① $y''' + 2y'' - 20y' + 24y = 0$

$\alpha^3 + 2\alpha^2 - 20\alpha + 24 = 0$

$(\alpha-2)(\alpha^2+4\alpha-12) = 0$

$(\alpha-2)^2(\alpha+6) = 0$

$y = c_1 e^{2x} + c_2 e^{2x} x + c_3 e^{-6x}$

$y(0) = c_1 + c_3 = 1/2$

$y' = 2c_1 e^{2x} + c_2(e^{2x}\cdot 2x + e^{2x}) - 6\cdot c_3 \cdot e^{-6x}$

$y'(0) = 2c_1 + c_2 - 6c_3 = 13/2$

$y'' = 4c_1 e^{2x} + c_2(2\cdot e^{2x}\cdot 2x + 2\cdot e^{2x}) + 2c_2\cdot e^{2x} + 36 c_3 e^{-6x}$

$y'''(0) = 4c_1 + 4c_2 + 36 c_3 = -8$

$$\begin{pmatrix} 1 & 0 & 1 & 1/2 \\ 2 & 1 & -6 & 13/2 \\ 4 & 4 & 36 & -8 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & 1 \\ 4 & 2 & -12 & 13 \\ 1 & 1 & 9 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 9 & -2 \\ 0 & -2 & -16 & 5 \\ 0 & -2 & -48 & 21 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 9 & -2 \\ 0 & -2 & -16 & 5 \\ 0 & 0 & -32 & 16 \end{pmatrix}$$

$\begin{cases} C_3 = -1/2 \\ C_2 = 3/2 \\ C_1 = 1 \end{cases}$

$y = e^{2x} + 3/2\, e^{2x}\cdot x - 1/2\, e^{-6x}$

$y(1/2) = e + 3/2 \cdot e \cdot 1/2 - 1/2 \cdot e^{-3} \approx 4.7321$

② $y^{(5)} - 2y^{(4)} + 2y^{(3)} - 4y'' + y' - 2y = 0$

$\alpha^5 - 2\alpha^4 + 2\alpha^3 - 4\alpha^2 + \alpha - 2 = 0$

$(\alpha-2)(\alpha^2+1)^2 = 0$

$\left[\begin{array}{l} \alpha = 2 \\ \alpha = \pm i \\ \alpha = \pm i \end{array}\right. \Rightarrow y = c_1 e^{2x} + e^{0x}(c_2 \cdot \cos(1x) + c_3 \cdot \sin(1x)) + e^{0x}\cdot x(c_4 \cdot \cos(1x) + c_5 \cdot \sin(1x)) =$

$y = c_1 e^{2x} + c_2 \cdot \cos(x) + c_3 \cdot \sin(x) + x(c_4 \cdot \cos(x) + c_5 \cdot \sin(x))$

$y(0) = c_1 + c_2 = 0$

$y' = 2c_1 \cdot e^{2x} - c_2 \cdot \sin(x) + c_3 \cdot \cos(x) + c_4(-x\cdot\sin(x) + \cos(x)) + c_5(x\cdot\cos(x) + \sin(x))$

$y'(0) = 2C_1 + C_3 + C_4 = 0$

$y'' = 4c_1 e^{2x} - c_2 \cdot \cos(x) - c_3 \cdot \sin(x) + c_4(-x\cdot\cos(x) - 2\sin(x)) + c_5(-x\cdot\sin(x) + 2\cos(x))$

$y''(0) = 4C_1 - C_2 + 2C_5 = -3$

$y''' = 8C_1 \cdot e^{2x} + c_2 \cdot \sin(x) - c_3 \cdot \cos(x) + c_4(x\cdot\sin(x) - 3\cos(x)) + c_5(-x\cdot\cos(x) - 3\sin(x))$

$y'''(0) = 8C_1 - C_3 - 3C_4 = 0$

$y^{IV} = 16 C_1 \cdot e^{2x} + c_2 \cdot \cos(x) + c_3 \cdot \sin(x) + c_4(x\cdot\cos(x) + 4\sin(x)) + c_5(x\cdot\sin(x) - 4\cos(x))$

$y^{IV}(0) = 16C_1 + C_2 - 4C_5 = 6$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 4 & -1 & 0 & 0 & 2 & -3 \\ 8 & 0 & -1 & -3 & 0 & 0 \\ 16 & 1 & 0 & 0 & -4 & 6 \end{array}\right) \Rightarrow (0\ 0\ 0\ 0\ -3/2) \quad ; \quad y = -3/2\, x \cdot \sin(x)$$

$y(4) = -6\sin(4) \approx 4.54$

⑤ $\begin{cases} \dot{x} = -2x - 3y & (1) \\ \dot{y} = 6x + 7y & (2) \end{cases}$

$(2)\quad x = \dot{y}/6 - 7/6\, y$

$\dot{x} = \ddot{y}/6 - 7/6\, \dot{y}$

$\Downarrow$ в (1)

