formula: $\beta_i = \rho_{i,M}\dfrac{\sigma_i}{\sigma_M}$, we have: $\beta_i = 0.80\dfrac{0.30}{0.20} = 1.2$.

2. If the covariance of Asset A's returns with the returns on the market index is 0.048, what is the beta of Asset A?

   Using the formula: $\beta_i = \dfrac{\text{Cov}_{i,M}}{\sigma_M^2}$, we have: $\beta_i = \dfrac{0.048}{0.2^2} = 1.2$.

In practice, we estimate asset betas by regressing returns on the asset on those of the market index. While regression is a concept discussed in Book 2, for the purposes of this topic, you can think of it as a mathematical estimation procedure that fits a line to a data plot. In Figure 2, we represent the excess returns on Asset *i* as the dependent variable and the excess returns on the market index as the independent variable. The *least squares regression line* is the line that minimizes the sum of the squared distances of the points plotted from the line (this is what is meant by the line of *best fit*). The slope of this line is our estimate of beta. In Figure 2, since the line is steeper than 45 degrees, the slope is greater than one, and the asset's estimated beta is greater than one. Our interpretation is that the returns on Asset *i* are more variable in response to systematic risk factors than the overall market, which has a beta of one.

**Figure 2: Regression of Asset Excess Returns against Market Asset Returns**




This regression line is referred to as the asset's security characteristic line. Mathematically, the slope of the security characteristic line is: $\text{Cov}_{i,M} / \sigma_M^2$, which is the same formula used to calculate beta.

### Portfolio Beta

In addition to individual assets, beta can also be computed for portfolios. The beta of a portfolio is the sum of the weighted individual asset betas within a portfolio.

**Example: Calculating portfolio beta**

Consider the following individual asset weights and betas for a 4-asset portfolio.

| *Asset* | *Portfolio Weight* | *Beta* |
|---|---|---|
| 1 | 25% | 1.2 |
| 2 | 15% | 1.8 |
| 3 | 35% | 0.9 |
| 4 | 25% | 1.4 |

**Calculate** the beta of this 4-asset portfolio.

**Answer:**

$$\beta_P = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + w_4\beta_4$$
$$\beta_P = (0.25 \times 1.2) + (0.15 \times 1.8) + (0.35 \times 0.9) + (0.25 \times 1.4)$$
$$\beta_P = 0.3 + 0.27 + 0.315 + 0.35 = 1.235$$

## Deriving the CAPM

### LO 10.1: Understand the derivation and components of the CAPM.

### A Straightforward Derivation

The procedure used to derive the equation for the capital asset pricing model requires an understanding of the characteristics of expected return, beta, the risk-free rate, and the security market line. The following steps illustrate how the CAPM is derived. The end result will be an equation where the expected return on a single security or portfolio of securities is equal to:

$$R_F + \text{Beta}_i[E(R_M) - R_F]$$

The first step in the derivation is to recognize that beta identifies the appropriate level of risk for which an investor should be compensated. An important concept in finance is that, as a portfolio becomes more diversified, idiosyncratic risk (i.e., unsystematic risk or asset-specific risk) in the portfolio becomes less of an issue as only systematic risk remains.

*Professor's Note: Starting with the formula for portfolio variance, and assuming n equally-weighted assets (e.g., each w = 1/n), it is possible to show that the portfolio variance for an equally-weighted portfolio is:*

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{Cov}$$

*where:*

$\overline{\sigma_i^2}$ = *average variance of all assets in the portfolio*

$\overline{Cov}$ = *average covariance of all pairings of assets in the portfolio*

*Note that the equally-weighted portfolio variance equals the sum of two components (unsystematic risk: the variance term and systematic risk: the covariance term), each of which is affected by the size of the portfolio:*

- $(1/n)\times\overline{\sigma_i^2}$ *gets closer to zero as n gets larger because 1/n approaches zero.*
- $[(n-1)/n]\times\overline{Cov}$ *gets closer to the average covariance as n gets larger because (n – 1) / n approaches 1.*

*Therefore, the following important result emerges: The variance of an equally-weighted portfolio approaches the average covariance as n gets large.*

Since diversification is costless and systematic risk is the only remaining risk in a diversified portfolio, an investor should only be compensated for systematic risk (or beta) exposure. Therefore, all assets with the same beta should earn the same return.

The next step in the derivation is to recognize that expected return is a linear function of beta. Since portfolio beta is the weighted average of the individual betas and expected portfolio return is a weighted average of the individual expected returns, the portfolio expected return is a linear function of beta.

$$E(R_P) = a + m \times \beta_P$$

where:

$\beta_P = \dfrac{Cov_{P,M}}{\sigma_M^2}$

$Cov_{P,M}$ = covariance between the returns for Stock P and the market portfolio

$\sigma_M^2$ = variance of the returns on the market portfolio

*Professor's Note: To show that portfolio return is a linear function of beta, start with the functions for expected portfolio return and portfolio beta.*

$$E(R_P) = w_1 E(R_1) + (1 - w_1) E(R_2)$$
$$\beta_P = w_1 \beta_1 + (1 - w_1) \beta_2$$

*Solve $w_1$ in the portfolio beta equation:*

$$w_1 = \frac{\beta_P - \beta_2}{\beta_1 - \beta_2}$$

*Substitute $w_1$ into the portfolio expected return equation:*

$$E(R_P) = \frac{\beta_P - \beta_2}{\beta_1 - \beta_2} E(R_1) + \left(1 - \frac{\beta_P - \beta_2}{\beta_1 - \beta_2}\right) E(R_2)$$
$$E(R_P) = \left| E(R_2) - \frac{\beta_2 [E(R_1) - E(R_2)]}{\beta_1 - \beta_2} \right| + \beta_P \frac{[E(R_1) - E(R_2)]}{\beta_1 - \beta_2}$$
$$E(R_P) = a + m\beta_P$$

*where:*

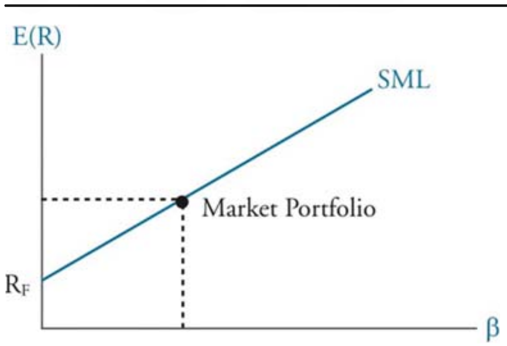
$$a = E(R_2) - \frac{\beta_2 [E(R_1) - E(R_2)]}{\beta_1 - \beta_2}$$
$$m = \frac{[E(R_1) - E(R_2)]}{\beta_1 - \beta_2}$$

Assets with equivalent betas should earn the same return because arbitrage will prevent assets with the same risk from earning different returns. So, if $\beta_i = \beta_P$ and $E(R_i) = E(R_P)$, then we can express the expected return for asset i as a linear function of its beta:

$$E(R_i) = a + m \times \beta_i$$

As shown in Figure 3, this equation plots a straight line, known as the security market line (SML) with an intercept of *a* and slope of *m*. Thus, the SML is a graphical representation of the CAPM.

**Figure 3: The Security Market Line**



The final step in this derivation is to find two points on the SML and solve for the CAPM. To solve for the equation of a line (which is known as identifying the line), we need to know two points on the line. Fortunately, we do know two of the points on this line: the risk-free asset and the market portfolio. Since it has no risk, the risk-free asset has a beta of zero; therefore, the intercept of the SML is $R_F$, and our first point is $(0, R_F)$. The market portfolio has a beta of one, so the second point is $[1, E(R_M)]$. With these two points, we can find the slope of the line, *m*:

$$E(R_i) = a + m \times \beta_i$$

$$E(R_M) = R_F + m \times 1$$

$$m = E(R_M) - R_F$$

*Professor's Note: It is relatively straightforward to see that the beta of the market is one. The covariance of the market with itself is equal to the variance of the market. Therefore, solving for market beta, we get:*

$$\beta_M = \frac{Cov_{M,M}}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

With information on both the intercept (a) and the slope (m), we are now able to display the well-known capital asset pricing model:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$



## Calculating Expected Return Using the CAPM

### LO 10.4: Apply the CAPM in calculating the expected return on an asset.

**Example: Expected return on a stock**

Assume you are assigned the task of evaluating the stock of Sky-Air, Inc. To evaluate the stock, you calculate its required return using the CAPM. The following information is available:

| | |
|---|---|
| expected market risk premium | 5% |
| risk-free rate | 4% |
| Sky-Air beta | 1.5 |

Using CAPM, **calculate** and **interpret** the expected return for Sky-Air.

**Answer:**

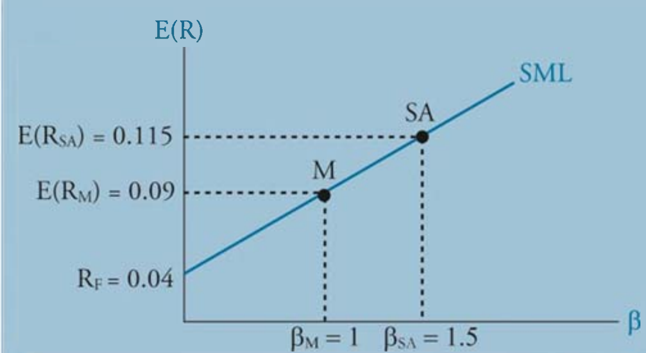
The expected return for Sky-Air is:

$$E(R_{SA}) = 0.04 + 1.5(0.05) = 0.115 = 11.5\%$$

The market risk premium is the expected market return minus the risk-free rate. The CAPM return can be viewed as the minimum return that investors should be willing to accept (i.e., the required rate of return), commensurate with the risk associated with the asset. For example, if investors predict that the return will exceed 11.5%, then they should buy Sky-Air stock. However, if investors predict that the return will be less than 11.5%, then they should sell Sky-Air stock (or short the stock).

Figure 4 illustrates the required return for Sky-Air on the SML.

**Figure 4: Sky-Air Plotted on the Security Market Line**




In the previous example, we calculated the required rate of return, which always lies on the security market line. If through the valuation of an asset an analyst determines that the expected return is different from the required rate of return implied by CAPM, then the security may be mispriced according to rational expectations. A mispriced security would not lie on the security market line. In general:

- An overvalued security would have a required rate of return (computed by CAPM) that is higher than its expected return (computed by the analyst's valuation). An overvalued security would lie below the security market line.
- An undervalued security would have a required rate of return (computed by CAPM) that is lower than its expected return (computed by the analyst's valuation). An undervalued security would lie above the security market line.

In addition to computing the required or expected return for an individual asset, it is possible to solve for the expected return on the market and/or the market risk premium given the risk-free rate, expected return on an asset, and the systematic risk for that asset.

**Example: Using CAPM to calculate the expected market return**

A stock has a beta of 0.75 and an expected return of 13%. The risk-free rate is 4%. **Calculate** the market risk premium and the expected return on the market portfolio.

**Answer:**

According to CAPM: $0.13 = 0.04 + 0.75[E(R_M) - R_F]$.

Therefore, the market risk premium is equal to: $[E(R_M) - R_F] = 0.12 = 12\%$.

The expected return on the market is calculated as: $[E(R_M) - 0.04] = 0.12$, or $E(R_M) = 0.16 = 16\%$.

# Key Concepts

## LO 10.1

There are three major steps in deriving the CAPM:

1. Recognize that since investors are only compensated for bearing systematic risk, beta is the appropriate measure of risk.

2. By knowing that portfolio expected return is a weighted average of individual expected returns and portfolio beta is a weighted average of the individual betas, we can show that portfolio return is a linear function of portfolio beta. Since arbitrage prevents mispricing of assets relative to systematic risk (beta), an individual asset's expected return is a linear function of its beta.

3. Use the risk-free asset and the market portfolio, which are two points on the security market line, to solve for the intercept and slope of the CAPM. The equation for CAPM is:

   $$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

## LO 10.2

The capital asset pricing model (CAPM), derived by Sharpe, Lintner, and Mossin, expresses the expected return for an asset as a function of the asset's level of systematic risk (measured by beta), the risk-free rate, and the market risk premium (the expected return of the market minus the risk-free rate). There are several assumptions underlying the CAPM.

- Investors face no transaction costs.
- Assets are infinitely divisible.
- There are no taxes.
- Investors are price takers whose individual buy and sell decisions have no effect on asset prices.
- Investors' utility functions are based solely on expected portfolio return and risk.
- Unlimited short-selling is allowed.
- Investors are only concerned about returns and risk over a single period, and the single period is the same for all investors.
- All investors have the same forecasts of expected returns, variances, and covariances.
- All assets are marketable.

## LO 10.3

The capital market line (CML) expresses the expected return of a portfolio as a linear function of its standard deviation, the market portfolio's return and standard deviation, and the risk-free rate.

