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Assignment 1 – Theory Questions

Q1

a) Big-O time complexity

```
Algorithm MyAlgorithm (A,B)
Input: Arrays A and B each storing n >= 1 integers.
Output: What is the output? (Refer to part b below)
Start:
count = 0 -> 1
① for i = 0 to n-1 do { -> 0 to n-1 is same as 1 to n
    sum = 0 -> 1
②   for j = 0 to n-1 do {
        sum = sum + A[0] -> 3
③       for k = 1 to j do -> 1 to j
            sum = sum + A[k] -> 3
    }
    if B[i] == sum then count = count + 1 -> 4
}
return count -> 1
```

$$1 + \sum_{i=0}^{n-1}\left[1 + \sum_{j=0}^{n-1}\left[3 + \sum_{k=1}^{j}[3]\right] + 4\right] + 1$$

$$= 1 + \sum_{i=1}^{n}\left[1 + \sum_{j=1}^{n}\left[3 + 3\underbrace{\sum_{k=1}^{j}[1]}_{j}\right] + 4\right] + 1$$

$$= 1 + \sum_{i=1}^{n}\left[1 + \sum_{j=1}^{n} 3 + \sum_{j=1}^{n} 3j + 4\right] + 1$$

$$= 1 + \sum_{i=1}^{n}\left[1 + 3n + \underbrace{\frac{3n(n+1)}{2}}_{\frac{3n^2}{2}+\frac{3n}{2}} + 4\right] + 1$$

$$= 2 + \sum_{i=1}^{n} 1 + 3\sum_{i=1}^{n} n + \frac{3n^2}{2}\sum_{i=1}^{n} 1 + \frac{3n}{2}\sum_{i=1}^{n} 1 + 4\sum_{i=1}^{n} 1$$

$$= 2 + n + \frac{3n(n+1)}{2} + \frac{3n^2(n)}{2} + \frac{3n(n)}{2} + 4n$$

$$= 2 + n + \frac{3n^2}{2} + \frac{3n}{2} + \frac{3n^3}{2} + \frac{3n^2}{2} + 4n$$

$$= \frac{3}{2}n^3 + 3n^2 + \frac{13}{2}n + 2 \Rightarrow \boxed{O(n^3)}$$

b) Hand-run

A = [1, 2, 5, 9]

B = [2, 29, 40, 57]

n is 4

Start:

Count = 0

loop 1 i=0

sum = 0

loop2 j=0

sum + 1 = 1

loop3 k=1

does not happen

j=1

sum + 1 = 2

loop3 k=1

sum + 2 = 4

j=2

sum + 1 = 5

loop3 k=1

sum + 2 = 7

k=2

sum + 5 = 12

j=3

sum+1 = 13

loop3 k=1

sum + 2 = 15

k=2

sum + 5 = 20

k=3

sum + 9 = 29

2 = 29? FALSE

count = 0

i=1

sum = 0

Loop 2 j=0

sum + 1 = 1

loop3 k=1

does not happen

j=1

sum + 1 = 2

loop3 k=1

sum + 2 = 4

j=2

sum + 1 = 5

loop3 k=1

sum + 2 = 7

k=2

sum + 5 = 12

j=3

sum+1 = 13

loop3 k=1

sum + 2 = 15

k=2

sum + 5 = 20

k=3

sum + 9 = 29

29 = 29? TRUE

Count = 1

i=2

sum = 0

Loop 2 j=0

sum + 1 = 1

loop3 k=1

does not happen

j=1

sum + 1 = 2

loop3 k=1

sum + 2 = 4

j=2

sum + 1 = 5

loop3 k=1

sum + 2 = 7

k=2

sum + 5 = 12

j=3

sum+1 = 13

loop3 k=1

sum + 2 = 15

k=2

sum + 5 = 20

k=3

sum + 9 = 29

40 = 29? FALSE

count = 1

i=3

sum = 0

Loop 2 j=0

sum + 1 = 1

loop3 k=1

does not happen

j=1

sum + 1 = 2

loop3 k=1

sum + 2 = 4

j=2

sum + 1 = 5

loop3 k=1

sum + 2 = 7

k=2

sum + 5 = 12

j=3

sum+1 = 13

loop3 k=1

sum + 2 = 15

k=2

sum + 5 = 20

k=3

sum + 9 = 29

57 = 29? FALSE

count = 1

OUTPUT:

