25B

① $\sin y = x y^2 + 5$

y' $\quad (\sin y)' = (x y^2 + 5)'$

$y' (\cos y) = 1 \cdot y^2 + 2y y' x$

$y' (\cos y - 2yx) = y^2$

$$y' = \frac{y^2}{\cos y - 2yx}$$

y'': $\quad \left(\frac{y^2}{\cos y - 2xy}\right)' = \frac{(y^2)'(\cos y - 2xy) - (\cos y - 2xy)' y^2}{(\cos y - 2xy)^2} = \frac{2y\, y' (\cos y - 2xy) - (-y' \sin y - 2(1 \cdot y + y' x)) y^2}{(\cos y - 2xy)^2} =$

$$= \frac{y\left(2y'(\cos y - 2xy) + (y' \sin y + 2y + 2x y')\, y\right)}{(\cos y - 2xy)^2} = \frac{y\left(\frac{2y^2(\cos y - 2yx)}{\cos y - 2xy} + \left(\frac{y^2 \cdot (\sin y + 2x)}{\cos y - 2xy} + 2y\right) y\right)}{(\cos y - 2xy)^2}$$

② $\begin{cases} x = e^{-3t} \\ y = e^{8t} \end{cases}$

$y' = \frac{y_t'}{x_t'}$ $\qquad x_t' = (e^{-3t})' = -3 e^{-3t}$

$y'' = \frac{y_t''}{x_t'}$ $\qquad y_t' = (e^{8t})' = 8 e^{8t}$

$$y' = \frac{8 e^{8t}}{-3 e^{-3t}} = -\frac{8}{3} e^{11t}$$

$$y_t'' = \left(-\frac{8}{3} e^{11t}\right)' = -\frac{88}{3} e^{11t} \quad \Rightarrow \quad y'' = \frac{-\frac{88}{3} e^{11t}}{-3 e^{-3t}} = 88 e^{14t}$$

③ $y = 2^{x^2}$, $x_0 = 1$

$y' = 2^{x^2} \ln 2 \cdot 2x = 2^{x^2+1} \cdot x \ln 2$,

$y'' = \ln 2 \left(x' 2^{x^2+1} + (2^{x^2+1})' x\right) = \ln 2 \left(2^{x^2+1} + 2x \cdot 2^{x^2+1} \ln 2 \cdot x\right) = \ln 2\, 2^{x^2+1} + \ln^2 2\, x^2\, 2^{x^2+1}$;

$y''' = \ln 2 \cdot 2x \ln 2\, 2^{x^2+1} + \ln^2 2 \left(2x\, 2^{x^2+2} + 2x \ln 2\, 2^{x^2+1} x^2\right) = \ln^2 2 \cdot x\, 2^{x^2+2} + \ln^2 2 \left(x\, 2^{x^2+3} + x^3 \ln 2\, 2^{x^2+3}\right)$

$y'''(x_0) = \ln^2 2 \cdot 8 + \ln^2 2 \left(8 + 16 + 1 \ln 2 \cdot 16\right) = \ln^2 2 (8 + 16(\ln 2 + 1)) = 8 \ln^2 2 (2 \ln 2 + 3)$

④ $y = \sqrt{x+7}$. $\qquad y' = \frac{1}{2} (x+7)^{-1/2}$,

$y'' = -\frac{1}{2} \cdot \frac{1}{2} (x+7)^{-3/2}$, $\qquad y''' = -\frac{1}{2} \cdot \frac{1}{2} \left(-\frac{3}{2}\right) (x+7)^{-5/2}$

$$y^{(n)} = \frac{1 \cdot (-1) \cdot (-3) \cdot \ldots \cdot (3-2n)}{2^n} \cdot (x+7)^{-\frac{2n-1}{2}}$$

⑤ $y = 5x^2 - 6x + 1$, $\qquad x - 6y_1 + 15 = 0 \Rightarrow y_1 = -\frac{x}{6} - \frac{15}{x}$

$g = y(x_0) + y'(x_0)(x - x_0)$. $\quad g = 5x_0^2 - 6x_0 + 1 + (10x_0 - 6)(x - x_0) \Rightarrow$

$\Rightarrow g = (10x_0 - 6)x - 5x_0^2 + 1$

$y_1 \perp g \Rightarrow (10x_0 - 6)\left(-\frac{1}{6}\right) = -1$

$10x_0 - 6 = 6$

$10x_0 = 12$

$x_0 = 6/5$

⑥ $x_1 = 2t^3 - 2t^2 + 6t - 7 \Rightarrow v_1 = x_1' = 6t^2 - 4t + 6$

$x_2 = \frac{5}{3}t^3 - 1t^2 + 14t + 9 \Rightarrow v_2 = x_2' = 5t^2 - 2t + 14$

$\Rightarrow v_1 = v_2 \Rightarrow t^2 - 2t - 8 = 0$

$\left[\begin{array}{l} t = 4 \\ t = -2 \end{array}\right. \rightarrow t = 4c$