PHYS 1200 Lab 4: Force balancing

This lab has four parts, each with a paired experimental and theoretical problem to solve. First, experimentally find the unknown force $\mathbf{F}_1$ that balances out two or three known forces $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$ by setting them up on the force table. Then, use N2L (Newton's 2nd Law) to solve for the same unknown force mathematically. Finally, compare the theoretical F_1 (N2L) to the experimental $F_1 = m_1 g$.

Force tables

The force table you'll use in this lab allows you to apply forces to a central ring by hanging masses from strings attached to it and running over pulleys. The mass on the string sets the force magnitude, and the direction is set by the pulley position on the table's edge.

Equipment

- Force table
- 4 mass hangers, assorted masses
- 4 pulleys
- String, scissors

Calculations

Since the entire experiment will be at rest, the tension force in each string is equal to the weight *mg* it suspends. But the masses are in grams, so you have to convert to kilograms before calculating the tension force in newtons. Example: if m_2 = 115 g, then

$$m_2 = 115\ \cancel{\text{g}} \times \frac{1\text{ kg}}{1000\ \cancel{\text{g}}} = 0.115\text{ kg}$$

$$F_2 = m_2 g = (0.115\,kg)(9.81\,m/s^2) = 1.13\,N$$

When doing your trig in this lab

- set your calculator to DEG (not RAD)

- only calculate with angles LESS than 90°

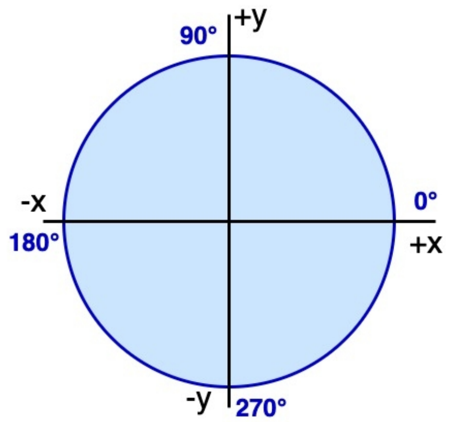
The theoretical F_1's and θ_1's you calculate in each part will come from N2L and trigonometry. Since the ring the strings are tied to will be at rest, N2L gives $\vec{F}_{\text{net}} = 0$. With up to four tension forces acting on the ring, this means $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$, though $F_4 = 0$ in Parts 1-3. Solving this gives $\vec{F}_1 = -(\vec{F}_2 + \vec{F}_3 + \vec{F}_4)$. So in terms of x and y components, you'll be working with $\vec{F}_{1x} = -(\vec{F}_{2x} + \vec{F}_{3x} + \vec{F}_{4x})$ and $\vec{F}_{1y} = -(\vec{F}_{2y} + \vec{F}_{3y} + \vec{F}_{4y})$. That is, to get the theoretical components $\vec{F}_{1x}$ and $\vec{F}_{1y}$, you add up the components of the other two or three forces, then flip the sign. The tables in the **Data** section should help facilitate these calculations. Keep in mind that all four parts of the lab have been designed so that $\vec{F}_{1x}$ and $\vec{F}_{1y}$ always come out positive.

Once you have $\vec{F}_{1x}$ and $\vec{F}_{1y}$, the theoretical force magnitude and direction are $F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$ and $\theta_1 = \tan^{-1}(F_{1y}/F_{1x})$, which can then be compared to the actual values $F_1 = m_1 g$ and the θ_1 read directly from the force table. They should match pretty closely, so if they don't, ask the instructor for help tracking down and fixing the error(s).

Setup

Tie 4 strings to the force table ring, each long enough to run over a pulley and suspend a mass hanger as shown in the photo above

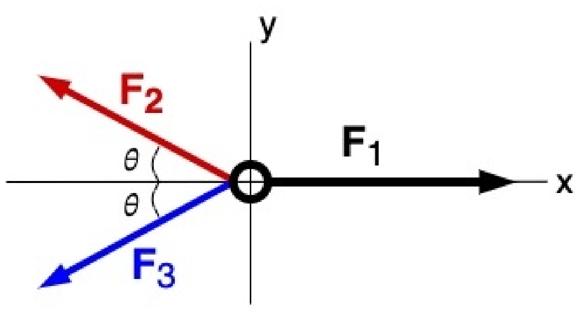
The force table is marked off in degrees around its rim. This image shows the relations between those markings and the x/y axes we'll be using:



x and y axes relative to degree marks on force table

Part 1: Balancing two symmetric forces

The known forces $\mathbf{F}_2$ and $\mathbf{F}_3$ have equal magnitudes, and make equal but opposite angles θ with the -x direction. The unknown balancing force $\mathbf{F}_1$ points in the +x direction. An incomplete FBD for the ring (force components not illustrated) looks like this:



