1) Область сходимости ряда?

$$\sum_{n=1}^{\infty} \ln^n(1+x^2).$$

$$\lim_{n\to\infty}\left|\frac{f_{n+1}(x)}{f_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\ln^{n+1}(1+x^2)}{\ln^n(1+x^2)}\right| =$$

$$= \lim_{n\to\infty}\left|\ln(1+x^2)\right| = \ln(1+x^2)$$

$$\ln(1+x^2) < 1$$

$$1+x^2 < e$$

$$x^2 < e-1$$

$$-\sqrt{e-1} < x < \sqrt{e-1}$$

1) $x = -\sqrt{e-1}$: $\sum_{n=1}^{\infty} \ln^n(1+e-1) = \sum_{n=1}^{\infty} \ln^n(e) \Rightarrow$

$\Rightarrow \lim_{n\to\infty} \ln^n(e) = 1 \; != 0 \to$ расход.

2.) $x = \sqrt{e-1}$: аналогично расход.

$$x \in (-\sqrt{e-1}; \sqrt{e-1})$$

$$l = 2\cdot\sqrt{e-1} = 2.62.$$

2) D – обл. сход., cnt чисел $\in D \cap [-1000; 1000]$

$$\sum_{k=1}^{\infty} \frac{1}{1+x^k}; \quad \lim_{k\to\infty}\left|\frac{\frac{1}{1+x^{k+1}}}{\frac{1}{1+x^k}}\right| = \lim_{k\to\infty}\left|\frac{1+x^k}{1+x^{k+1}}\right| =$$

$$= \lim_{k\to\infty}\left|\frac{1+x^k}{x\left(\frac{1}{x}+x^k\right)}\right| = \frac{1}{|x|}.$$

$$\frac{1}{|x|} < 1$$

$$|x| > 1 \iff \left[\begin{array}{l} x < -1 \\ x > 1 \end{array}\right.$$

1. $x = -1$: $\sum_{k=1}^{\infty} \frac{1}{1+(-1)^k}$ – иногда $1/0 \Rightarrow$ расход.

2. $x = 1$: $\sum_{k=1}^{\infty} \frac{1}{2} \Rightarrow \lim_{k\to\infty} \neq 0 \Rightarrow$ расход.

$$x \in (-\infty; -1) \cup (1; +\infty)$$

$$cnt = \underbrace{(1000+1000+1)}_{D} - (\underbrace{1}_{-1}+\underbrace{1}_{0}+\underbrace{1}_{1}) = 1998$$

3) Промежуток сходимости?

$$\sum_{n=0}^{\infty} \frac{n\cdot x^n}{4^n(n^2+1)};$$

$$\lim_{n\to\infty}\left|\frac{f_{n+1}(x)}{f_n(x)}\right| : \lim_{n\to\infty}\left|\frac{(n+1)\cdot x^{n+1}\cdot 4^n(n^2+1)}{n\cdot x^n\cdot 4^{n+1}((n+1)^2+1)}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)\cdot x\,(n^2+1)}{4n\,(n^2+2n+3)}\right| = \frac{|x|}{4};$$

$$\frac{|x|}{4} < 1 \to |x| < 4 \to x \in (-4; 4)$$

1. $x = 4$: $\sum_{n=0}^{\infty} \frac{n\cdot 4^n}{4^n(n^2+1)} = \sum_{n=0}^{\infty} \frac{n}{n^2+1} < \sum \frac{1}{n}$ – расходится.

2. $x = -4$: $\sum_{n=0}^{\infty} \frac{n\cdot(-4)^n}{4^n\cdot(n^2+1)}$;

Лейбниц: * знакочеред.

1. $\lim_{n\to\infty}\left|\frac{n(-4)^n}{4^n(n^2+1)}\right| = 0$ – убыв. по модулю.

2. $|a_{n+1}| < |a_n|$: $\frac{(n+1)\,4^{n+1}}{4^{n+1}\cdot((n+1)^2+1)} < \frac{n\cdot 4^n}{4^n(n^2+1)}$ ✓

сходится.

$$x \in [-4; 4)$$

4) $\sum_{n=0}^{\infty} (-2)^n(n+1)(x-1)^n$ like 3).

$$\lim_{n\to\infty}\left|\frac{(-2)^{n+1}(n+2)(x-1)^{n+1}}{(-2)^n(n+1)(x-1)^n}\right| = |2(x-1)|$$

$$|2(x-1)| < 1$$

$$|x-1| < 1/2$$

$$x \in (1/2; 3/2)$$

1. $x = 1/2$: $\sum_{n=0}^{\infty} (-2)^n(n+1)(-1/2)^n =$

$= \sum_{n=0}^{\infty} (n+1) \Rightarrow \lim_{n\to\infty} != 0 \Rightarrow$ расход.

2. $x = 3/2$: $\sum_{n=0}^{\infty} (-2)^n(n+1)(1/2)^n =$

$= \sum_{n=0}^{\infty} (-1)^n(n+1) \Rightarrow \lim != 0$ – расход.

$$x \in (1/2; 3/2)$$

5) $f(x) = (1+2x)^{1/5}$; $k\,x^4$ – ?

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$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k, \quad x_0 = 0 \iff \sum \frac{f^{(k)}(0)}{k!} x^k$$

$$\sum_{k=0}^{\infty} = 1 + \ldots - \frac{16\cdot 18\cdot 28}{625\cdot 4!}x^4 + \ldots$$

$$-\frac{\cancel{16}^{\,4}\cdot\cancel{18}^{\,3}\cdot 28}{625\cdot\cancel{4}\cdot\cancel{3}\cdot 2\cdot 1} = -\frac{336}{625}$$

$$f'(0) = \frac{1}{5}\cdot\frac{1}{(1+2x)^{4/5}}\cdot 2 = 2/5$$

$$f''(0) = 2/5\cdot(-4/5)\,\frac{1}{(1+2x)^{5/5}}\cdot 2 =$$

$$= -\frac{16}{25}$$

$$f'''(0) = -\frac{16}{25}\cdot(-9/5)\cdot 2\cdot\frac{1}{(1+2x)^{14/5}} =$$

$$= \frac{16\cdot 18}{125}$$

$$f^{(4)}(0) = \frac{16\cdot 18}{125}\cdot(-14/5)\cdot 2\cdot\frac{1}{(1+2x)^{19/5}} =$$

$$= -\frac{14\cdot 16\cdot 18\cdot 2}{625}$$