① $f(x) = e^{\operatorname{tg} x}$; $S_3(x) - ?$; $|S_3(x) - f(x)|$, $x = \pi/8$, $\varepsilon = 10^{-3}$

$$\sum \frac{f^n(x_0)}{n!}(x - x_0)^n \qquad \# f(0) = 1$$

$$f'(x) = \left(e^{\operatorname{tg} x}\right)' = e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x} ; \qquad \# f'(0) = 1$$

$$f^2(x) = \left(e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}\right)' = \left(e^{\operatorname{tg} x}\right)' \cdot \frac{1}{\cos^2 x} + \left(\frac{1}{\cos^2 x}\right)' \cdot e^{\operatorname{tg} x} =$$

$$= e^{\operatorname{tg} x}\left(\frac{1}{\cos^4 x} + 2 \cdot \left(\frac{1}{\cos x}\right)' \cdot \frac{1}{\cos x}\right)$$

$$= e^{\operatorname{tg} x}\left(\frac{1}{\cos^4 x} + 2 \cdot \frac{\sin x}{\cos^2 x} \cdot \frac{1}{\cos x}\right) = e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}\left(\frac{1}{\cos^2 x} + 2\operatorname{tg} x\right) \qquad \# f^2(0) = 1 \cdot 1 = 1$$

$$f^3(x) = \left(e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x} \cdot \left(\frac{1}{\cos^2 x} + 2\operatorname{tg} x\right)\right)' = \left(e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}\right)' \cdot \left(\frac{1}{\cos^2 x} + 2\operatorname{tg} x\right) + \left(e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}\right)\left(\frac{1}{\cos^2 x} + 2\operatorname{tg} x\right)' =$$

$$= e^{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}\left(\left(\frac{1}{\cos^2 x} + 2\operatorname{tg} x\right)^2 + \frac{2\operatorname{tg} x}{\cos^2 x} + \frac{2}{\cos^2 x}\right) \qquad \# f^3(0) = 1 \cdot (1^2 + 2) = 3$$

$$S_3 = 1 + 1 \cdot x + \frac{1 \cdot x^2}{2!} + \frac{3x^3}{3!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{2}$$

$$|S_3(x) - f(x)| = \left|1 + x + \frac{x^2}{2} + \frac{x^3}{2} - e^{\operatorname{tg} x}\right|, \quad x = \pi/8$$

$$\left|1 + \pi/8 + \pi^2/128 + \pi^3/512 - e^{\operatorname{tg} \pi/8}\right| = \underline{0.013}$$

② $f(x) = \int_0^x \frac{e^{-t^2} - 1}{t}\, dt$

$$e^x = 1 + \frac{x}{1!} + \ldots + \frac{x^n}{n!} ;$$

$$e^{-t^2} = 1 + \frac{-t^2}{1!} + \ldots + \frac{(-t^2)^n}{n!}$$

$$e^{-t^2} - 1 = \frac{-t^2}{1!} + \ldots + \frac{(-t^2)^n}{n!}$$

$$\frac{e^{-t^2} - 1}{t} = \frac{1}{t}\left(\frac{-t^2}{1!} + \ldots + \frac{(-t^2)^n}{n!}\right) = f'(x)$$

$$\int_0^x \frac{1}{t}\left(\frac{-t^2}{1!} + \ldots + \frac{(-t^2)^n}{n!}\right) dt = f(x) =$$

$$n = 9 : \int_0^x \frac{1}{t}\frac{(-t^2)^9}{9!}\, dt = \int_0^x \frac{-t^{17}}{9!}\, dt = \frac{-1}{9!}\int_0^x t^{17}\, dt = \underline{\frac{-1}{9! \cdot 18}}\, x^{18}$$

③ $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx ; \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot \cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

$f(x) = |\cos x|$;

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x|\, dx = \frac{2}{\pi}\int_0^{\pi} |\cos x| = \frac{4}{\pi}\int_0^{\pi/2} |\cos x|\, dx = \frac{4}{\pi}\int_0^{\pi/2} \cos x\, dx = \frac{4}{\pi}\sin x \Big|_0^{\pi/2} = \frac{4}{\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x| \cdot \sin nx\, dx \quad - \text{ четная } \Rightarrow b_n = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x| \cdot \cos nx\, dx = \frac{2}{\pi}\int_0^{\pi} |\cos x| \cdot \cos nx = \frac{2}{\pi}\left(\int_0^{\pi/2} \cos x \cdot \cos nx\, dx - \int_{\pi/2}^{\pi} \cos x \cdot \cos nx\, dx\right) =$$

$$= \frac{2}{\pi}\left(\int_0^{\pi/2} \frac{1}{2}(\cos(x - nx) + \cos(x + nx))\, dx - \int_{\pi/2}^{\pi} \frac{1}{2}(\cos(x - nx) + \cos(x + nx))\, dx\right) =$$

$$= \frac{1}{\pi}\left(\int_0^{\pi/2} (\cos(x - nx) + \cos(x + nx))\, dx - \int_{\pi/2}^{\pi} (\cos(x - nx) + \cos(x + nx))\, dx\right) =$$

$$= \frac{1}{\pi}\left(\int_0^{\pi/2} \cos(x - nx)\, dx + \int_0^{\pi/2} \cos(x + nx)\, dx - \int_{\pi/2}^{\pi} \cos(x - nx)\, dx - \int_{\pi/2}^{\pi} \cos(x - nx)\, dx\right) =$$

