

Assignment 1 – Theory Questions

Q1

a) def generateRecursively(n):  
    arr ← new int[n]  
    for i, 0 to n-1:  
        arr[i] ← i+1  
    RecursiveP(arr, n) → calling recursively  
                            dominates the rest

def RecursiveP(arr, n):  
    if n is 1:  
        print(arr)  
        return } constant  
    for i, 0 to n-1: →  $O(n)$   
        constant ← arr.swap(i, n-1)  
        RecursiveP(arr, n-1) →  $O(n)$   
        constant ← arr.swap(i, n-1)  
    →  $O(n) * O(n)$  ⇒  $O(n^2)$

```

b) def generateIterative(n):
    s1 ← new Stack(n) // Stacks with
    s2 ← new Stack(n) // length n
    for i, 0 to n:
        s1.push(i+1)
    iterativeP(s1, s2, n) → will dominate
                           the rest

```

```

def iterativeP(s1, s2, n):
    for i, 1 to n! : → O(n!)
        temp ← i % n } constant
        if i % 2 == 0:
            while s1 is not Empty: → O(n)
                if temp == 0:
                    val ← s1.pop()
                else:
                    temp ← temp - 1
                    s2.push(s1.pop()) } constant
                s2.push(val)
                print(s2)
            else:
                while s2 is not Empty: → O(n)
                    if temp == 0:
                        val ← s2.pop()
                    else:
                        temp ← temp - 1
                        s1.push(s2.pop()) } constant
                    s1.push(val)
                    print(s1)

```

$(n!)(n)$   
 $\Rightarrow O(n(n!))$

a)

def powerSet(T):

start ← [] // Empty Set

s ← new stack

q ← new queue

for i, all elements of T // Puts all the numbers

s.push(i) // from 1 to n into stack

q.enqueue(start)

while s is not Empty:

val ← s.pop() // element to add  
// this iteration

while TRUE:

begin ← q.dequeue()

q.enqueue(begin + [val]) // concatenating starting set  
// and value to add

q.enqueue(begin)

if begin is []: // keeps previously found set  
breaklike a  
do while

return q

b)  $n$  is Nb of elements in  $T$

```
def powerSet(T):
    start ← []
    s ← new stack
    q ← new queue
    for i, all elements of T
        s.push(i)
    q.enqueue(start)
    while s is not Empty:
        val ← s.pop()
        while TRUE:
            begin ← q.dequeue()
            q.enqueue(begin + [val])
            q.enqueue(begin)
            if begin is []:
                break
        return q
```

Annotations in the code:

- $\{ \text{constant} \}$  for the initialization of `start`, `s`, and `q`.
- $\{ O(n) \}$  for the `for` loop over elements of `T`.
- $\{ O(n) \}$  for the `q.enqueue(start)` operation.
- $\{ O(n) \}$  for the `while s is not Empty:` loop.
- $\{ \text{constant} \}$  for the `val ← s.pop()` operation.
- $\{ \text{constant} \}$  for the `while TRUE:` loop.
- $\{ \text{constant} \}$  for the `begin ← q.dequeue()` operation.
- $\{ O(n) \}$  for the `q.enqueue(begin + [val])` and `q.enqueue(begin)` operations.
- $\{ \text{constant} \}$  for the `if begin is []:` condition and `break` statement.
- $\{ \text{constant} \}$  for the `return q` statement.

→ happens

$$\left. \begin{array}{l} 1 = 2^0 \\ 2 = 2^1 \\ 4 = 2^2 \\ \vdots \\ 2^k \end{array} \right\}$$

where  $k = n$   
↳ Nb of iterations for outer loop

$$(1 + 2 + 4 + \dots + 2^k)$$

geometric series

$$2^{k+1} - 1$$

$$2^{n+1} - 2 \Rightarrow O(2^n)$$

Q3

$$n^4 + \log^2 n \Rightarrow O(n^4) \quad (8)$$

$$\log \log n \Rightarrow O(\log \log n) \quad (1)$$

$$\sqrt{n} \Rightarrow O(\sqrt{n}) \quad (2)$$

$$n! + n \Rightarrow O(n!) \quad (9)$$

$$n/2 \Rightarrow O(n) \quad (4)$$

$$\binom{n}{2} = \frac{n!}{2(n-2)!} = \frac{\cancel{1 \times 2 \times 3 \times \dots \times n-2} \times n-1 \times n}{1 \times 2 \times 3 \times \dots \times n-2} = \frac{(n-1)n}{2} \Rightarrow O(n^2) \quad (6)$$

$$2^n \Rightarrow O(2^n) \quad (7)$$

$$n \log n \Rightarrow O(n \log n) \quad (3)$$

$$n^n \Rightarrow O(n^n) \quad (10)$$

$$2^{\log n} = n \Rightarrow O(n) \quad (5)$$

$$2^{n!} + n^2 \Rightarrow O(2^{n!}) \quad (12)$$

$$2^{2^n} \Rightarrow O(2^{2^n}) \quad (11)$$

$$\Rightarrow \log \log n \leq \sqrt{n} \leq n \log n \leq n/2 \leq \\ 2^{\log n} \leq \binom{n}{2} \leq 2^n \leq n^4 + \log^2 n \leq \\ n! + n \leq n^n \leq 2^{2^n} \leq 2^{n!} + n^2.$$