Part 1

Experiment

- $\mathbf{F}_1$ pulls in the +x direction: set pulley 1 at the 0° mark
  - This means that F_{1y} = 0 N, and θ_1 = 0°; the only theoretical value to find is $F_{1x} = F_1$
- Choose an angle θ: 23°, 27°, 29°, 31°, 33°, 37°, or 39°
- $\mathbf{F}_2$ pulls θ degrees above the –x direction: set pulley 2 at the 180° – θ mark
- $\mathbf{F}_3$ pulls θ degrees below the –x direction: set pulley 3 at the 180° + θ mark
- Load up hangers 2 & 3 with the same amount of mass, so that: $m_2 = m_3$
  - 120 g, 130 g, 140 g, or 150 g in disks, for BOTH hangers
  - record $m_2 = m_3$ = disks + 5 g for the hanger
- Load up hanger 1 until the ring is centered and at rest
  - for final adjustments, use increments of 1-2 g
- **Adjust** strings and pulley wheel height and alignment as needed until
  - all strings follow radii of the force table (lines from the center to the edge)
  - each string is parallel to it's pulley wheel's groove (pulleys are not slanted)
  - each string is as horizontal as you can make it

- Record: θ, m_1 (<u>total</u> mass hanging from string 1 = mass of all disks + 5 g for the hanger)

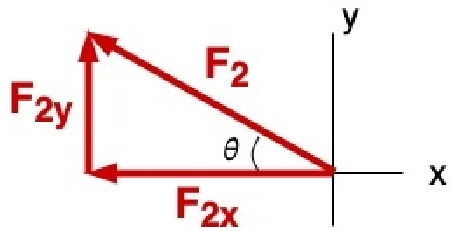
Actual force magnitudes

- Calculate $F_2 = F_3 = m_2 g$
- Calculate the actual value of $F_1 = m_1 g$ in N from your measured value of m_1

Theoretical F_1 from N2L

This is the test of your setup and measurements: using the actual values of F_2, F_3, and θ, do N2L and trig give you just about the same magnitude F_1 for the balancing force as you got by calculating it directly from $F_1 = m_1 g$?

- Use CAH to calculate the magnitude of the horizontal tension components $F_{2x} = F_{3x}$
  - See the illustration of $\mathbf{F}_2$ and its components here:
  - **NOTE: θ is the acute angle you chose above - it's LESS than 40°**
- Since there's no F_4, and we already know that $F_{1y} = 0$ and $\theta_1 = 0°$, the only theoretical quantity to calculate is $F_1 = \vec{F}_{1x} = -(\vec{F}_{2x} + \vec{F}_{3x})$



Filling out the tables

Note that when filling out the data tables for the force components, you can figure out the signs for the "+/-" column from the diagram: if a force points up and left, then its *x* component points left (negative) and its *y* component points up (positive). Left is negative, right is positive, up is positive, down is negative.

The entries in the "Magnitude (N)" column are all positive, since magnitudes are *always* positive.

For example, if you calculate the magnitude F_{2x} = 3.88 N in Part 1, then since $\vec{F}_2$ points up and left, $\vec{F}_{2x} = -F_{2x} = -3.88\,N$. So you'd put "–" in the "+/-" column and "3.88" in the "Magnitude (N)" column for the $\vec{F}_{2x}$ row.

Double-check Part 1

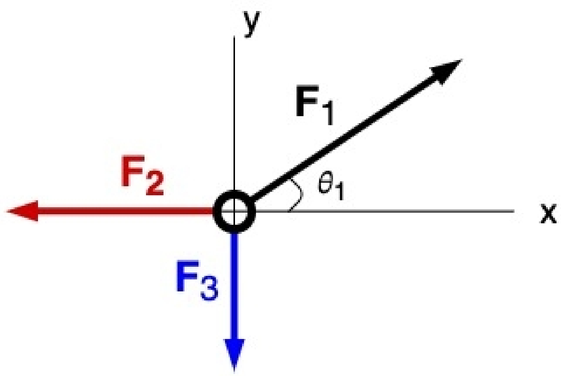
Before you move on to Parts 2-4, finish the analysis described above and check to see if the actual and theoretical F_1's are within about 0.1 N of each other. If not, run through your calculations again, have the instructor check them. If the calculations seem solid, then check the your setup for misaligned strings, pulleys, incorrect masses.