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_C$$

## LO 10.4

The expected return for an asset can be computed using the CAPM given the risk-free rate, the market risk premium, and an asset's systematic risk.

## LO 10.5

Beta can be calculated using the following equation:

$$\beta_i = \frac{Cov_{i,M}}{\sigma_M^2}$$

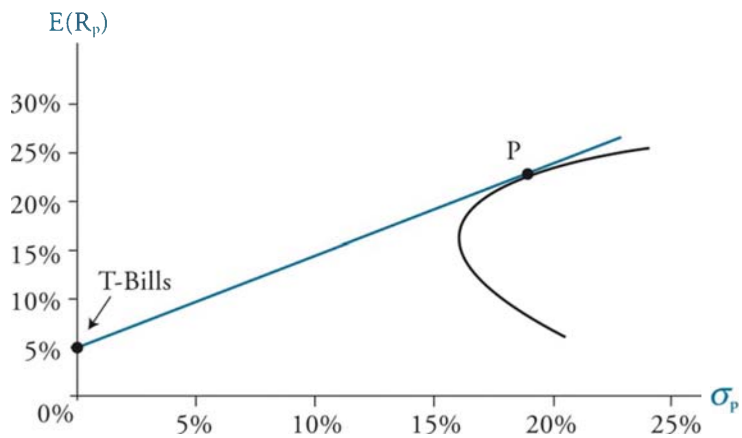
Portfolio beta is the weighted average of the asset betas in a portfolio.

## Concept Checkers

1. Which of the following statements is most likely an assumption of the capital asset pricing model (CAPM)?
   A. Investors only face capital gains taxes.
   B. Investors' actions affect the prices of assets.
   C. Transaction costs are constant across all assets.
   D. All assets including human capital are marketable.

**Use the following graph to answer Question 2.**

**Mean-Variance Analysis**




2. Portfolio P in the mean variance analysis represents the tangency point between the capital market line and the portfolio possibilities curve. In this analysis, the market price of risk would be the:
   A. standard deviation of Portfolio P.
   B. expected return on the minimum-variance portfolio.
   C. slope of the line connecting T-bills and Portfolio P.
   D. point at which the straight line intersects the expected return axis.

3. At a recent analyst meeting at Invest Forum, analysts Michelle White and Ted Jones discussed the use of the capital market line (CML). White states that the CML assumes that investors hold two portfolios: 1) a risky portfolio of all assets weighted according to their relative market value capitalizations; and 2) the risk-free asset. Jones states that the CML is useful in determining the required rate of return for individual securities.

   Are the statements of White and Jones correct?
   A. Only Jones's statement is correct.
   B. Only White's statement is correct.
   C. Both statements are correct.
   D. Neither statement is correct.

4. Patricia Franklin makes buy and sell stock recommendations using the capital asset pricing model. Franklin has derived the following information for the broad market and for the stock of the CostSave Company (CS):

- Expected market risk premium 8%
- Risk-free rate 5%
- Historical beta for CostSave 1.50

Franklin believes that historical betas do not provide good forecasts of future beta, and therefore uses the following formula to forecast beta:

forecasted beta = 0.80 + 0.20 × historical beta

After conducting a thorough examination of market trends and the CS financial statements, Franklin predicts that the CS return will equal 10%. Franklin should derive the following required return for CS along with the following valuation decision (undervalued or overvalued):

| | Valuation | CAPM required return |
|---|---|---|
| A. | overvalued | 8.3% |
| B. | overvalued | 13.8% |
| C. | undervalued | 8.3% |
| D. | undervalued | 13.8% |

5. Albert Dreiden wants to estimate the expected return on the market. He believes that the stock of the Hobart Materials Company is fairly valued, and gathers the following information:

- Expected return for Hobart 7.50%
- Risk-free rate 4.50%
- Beta for Hobart 0.80

Based on this information, the estimated expected return for the market portfolio is closest to:

A. 3.00%.
B. 3.75%.
C. 6.90%.
D. 8.25%.

## Concept Checker Answers

1. **D** The capital asset pricing model (CAPM) assumes that all assets including human capital are marketable. Additionally, CAPM assumes no taxes, no transaction costs, and that investor actions do not affect market prices.

2. **C** The CML is the line connecting T-bills and Portfolio P. The market price of risk is the slope of the CML. Had risk been measured on the graph with beta, the graph would represent the SML. The market price of risk would still be the slope of the line.

3. **B** The CML assumes all investors have identical expectations and all use mean-variance analysis, implying that they all identify the same risky tangency portfolio (the "market portfolio") and combine that risky portfolio with the risk-free asset when creating their portfolios. Because all investors hold the same risky portfolio, the weight on each asset must be equal to the proportion of its market value to the market value of the entire portfolio. Therefore, White is correct. The CML is useful for determining the rate of return for efficient portfolios, but it cannot be used to determine the required rate of return for inefficient portfolios or individual securities. The capital asset pricing model (CAPM) is used to determine the required rate of return for inefficient portfolios and individual securities. Therefore, Jones is incorrect.

4. **B** The CAPM equation is:

   $E(R_i) = R_F + \beta_i[E(R_M - R_F)]$

   Franklin forecasts the beta for CostSave as follows:

   beta forecast = 0.80 + 0.20 (historical beta)

   beta forecast = 0.80 + 0.20(1.50) = 1.10

   The CAPM required return for CostSave is:

   0.05 + 1.1(0.08) = 13.8%

   Note that the market premium, $E(R_M) - R_F$ , is provided in the question (8%).

   Franklin should decide that the stock is overvalued because she forecasts that the CostSave return will equal only 10%, whereas the required return (minimum acceptable return) is 13.8%.

5. **D** The CAPM equation is:

   $E(R_i) = R_F + \beta_i[E(R_M - R_F)]$

   Using the given information, we can solve for the expected return for the market portfolio as follows:

   $7.50\% = 4.50\% + 0.80[E(R_M) - 4.50\%]$

   $E(R_M) = (7.50\% - 4.50\%) / 0.80 + 4.50\% = 8.25\%$

   Based on the information given and using the CAPM, the expected return on the market is 8.25%.

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# Applying the CAPM to Performance Measurement: Single-Index Performance Measurement Indicators

Topic 11

## Exam Focus

This topic further expands on the concepts of the capital market line and the security market line by examining measures used to assess portfolio performance on a risk-adjusted basis. In the previous topic, we mentioned that the risk-to-reward ratio for the capital market line (i.e., its slope) is known as the Sharpe ratio. In addition, we discussed how to assess a portfolio's alpha return when comparing actual performance to expected performance based on the CAPM. The formal expression for this calculation is known as Jensen's alpha. The Treynor measure is another popular performance metric, and is similar to the Sharpe ratio but uses beta as the risk measure instead of standard deviation. Toward the end of this topic, we examine additional risk-return assessment measures such as the information ratio and the Sortino ratio. In general, all of the performance measures introduced evaluate excess return over some form of risk. For the exam, memorize these measures of performance since they are popular concepts to test.

## Measures of Performance

### LO 11.1: Calculate, compare, and evaluate the Treynor measure, the Sharpe measure, and Jensen's alpha.

Modern portfolio theory and the CAPM are built upon the link between risk and return. Three measures exist to assess an asset's or portfolio's return with respect to its risk.

- The **Treynor measure** is equal to the risk premium divided by beta, or systematic risk:

$$\text{Treynor measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

- The **Sharpe measure** is equal to the risk premium divided by the standard deviation, or total risk:

$$\text{Sharpe measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

- The **Jensen measure** (or Jensen's alpha or just **alpha**), is the asset's excess return over the return predicted by the CAPM:

$$\text{Jensen measure of a portfolio} = \alpha_P = E(R_P) - [R_F + [E(R_M) - R_F]\beta_P]$$

In all three cases, for a given portfolio, the higher, the better. The two that are most similar are the Treynor and Sharpe measures. They both normalize the risk premium by dividing by a measure of risk. Investors can apply the Sharpe measure to all portfolios because it uses total risk, and it is more widely used than the other two measures. The Treynor measure is more appropriate for comparing well-diversified portfolios. Jensen's alpha is the most appropriate for comparing portfolios that have the same beta.

Some consider the Sharpe measure a better method for measuring historical performance. Since betas must be estimated and the portfolio beta is the weighted average of the betas of assets in a portfolio, the Treynor measure is considered a more forward-looking measure.

In addition to these comparisons, it is useful to realize that some relationships exist between the measures. For instance:

$$\text{Treynor measure} = \frac{\alpha_P}{\beta_P} + [E(R_M) - R_F]$$

For a well-diversified portfolio we can use the following approximation: $\beta_P \approx \frac{\sigma_P}{\sigma_M}$.

Substituting this into the expression for Jensen's alpha and applying some algebra gives us:

$$\text{Sharpe measure} \approx \frac{\alpha_P}{\sigma_P} + \frac{E(R_M) - R_F}{\sigma_M}$$

Applying the approximation $\beta_P \approx \frac{\sigma_P}{\sigma_M}$ again gives us:

$$\text{Sharpe measure} \approx \left[\frac{\text{Treynor measure}}{\sigma_M}\right]$$

*Professor's Note: Do not focus too much attention on these approximations. The key is understanding how to calculate the three measures of performance as is shown in the following example.*

**Example: Calculating performance measures**

For a portfolio of ten stocks, we may find, via fundamental analysis estimates of the individual stocks, that the portfolio's expected return is 14% with a standard deviation of 25%. The beta of the portfolio is 1.1. The expected return of the market is 12.5% with a standard deviation of 20.2%. The risk-free rate is 2.6%. **Calculate** the Treynor, Sharpe, and Jensen measures.

**Answer:**

$$\text{Treynor measure} = \left[\frac{E(R_P) - R_F}{\beta_P}\right] = \left[\frac{0.14 - 0.026}{1.1}\right] = 0.1036$$

$$\text{Sharpe measure} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right] = \left[\frac{0.14 - 0.026}{0.25}\right] = 0.456$$

$$\text{Jensen measure} = \alpha_P = E(R_P) - \left[R_F + [E(R_M) - R_F]\beta_P\right]$$

$$= 0.14 - [0.026 + (0.125 - 0.026)(1.1)] = 0.0051$$

We can compare these to the corresponding measures of the market portfolio:

$$\text{Treynor measure of the market} = \left[\frac{0.125 - 0.026}{1}\right] = 0.099$$

$$\text{Sharpe measure of the market} = \left[\frac{0.125 - 0.026}{0.202}\right] = 0.4901$$

$$\text{Jensen measure of a portfolio} = 0.125 - [0.026 + (0.125 - 0.026)(1)] = 0.0$$

Based upon the Treynor measure and the Jensen measure of the preceding example, the portfolio of ten stocks is superior to the market. However, the relationship is reversed using the Sharpe measure. This implies that the manager has selected ten stocks that offer superior returns relative to their systematic risk; however, a 10-stock portfolio is much less diversified than the market. The standard deviation for the 10-stock portfolio ($\sigma_P$ = 25%), when compared to $\sigma_M$ = 20.2%, reflects the lower level of diversification.

## Extensions to Jensen's Alpha

There are several ways to modify or extend the Jensen measure. Since Jensen's measure is simply a raw return in excess of some reference (i.e., that implied by the CAPM in the case of the standard Jensen's measure) we can simply replace that reference with a value that we feel is more appropriate. One reference would be the required return based on the CML. The manager has created a portfolio with risk $\sigma_P$, which then has a reference return equal to $E(R_{reference})$ as given by the equation:

$$E(R_{reference}) = R_F + [E(R_M) - R_F]\left[\frac{\sigma_P}{\sigma_M}\right]$$

The alpha in this case would be the portfolio's return minus the reference return:

$$\text{alpha} = E(R_P) - E(R_{\text{reference}})$$

Other extensions of Jensen's measure would use a measure of $E(R_{\text{reference}})$ derived from a multifactor model (i.e., more than one independent variable). Another value of $E(R_{\text{reference}})$ could be derived from a variation of the CAPM called the Black model, which uses the return on a "zero-beta" portfolio in place of the risk-free rate. In all cases, the idea is the same: measure the raw return difference of the managed portfolio against the required return given its level of risk.

## LO 11.2: Compute and interpret tracking error, the information ratio, and the Sortino ratio.

If a manager is trying to earn a return higher than the market portfolio or any other reference or benchmark, the difference will have some variability over time. In other words, even if the manager is successful in generating a positive alpha, the alpha will vary over time. **Tracking error** is the term used to describe the standard deviation of the difference between the portfolio return and the benchmark return. This source of variability is another source of risk to use in assessing the manager's success. Typically, the manager must keep the tracking error below a stated threshold. The manager must weigh transactions and other costs in managing the portfolio to reduce tracking error against the extra risk it introduces into the management process.

