① $y' = \frac{2xy^2}{1-x^2}$; $y(0)=1$; $y(0.9)$ - ?

$\frac{dy}{dx} = \frac{2xy^2}{1-x^2}$; $\frac{dy}{y^2} = \frac{2x \cdot dx}{1-x^2}$; $\int \frac{1}{y^2} dy = \int \frac{2x}{1-x^2} dx$; $/ u = 1-x^2$; $du = -2x \cdot dx$ /

$-\frac{1}{y} = \int -\frac{1}{u} du$

$-\frac{1}{y} = -\ln(|1-x^2|)$;

$\frac{1}{y} = \ln(|x^2-1|) + C$

$y = \frac{1}{\ln(|x^2-1|) + C}$

$y(0) = \frac{1}{\ln(1) + C} \rightarrow C = 1$

$y(0.9) = \frac{1}{\ln(1-0.81)+1} = -1.51$

② $y' = y^3$

$\frac{dy}{dx} = y^3$

$\frac{dy}{y^3} = 1/dx$

$\int \frac{dy}{y^3} = \int 1/dx$

$-\frac{1}{2y^2} = x + C$

$y = \sqrt{\frac{-1}{2(x+C)}} \rightarrow C = -1 \rightarrow \varphi(-1) = 1/2$

③ $dx - (2+x)xy\,dy = 0$

$\frac{dx}{(2+x)x} = y\,dy$

$\int \frac{1}{(2+x)x} dx = \int y \cdot dy$

$\int \frac{1}{(x+1)^2-1} dx = \frac{y^2}{2}$

$\frac{-\ln\left|\frac{x+2}{x}\right|}{2} = \frac{y^2}{2} + C$

$-y^2 + C = \ln\left|\frac{x+2}{x}\right|$

$e^{-y^2+C} = \left|\frac{x+2}{x}\right|$

$e^{-y^2+C} - 1 = 2/x \Rightarrow x = \frac{2}{e^{-y^2+C} - 1} = 2$

$e^C = 2 \rightarrow C = \ln(2)$

$x(1/2) = \frac{2}{e^{-1/4+\ln(2)} - 1} =$

④ $3e^x \cdot \sin(y)\,dx + \frac{(2-e^x)dy}{\cos(y)} = 0$

$\frac{3e^x}{2-e^x} dx = \frac{-dy}{\sin(y)\cos(y)}$

$\int \frac{3e^x}{2-e^x} dx = 3\int \frac{e^x}{2-e^x} dx = -3 \int \frac{d(2-e^x)}{2-e^x} = -3\ln(|2-e^x|)$

$\int \frac{-2dy}{\sin(2y)} = -2\int \frac{dy}{\sin(2y)} = -2\int \frac{\sin^2(y)+\cos^2(y)}{2\sin(y)\cos(y)} dy =$ $/ t = \sin y$; $dt = \cos y\,dy$ /

$= -2\int\left(\frac{1}{2}tg(y) + 1/2\,ctg(y)\right)dy = -\int(tg(y)+ctg(y))dy =$ $/ u = \cos y$; $du = -\sin y \cdot dy$ / $= -\left(\int -1/u\,du + \int 1/t\,dt\right) =$

$= -\left(-\ln|\cos y| + \ln|\sin y|\right) = \ln|\cos(y)| - \ln|\sin(y)| = \ln|ctg(y)|$

$-3\ln(|2-e^x|) = \ln(|ctg(y)|) \Rightarrow \ln|2-e^x|^3 = \ln|tg(y)| + C$

$\ln|2-e^{\ln 3/2}|^3 = \ln|tg(arctg(1/2))| + C \rightarrow C = 0$

$x = 0$: $0 = \ln|tg(y)| + 0$

$\ln|tg(y)| = 0$

$|tg(y)| = 1$; $y = \frac{\pi/4 + \pi k}{2}$

$y = 1/4 + k/2$

⑤ $y' = \sqrt{y-6}$

$\frac{dy}{dx} = \sqrt{y-6}$

$\frac{dy}{\sqrt{y-6}} = dx$

$2\sqrt{y-6} = x + C$

$y = \frac{(x+C)^2}{4} + 6$;

$y(4)$: $\left[\begin{array}{l} C = -4(1+\sqrt{2}) \\ C = 4(\sqrt{2}-1) \end{array}\right.$ $\quad -4 - 4\sqrt{2}$; $4\sqrt{2} - 4$

$y(-3)$: $\frac{(-3-4(1+\sqrt{2}))^2}{4} + 6 = 17/4 - \sqrt{2}$

$\ldots = 17/4 + \sqrt{2}$