$\ddot{y}/6 - 7/6\,\dot{y} = -\dot{y}/3 + 7/3\, y - 3y$

$\ddot{y} - 7\dot{y} = -2\dot{y} + 14y - 18y$

$\ddot{y} - 5\dot{y} + 4y = 0$

$\alpha^2 - 5\alpha + 4\alpha = 0$

$y = c_1 e^t + c_2 e^{4t}$

$x = (c_1 e^t + 4c_2 e^{4t})/6 - 7/6\, y =$

$= \frac{-6c_1 e^t - 3c_2 e^{4t}}{6} = -c_1 e^t - 1/2\, c_2 e^{4t}$

$x(0) = -c_1 - 1/2\, c_2 = -3 \qquad \begin{cases} c_1 = 2 \\ c_2 = 2 \end{cases}$

$y(0) = c_1 + c_2 = 4$

$x(\ln(2)) = -2\cdot 2 - 1/2 \cdot 2 \cdot 2^4 = -20$

$y(\ln(2)) = 2\cdot 2 + 2\cdot 2^4 = 36$

③ $2y'' - 11y' - 6y = -48x^2 - 170x + 43$

$2\alpha^2 - 11\alpha - 6 = 0 \qquad \left[\begin{array}{l} \alpha = -1/2 \\ \alpha = 6 \end{array}\right.$

$y_o = c_1 e^{-1/2 x} + c_2 e^{6x}$

$\begin{cases} y = Ax^2 + Bx + C \\ y' = 2Ax + B \\ y'' = 2A \end{cases} \Rightarrow 4A - 22Ax - 11B - 6Ax^2 - 6Bx - 6C = -48x^2 - 170x + 43$

$x^2(48 - 6A) + x(-22A - 6B + 170) + (4A - 11B - 6C - 43) = 0$

$\begin{cases} A = 8 \\ B = -1 \\ C = 0 \end{cases} \qquad y_ч = 8x^2 - x$

$y = 8x^2 - x + c_1 e^{-1/2 x} + c_2 e^{6x}$

$y(0) = c_1 + c_2 = 1$

$y/x^2$ — ограничена при $x \to \infty$

$C_2 = 0$, $C_1 = 1 \Rightarrow y = 8x^2 - x + e^{-1/2 x}$

$y(2) = 30.36$

④ $y'' + y = -ctg^2(x)$; $y(\pi/2) = 2$, $y'(\pi/2) = 0$; $y(\pi/3)$ − ?

$\alpha^2 + 1 = 0$

$y = (C_1 \cdot \sin(x) + C_2 \cdot \cos(x)) \cdot e^0 = c_1 \cdot \sin(x) + c_2 \cdot \cos(x)$

$y = z_1(x) \cdot \sin(x) + z_2(x) \cdot \cos(x)$

$\begin{cases} z_1'(x)\cdot\sin(x) + z_2'(x)\cdot\cos(x) = 0 \\ z_1' \cdot \cos(x) - z_2' \cdot \sin(x) = \frac{-ctg^2(x)}{1} \end{cases}$

$W = \begin{vmatrix} \sin(x) & \cos(x) \\ \cos(x) & -\sin(x) \end{vmatrix} = 1 \qquad W_2 = \begin{vmatrix} \sin(x) & 0 \\ \cos(x) & -ctg^2(x) \end{vmatrix} =$

$W_1 = \begin{vmatrix} 0 & \cos(x) \\ -ctg^2(x) & -\sin(x) \end{vmatrix} = -\frac{\cos^3(x)}{\sin^2(x)} \qquad = -\frac{\cos^2(x)}{\sin(x)}$

$z_1' = \frac{W_1}{W} = -\frac{\cos^3(x)}{\sin^2(x)}; \quad z_1 = -\int \frac{\cos^3(x)}{\sin^2(x)}dx = \sin(x) + 1/\sin(x) + C_1$

$z_2' = \frac{W_2}{W} = -\frac{\cos^2(x)}{\sin(x)}; \quad z_2 = -\int \frac{\cos^2(x)}{\sin(x)}dx =$

$= \frac{\ln(\cos(x)+1)}{2} - \frac{\ln(1-\cos(x))}{2} - \cos(x) + C_2$

$y = (\sin(x) + 1/\sin(x) + C_1)\cdot\sin(x) +$

$+ \left(\frac{\ln(\cos(x)+1)}{2} - \frac{\ln(1-\cos(x))}{2} - \cos(x) + C_2\right)\cdot\cos(x);$

$\begin{cases} C_1 = 0 \\ C_2 = 0 \end{cases}$

$y(\pi/3) = 3/4 + 1 + \left(\frac{\ln 3/2}{2} - \frac{\ln 1/2}{2} - 1/2\right)\cdot 1/2 =$

= тут ошибка мне лень

1.73 ответ