The **information ratio** is essentially the alpha of the managed portfolio relative to its benchmark divided by the tracking error. If we let $R_B$ denote the return of the benchmark we can write:

$$e_P = R_P - R_B$$

$$\text{tracking error} = \sigma_{e_P}$$

$$\text{information ratio} = \left[\frac{E(R_P) - E(R_B)}{\sigma_{e_P}}\right] = \frac{\alpha_P}{\sigma_{e_P}}$$

This is a measure used to assess if the manager's deviation from the benchmark has reaped an appropriate return. It is called an "information ratio" because it is essentially a measure of how well the manager has acquired and used information compared to the average manager.

**Example: Calculating the information ratio**

A manager typically generates an alpha of 1.5% with a tracking error of 2.25%. **Calculate** the information ratio.

**Answer:**

$$\text{information ratio} = \left[\frac{1.5}{2.25}\right] = 0.667$$

The **Sortino ratio** is similar to the Sharpe ratio except for two changes. We replace the risk-free rate with a minimum acceptable return, denoted $R_{min}$, and we replace the standard deviation with a type of semi-standard deviation. A semi-standard deviation measures the variability of only those returns that fall below the minimum acceptable return. The measure of risk in the Sortino ratio is the square root of the mean squared deviation from $R_{min}$ of those observations in time periods *t* where $R_{Pt} < R_{min}$, else zero. Letting $R_{min}$ denote the minimal acceptable return and $MSD_{min}$ the risk measure:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\sqrt{MSD_{min}}}$$

where:

$$MSD_{min} = \frac{\sum_{R_{Pt}<R_{min}} (R_{Pt} - R_{min})^2}{N}$$

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

**Example: Calculating the information ratio and the Sortino ratio**

Over a 10-year period, a manager uses a covered call strategy to enhance the return of the index fund she manages. The record of the fund's returns is (0.095, 0.08, –0.022, 0.11, 0.09, –0.05, –0.035, 0.124, 0.072, 0.055). The corresponding benchmark returns record is (0.087, 0.078, –0.034, 0.124, 0.10, –0.064, –0.042, 0.131, 0.062, 0.059). The minimum acceptable return is 4%. **Calculate** the information ratio and the Sortino ratio. Assume tracking error = 0.00992 and mean squared deviation (min) = 0.0017569.

**Answer:**

If we were to compute the tracking error, the first step would be to compute the differences between the portfolio and the benchmark. Those differences are: (0.008, 0.002, 0.012, –0.014, –0.01, 0.014, 0.007, –0.007, 0.01, –0.004). The tracking error is the standard deviation of these numbers.

*Professor's Note: In Book 2, we will examine the formula for standard deviation that is used for the tracking error calculation. It is based on the sum of the squared differences between each data point and the mean. This sum is divided by the number of observations adjusted for degrees of freedom (in this case n – 1). The square root of the computed value will be the standard deviation.*

To compute the information ratio, divide the mean of the differences by the tracking error.

information ratio = 0.0018 / 0.00992 = 0.1815

The Sortino ratio is the mean of the ten portfolio returns minus 4%, which is 0.0519 – 0.04 = 0.0119, divided by the square root of $MSD_{min}$.

$MSD_{min}$ = 0.0017569

Sortino ratio = 0.0119 / 0.0419 = 0.2840

## Key Concepts

### LO 11.1

Three commonly used risk/return measures are:

- Treynor measure of a portfolio = $\left[\dfrac{E(R_P) - R_F}{\beta_P}\right]$
- Sharpe measure of a portfolio = $\left[\dfrac{E(R_P) - R_F}{\sigma_P}\right]$
- Jensen measure of a portfolio = $\alpha_P = E(R_P) - [R_F + [E(R_M) - R_F]\beta_P]$

These three risk measures give different perspectives and may give different rankings for portfolios. A portfolio with low diversification may have a higher Treynor measure, a higher alpha, but a lower Sharpe measure than another portfolio.

Alpha can be modified by the use of other reference portfolios.

### LO 11.2

Tracking error and the information ratio build upon Jensen's alpha. Tracking error is the standard deviation of alpha over time. The information ratio is the average alpha over time divided by the tracking error.

The Sortino ratio should be used when there is more focus on the likelihood of loss:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{min}}{\sqrt{MSD_{min}}}$$

The $MSD_{min}$ is a semi-variance that only measures the variability of the portfolio's return observations below $R_{min}$.

## Concept Checkers

1. For a given portfolio, having a Treynor measure greater than the market but a Sharpe measure that is less than the market would most likely indicate that the portfolio is:
   A. not well diversified.
   B. generating a negative alpha.
   C. borrowing at the risk-free rate.
   D. not borrowing at the risk-free rate.

2. With respect to performance measures, the use of the standard deviation of portfolio returns is a distinguishing feature of the:
   A. beta measure.
   B. Jensen measure.
   C. Sharpe measure.
   D. Treynor measure.

3. For a given portfolio, the expected return is 9% with a standard deviation of 16%. The beta of the portfolio is 0.8. The expected return of the market is 12% with a standard deviation of 20%. The risk-free rate is 3%. The portfolio's alpha is:
   A. –1.2%.
   B. –0.6%.
   C. +0.6%.
   D. +1.2%.

4. You are given the following information:

| | |
|---|---|
| Risk-free rate | 4% |
| Minimum acceptable return | 6% |
| Benchmark return | 10% |
| Expected return on portfolio | 12% |
| Expected return on market | 9% |
| Beta | 1.25 |
| Standard deviation (portfolio) | 7.3% |
| Semi-standard deviation (portfolio) | 8.2% |

   The Sortino ratio of the portfolio is closest to:
   A. 0.24.
   B. 0.73.
   C. 0.82.
   D. 0.98.

5. An analyst has compiled the following data on Stock P:

| | |
|---|---|
| $\text{Covariance}_{\text{P, market}}$ | 0.0315 |
| $\sigma_{\text{Stock P}}$ | 16.50% |
| $\sigma_{\text{market}}$ | 15.00% |
| Expected market return | 11.80% |
| Risk-free rate | 4.50% |
| Stock P actual return | 13.25% |

Calculate and interpret Jensen's Alpha for Stock P.

A. +1.47% overperformed.
B. –1.47% underperformed.
C. +1.45% overperformed.
D. –1.45% underperformed.

## Concept Checker Answers

1. **A** Low diversification can produce this result because it will likely increase the standard deviation of the portfolio's returns, thus decreasing its Sharpe ratio. Using margin is not directly related to the risk-adjusted performance because adjusting for risk removes the effect of leverage. A Treynor ratio greater than the market Treynor ratio would result in a positive alpha (not a negative alpha).

2. **C** The Sharpe measure is the portfolio return minus the risk-free rate divided by the standard deviation of the return. The Treynor and Jensen measures use beta. The answer "beta measure" is a nonsensical choice for this question.

3. **A** The alpha is 9% – [3% + 0.8 × (12% – 3%)] = –1.2%.

4. **B** (portfolio return – minimum acceptable return) / semi-standard deviation

   (0.12 – 0.06) / 0.082 = 0.7317

   Choice A is incorrect because it uses the benchmark return in the numerator instead of the minimum acceptable return.

   Choice C is incorrect because it uses the standard deviation in the denominator instead of the semi-standard deviation.

   Choice D is incorrect because it uses the risk-free rate in the numerator instead of the minimum acceptable return.

5. **B** Jensen's Alpha = actual return – CAPM expected return

   CAPM: $E(R) = R_F + \beta(R_M - R_F)$

   $$\beta = \frac{\text{covariance}_{P,\ \text{market}}}{\text{variance}_{\text{market}}}$$

   *Step 1:* Calculate $\beta$

   $\beta = 0.0315 / 0.15^2$ $\qquad$ $\beta = 1.4$

   *Step 2:* Calculate the CAPM expected return

   $E(R) = 4.5 + 1.4(11.80 - 4.5) = 14.72\%$

   *Step 3:* Calculate Jensen's Alpha

   Jensen's Alpha = actual return – CAPM E(R) = 13.25% – 14.72% = –1.47%

   Stock P has underperformed the market by 1.47% when taking into account its level of systematic risk as measured by beta.

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Topic 12

## Exam Focus

The relationship between risk and return is one of the most important concepts in finance. The capital asset pricing model (CAPM) asserts that the expected return on any asset is solely determined by its exposure to the market portfolio. The risk exposure in the CAPM is known as beta. In contrast, the arbitrage pricing theory (APT) asserts that expected returns are determined by exposures to economy-wide risk factors. The risk exposures in the APT are known as factor betas. For the exam, be able to calculate expected returns using single-factor and multifactor models. In addition, know how to construct the security market line (SML) for a well-diversified portfolio using a single-factor model. Also, be able to explain the APT, and know how to construct a portfolio to hedge exposure to multiple risk factors.

## The Multifactor Model of Risk and Return

### LO 12.1: Describe the inputs, including factor betas, to a multifactor model.

The inputs to a **multifactor model**, for any stock, are as follows:

- Expected return for the stock.
- Factor betas, also known as factor sensitivities or factor loadings.
- Deviation of macroeconomic factors from their expected values.
- Firm-specific return.

The equation for a multifactor model for stock *i* can be expressed as follows:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$$

where:

$R_i$ = return on stock *i*
$E(R_i)$ = expected return for stock *i*
$\beta_{ij}$ = $j^{th}$ factor beta for stock *i*
$F_j$ = deviation of macroeconomic factor *j* from its expected value
$e_i$ = firm-specific return for stock *i*

Regarding macroeconomic factors, assume that one of the macro factors is gross domestic product (GDP). In this case, $F_{GDP}$ will represent the deviation of GDP from its expected value. If we assume that the consensus forecast for GDP equals 3%, and GDP for the period ended up being 4%, $F_{GDP}$ would equal 0.01 (0.04 – 0.03 = 0.01).

The **factor beta**, $\beta_{ij}$, equals the sensitivity of the stock return to a 1-unit change in the factor. For example, for stock *i*, assume $\beta_{i,GDP} = 2$. Therefore, for every one percentage point change in GDP, this stock's return changes, on average, by two percentage points.

The **firm-specific return**, $e_i$, is the portion of the stock's return that is unexplained by macro factors (i.e., the F terms in the equation). The firm-specific return will be a nonzero value whenever unexpected firm-specific events take place (e.g., a strike that impacts a single firm). However, the expected value of the firm-specific return equals zero, because, by definition, firm-specific events are random.

*Professor's Note: For the moment, we will assume that the expected return, $E(R_i)$, is known. Later, we will use the arbitrage pricing theory to derive the expected return. In that case, the factor betas will be estimated as the slope coefficients in a multiple linear regression.*

## LO 12.2: Calculate the expected return of an asset using a single-factor and a multifactor model.

The factor model just described can be used to revise the estimate of a stock's expected rate of return. The number of factors to include in a factor model should be as small as possible, yet still capture the priced sources of nondiversifiable (or *systematic*) risk. The simplest version of the model consists of just one macro factor: the **single-factor model**. We will consider examples of a single-factor model first, and, then, will consider a 2-factor model.

For the first example, assume the common stock of HealthCare Inc. (HCI) is examined with a single-factor model, using unexpected percent changes in GDP as the single factor. Assume the following data is provided:

| | |
|---|---|
| Expected return for HCI | = 10% |
| GDP factor beta | = 2.00 |
| Expected GDP growth | = 3% |

Given this data, we can see that the stock return for HCI is strongly impacted by GDP. On average, the stock price changes by two percentage points for every one percentage point change in GDP.

Suppose new macroeconomic information indicates that GDP growth will equal 4% rather than the original consensus forecast of 3%. Also assume there's no new information regarding firm-specific events. The revised expected return for HCI using a single-factor model can be calculated as follows:

$$R_{HCI} = E(R_{HCI}) + \beta_{HCI,GDP}F_{GDP} + e_{HCI}$$

$$R_{HCI} = 0.10 + 2(0.01) = 0.12 = 12\%$$

Therefore, based on the single-factor model, the analyst should revise the expected return for HCI from 10% to 12%, because GDP was revised above its original expected value. The additional two percentage points resulted from the one percentage point deviation of GDP from its expected value, combined with the GDP factor beta of 2: $2 \times 0.01 = 0.02 = 2\%$.