Parts 2-4: Experiment first

Once you're confident that you've done the Part 1 experiment correctly, proceed through the *Experiment* portions of Parts 2-4. Hold off on theoretical calculations until you've completed all measurements.

Part 2: Balancing two perpendicular forces

The known forces $\mathbf{F}_2$ and $\mathbf{F}_3$ have different magnitudes. $\mathbf{F}_2$ points in the –x direction, and $\mathbf{F}_3$ points in the –y direction. Both the magnitude F_1 and direction θ_1 of the balancing force $\mathbf{F}_1$ are unknown and to be found experimentally, then calculated from N2L



Part 2

Experiment

- $\mathbf{F}_2$ pulls in the –x direction: set pulley 2 at the 180° mark
- $\mathbf{F}_3$ pulls in the –y direction: set pulley 3 at the 270° mark
- Load up hanger 2 with 75 g, 95 g, 115 g, 135 g, 155 g or 175 g of disks
- Load up hanger 3 with 85 g, 105 g, 125 g, 145 g, or 165 g of disks
  - **BUT** make sure the difference between m_2 and m_3 is at least 40 g
- Make initial adjustments to get strings 2 and 3 as level and radial as possible
- Pick up string 1 and find out which direction you have to pull to center the ring

You're not picking a random direction, here, you're discovering which direction you have to pull to balance out the forces $\mathbf{F}_2$ and $\mathbf{F}_3$. Once you've located the direction that works,

- Attach pulley 1 at that degree mark
- Set the string down in pulley 1 and load up hanger 1 until the ring is centered
  - for final adjustments, use increments of 1-2 g
- Double-check and adjust as needed – see Part 1 ("**Adjust** …") for details
- Record: m_1, m_2, m_3, θ_1

Actual force magnitudes

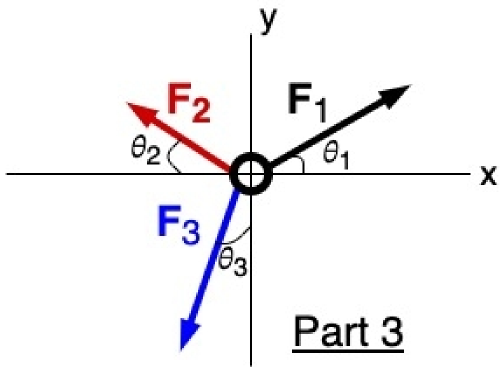
- Calculate F_1, F_2, F_3 in N from m_1, m_2, m_3

Theoretical F₁ , θ₁

- Fill out the data table to determine $\vec{F}_{1x} = -(\vec{F}_{2x} + \vec{F}_{3x})$ and $\vec{F}_{1y} = -(\vec{F}_{2y} + \vec{F}_{3y})$
  - No trig needed; F_2 and F_3 are each parallel to an axis. Which components are zero?
- Calculate $F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$
- Calculate $\theta_1 = \tan^{-1}(F_{1y} / F_{1x})$ or $\theta_1 = \arctan(F_{1y} / F_{1x})$

Part 3: Balancing two asymmetric forces

The known forces $\mathbf{F}_2$ and $\mathbf{F}_3$ have different magnitudes and point in different directions. Both the magnitude F_1 and direction θ_1 of the balancing force $\mathbf{F}_1$ are unknown and to be found experimentally, then calculated from N2L. An incomplete FBD for the ring:



Experiment

- $\mathbf{F}_2$ pulls **θ_2 = 27°** above the –x direction: set pulley 2 at the 180° – 27° = 153° mark
- $\mathbf{F}_3$ pulls **θ_3 = 18°** left of the –y direction: set pulley 3 at the 270° – 18° = 252° mark
- Load up
  - hanger 2 with 105 g, 115 g, 125 g, 135 g, or 145 g of disks
  - hanger 3 with 165 g, 175 g, 185 g, 195 g, 205 g or 215 g of disks
- Record m_2 and m_3 (disks + 5 g for the hanger)
- Make initial adjustments to get strings 2 and 3 as level and radial as possible
- Pick up string 1 and find out which direction you have to pull to center the ring
- Attach pulley 1 at that degree mark
- Set the string down in the pulley and load up hanger 1 until the ring is centered
  - for final adjustments, use increments of 1-2 g
- Double-check and adjust as needed – see Part 1 ("**Adjust** …") for details
- Record: m_1, θ_1

