

Relationships of Impedance, Gamma, Return Loss, and SWR

Written by Bill Ashley for
AEA Technology Inc.
www.aeatechnology.com

Abstract:

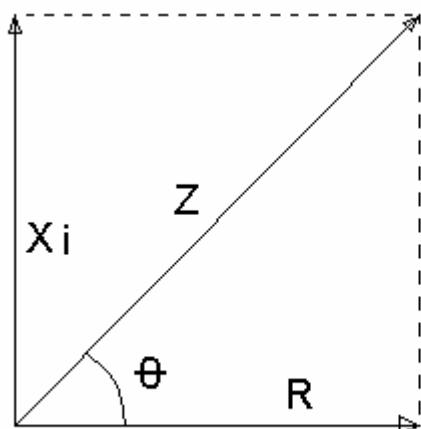
This paper shows how the various formats of impedance readings relate to each other.

Introduction:

The AEA network analyzers display impedance information in several formats. You select which two to plot. The different formats still convey the same information; you just pick the two you want to see. Most of the formats are “vector”, which means that two independent quantities make up one impedance measurement. The exceptions are SWR and Return Loss which are “scalar” quantities. Once you calculate impedance to a scalar format, you cannot recalculate back to a vector format. Any vector format can be calculated to another vector format, or to a scalar.

Discussion:

Let's start with resistance (R) and reactance (X). When the impedance is purely resistive, the voltage (V) and current (I) waveforms are in phase. When purely reactive the current leads by 90 degrees (capacitive), or lags by 90 degrees (inductive). The old saying “ELI the ICE man” is used to remember this relationship. Most loads combine resistive and reactive impedances. To illustrate a complex impedance we draw the R as a horizontal vector, pointing to the right , and the X as a vertical vector, up for inductive and down for capacitive (R always has an angle of 0, X always has an angle of +or- 90 degrees). The total impedance (Z) equals the vector sum of the R and X. This Z has a direction described by the impedance angle (θ), which is also equal to the phase shift between the V and I waveforms respectively. Reactance can also be calculated into Capacitance (C) or Inductance (L). Note that this diagram depicts two components in series.



$$Z = R + X \cdot i$$

$$i = \sqrt{-1}$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta = \text{ATAN}(X/R)$$

R=Resistance
 X=Reactance
 Z=Total Impedance
 θ =Z angle

The reverse operations:

$$R = |Z| \cos(\theta)$$

$$X = |Z| \sin(\theta)$$

To convert to parallel components:

$$R_p = |Z|^2 / R$$

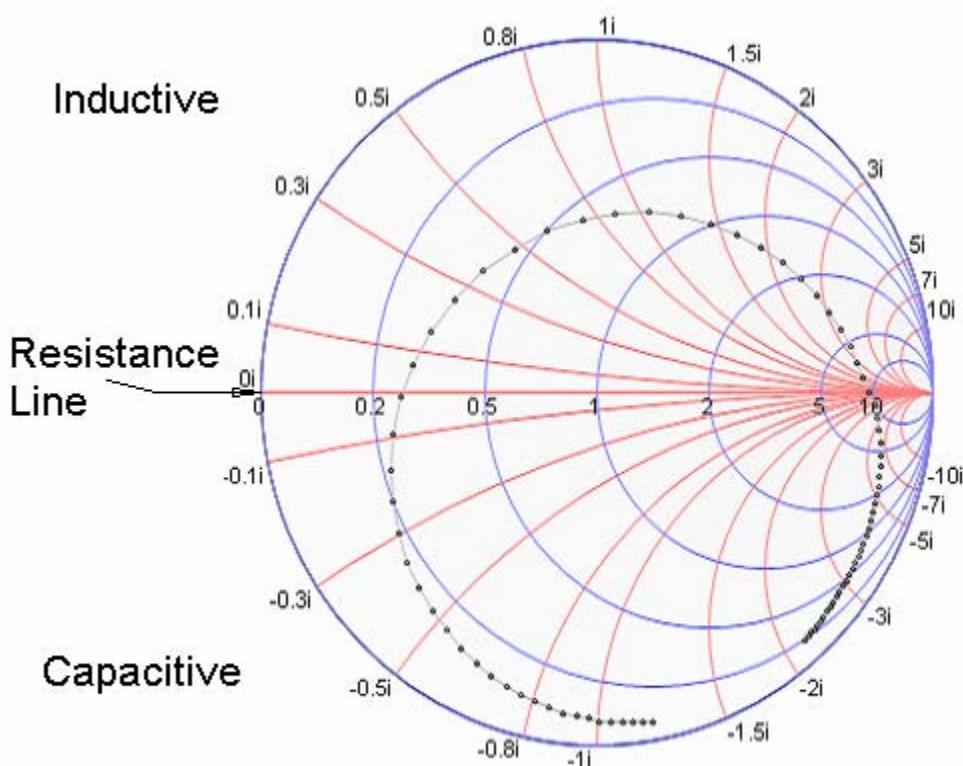
$$X_p = |Z|^2 / X$$

Now we move on to the Smith Chart vector. This vector is known as Gamma or Rho, depending upon who your professor was. Since Gamma is a vector, it has a magnitude and an angle. The Smith Chart covers a circular area with a radius of 1; the center of this circle represents the Gamma starting point. The equation that converts Z to the gamma will produce a vector with magnitude ≤ 1 at some angle between $+\/- 180$ degrees (for Z with non negative R).

Use the internet to find some excellent Smith Charts that can be downloaded. The Smith Chart below came from AEA's PC Vision software program. The plot shows the gamma of a series-parallel tank circuit over a sweep of 80 different frequencies. There is a series resonance near 15 ohms and a parallel resonance near 500 ohms. See AN104 for details on using the Smith Chart.

$$\Gamma = \frac{1 - \frac{Z_0}{Z}}{1 + \frac{Z_0}{Z}}$$

Use vector math (since Z is a vector), resulting in a vector for Gamma. Note this chart is normalized to Z_0 .



Sample of a Smith Chart



Since Gamma is a vector, we can calculate back to R and X:

$$R+Xi = (1-\Gamma)/(1+\Gamma)*Z_0 \text{ where } i = \text{SQRT}(-1)$$

Again, use vector math for this equation.

From Gamma we can calculate the scalar quantities SWR and Return Loss:

$$\text{SWR} = (1+|\Gamma|)/(1-|\Gamma|) \text{ If SWR}<1.0 \text{ then invert it:}$$

$$\text{SWR} = (1-|\Gamma|)/(1+|\Gamma|)$$

$$RL = -20*\log(|\Gamma|) \text{ and } |\Gamma|=10^{RL/20}$$

Since SWR and return loss are scalars, they cannot be converted back to vectors.

Reactance can be converted to a component value:

$$C = -1 / (\omega * X) \text{ if } X < 0 \text{ and } L = X/\omega \text{ if } X > 0 \text{ where } \omega = 2 * \pi * \text{frequency}$$

The S21 equations are very simple:

$$\text{LogGain} = 20*\log(\text{LinGain}) \text{ and } \text{LinGain} = 10^{\text{LogGain}/20}$$

$$\text{GainPhase (degrees)} = \text{Delay} * \text{Frequency} * 360$$

These equations describe the gain or loss of the system, and are not tied into the impedance formats.

Conclusion:

Impedance may be expressed in many formats, choose the one that best suits your application.