For the second example, assume the common stock of HealthCare Inc. (HCI) is examined using a multifactor model, based on two factors: unexpected percent changes in GDP and unexpected percent changes in consumer sentiment. Assume the following data is provided:

| | |
|---|---|
| Expected return for HCI | = 10% |
| GDP factor beta | = 2.00 |
| Consumer sentiment (CS) factor beta | = 1.50 |
| Expected growth in GDP | = 3% |
| Expected growth in consumer sentiment | = 1% |

Suppose new macroeconomic information indicates that GDP will grow 4% rather than 3%, and that consumer sentiment will grow 3% rather than 1%. A 2-factor model to calculate the return for HCI can be expressed as follows:

$$R_{HCI} = E(R_{HCI}) + \beta_{HCI,GDP}F_{GDP} + \beta_{HCI,CS}F_{CS} + e_{HCI}$$

$$R_{HCI} = 0.10 + 2(0.01) + 1.5(0.02) = 0.15 = 15\%$$

Therefore, based on this multifactor model, the revised estimate of the expected return for HCI equals 15%, which is five percentage points higher than initially expected. The sources of the revised return are the macroeconomic revisions to GDP and consumer sentiment.

## The Law of One Price and Arbitrage Opportunities

According to the **Law of One Price**, identical assets selling in different locations should be priced identically in the different locations. For example, assume the common stock of GH Inc. is listed on both the NYSE and NASDAQ. According to the Law of One Price, there should be no difference in the stock prices of GH's transactions on the NYSE versus NASDAQ. If there were different prices available, an arbitrage opportunity would exist. For example, assume the stock price for GH on the NYSE is $60, and assume the stock price for GH on NASDAQ is $58. In this case, investors could buy GH's stock on NASDAQ for $58 and simultaneously sell the stock for $60 on the NYSE. These actions would continue until the stock prices are identical on the two exchanges.

The action of buying an asset in the cheaper market and simultaneously selling that asset in the more expensive market is called **arbitrage**. The actions of arbitrageurs cause prices to rise in the cheaper market and fall in the expensive market. The simultaneous trades will continue until the asset trades at one price in both markets, at which point the arbitrage opportunity will be fully exploited.

The Law of One Price can be extended to different assets that have identical risks. The following example illustrates whether an arbitrage situation exists using a multifactor model. Assume a 2-factor model is used to examine the returns for two assets, A and B. Also, assume the two factors are unexpected percentage changes in GDP and consumer sentiment (CS).

$$R_A = E(R_A) + \beta_{A,GDP}F_{GDP} + \beta_{A,CS}F_{CS} + e_A$$

$$R_B = E(R_B) + \beta_{B,GDP}F_{GDP} + \beta_{B,CS}F_{CS} + e_B$$

Additional data:

$E(R_A) = 0.12$
$E(R_B) = 0.10$
$\beta_{A,GDP} = \beta_{B,GDP} = 2$
$\beta_{A,CS} = \beta_{B,CS} = 1.5$

Notice that the factor betas are identical for Asset A and Asset B. Therefore, the systematic risks are identical between these assets. If the systematic risks of two assets are identical, then the expected returns for the two assets should also be identical.

In this example, an arbitrage opportunity exists because the assets are priced based on different expected returns even though the systematic risks of the two assets are identical. An investor can exploit the arbitrage opportunity by shorting Asset B and using the proceeds from the short position to take a long position in Asset A. The expected return and factor betas for the long-short hedge portfolio, H, are calculated as follows:

$E(R_H) = E(R_A) - E(R_B) = 0.12 - 0.10 = 0.02$
$\beta_{H,GDP} = \beta_{A,GDP} - \beta_{B,GDP} = 0$
$\beta_{H,CS} = \beta_{A,CS} - \beta_{B,CS} = 0$

Therefore, the net investment in Portfolio H is zero (invest $1 in Asset A for every $1 sold short in Asset B), the net position for Portfolio H is riskless (i.e., factor betas are zero), but the expected profit from Portfolio H is positive (equal to 2%). These conditions constitute an arbitrage situation.

## Well-Diversified Portfolios

**LO 12.3: Describe properties of well-diversified portfolios and explain the impact of diversification on the residual risk of a portfolio.**

The part of an individual security's risk that is uncorrelated with the volatility of the market portfolio is that security's **nonsystematic risk** (or *diversifiable risk*). The part of an individual security's risk that arises because of the positive covariance of that security's returns with overall market returns is called its **systematic risk**. As the number of securities in a portfolio becomes large, the portfolio's nonsystematic risk approaches zero. In other words, portfolio risk reduction through diversification comes from reducing nonsystematic risk. Therefore, when a risky security is added to a well-diversified (efficient) portfolio, the portfolio's risk is only affected by the systematic risk of that security.

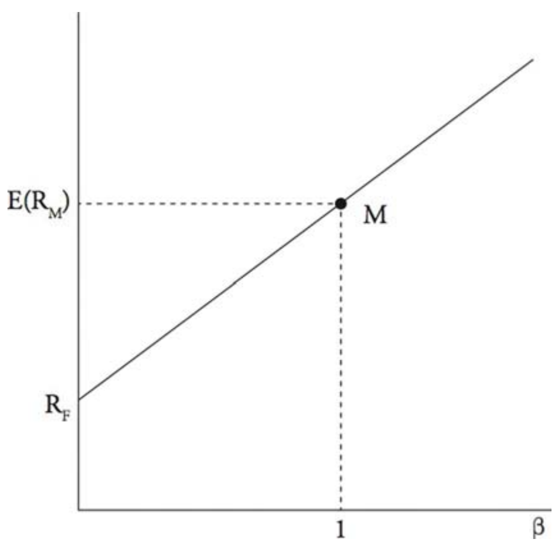
The standardized measure of systematic risk is **beta**, standardized into units of market risk, which is the risk of holding the market portfolio (often represented by the S&P 500 Index). Recall from Topic 10 that beta is calculated as the covariance of the returns on security or portfolio *i* with the returns on the market portfolio, divided by the variance of the returns on the market portfolio.

Because nonsystematic (diversifiable) risk can be avoided (without cost) by efficient diversification, there is no added expected return for bearing nonsystematic risk. This result is the crucial point of the **capital asset pricing model** (CAPM) and is expressed in the equation of the **security market line** (SML). As we saw in previous topics, this relation says that, for each unit of market risk (beta), investors can expect to receive a premium over the risk-free rate equal to the market risk premium.

## The Single-Factor Security Market Line

Assume we create a well-diversified, observable, market index portfolio, M. Index M can be any well-diversified portfolio thought to be highly correlated with the systematic factor that affects the returns of assets. Also, assume that we use a single-factor model in which the factor is the unexpected return on the market index portfolio. In this case, the beta of the market index is equal to one. Portfolio M must lie on the line illustrated in Figure 1, which is known as the **single-factor security market line** (SML).

**Figure 1: The Single-Factor Security Market Line**




The implication of the single-factor security market line is that, when no arbitrage opportunities exist, all well-diversified portfolios must lie on the line.

To derive the equation for the single-factor SML, note that the intercept of the line is the risk-free rate, $R_F$, and the slope of the line is the market index risk premium, $E(R_M) - R_F$.

The equation for the line (for any well-diversified Portfolio P) is:

$$E(R_P) = R_F + \beta_P[E(R_M) - R_F]$$

where M is an observable, well-diversified, market index, and $\beta_P$ is the beta of any portfolio, P, relative to the market index, M.

The equation is analogous to the CAPM. The key difference is that the CAPM relies on the existence of the mean-variance efficient market portfolio (i.e., an unobservable portfolio that lies on the efficient frontier, and consists of all marketable assets). In contrast, the equation for the single-factor security market line merely relies on the assumptions that security returns can be explained by a single-factor model, that well-diversified portfolios can be created, and that no arbitrage opportunities exist.

## Hedging Exposures to Multiple Factors

### LO 12.4: Explain how to construct a portfolio to hedge exposure to multiple factors.

Consider an investor who manages a portfolio with the following factor betas:

| | |
|---|---|
| GDP beta | = 0.50 |
| Consumer sentiment beta | = 0.30 |

Assume the investor wishes to pursue strategies to hedge exposure to GDP risk, or to consumer sentiment risk, or to both factor risks. The following explanation makes use of what are called **factor portfolios**, which are well-diversified portfolios with betas equal to one for a single risk factor and betas equal to zero on all remaining factors. For example, the GDP factor portfolio is a well-diversified portfolio designed to have a GDP beta of one and a consumer sentiment beta of zero. And, the consumer sentiment factor portfolio is a well-diversified portfolio designed to have a consumer sentiment beta of one and a GDP beta of zero.

Now, assume the investor wishes to hedge away GDP factor risk, yet maintain the 0.30 exposure to consumer sentiment. To do so, the investor should combine the original portfolio with a 50% short position in the GDP factor portfolio. The GDP factor beta on the 50% short position in the GDP factor portfolio equals –0.50, which perfectly offsets the 0.50 GDP factor beta on the original portfolio. The combined long and short positions hedge away GDP risk but retain the consumer sentiment exposure.

Alternatively, the investor might want to hedge away consumer sentiment (CS) factor risk, yet maintain the 0.50 exposure to GDP. To do so, the investor should combine the original portfolio with a 30% short position in the CS factor portfolio. The CS factor beta on the 30% short position in the CS factor portfolio equals –0.30, which perfectly offsets the 0.30 CS factor beta on the original portfolio. The combined long and short positions hedge away CS risk but retain the GDP exposure.

Finally, what if the investor wants to hedge away both factor risks? To do so, first, the investor should form a portfolio invested 50% in the GDP factor portfolio and 30% in the CS factor portfolio (and the remaining 20% in the risk-free asset). This portfolio can be referred to as Portfolio H, which can be used to hedge away the factor risks of the original portfolio. To hedge away the factor risks of the original portfolio, the investor can combine the original portfolio with a short position in the hedge portfolio, H. That way, the original portfolio factor betas (0.50 and 0.30, respectively) are perfectly offset by the short position in the hedge portfolio.

The hedging process can also be used to exploit an arbitrage opportunity. Assume that the original portfolio expected return is 12% and that the hedge portfolio return equals 10%. In this case, the investor can take a long position in the original portfolio combined with a short position in the hedge portfolio, resulting in a 2% arbitrage profit. Conversely, if the hedge portfolio return instead equals 14%, the investor can take a long position in the hedge portfolio combined with a short position in the original portfolio, resulting in a 2% arbitrage profit.

## The Arbitrage Pricing Theory

### LO 12.5: Describe and apply the Fama-French three factor model in estimating asset returns.

The arbitrage pricing theory (APT) describes expected returns as a linear function of exposures to common (i.e., macroeconomic) risk factors:

$$E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \ldots + \beta_{ik}RP_k$$

where $RP_j$ is the risk premium associated with risk factor *j*. The risk premiums are derived as follows:

Step 1: Create factor portfolios. Each factor portfolio is a well-diversified portfolio (e.g., nonsystematic risk is zero) that has a beta equal to one for a single risk factor, and betas equal to zero on the remaining factors. Repeat the process for all *k* factors in the multifactor model so that a factor portfolio is derived for each of the risk factors.

Step 2: Derive returns for each factor portfolio. For instance, define $E(R_1)$ as the expected return on Factor Portfolio 1, $E(R_2)$ as the expected return on Factor Portfolio 2, and so on.

Step 3: Calculate risk premiums for each factor portfolio. For example, the risk premium for Factor Portfolio 1 equals $E(R_1) - R_F$, the risk premium for Factor Portfolio 2 equals $E(R_2) - R_F$, and so on.

Given the derivation of the risk premiums, the APT model can be rewritten as:

$$E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + \ldots + \beta_{ik}[E(R_k) - R_F]$$

The assumptions underlying the APT model are as follows:

- Returns follow a *k*-factor process: $R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$.
- Well-diversified portfolios can be formed.
- No arbitrage opportunities exist.

*Professor's Note: The main conclusion of the APT is that if these three assumptions hold, then expected returns on well-diversified portfolios are proportional to their factor betas. However, we cannot conclude that the APT relationship will hold for ALL securities. For example, if the APT relation is violated for, say, one security in the portfolio, its effect will be too small to produce meaningful arbitrage opportunities for the portfolio. Therefore, we can conclude that the APT relation can hold for well-diversified portfolios even if it does not hold for all securities in the portfolio. But, the APT relation must hold for nearly all securities in a well-diversified portfolio, or else arbitrage opportunities will become available for the portfolio. Therefore, we can conclude that the APT relation must hold for NEARLY all securities.*

**Example: Compute expected return using the APT model**

Assume the following data for Asset Z:

| | |
|---|---|
| Risk-free rate | = 5% |
| GDP factor beta | = 0.50 |
| Consumer sentiment factor beta | = 0.30 |
| GDP risk premium | = 4% |
| Consumer sentiment risk premium | = 3% |

**Calculate** the expected return for Asset Z using a 2-factor APT model.

**Answer:**

$$E(R_Z) = 0.05 + 0.5(0.04) + 0.3(0.03) = 0.079 = 7.9\%$$

Both the arbitrage pricing theory model and the capital asset pricing model describe equilibrium expected returns for assets. In fact, the CAPM can be considered a special restrictive case of the APT in which there is only one risk factor (the market risk factor).