Count = 1

<u>Q2</u>

a)

```
for (int i = 0; i < n; i = i + C)
    for (int j = 1; j < 1024; j = j*2)
        Sum[i] += j * Sum[i];
```

$\sum_{i=0}^{n-1} \Rightarrow \sum_{i=1}^{n}$

k = Nb of iterations for j

$\log 1024 = k = 10$

$\sum_{i=1}^{n}\sum_{j=1}^{k} 4 = 4\sum_{i=1}^{n}\sum_{j=1}^{10} 1$

$= 4\sum_{i=1}^{n} 10$

$= 4\cdot 10n = 40n \Rightarrow \boxed{O(n)}$

b)

```
for (int i = 1; i < n; i = i*2)
    for (int j = 0; j < i; j = j+2)
        Sum[i] += j * Sum[i];
```

k: Nb of iterations for outer

$k = \log n$

j: $0, 2, 4, 6, \ldots, i-1$ $= 2 \times$ $\left(0, 1, 2, \ldots, \frac{i-1}{2}\right)$

highest $\frac{n+1}{2}$ → Nb of iterations

$\sum_{i=1}^{k}\sum_{j=0}^{n} 4 = 4\sum_{i=1}^{k}\sum_{j=0}^{\frac{n+1}{2}} 1$

$= 4\sum_{i=1}^{k} \frac{n+1}{2} - \cancel{0} + 1$

$= 4\sum_{i=1}^{k} \frac{n+1}{2} + 1$

$= 4\left[\frac{1}{2}\sum_{i=1}^{k} n+1 + \sum_{i=1}^{k} 1\right]$

$= 4\left[\frac{n}{2}\sum_{i=1}^{k} 1 + \frac{1}{2}\sum_{i=1}^{k} 1 + \sum_{i=1}^{k} 1\right]$

$= 4\left[\frac{nk}{2} + \frac{k}{2} + k\right]$

$= 2nk + 2k + 4k = 2nk + 6k$

$= 2n\log n + 6\log n$

$\Rightarrow \boxed{O(n \log n)}$

c)

```
for (int i = 1; i < n; i = i * 2)
    for (int j = 1; j < i; j = j * 2)
        Sum[i] += j * Sum[i];
```

K: Nb of iterations for outer $K = \log(n)$

L: Nb of iterations for inner $L = \log(i)$

$$\sum_{i=1}^{K} \sum_{j=1}^{L} 4 = 4 \sum_{i=1}^{K} \sum_{j=1}^{L} 1$$

$$= 4 \sum_{i=1}^{K} L$$

$$= 4L \sum_{i=1}^{K} 1$$

$$= 4LK = 4 \log n \log n$$

$$\Rightarrow O((\log n)^2)$$

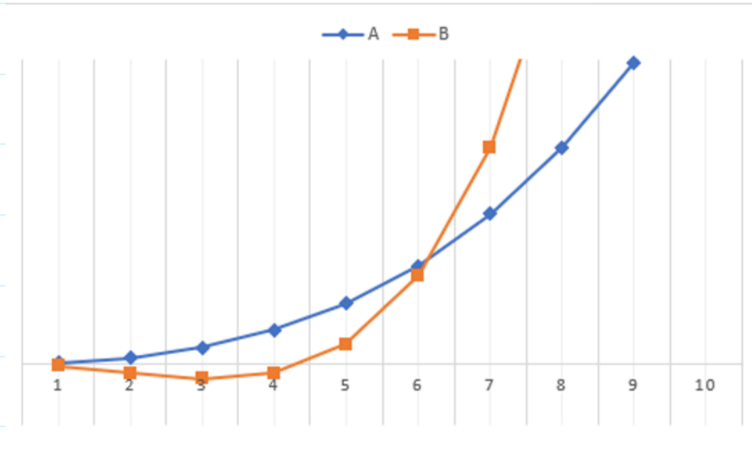
Q3

Q3

A: $12n^3 + 40n \log n$

B: $5n^4 - 100n^2$

Finding n_0



n	A	B
1	12	-95
2	176	-320
3	514	-495
4	1088	-320
5	1964	625
6	3212	2880
7	4902	7105
8	7104	14080
9	9889	24705