Actual force magnitudes

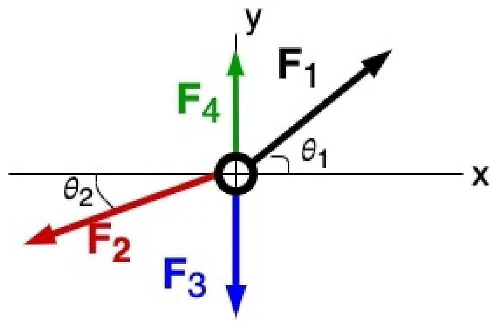
- Calculate F_1, F_2, F_3 in N from m_1, m_2, m_3

Theoretical F_1 , θ_1

- Use SOH CAH to calculate the x and y components of $\mathbf{F}_2$ and $\mathbf{F}_3$
  - **θ_2 = 27° and θ_3 = 18°, <u>not</u> the angles on the edge of the force table**
- Fill out the data table to determine $\vec{F}_{1x} = -(\vec{F}_{2x} + \vec{F}_{3x})$ and $\vec{F}_{1y} = -(\vec{F}_{2y} + \vec{F}_{3y})$
- Calculate $F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$
- Calculate $\theta_1 = \tan^{-1}(F_{1y} / F_{1x})$ or $\theta_1 = \arctan(F_{1y} / F_{1x})$

Part 4: Balancing three forces

The known forces **F**$_2$, **F**$_3$, **F**$_4$ have different magnitudes. **F**$_3$ and **F**$_4$ pull in the –y and +y directions, respectively, while **F**$_2$ pulls at an angle θ_2 below the –x axis. Both the magnitude F_1 and direction θ_1 of the balancing force **F**$_1$ are unknown and to be found experimentally, then theoretically.



Part 4

Experiment

- Choose an angle θ_2: 12°, 13°, 14°, 16°, 17°, 18°, 19°, 21°, or 22°
- **F**$_2$ pulls θ_2 degrees below the –x direction: set pulley 2 at the 180° + θ_2 mark
- **F**$_3$ pulls in the –y direction: set pulley 3 at the 270° mark
- **F**$_4$ pulls in the +y direction: set pulley 4 at the 90° mark
- Load up
  - hanger 2 with 170 g, 180 g, 190 g, 210 g, or 220 g of disks
  - hanger 3 with 110 g, 120 g, 130 g, or 140 g of disks
  - hanger 4 with 50 g, 60 g, or 70 g of disks
- Record total masses m_2, m_3, m_4 (disks + 5 g for the hanger)
- Make initial adjustments to get strings 2-4 as level and radial as possible
- Pick up string 1 and find out which direction you have to pull to center the ring
- Attach pulley 1 at that degree mark
- Set the string down in the pulley and load up hanger 1 until the ring is centered
  - for final adjustments, use increments of 1-2 g
- Double-check and adjust as needed – see Part 1 (“**Adjust** …”) for details
- m_1, θ_2 (<90°!), θ_1

Actual force magnitudes

- Calculate F_1, F_2, F_3, F_4 in N from m_1, m_2, m_3, m_4

Theoretical F$_1$, θ_1

- Use SOH CAH to calculate the *x* and *y* components of **F**$_2$
  - **θ_2 is the angle you chose**, <u>not</u> the degree mark you set pulley 2 at
- Fill out the data table to get $\vec{F}_{1x} = -(\vec{F}_{2x} + \vec{F}_{3x} + \vec{F}_{4x})$ and $\vec{F}_{1y} = -(\vec{F}_{2y} + \vec{F}_{3y} + \vec{F}_{4y})$
- Calculate $F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$
- Calculate $\theta_1 = \tan^{-1}(F_{1y} / F_{1x})$ or $\theta_1 = \arctan(F_{1y} / F_{1x})$

Prelab

Suppose a team works through Part 1 but misreads the required masses, using $m_2 = m_3 = 194$ g, and obtaining $m_1 = 269$ g. They also misread the allowed angles and use $\theta = 47°$. Use their data to fill out the table below. The entries in the "Magnitude (N)" column should all be positive, since magnitudes are positive by definition; convert masses to kg before calculating force magnitudes. The entries in the "+/-" column should be + for vectors that point right, and – for vectors that point left.