## The Fama-French Three-Factor Model

A major weakness of the APT is that it offers no guidance as to the identification of the appropriate risk factors. More recently, a competing model has been developed by Professors Eugene Fama and Kenneth French. In contrast to the APT model, the Fama-French model identifies the factors. In addition to the market return factor ($R_M - R_F$), the Fama-French three-factor model specifies the following two factors:

- SMB (small minus big) is the firm size factor equal to the difference in returns between portfolios of small and big firms ($R_S - R_B$).
- HML (high minus low) is the book-to-market (i.e., book value per share divided by stock price) factor equal to the difference in returns between portfolios of high and low book-to-market firms ($R_H - R_L$).

Notice that SMB is a hedge strategy long small firms and short big firms. Likewise, HML is a hedge strategy long high book-to-market firms and short low book-to-market firms.

The equation for the Fama-French three-factor model is:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

The intercept term (i.e., alpha) equals the abnormal performance of the asset after controlling for its exposures to the market, firm size, and book-to-market factors. In equilibrium, the intercept should equal zero, assuming the Fama-French three factors adequately capture all systematic risks.

The SMB and HML factors are chosen because history shows that returns are higher on small versus big firms and on high versus low book-to-market firms. Fama and French argue that these differences exist because small firms are inherently riskier than big firms, and that high book-to-market firms are inherently riskier than low book-to-market firms.

# KEY CONCEPTS

## LO 12.1

The inputs to a multifactor model, for any stock, are:

- Expected return for the stock.
- Factor betas.
- Deviation of macroeconomic factors from their expected values.
- Firm-specific return.

The factor beta equals the sensitivity of the stock return to a 1-unit change in the factor. The firm-specific return is that portion of the stock's return that is unexplained by the macro factors. The expected value of the firm-specific return equals zero, because, by definition, firm-specific events are random.

## LO 12.2

The equation for a *k*-factor model is: $R = E(R) + \beta_1F_1 + \beta_2F_2 + \ldots + \beta_kF_k + e$. The return equals its expected value if none of the macro factors deviate from their expected values and if the firm-specific return equals zero. If macro factor $F_j$ deviates from its expected value, then $F_j$ is nonzero. If the firm experiences a nonfactor related surprise, then the firm-specific component, e, will be nonzero. The multifactor model can be used to calculate the expected return after new macroeconomic and/or firm-specific information is released.

## LO 12.3

Risk reduction benefits achieved through diversification come from reducing nonsystematic risk. Therefore, the expected return on a well-diversified portfolio is determined by systematic risk as measured by beta.

A single-factor security market line (SML) is analogous to the capital asset pricing model (CAPM). In the single-factor SML, systematic risk is measured as the exposure of the asset to a well-diversified market index portfolio. The index portfolio can be any well-diversified portfolio thought to be highly correlated with the systematic factor that affects the returns of assets. The equation for the single-factor SML for any well-diversified portfolio is: $E(R_P) = R_F + \beta_P[E(R_M) - R_F]$ , where $R_F$ is the risk-free rate, M is an observable well-diversified market index, and $\beta_P$ is the beta of any portfolio, P, relative to the market index.

## LO 12.4

A multifactor model can be used to hedge away multiple factor risks. To do so, the investor can create factor portfolios, which are well-diversified portfolios with beta equal to one for a single risk factor, and betas equal to zero on the remaining risk factors. Factor portfolios can be used to hedge multiple risk factors by combining the original portfolio with offsetting positions in the factor portfolios.

## LO 12.5

The arbitrage pricing theory describes expected returns as a linear function of exposures to common (i.e., macroeconomic) risk factors: $E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \ldots + \beta_{ik}RP_k$, where $RP_j$ is the risk premium associated with risk factor *j*.

The CAPM is a special case of the APT where there is only one priced risk factor (market risk).

The Fama-French three-factor model describes returns as a linear function of the market index return, firm size, and book-to-market factors. The firm size factor, SMB, equals the difference in returns between portfolios of small and big firms. The book-to-market factor, HML, equals the difference in returns between portfolios of high and low book-to-market firms.

## Concept Checkers

1. Which of the following is least likely to be one of the inputs to a multifactor model?
   A. The mean-variance efficient market portfolio.
   B. Factor betas.
   C. Deviations of factor values from their expected values.
   D. Firm-specific returns.

2. Suppose an analyst examines expected returns for the BroadBand Company (BBC) based on a 2-factor model. Initially, the expected return for BBC equals 10%. The analyst identifies GDP and 10-year interest rates as the two factors for the factor model. Assume the following data is used:

| | |
|---|---|
| GDP growth consensus forecast | = 6% |
| Interest rate consensus forecast | = 3% |
| GDP factor beta for BBC | = 1.50 |
| Interest rate factor beta for BBC | = –1.00 |

   Suppose GDP ends up growing 5% and the 10-year interest rate ends up equaling 4%. Also assume that during the period, the BroadBand Company unexpectedly experiences shortages of key inputs, causing its revenues to be less than originally expected. Consequently, the firm-specific return is –2% during the period. Using the 2-factor model with the revised data, which of the following expected returns for BBC is correct?
   A. 1.5%.
   B. 3.5%.
   C. 5.5%.
   D. 6.5%.

3. Which of the following statements is least likely a requirement for an arbitrage opportunity? The arbitrage situation leads to a:
   A. risk-free opportunity.
   B. zero net investment opportunity.
   C. profitable opportunity.
   D. return in excess of the risk-free rate opportunity.

4. Which of the following assumptions is not made when forming a single-factor security market line?
   A. Security returns are described by a factor model.
   B. A mean-variance efficient market portfolio exists.
   C. Well-diversified portfolios can be formed.
   D. No arbitrage opportunities exist.

5. Suppose Portfolio P has factor betas of 0.40 and 0.50 on two risk factors (risk factors 1 and 2, respectively). Assume a portfolio manager wishes to hedge away all of the exposure to the two risk factors, yet does not want to sell the portfolio. Which of the following strategies is expected to achieve the desired result?
    A. Short sell a hedge portfolio that allocates 40% to the first factor portfolio, 50% to the second factor portfolio, and 10% to the risk-free asset.
    B. Short sell a hedge portfolio that allocates 90% to the market portfolio and 10% to the risk-free asset.
    C. Buy a hedge portfolio that allocates 40% to the first factor portfolio, 50% to the second factor portfolio, and 10% to the risk-free asset.
    D. Buy a hedge portfolio that allocates 90% to the market portfolio and 10% to the risk-free asset.

## Concept Checker Answers

1. **A** The mean-variance efficient market portfolio is essential to the capital asset pricing model, but is not required in multifactor models.

2. **C** $R_{BBC} = E(R_{BBC}) + \beta_{BBC,GDP}F_{GDP} + \beta_{BBC,IR}F_{IR} + e_{BBC}$
$R_{BBC} = 0.10 + 1.5(-0.01) - 1(0.01) - 0.02 = 0.055 = 5.5\%$

3. **D** An arbitrage situation exists if a risk-free, zero net investment can be created that produces a positive profit. The arbitrage return need not exceed the risk-free rate.

4. **B** The derivation of the single-factor security market line does not rely on the assumption that a mean-variance efficient market portfolio exists. This is in contrast with the capital asset pricing model, which relies on the existence of the mean-variance efficient market portfolio.

5. **A** A factor portfolio is a well-diversified portfolio that has a factor beta equal to one for a single risk factor, and factor betas equal to zero on the remaining factors. By shorting the hedge portfolio, the investor will offset the factor risks of the original portfolio. In this case, the 0.40 and 0.50 exposures to the two risk factors are offset by the short position in the hedge portfolio that also has 0.40 and 0.50 exposures to the two risk factors.

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# Principles for Effective Risk Data Aggregation and Risk Reporting

Topic 13

## Exam Focus

This is a highly qualitative topic that explores the Basel Committee's principles for effective risk data aggregation and reporting. Much of this topic is practical, in terms of the need for the data to be accurate, complete, timely, comprehensive, and adaptable. Governance principles are important, and the committee notes that risk data aggregation and reporting is expensive, and, as a result, senior management and the board of directors should be fully invested in the process so that adequate resources are devoted to the effort. Risk reporting should also be accurate, comprehensive, clear, and useful. For the exam, understand how data aggregation principles interact and know that the committee implores banks to meet the requirements of each principle while still meeting the other principles. In other words, the bank should not put one principle ahead of another.

## Benefits of Risk Data Aggregation

### LO 13.1: Explain the potential benefits of having effective risk data aggregation and reporting.

According to the Basel Committee on Banking Supervision, **risk data aggregation** means "defining, gathering and processing risk data according to the bank's risk reporting requirements to enable the bank to measure its performance against its risk tolerance/appetite." The aggregation process includes breaking down, sorting, and merging data and datasets. Risk management reports should reflect risks in a reliable way.

Several benefits accrue to banks that have effective risk data aggregation and reporting systems in place. These benefits include:

- An increased ability to **anticipate problems**. Aggregated data allows risk managers to understand risks holistically. It is easier to see problems on the horizon when risks are viewed as a whole rather than in isolation.
- In times of financial stress, effective risk data aggregation enhances a bank's ability to **identify routes to return to financial health.** For example, a bank may be better able to identify a suitable merger partner in order to restore the bank's financial viability.
- **Improved resolvability** in the event of bank stress or failure. Regulatory authorities should have access to aggregated risk data to resolve issues related to the health and viability of banks. This is especially important for global systemically important banks (G-SIBs).
- By strengthening a bank's risk function, the bank is better able to make strategic decisions, **increase efficiency, reduce the chance of loss, and ultimately increase profitability.**

## Governance

### LO 13.2: Describe key governance principles related to risk data aggregation and risk reporting practices.

During the global financial crisis that began in 2007, many banks were unable to quickly and accurately identify concentrations of risk across business lines and at the bank group level due, in part, to an inability to aggregate risk exposures and report bank-wide risks effectively. As part of the Basel Committee's push for greater corporate governance, the committee issued supplemental Pillar 2 guidance regarding capital models and other key risk management models (e.g., value at risk) to improve banks' capabilities regarding the recognition and management of bank-wide risks.



#### Principle 1—Governance

According to the committee, "a bank's risk data aggregation capabilities and risk reporting practices should be subject to strong governance arrangements consistent with the other principles and guidance established by the Basel Committee."

The governance principle suggests that risk data aggregation should be part of the bank's overall risk management framework. To ensure that adequate resources are devoted to data aggregation and reporting, senior management should approve the framework before implementation.

Data aggregation and risk reporting practices should be:

- Fully documented.
- Independently reviewed and validated by individuals with expertise in information technology (IT) and data and risk reporting functions.
- Considered when the firm undergoes new initiatives, including new product development, acquisitions, and/or divestitures. As part of an acquisition, the bank should assess the risk data aggregation and reporting capabilities of the target firm and explicitly evaluate those capabilities when deciding whether to make the acquisition. In addition, a time frame should be established to integrate the risk data aggregation and reporting processes of the two firms.
- Unaffected by the bank's structure. Specifically, decisions regarding data aggregation and reporting should be independent of the bank's physical location or geographical presence and/or legal organization.
- A priority of senior management, who should support risk data aggregation and reporting processes with financial and human resources. Senior management should include risk data aggregation and reporting in strategic IT planning and ensure that the implementation of these processes is not impeded.
- Supported by the board of directors, which should remain aware of the bank's implementation of and compliance with the key governance principles set out by the Basel Committee.

## Data Architecture and IT Infrastructure

**LO 13.3: Identify the data architecture and IT infrastructure features that can contribute to effective risk data aggregation and risk reporting practices.**

IT systems are expensive, and risk aggregation and reporting systems require significant commitments of financial and human resources. Benefits from these investments are generally realized over the long-term, not the short-term. As the memories of the recent financial crisis fade, banks may not give priority to the needed IT investment. The Basel Committee believes that the long-term benefits of improving risk aggregation and reporting processes will outweigh the banks' investments.

### Principle 2—Data Architecture and Infrastructure

According to the committee, "a bank should design, build and maintain data architecture and IT infrastructure which fully supports its risk data aggregation capabilities and risk reporting practices not only in normal times but also during times of stress or crisis, while still meeting the other Principles."

Principle 2, as referenced in Principle 1, implores the bank to devote financial and human resources to risk data aggregation and reporting, both when the bank is financially sound and when the bank is struggling due to financial stresses. Principle 2 requires that:

- Risk data aggregation and reporting practices should be a part of the bank's planning processes and subject to business impact analysis.
- Banks establish integrated data classifications and architecture across the banking group. Multiple data models may be used as long as there are robust automated reconciliation measures in place. Data architecture should include information on data characteristics (metadata) and naming conventions for legal entities, counterparties, customers, and account data.
- Accountability, roles, responsibilities, and ownership should be defined relative to the data. Adequate controls should be in place throughout the lifecycle of the data for all aspects of the technology infrastructure. Risk managers, business managers, and/or IT functions are responsible for data, ensuring that it is entered correctly, is relevant and current, is aligned with data taxonomies, and is consistent with bank policies.

