① $T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $x = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$x' = T^{-1} \cdot x$; $T^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$x' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

② $A = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{pmatrix}$; $|A - \lambda E| = \begin{pmatrix} 6-\lambda & 0 & 0 \\ 0 & 6-\lambda & 2 \\ 0 & 2 & 6-\lambda \end{pmatrix} =$

$$= (6-\lambda)\left((6-\lambda)^2 - 2^2\right) = (4-\lambda)(6-\lambda)(8-\lambda)$$

$\sigma_A = \{4; 6; 8\}$

③ $E_1 \to E_1 - E_2$; $E_2 \to -2E_1 + 3E_2$

$\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

④ $A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ -1 & -1 & 1 \end{pmatrix}$; $\begin{pmatrix} 1 & -1 & 3 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ -1 & -1 & 1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 3 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ 0 & -2 & 4 & | & 0 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 1 & 0 & 1 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

$\begin{cases} x_1 + x_3 = 0 \\ x_2 - 2x_3 = 0 \end{cases}$ $\begin{cases} x_1 = -x_3 \\ x_2 = 2x_3 \end{cases}$

$\begin{pmatrix} -1/2 \\ 1 \\ 1/2 \end{pmatrix}$

⑤ $A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 8 \end{pmatrix}$; $|A - \lambda E| = \begin{vmatrix} 4-\lambda & 0 & 0 \\ 0 & 8-\lambda & 4 \\ 0 & 4 & 8-\lambda \end{vmatrix} =$

$$= (4-\lambda)\left((8-\lambda)^2 - 4^2\right) = (4-\lambda)^2(12-\lambda)$$

$\sigma_A = \{4^{(2)}, 12^{(1)}\}$

$\lambda = 4:$ $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 8 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $x_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\lambda = 12$:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 8 \end{pmatrix} - \begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix} = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

⑥ $A = \begin{pmatrix} 1 & -2 & 2 & -6 & -1 \\ 1 & -1 & 1 & -3 & -1 \\ 0 & -1 & 2 & -5 & -2 \\ -1 & -1 & 2 & -5 & -1 \\ -4 & 5 & -4 & 13 & 2 \end{pmatrix}$ = к треугольному =

$$= \begin{pmatrix} 1 & -2 & 2 & -6 & -1 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

MAX невырожденная =

$$= \sum_{i=0}^{5} if(\,diagonal[i] \,!=0) =$$

$$= 3.$$

⑦ $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \iff A \cdot X = B. \rightarrow X = A^{-1} \cdot B$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}; \quad X = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$$