$m_1 = 269\ g$ $\qquad m_2 = m_3 = 194$ g $\qquad \theta = 47°$

$F_2 = F_3 = m_2 g =$ ________

	+/-	Magnitude (N)
$\boldsymbol{F}_{2x}$		
$\boldsymbol{F}_{3x}$		
$\boldsymbol{F}_{2x} + \boldsymbol{F}_{3x}$		
$\boldsymbol{F}_{1x}$		

Actual F_1 (m_1g) = ________ Theoretical F_1 = ________

Report

Data (Parts 1-4, 2 points each)

Actual and theoretical F_1's should be within about 0.1 N of each other, and actual and theoretical θ_1's should be within about 2° of each other. For full credit, ask for help if they're farther apart and you can't find and fix the errors on your own.

Work (2 points)

For this section, pick either Part 3 OR Part 4, and show the work described below.

- ☐ Title this section according to your choice: *Part 3* or *Part 4*

- ☐ Force components: write out the SOH CAH equations and show how you calculated
  - *Part 3:* F_{2x}, F_{2y}, F_{3x}, F_{3y} (angles < 90°!!)
  - *Part 4:* F_{2x}, F_{2y} (θ_2 < 90°!!)

- ☐ Theoretical calculations: starting with equations, show how you calculated
  - F_1
  - θ_1

Graded Prelab

LAB 4 DATA

Part 1

m_1 = _________ $m_2 = m_3$ = _________ θ = ____

$F_2 = F_3 = m_2 g$ = _________

	+/-	Magnitude (N)
$\boldsymbol{F}_{2x}$		
$\boldsymbol{F}_{3x}$		
$\boldsymbol{F}_{2x} + \boldsymbol{F}_{3x}$		
$\boldsymbol{F}_{1x}$		

Actual F_1 (m_1g) = _________ Theoretical F_1 = _________

Part 2

m_1 = _________ m_2 = _________ m_3 = _________

$F_2 = m_2 g$ = _________ $F_3 = m_3 g$ = _________

	+/-	Magnitude (N)		+/-	Magnitude (N)
$\boldsymbol{F}_{2x}$			$\boldsymbol{F}_{2y}$		
$\boldsymbol{F}_{3x}$			$\boldsymbol{F}_{3y}$		
$\boldsymbol{F}_{2x} + \boldsymbol{F}_{3x}$			$\boldsymbol{F}_{2y} + \boldsymbol{F}_{3y}$		
$\boldsymbol{F}_{1x}$			$\boldsymbol{F}_{1y}$		

Actual F_1 (m_1g) = _________ Theoretical F_1 = _________

Actual θ_1 = ____ Theoretical θ_1 = ____

Part 3

m_1 = __________ m_2 = __________ m_3 = __________

$F_2 = m_2g$ = __________ $F_3 = m_3g$ = __________

	+/-	Magnitude (N)		+/-	Magnitude (N)
$\boldsymbol{F}_{2x}$			$\boldsymbol{F}_{2y}$		
$\boldsymbol{F}_{3x}$			$\boldsymbol{F}_{3y}$		
$\boldsymbol{F}_{2x} + \boldsymbol{F}_{3x}$			$\boldsymbol{F}_{2y} + \boldsymbol{F}_{3y}$		
$\boldsymbol{F}_{1x}$			$\boldsymbol{F}_{1y}$		

Actual F_1 (m_1g) = __________ Theoretical F_1 = __________

Actual θ_1 = _____ Theoretical θ_1 = _____

Part 4

m_1 = ________ m_2 = ________ m_3 = ________ m_4 = ________

$F_2 = m_2g$ = ______ $F_3 = m_3g$ = ________ $F_4 = m_4g$ = ________ θ_2 = _____

	+/-	Magnitude (N)		+/-	Magnitude (N)
$\boldsymbol{F}_{2x}$			$\boldsymbol{F}_{2y}$		
$\boldsymbol{F}_{3x}$			$\boldsymbol{F}_{3y}$		
$\boldsymbol{F}_{4x}$			$\boldsymbol{F}_{4y}$		
$\boldsymbol{F}_{2x} + \boldsymbol{F}_{3x} + \boldsymbol{F}_{4x}$			$\boldsymbol{F}_{2y} + \boldsymbol{F}_{3y} + \boldsymbol{F}_{4y}$		
$\boldsymbol{F}_{1x}$			$\boldsymbol{F}_{1y}$		

Actual F_1 (m_1g) = __________ Theoretical F_1 = __________

Actual θ_1 = _____ Theoretical θ_1 = _____