## Risk Data Aggregation Capabilities

**LO 13.4: Describe characteristics of a strong risk data aggregation capability and demonstrate how these characteristics interact with one another.**

### Principle 3—Accuracy and Integrity

According to the committee, "a bank should be able to generate accurate and reliable risk data to meet normal and stress/crisis reporting accuracy requirements. Data should be aggregated on a largely automated basis so as to minimize the probability of errors."

Principle 3 requires that:

- Data aggregation and reporting should be accurate and reliable.
- Controls applied to risk data should be as robust as those surrounding accounting data.
- To ensure the quality of the data, effective controls should be in place when the bank relies on manual processes and desktop applications such as spreadsheets and databases.
- Data should be reconciled with other bank data, including accounting data, to ensure its accuracy.
- A bank should endeavor to have a single authoritative source for risk data for each specific type of risk.
- Risk personnel should have access to risk data to effectively aggregate, validate, reconcile, and report the data in risk reports.
- Data should be defined consistently across the bank. The bank may maintain a dictionary of risk data concepts and terms.
- While data should be aggregated on a largely automated basis to reduce the risk of errors, human intervention is appropriate when professional judgments are required. There should be balance between manual and automated risk management systems.
- Bank supervisors expect banks to document manual and automated risk data aggregation systems and explain when there are manual workarounds, why the workarounds are critical to data accuracy, and propose actions to minimize the impact of manual workarounds.
- Banks monitor the accuracy of risk data and establish plans to correct poor data quality.

## Principle 4—Completeness

According to the committee, "a bank should be able to capture and aggregate all material risk data across the banking group. Data should be available by business line, legal entity, asset type, industry, region and other groupings, as relevant for the risk in question, that permit identifying and reporting risk exposures, concentrations and emerging risks."

Principle 4 requires that:

- Both on- and off-balance sheet risks should be aggregated.
- Risk measures and aggregation methods should be clear and specific enough that senior managers and the board of directors can properly assess risk exposures. However, not all risks need to be expressed in the same metric.
- Bank risk data should be complete. If risk data is not complete, the bank should identify and explain areas of incompleteness to bank supervisors.

## Principle 5—Timeliness

According to the committee, "a bank should be able to generate aggregate and up-to-date risk data in a timely manner while also meeting the principles relating to accuracy and integrity, completeness and adaptability. The precise timing will depend upon the nature and potential volatility of the risk being measured as well as its criticality to the overall risk profile of the bank. The precise timing will also depend on the bank specific frequency requirements for risk management reporting, under both normal and stress/crisis situations, set based on the characteristics and overall risk profile of the bank."

Principle 5 requires that:

- Risk data aggregation should be timely and should meet all requirements for risk management reporting. Bank supervisors will review the timeliness and specific frequency requirements of bank risk data in normal and stress/crisis periods.
- Systems should be in place to produce aggregated risk data quickly in stress/crisis situations for all critical risks. Critical risks include, but are not limited to:
    - Aggregated credit exposures to large corporate borrowers.
    - Counterparty credit risk exposures, including derivatives.
    - Trading exposures, positions, and operating limits.
    - Market concentrations by region and sector.
    - Liquidity risk indicators.
    - Time-critical operational risk indicators.

### Principle 6—Adaptability

According to the committee, "a bank should be able to generate aggregate risk data to meet a broad range of on-demand, ad hoc risk management reporting requests, including requests during stress/crisis situations, requests due to changing internal needs and requests to meet supervisory queries."

Principle 6 requires that:

- Data aggregation capabilities should be adaptable and flexible. Adaptable data makes it easier for managers and the board of directors to conduct stress tests and scenario analysis. Data should be available for ad hoc data requests to assess emerging risks. Adaptability includes:
    - Aggregation processes should be flexible and should allow bank managers to assess risks quickly for decision-making purposes.
    - Data should be customizable (e.g., anomalies, dashboards, and key takeaways) and should allow the user to investigate specific risks in greater detail.
    - It should be possible to include new aspects of the business or outside factors that influence overall bank risk in the risk data aggregation process.
    - Regulatory changes should be incorporated in risk data aggregation.
- A bank should be able to pull out specifics from aggregated risk data. For example, a bank should be able to aggregate risks of a certain country or region. Credit risk exposures (e.g., corporate, bank, sovereign and retail exposures) for a specific country should be readily accessible. Data regarding risks across geographic areas or business lines should be available as needed.

The principles of accuracy and integrity, completeness, timeliness, and adaptability interact; a bank may choose to put one principle ahead of another, or the data is aggregated with one principle in mind while ignoring another. For example, in the interest of speed and timeliness, a bank could take shortcuts with respect to completeness. Also, the accuracy and integrity of the data may suffer if a bank is in a hurry to comply with the timeliness standard. In addition, the data could be compiled in such a way that supports accuracy and integrity, but makes the data inflexible and not easily adaptable for specific needs. The bank should consider all the standards when creating and maintaining a risk data aggregation framework.

## Effective Risk Reporting Practices

### LO 13.5: Describe characteristics of effective risk reporting practices.

Effective **risk reporting practices** include:

- Clear, complete, timely, and accurate data.
- Risk data is reported to the right people at the right time. In other words, the key decision-makers should have access to the data in a timely fashion to allow for good decision-making.

### Principle 7—Accuracy

According to the committee, "risk management reports should accurately and precisely convey aggregated risk data and reflect risk in an exact manner. Reports should be reconciled and validated."

Principle 7 requires that:

- Risk reports should be accurate and precise. Senior managers and board members should be able to use the reports to make critical decisions about bank risks.
- To ensure the accuracy of risk reports the bank should:
    - Define the processes used to create risk reports.
    - Create reasonableness checks of the data.
    - Include descriptions of mathematical and logical relationships in the data that should be verified.
    - Create error reports that identify, report, and explain weaknesses or errors in the data.
- The bank should ensure the reliability, accuracy, and timeliness of risk approximations (e.g., scenario analysis, sensitivity analysis, stress testing, and other risk modeling approaches).
- The board of directors and senior managers should establish precision and accuracy requirements for regular and stress/crisis risk reports. The reports should include information on positions and exposures in the market. The criticality of decisions made using the data should be clearly stated.
- Bank supervisors expect banks to impose accuracy requirements on risk data (both regular and stress/crisis) commensurate with and analogous to accounting materiality. For example, if an omission influences risk decision-making, then it is deemed material.

### Principle 8—Comprehensiveness

According to the committee "risk management reports should cover all material risk areas within the organization. The depth and scope of these reports should be consistent with the size and complexity of the bank's operations and risk profile, as well as the requirements of the recipients."

Principle 8 requires that:

- Reports should contain position and risk exposure information for all relevant risks, such as credit risk, liquidity risk, market risk, and operational risk. The report should also include detailed information for specific risks. For example, credit risk reports should include information on the country, region, sector, industry, and/or single name exposures. Risk related measures such as the bank's regulatory capital should also be included in risk reports.
- Risk reports should be forward-looking and should include forecasts and stress tests. The bank's risk appetite/tolerance should be discussed in the context of emerging risks. Recommendations for reducing risk should be included where appropriate. Senior managers and the board of the directors should gain a sense of the bank's future capital and risk profiles from reports.
- Bank supervisors should be satisfied that the bank's risk reporting is sufficient in terms of coverage, analysis, and comparability across institutions. A risk report should include, but not be limited to, information regarding:
    - Credit risk.
    - Market risk.
    - Liquidity risk.
    - Operational risk.
    - Results of stress tests.
    - Capital adequacy.
    - Regulatory capital.
    - Liquidity projections.
    - Capital projections.
    - Risk concentrations.
    - Funding plans.

### Principle 9—Clarity and Usefulness

According to the committee "risk management reports should communicate information in a clear and concise manner. Reports should be easy to understand yet comprehensive enough to facilitate informed decision-making. Reports should include meaningful information tailored to the needs of the recipients."

Principle 9 requires that:

- Reports be tailored to the end user (e.g., the board, senior managers, and risk committee members) and should assist them with sound risk management and decision-making.
- Reports will include:
    - Risk data.
    - Risk analysis.
    - Interpretation of risks.
    - Qualitative explanations of risks.

    Different members of the organization have different needs in terms of reporting. For example, information relevant to the risk committee may not be specifically relevant to the board of directors. Aggregation increases as the report moves up the organizational hierarchy (i.e., to senior managers and to the board). There is a greater need for qualitative interpretation and explanation as aggregation increases.

- The board of directors should ensure the bank is operating within its risk tolerance/appetite, and should therefore make sure that it is asking for and receiving relevant risk information to make the determination. The board should tell senior management if the amount/type of data it is receiving is insufficient or redundant. The mix of quantitative data versus qualitative data is important. Senior management should also ensure that it is getting the right mix of information to foster good risk decisions.
- Risk data should be classified, and the bank should develop an inventory of terms used in risk reports.
- Bank supervisors will confirm periodically that the risk data is clear, relevant, and useful for decision-making.

## Principle 10—Frequency

According to the committee, "the board and senior management (or other recipients as appropriate) should set the frequency of risk management report production and distribution. Frequency requirements should reflect the needs of the recipients, the nature of the risks reported, and the speed at which the risks can change, as well as the importance of reports in contributing to sound risk management and effective and efficient decision-making across the bank. The frequency of reports should be increased during times of stress/crisis."

Principle 10 requires that:

- The frequency of reports will vary depending on the recipient (e.g., the board, senior managers, and risk committee members), the type of risk, and the purpose of the report. The bank should periodically test whether reports can be accurately produced in the established time frame both in normal and stress/crisis periods.
- In stress/crisis periods, liquidity, credit, and market risk reports may be required immediately in order to react to the mounting risks.

## Principle 11—Distribution

According to the committee, "risk management reports should be distributed to the relevant parties while ensuring confidentiality is maintained."

Principle 11 requires that:

- Reports should be disseminated in a timely fashion while maintaining confidentiality where required. Supervisors expect banks to confirm that recipients receive reports in a timely manner.

*Professor's Note: There are a total of 14 principles for effective risk data aggregation and risk reporting set out by the Basel Committee on Banking Supervision. Principles 12–14 describe the role of bank supervisors in reviewing and evaluating a bank's risk aggregation and reporting practices and are not discussed here, because the LOs for this topic focus on Principles 1–11.*

## Key Concepts

### LO 13.1

Benefits that accrue from effective risk data aggregation and reporting include: (1) an increased ability of managers and the board to anticipate problems, (2) enhanced ability to identify alternative routes to restore financial health in times of financial stress, (3) improved resolvability in the event of bank stress or failure, and (4) an enhanced ability to make strategic decisions, increasing the bank's efficiency, reducing the chance of loss and ultimately increasing bank profitability.

### LO 13.2

The governance principle (Principle 1) suggests that risk data aggregation should be part of the bank's overall risk management framework. The board and senior management should assure that adequate resources are devoted to risk data aggregation and reporting.

### LO 13.3

The data architecture and IT infrastructure principle (Principle 2) states that a bank should design, build, and maintain data architecture and IT infrastructure which fully supports its risk data aggregation capabilities and risk reporting practices not only in normal times but also during times of stress or crisis, while still meeting the other principles. It stresses that banks should devote considerable financial and human resources to risk data aggregation and reporting.

### LO 13.4

Principles 3–6 specify standards and requirements for effective risk data aggregation. Banks should ensure that the data is accurate and has integrity (Principle 3), is complete (Principle 4), is timely (Principle 5), and is adaptable to the end user (Principle 6). In addition, the bank should not have high standards for one principle at the expense of another. Aggregated risk data should exhibit all of the features together, not in isolation.

### LO 13.5

Principles 7–11 specify standards and requirements for effective risk reporting practices. Risk reports should be accurate (Principle 7), comprehensive (Principle 8), and clear and useful (Principle 9). Principle 10 states that reports should be "appropriately frequent" (i.e., frequency depends on the role of the recipient—board members need reports less frequently than risk committee members). Reports should be distributed to relevant parties in a timely fashion while maintaining confidentially (Principle 11).

## Concept Checkers

1. Jeffrey Gibson, a bank supervisor with a national regulatory agency, has requested as part of a bank examination, that Star Bank, a global systemically important bank (G-SIB), improve its aggregation and reporting of risk data. Star Bank has experienced significant losses resulting from multiple causes, ranging from poor lending decisions to bad decisions regarding the use of derivatives. The bank is now undercapitalized because of losses. Gibson refers Star Bank's risk managers to the Basel Committee's recommendations for effective risk data aggregation. He informs risk committee members and senior management that one of the potential direct benefits of effective risk data aggregation, particularly in light of Star Bank's current troubles, is:
   A. increased bank efficiency.
   B. more effective IT infrastructure.
   C. improved resolvability of bank problems.
   D. a clearer definition of the bank's risk appetite.