→ B becomes bigger between $n=6$ & $n=7$

$$\boxed{\begin{array}{l} n_0 = 7 \text{ such that} \\ B > A \text{ for } n \geq n_0 \\ \qquad\qquad\quad n \geq 7 \end{array}}$$

Q4

a) if → $d(n)$ is $O(f(n))$ &

→ $e(n)$ is $O(g(n))$

prove that $d(n)+e(n)$ is $O(f(n)+g(n))$

→ given that $d(n)$ is $O(f(n))$

$$d(n) \le c_1 f(n)$$

→ given that $e(n)$ is $O(g(n))$

$$e(n) \le c_2 g(n)$$

→ $e(n) + d(n) \le c_1 f(n) + e(n)$

$$d(n)+e(n) \le c_1 f(n) + e(n)$$

→ $e(n)$ can be $= c_2 g(n)$

$$d(n)+e(n) \le c_1 f(n) + c_2 g(n)$$

→ let $c_1 = c_2 = c$

$$d(n)+e(n) \le c f(n) + c g(n)$$

$$d(n)+e(n) \le c[f(n)+g(n)]$$

$\therefore$ $d(n)+e(n)$ is $O(f(n)+g(n))$

QED.

b) Prove/Disprove

$2^{n+1}+n^3$ is $O(2^n)$ → $2^{n+1}+n^3 \le c\, 2^n$

$$2^n \cdot 2 + n^3 \le c\, 2^n$$

$$\cancel{2^n}\left[2+\frac{n^3}{2^n}\right] \le c\, \cancel{2^n}$$

$$2+\frac{n^3}{2^n} \le c$$

$2^{n+1}+n^3$ is $O(2^n)$

for any n value

such that $c \ge 2+\frac{n^3}{2^n}$

c) Prove

2^n is $O(n!)$ $\rightarrow$ $2^n \leq c\, n!$

let $c = 1$

$n=0$ $\quad 2^0 \leq 1 \cdot 0!$

$1 \leq 1$ TRUE

$n=1$ $\quad 2^1 \leq 1 \cdot 1!$

$2 \leq 1$ FALSE

only true in 1 case
let's try another
value for c

let $c = 2$

$n=0$ $\quad 2^0 \leq 2 \cdot 0!$

$1 \leq 2$ TRUE

$n=1$ $\quad 2^1 \leq 2 \cdot 1!$

$2 \leq 2$ TRUE

$n=2$ $\quad 2^2 \leq 2 \cdot 2!$

$4 \leq 4$ TRUE

$n=3$ $\quad 2^3 \leq 2 \cdot 3!$

$9 \leq 12$ TRUE

$n=4$ $\quad 2^4 \leq 2 \cdot 4!$

$16 \leq 48$ TRUE

$n=10$ $\quad 2^{10} \leq 2 \cdot 10!$

$1024 \leq 7257600$ TRUE

$$\boxed{2^n \text{ is } O(n!) \text{ for } c=2 \quad n \geq 0}$$

d) $\log(n!)$ is $O(n \log n)$ $\longrightarrow$ $\log(n!) \leq c\, n \log n$

Prove or Disprove

$c = 1$

$n=1$ $\quad \log(1!) \leq 1(1) \log(1)$

$0 \leq 0$ True

$n=2$ $\quad \log(2!) \leq 1(2) \log(2)$

$1 \leq 2$ True

$n=3$ $\quad \log(3!) \leq 1(3) \log(3)$

$2.584 \leq 3.169$ True

$n=10$ $\quad \log(10!) \leq 1(10) \log(10)$

$21.79 \leq 33.22$ True

$$\boxed{\log(n!) \text{ is } O(n \log n) \text{ for } c=1 \text{ \& } n \geq 1}$$