(1) $a_{ijk} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right)$ $\qquad a_{ikj} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right)$

$a_{i(jk)}$ - ?

$$a_{i(jk)} = \frac{1}{2}\left(a_{ijk} + a_{ikj}\right) = a_{ijk}$$

(2) $a_{ijk} \sim \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 0 & -2 & 1 \end{array}\right)$

$a_{(ijk)}$ - ?

$$a_{(ijk)} = \frac{1}{3!}\left(a_{ijk} + a_{ikj} + a_{jki} + a_{jik} + a_{kij} + a_{kji}\right)$$

$a_{ikj} \sim \left(\begin{array}{cc|cc} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & 1 \end{array}\right)$ $\qquad a_{jik} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ -1 & 0 & 0 & 1 \end{array}\right)$

$a_{jki} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ -1 & 0 & 0 & 1 \end{array}\right)$ $\qquad a_{kij} \sim \left(\begin{array}{cc|cc} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & 1 \end{array}\right)$

$$a_{kji} \sim \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 0 & -2 & 1 \end{array}\right)$$

$$S = \left(\begin{array}{cc|cc} 1 & 1/3 & 1/3 & -2/3 \\ 1/3 & -2/3 & -2/3 & 1 \end{array}\right)$$

(3) $a^{ij}_{kl} \sim \left(\begin{array}{cc|cc} 1 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ \hline 1 & -2 & 1 & 0 \\ -1 & 3 & -2 & 1 \end{array}\right)$; $\qquad a^{ji}_{kl} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -1 \\ -1 & 0 & 1 & 0 \\ \hline 1 & -1 & 1 & -2 \\ -2 & 3 & 0 & 1 \end{array}\right)$

$$a^{(ij)}_{kl} = \frac{1}{2}\left(a^{ij}_{kl} + a^{ji}_{kl}\right) = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 1 & -3/2 & 1 & -1 \\ -3/2 & 3 & -1 & 1 \end{array}\right)$$

$$a^{(ij)}_{ki} = a^{1j}_{k1} + a^{2j}_{k2} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix}$$

④ $a_{ijk} \sim \left(\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 1 & 1 & 0 & 1 \end{array}\right)$ $\quad a_{ikj} \sim \left(\begin{array}{cc|cc} 1 & 1 & 0 & -2 \\ 1 & 0 & 1 & 1 \end{array}\right)$

$a_{i[jk]}$ – ?

$$a_{i[jk]} = \frac{1}{2}\left(a_{ijk} - a_{ikj}\right) = \left(\begin{array}{cc|cc} 0 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \end{array}\right)$$

⑤ $a_{ijk} \sim \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 1 \end{array}\right)$

$a_{[ijk]}$ – ?

$$a_{[ijk]} = \frac{1}{6}\left(a_{ijk} + a_{jki} + a_{kij} - a_{jik} - a_{kji} - a_{ikj}\right)$$

$a_{ikj} = \left(\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 \end{array}\right)$; $a_{jki} = \left(\begin{array}{cc|cc} 1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$

$a_{kij} = \left(\begin{array}{cc|cc} 1 & -1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{array}\right)$; $a_{jik} = \left(\begin{array}{cc|cc} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right)$

$$a_{kji} = \left(\begin{array}{cc|cc} 1 & 0 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right)$$

$$S = \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

⑥ $a^{ij}_{kl} \sim \left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ -1 & 3 & -1 & 1 \\ \hline 1 & -1 & 1 & -1 \\ -1 & 2 & -2 & 3 \end{array}\right)$; $a^{ji}_{kl} \sim \left(\begin{array}{cc|cc} 1 & -1 & 1 & -1 \\ -2 & 3 & 0 & 1 \\ \hline 1 & -1 & 1 & -2 \\ -1 & 2 & -1 & 3 \end{array}\right)$

$$a^{[ij]}_{kl} = \frac{1}{2}\left(a^{ij}_{kl} - a^{ji}_{kl}\right) = \left(\begin{array}{cc|cc} 0 & -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 & 0 \\ \hline 0 & 0 & 0 & 1/2 \\ 0 & 0 & -1/2 & 0 \end{array}\right)$$

$$a^{ij}_{ki} = a^{1j}_{k1} + a^{2j}_{k2}$$

$$\begin{pmatrix} -1/2 & -1/2 \\ -1/2 & 0 \end{pmatrix}$$