2. Donna Grinstead is the risk management officer at Republic Bank. She is establishing governance principles for effective risk data aggregation. The bank has historically been lenient with respect to risk management processes, and Grinstead has been hired to remedy the situation. Which of the following statements regarding governance principles is false?
   A. The overall risk management framework of the bank should include risk data aggregation.
   B. Human and financial resources should be devoted to risk data aggregation, and thus senior management should approve the framework.
   C. A bank should have multiple sources for risk data for each type of risk to improve reliability.
   D. Risk data aggregation should be considered when the firm undergoes new initiatives, including acquisitions and divestitures.

3. A bank should include information on data characteristics (metadata) and naming conventions for legal entities, counterparties, customers, and account data in aggregated risk data. This is suggested by the Basel Committee on Banking Supervision in the principle related to:
   A. accuracy.
   B. completeness.
   C. clarity and usefulness.
   D. data architecture and infrastructure.

4. Emily Lister, a risk management specialist at American Bank and Trust, has been asked, as part of Principle 3 on the accuracy and integrity of aggregated risk data, to provide a report to bank supervisors on why a bank employee decided to forgo the automated processes put in place by the risk management team and write data entries by hand. Lister believes it was necessary after discussing the action with the employee. In her report, she details why it was necessary for the employee to forgo automated processes and why she believes the integrity of the data is still intact. In the report, she is describing a(n):
   A. breach of protocol.
   B. manual workaround.
   C. reliability exception to Principle 3.
   D. unexcused exception to risk data aggregation principles.

5. Senior management and the board of directors should receive accurate and timely aggregated risk data reports for all of the following reasons except:
   A. bank supervisors request risk reports from board members, who should be prepared to provide this information during bank examinations.
   B. senior management and board members use risk reports to make decisions regarding bank risks.
   C. senior management and board members should react in times of financial stress and/or crisis and need reliable risk reports to make good decisions.
   D. the board should ensure that the bank is operating within its risk tolerance/appetite and should therefore make sure that it receives relevant risk information.

## Concept Checker Answers

1. **C** There are several benefits that accrue to banks that have effective risk data aggregation and reporting systems in place. These benefits include an increased ability to anticipate problems. Also, in times of severe financial stress, effective risk data aggregation enhances a bank's ability to identify alternative routes to restore financial health. Regulatory authorities should have access to aggregated risk data to resolve issues related to bank health and viability. This aids regulators in resolving problems in the event of financial stress. By strengthening a bank's risk function, the bank is better able to make strategic decisions, increase efficiency, reduce the probability of loss and ultimately increase profitability. In this case, the bank appears to be in financial stress, so the most relevant benefit is improved resolvability.

2. **C** Governance principles for risk data aggregation relate to overall bank processes and the roles of senior management and the board in supporting risk data aggregation and reporting. Data sources relate to the accuracy and integrity of the data, not governance. In addition, the bank should strive to have a single source for risk data, not multiple sources.

3. **D** Principle 2, data architecture and infrastructure, requires that risk data aggregation and reporting practices should be a part of the bank's planning processes and subject to business impact analysis. Banks should establish integrated data classifications and architecture across the banking group. Multiple data models may be used as long as there are robust automated reconciliation measures in place. In addition, data architecture should include information on data characteristics (metadata) and naming conventions for legal entities, counterparties, customers, and account data.

4. **B** As part of Principle 3 on the accuracy and integrity of aggregated risk data, bank supervisors expect banks to document manual and automated risk data aggregation systems and explain when there are manual workarounds, why the workarounds are critical to data accuracy, and propose actions to minimize the impact of a manual workaround.

5. **A** It is important for the board and senior management to have accurate and timely risk reports to oversee the bank's risk-taking activities. The bank's risk tolerance/appetite is monitored by the board. The board and senior managers should be prepared to make decisions in times of financial stress and crisis. The board does not provide reports to regulators. Information requests from supervisors would be made at the bank level, not the board level.

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# GARP Code of Conduct

Topic 14

## Exam Focus

This topic addresses the GARP Code of Conduct which sets forth principles related to ethical behavior within the risk management profession. FRM candidates are expected to know all Member responsibilities as well as sanctions that could result if violations of the Code occur. The material in this topic is relatively easy to understand; however, for the exam, you should expect complex questions related to these ethical standards that test whether or not a violation has occurred.

The GARP Code of Conduct contains a set of key principles designed to support financial risk management practices. The Code was developed for the Financial Risk Manager (FRM) program as well as other certification programs administered by the Global Association of Risk Professionals (GARP). All GARP Members (including FRM candidates) are expected to abide by the principles outlined in the Code and are subject to consequences, such as suspensions, for violating any parts of the Code.

A GARP Member should understand that high ethical behavior goes beyond the principles addressed in this topic. When encountering a situation not specifically outlined in the Code, Members are always expected to act in an ethical fashion. Acting with prudence in all situations related to the profession will uphold the integrity of the risk management field as well as risk management practitioners.



## The Code of Conduct[1]

The Code of Conduct stresses ethical behavior in two areas: (1) Principles and (2) Professional Standards. The Principles section addresses: (1) professional integrity and ethical conduct, (2) conflicts of interest, and (3) confidentiality. The Professional Standards section addresses: (1) fundamental responsibilities and (2) adherence to generally accepted practices in risk management. The responsibilities listed in each section are examined in the following LO.

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## LO 14.1: Describe the responsibility of each GARP member with respect to professional integrity, ethical conduct, conflicts of interest, confidentiality of information, and adherence to generally accepted practices in risk management.

### 1. Professional Integrity and Ethical Conduct

GARP Members:

1.1. shall act professionally, ethically and with integrity in all dealings with employers, existing or potential clients, the public, and other practitioners in the financial services industry.

1.2. shall exercise reasonable judgment in the provision of risk services while maintaining independence of thought and direction. GARP Members must not offer, solicit, or accept any gift, benefit, compensation, or consideration that could be reasonably expected to compromise their own or another's independence and objectivity.

1.3. must take reasonable precautions to ensure that the Member's services are not used for improper, fraudulent or illegal purposes.

1.4. shall not knowingly misrepresent details relating to analysis, recommendations, actions, or other professional activities.

1.5. shall not engage in any professional conduct involving dishonesty or deception or engage in any act that reflects negatively on their integrity, character, trustworthiness, or professional ability or on the risk management profession.

1.6. shall not engage in any conduct or commit any act that compromises the integrity of GARP, the FRM® designation, or the integrity or validity of the examinations leading to the award of the right to use the FRM designation or any other credentials that may be offered by GARP.

1.7. shall be mindful of cultural differences regarding ethical behavior and customs, and avoid any actions that are, or may have the appearance of being unethical according to local customs. If there appears to be a conflict or overlap of standards, the GARP Member should always seek to apply the highest standard.

### 2. Conflict of Interest

GARP Members shall:

2.1. act fairly in all situations and must fully disclose any actual or potential conflict to all affected parties.

2.2. make full and fair disclosure of all matters that could reasonably be expected to impair independence and objectivity or interfere with respective duties to their employer, clients, and prospective clients.

### 3. Confidentiality

GARP Members:

3.1. shall not make use of confidential information for inappropriate purposes and unless having received prior consent shall maintain the confidentiality of their work, their employer or client.

3.2. must not use confidential information for personal benefit.

### 4. Fundamental Responsibilities

GARP Members shall:

4.1. comply with all applicable laws, rules, and regulations (including this Code) governing the GARP Members' professional activities and shall not knowingly participate or assist in any violation of such laws, rules, or regulations.

4.2. have ethical responsibilities and cannot outsource or delegate those responsibilities to others.

4.3. understand the needs and complexity of their employer or client, and should provide appropriate and suitable risk management services and advice.

4.4. be diligent about not overstating the accuracy or certainty of results or conclusions.

4.5. clearly disclose the relevant limits of their specific knowledge and expertise concerning risk assessment, industry practices, and applicable laws and regulations.

### 5. Best Practices

GARP Members shall:

5.1. execute all services with diligence and perform all work in a manner that is independent from interested parties. GARP Members should collect, analyze and distribute risk information with the highest level of professional objectivity.

5.2. be familiar with current generally accepted risk management practices and shall clearly indicate any departure from their use.

5.3. ensure that communications include factual data and do not contain false information.

5.4. make a distinction between fact and opinion in the presentation of analysis and recommendations.

## Violations of the Code of Conduct

### LO 14.2: Describe the potential consequences of violating the GARP Code of Conduct.

All GARP Members are expected to act in accordance with the GARP Code of Conduct as well as any local laws and regulations that pertain to the risk management profession. If the Code and certain laws conflict, then laws and regulations will take priority.

Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership. In addition, violations could lead to a revocation of the right to use the FRM designation. Sanctions would be issued after a formal investigation is conducted by GARP.

*Professor's Note: There are no Key Concepts for this short topic.*

## Concept Checkers

1. Over the past two days, Lorraine Quigley, FRM, manager of a hedge fund, has been purchasing large quantities of Craeger Industrial Products' common stock while at the same time shorting put options on the same stock. Quigley did not notify her clients of the trades although they are aware of the fund's general strategy to generate returns. Which of the following statements is most likely correct? Quigley:
   A. did not violate the Code.
   B. violated the Code by manipulating the prices of publicly traded securities.
   C. violated the Code by failing to disclose the transactions to clients before they occurred.
   D. violated the Code by failing to establish a reasonable and adequate basis before making the trades.

2. Jack Schleifer, FRM, is an analyst for Brown Investment Managers (BIM). Schleifer has recently accepted an invitation to visit the facilities of ChemCo, a producer of chemical compounds used in a variety of industries. ChemCo offers to pay for Schleifer's accommodations in a penthouse suite at a luxury hotel and allow Schleifer to use the firm's private jet to travel to its three facilities located in New York, Hong Kong, and London. In addition, ChemCo offers two tickets to a formal high-society dinner in New York. Schleifer declines to use ChemCo's corporate jet or to allow the firm to pay for his accommodations but accepts the tickets to the dinner (which he discloses to his employer) since he will be able to market his firm's mutual funds to other guests at the dinner. Has Schleifer violated the GARP Code of Conduct?
   A. Yes.
   B. No, since he is using the gifts accepted to benefit his employer's interests.
   C. No, since the gifts he accepted were fully disclosed in writing to his employer.
   D. No, since the gift he accepted is of nominal value and he declined to accept the hotel accommodations and the use of ChemCo's jet.

3. Beth Bixby, FRM, oversees a mid-cap fund that is required to invest in a minimum of 40 and a maximum of 60 different issues. Bixby uses a quantitative approach to actively manage the assets. In promotional materials, she states that "through our complex quantitative approach, securities are selected that have similar exposures to a number of risk factors that are found in the S&P 500 Index. Thus the fund is designed to track the performance of the S&P 500 Index but will receive a return premium of between 2% and 4% according to our model's risk-return measures." This statement is:
   A. permissible since the assertion is supported by modern portfolio theory and estimates from the firms' model.
   B. not permissible since Bixby is misrepresenting the services that she and/or her firm are capable of performing.
   C. not permissible since Bixby is misrepresenting the investment performance she and/or her firm can reasonably expect to achieve.
   D. permissible since the statement describes the basic characteristics of the fund's risk and return objectives.

4. Gail Stefano, FRM, an analyst for a U.S. brokerage firm that serves U.S. investors, researches public utilities in South American emerging markets. Stefano makes the following statement in a recent report: "Based on the fact that the South American utilities sector has seen rapid growth in new service orders, we expect that most companies in the sector will be able to convert the revenue increases into significant profits. We also believe the trend will continue for the next three to five years." The report goes on to describe the major risks of investing in this market, in particular the political and exchange rate instability associated with South American countries. Stefano's report:
   A. has not violated the Code.
   B. violated the Code by failing to properly distinguish factual information from opinions.
   C. violated the Code by recommending an investment which would not be suitable for all of its clients.
   D. violated the Code by failing to properly identify details related to the operations of South American utilities.

5. Beth Anderson, FRM, is a portfolio manager for several wealthy clients including Reuben Carlyle. Anderson manages Carlyle's personal portfolio of stock and bond investments. Carlyle recently told Anderson that he is under investigation by the IRS for tax evasion related to his business, Carlyle Concrete (CC). After learning about the investigation, Anderson proceeds to inform a friend at a local investment bank so that they may withdraw their proposal to take CC public. Which of the following is most likely correct? Anderson:
   A. violated the Code by failing to immediately terminate the client relationship with Carlyle.
   B. violated the Code by failing to maintain the confidentiality of her client's information.
   C. violated the Code by failing to detect and report the tax evasion to the proper authorities.
   D. did not violate the Code since the information she conveyed pertained to illegal activities on the part of her client.