$/ * \int_0^{\pi/2} \cos(x - nx)\, dx$;

$t = x(n-1)$

$dt = (n-1)\, dx$

$dx = \frac{dt}{n-1}$ $*/$

$$= \frac{1}{\pi}\left(\frac{\sin(x(n-1))}{n-1}\Big|_0^{\pi/2} + \frac{\sin(x(n+1))}{n+1}\Big|_0^{\pi/2} - \frac{\sin(x(n-1))}{n-1}\Big|_{\pi/2}^{\pi} - \frac{\sin(x(n+1))}{n+1}\Big|_{\pi/2}^{\pi}\right) =$$

$$= \frac{1}{\pi}\left(\frac{\sin\left(\frac{\pi}{2}(n-1)\right)}{n-1} + \frac{\sin\left(\frac{\pi}{2}(n+1)\right)}{n+1} - \left(\frac{\sin(\pi(n-1))}{n-1} - \frac{\sin\left(\frac{\pi}{2}(n-1)\right)}{n-1}\right)\right.$$

$$\frac{\sin\left(\frac{\pi}{2}(n-1)\right)}{n-1} = -\frac{\cos\left(\frac{\pi n}{2}\right)}{n-1} \qquad \frac{\sin\left(\frac{\pi}{2}(n+1)\right)}{n+1} = \frac{\cos\left(\frac{\pi n}{2}\right)}{n+1} \qquad \sin(\pi(n-1)) = 0 : \pi \qquad *(1)$$

$$\left.- \left(\frac{\sin(\pi(n+1))}{n+1} - \frac{\sin\left(\frac{\pi}{2}(n+1)\right)}{n+1}\right)\right) =$$

$$\sin(\pi(n+1)) = 0 : \pi \qquad *(2)$$

$$= \frac{1}{\pi}\left(-\frac{2\cos\left(\frac{\pi n}{2}\right)}{n-1} + \frac{2\cos\left(\frac{\pi n}{2}\right)}{n+1}\right) = \frac{2}{\pi}\left(\frac{\cos\left(\frac{\pi n}{2}\right)}{n+1} - \frac{\cos\left(\frac{\pi n}{2}\right)}{n-1}\right) =$$

$$= \frac{2}{\pi} \cdot \frac{n\cos\left(\frac{\pi n}{2}\right) - \cos\frac{\pi n}{2} - n\cos\left(\frac{\pi n}{2}\right) - \cos\frac{\pi n}{2}}{(n+1)(n-1)} =$$

$$= \frac{-4\cos\left(\frac{\pi n}{2}\right)}{\pi(n+1)(n-1)}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cdot \cos(nx) + b_n \sin(nx) \right) =$$

$$= \frac{2}{\pi} + \sum_{n=1}^{\infty} -\frac{4\cos\frac{\pi n}{2}}{\pi(n+1)(n-1)} \cdot \cos(nx) = \frac{2}{\pi} - \frac{4}{\pi} \overbrace{\sum_{n=1}^{\infty} \frac{\cos\frac{\pi n}{2}}{(n+1)(n-1)} \cdot \cos(nx)}^{S_6} =$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left(0 + \frac{-1}{3} \cdot \cos(2x) + 0 + \frac{1}{5 \cdot 3} \cdot \cos(4x) + 0 + \frac{-1}{7 \cdot 5} \cdot \cos(6x)\right) =$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{-\cos(2x)}{3} + \frac{\cos 4x}{15} - \frac{\cos 6x}{35} \right);$$

$$\left| S_6(0.5) - f(0.5) \right| = \left| \frac{2}{\pi}\left(1 - 2\left(\frac{-\cos 1}{3} + \frac{\cos 2}{15} - \frac{\cos 3}{35}\right)\right) - \cos(0.5) \right| =$$

$$= 0.012$$

$$\pi \cdot a_{20}(f) = -\frac{4}{\pi} \cdot \frac{\cos(10\pi) \cdot \cos(20x)}{21 \cdot 19} = -\frac{4}{\pi} \cdot \frac{\cos(20x)}{21 \cdot 19}$$