## Concept Checker Answers

1. **A** Quigley's trades are most likely an attempt to take advantage of an arbitrage opportunity that exists between Craeger's common stock and its put options. She is not manipulating the prices of securities in an attempt to mislead market participants. She is pursuing a legitimate investment strategy. Participants in her hedge fund are aware of the fund's investment strategy, and thus Quigley did not violate the Code by not disclosing this specific set of trades in advance of trading (Standards 2.1 and 5.1).

2. **A** GARP Members must not offer, solicit, or accept any gift, benefit, compensation, or consideration that could be reasonably expected to compromise their own or another's independence and objectivity. Schleifer has appropriately rejected the offer of the hotel accommodations and the use of ChemCo's jet. However, Schleifer cannot accept the tickets to the dinner. Since it is a formal high-society dinner, the tickets are most likely expensive or hard to come by. Even though he has disclosed the gift to his employer and he plans to use the dinner as a marketing opportunity for his firm, the gift itself may influence Schliefer's future research in favor of ChemCo. Allowing such potential influence is a violation of Professional Integrity and Ethical Conduct (Standard 1.2).

3. **C** It is not reasonable for Bixby to expect a 40-to-60 stock mid-cap portfolio to track the entire S&P 500 Index, which is a large-cap index. She should know that there will be periods of wide variance between the performance of the portfolio and the S&P 500 Index. There is no assurance that a premium of 2% to 4% will consistently be obtained. Bixby is in violation of Standard 1.4: "GARP Members shall not knowingly misrepresent details relating to analysis, recommendations, actions, or other professional activities," since she has made an implicit guarantee of the fund's expected performance.

4. **A** Historical growth can be cited as a fact since it actually happened. Stefano states that her firm expects further growth and profitability which is an opinion. She does not claim that these are facts. Thus, she is not in violation of Standard 5.4. In addition, Stefano identifies relevant factors and highlights in particular the most significant risks of investing in South American utilities. She has fully complied with Standard 5.3.

5. **B** Anderson must maintain the confidentiality of client information according to Standard 3.1. Confidentiality may be broken in instances involving illegal activities on the part of the client, but the client's information may only be relayed to proper authorities. Anderson did not have the right to inform the investment bank of her client's investigation.

# Self-Test: Foundations of Risk Management

**10 Questions: 24 Minutes**

1. A firm has determined that the value at risk (VaR) of its investment portfolio is $18 million for one day at a 95% confidence level. Which of the following statements regarding this VaR measure is correct?
   A. There is a 95% probability that the portfolio will lose $18 million on a given day.
   B. There is a 95% probability that the portfolio will lose no more than $18 million on a given day.
   C. There is a 5% probability that the portfolio will lose $18 million on a given day.
   D. There is a 5% probability that the portfolio will lose no more than $18 million on a given day.

2. A risk management consultant is considering the feasibility of hedging several risk exposures at Firm XYZ. Regarding the advantages and disadvantages of hedging risk exposures in practice, which of the following actions will least likely result from hedging activities for the firm?
   A. Lower cost of equity capital.
   B. Operational improvements.
   C. Lower cost of debt capital.
   D. Lower compliance costs.

3. Over a decade ago, Yasuo Hamanaka, the lead copper trader for Sumitomo, attempted to corner the copper market in a classic market manipulation strategy. Such lack of supervision over his trading activities resulted from poor internal controls. Because of that lack of supervision, which of the following series of transactions was he able to engage in that ultimately resulted in a $2.6 billion trading loss for Sumitomo?
   A. Long physical copper, short futures contracts, bought put options.
   B. Short physical copper, long futures contracts, sold put options.
   C. Long physical copper, long futures contracts, sold put options.
   D. Short physical copper, short futures contracts, bought put options.

4. In 1976, Drysdale Securities was able to borrow $300 million in unsecured funds from Chase Manhattan Bank. Drysdale used the borrowed funds to take bond positions, which eventually declined in value. Given the loss in market value, Drysdale was unable to repay the borrowed funds and was forced into bankruptcy. Which firm was to blame for this financial disaster?
   I. Drysdale Securities
   II. Chase Manhattan

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. Risk metrics aid the management process by providing managers a target to achieve (e.g., a particular VaR level). Monitoring these risk metrics allows managers to appropriately manage risk. However, risk metrics may be too narrow in scope, which can make it more difficult to achieve the overall objective of managing risk in an effort to create value. Which of the following statements represents a shortcoming/misuse of VaR?
   I. VaR assumes the distributions of losses are correlated over time.
   II. Choosing a VaR time horizon that does not correspond to the liquidity of the assets in the portfolio will likely lead to risk mismeasurement.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

6. An analyst has estimated that the returns for an asset, conditional on the performance of the overall economy, are:

| *Return* | *Probability* | *Economic Growth* |
|---|---|---|
| 5% | 20% | Poor |
| 10% | 40% | Average |
| 14% | 40% | Good |

Also, the conditional expected returns on the market portfolio are:

| *Return* | *Probability* | *Economic Growth* |
|---|---|---|
| 2% | 20% | Poor |
| 10% | 40% | Average |
| 15% | 40% | Good |

According to the CAPM, if the risk-free rate is 5% and the risky asset has a beta of 1.1, with respect to the market portfolio, the analyst should:
   A. sell (or sell short) the risky asset because its expected return is less than equilibrium expected return on the market portfolio.
   B. buy the risky asset because the analyst expects the return on it to be higher than its required return in equilibrium.
   C. sell (or sell short) the risky asset because its expected return is not sufficient to compensate for its systematic risk.
   D. buy the risky asset because the analyst expects the return on it to be lower than its required return in equilibrium.

7. Chrome Fund has an expected return of 12%. Nickel Fund is expected to provide an excess return of 8%. Standard deviations of returns are 5% for Chrome Fund and 4% for Nickel Fund. The risk-free rate is 2%. Based on the Sharpe ratio, a rational investor should:
   A. prefer Chrome Fund to Nickel Fund.
   B. prefer Nickel Fund to Chrome Fund.
   C. be indifferent between Chrome Fund and Nickel Fund.
   D. not invest in either Chrome Fund or Nickel Fund.

8. A portfolio manager employs a 2-factor APT model to calculate expected returns for Portfolio P. The two factors are percentage changes in GDP and changes in the term structure of interest rates, defined as the difference between yields in 30-year Treasury bonds and 1-year Treasury bills. Assume the following data:

| | | |
|---|---|---|
| Risk-free rate | = | 4% |
| GDP factor beta | = | 2.00 |
| Term structure factor beta | = | 0.50 |
| GDP risk premium | = | 6% |
| Term structure risk premium | = | 5% |

Using the 2-factor APT model for Portfolio P, which of the following expected returns is correct?
A. 8.5%.
B. 12.5%.
C. 14.5%.
D. 18.5%.

9. Alan Walters is a risk manager at OneFirst Bank. He has recently been asked to prepare a risk management report for his bank's board of directors. The board members are planning to utilize this report to make critical risk management decisions regarding relevant bank risks. Walters assures the board that the report will be accurate and complete by covering all material risk areas within the organization. He also points out that the report will contain meaningful information that will be applicable to all employees and easily understood at all levels of the organization. Walters plans on distributing the risk management report to all relevant parties while ensuring confidentiality. Which of the following effective risk data aggregation and risk reporting principles set forth by the Basel Committee on Banking Supervision did Walters most likely violate?
A. Principle 7—Accuracy.
B. Principle 8—Comprehensiveness.
C. Principle 9—Clarity and Usefulness.
D. Principle 11—Distribution.

10. Will Lambert, FRM, is a financial risk analyst for Offshore Investments. He is preparing a purchase recommendation on Burch Corporation. According to the GARP Code of Conduct, which of the following statements about disclosure of conflicts is most correct? Lambert would have to disclose that:
A. his wife owns 2,000 shares of Burch Corporation.
B. Offshore is an over-the-counter market maker for Burch Corporation's stock.
C. he has a material beneficial ownership of Burch Corporation through a family trust.
D. All of these choices require disclosure.

# Self-Test Answers: Foundations of Risk Management

1. **B** The VaR of this investment can be interpreted as either (1) there is a 95% probability that the portfolio will lose no more than $18 million on a given day or (2) there is a 5% probability that the portfolio will lose more than $18 million on a given day.

   (See Topic 1)

2. **D** Hedging activities incur compliance costs related to disclosure and accounting.

   (See Topic 2)

3. **C** Hamanaka established a dominant long position in futures contracts and simultaneously purchased large quantities of physical copper. As well, to help finance his long copper positions, he even sold put options on copper. In essence, here was a "triple long" strategy that would only pay off if the price of copper or copper futures increased. At the same time, there was a huge risk of losses should the prices fall. Unfortunately, there was a continuation of plummeting copper prices after other copper traders began selling their copper holdings in anticipation of Sumitomo doing the same. The end result was total losses of $2.6 billion for Sumitomo.

   (See Topic 6)

4. **C** Clearly, misleading reporting was used by Drysdale in order to obtain the borrowed funds. However, Chase Manhattan was partially to blame for assuming that it was simply the middleman in the transactions and the positions taken had a low level of risk. The inexperienced managers at Chase failed to realize that the contract wording with Drysdale indicated that Chase would be held responsible for any payments due.

   (See Topic 6)

5. **B** One misuse of VaR is choosing a time period (e.g., daily or weekly) that does not correspond to the liquidity of the assets in the portfolio. Using daily VaR on a portfolio where the assets cannot be effectively traded within a day is clearly not appropriate. Taking a longer term horizon to account for liquidity of the assets may not be sufficient either. VaR assumes the distributions of losses are *not* correlated over time, so Statement I is incorrect. A crisis can change the nature of a return distribution for a given period as well as across periods.

   (See Topic 9)

6. **C** The analyst's forecast of the expected return on the risky asset is $5(0.2) + 10(0.4) + 14(0.4) = 10.6\%$. The expected/equilibrium return on the market portfolio is $2(0.2) + 10(0.4) + 15(0.4) = 10.4\%$.

   The CAPM equilibrium expected return (required return in equilibrium) on the risky asset is $5 + 1.1(10.4 - 5) = 10.94\%$. Since the analyst's forecast return on the risky asset is less than its required return in equilibrium, the asset is overpriced and the analyst would sell if he owned it and possibly sell it short.

   (See Topic 10)

7. **C** Excess return for Chrome is 12% – 2% = 10%. Chrome's Sharpe ratio is 10% / 5% = 2.0. Excess return for Nickel is given as 8%. Nickel's Sharpe ratio is 8% / 4% = 2.0. An investor should be indifferent between these two funds because they provide the same expected excess return per unit of risk.

   (See Topic 11)

8. **D** $E(R_P) = 0.04 + 2(0.06) + 0.5(0.05) = 0.185 = 18.5\%$.

   (See Topic 12)

9. **C** Principle 9 requires that reports be tailored to the end user (e.g., the board, senior managers, and risk committee members) and should assist them with sound risk management and decision making. Walters suggested that the report would not be tailored to the board because the report was going to be applicable to all employees and easily understood at all levels of the organization. Principle 7 requires that risk reports should be accurate and precise. Board members should be able to use the reports to make critical decisions about bank risks. Principle 8 requires that reports should contain position and risk exposure information for all relevant risks. Principle 11 requires that reports should be disseminated in a timely fashion while maintaining confidentiality where required.

   (See Topic 13)

10. **D** According to Standard 2.2, GARP Members shall make full and fair disclosure of all matters that could reasonably be expected to impair independence and objectivity or interfere with respective duties to their employer, clients, and prospective clients.

    (See Topic 14)

# Formulas



## Topic 10

$$\text{Beta}_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

capital asset pricing model: $E(R_i) = R_F + \text{Beta}_i[E(R_M) - R_F]$

capital market line: $E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_P$

## Topic 11

Treynor measure: $\left[\frac{E(R_P) - R_F}{\beta_P}\right]$

Sharpe measure: $\left[\frac{E(R_P) - R_F}{\sigma_P}\right]$

Jensen's alpha: $\alpha_P = E(R_P) - R_F - [E(R_M) - R_F]\beta_P$

Sortino ratio: $\frac{E(R_P) - R_{min}}{\sqrt{MSD_{min}}}$

information ratio = $\left[\frac{E(R_P) - E(R_B)}{\sigma_{ep}}\right] = \frac{\alpha_P}{\sigma_{ep}}$

## Topic 12

multifactor model: $R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$

where:
$R_i$ = return on stock *i*
$E(R_i)$ = expected return for stock *i*
$\beta_{ij}$ = $j^{th}$ factor beta for stock *i*
$F_j$ = deviation of macroeconomic factor *j* from its expected value
$e_i$ = firm-specific return for stock *i*

APT model: $E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + \ldots + \beta_{ik}[E(R_k) - R_F]$

Fama-French three-factor model:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

# Index