ELSEWEDY UNIVERSITY OF TECHNOLOGY
- POLYTECHNIC OF EGYPT -

Principles of Physics

Lecture 6: Vectors and Scalars

References

- University Physics Volume 1, 2016
  - https://cnx.org/content/col12031/1.10
- University Physics Volume 2, 2016
  - https://cnx.org/content/col12074/1.9

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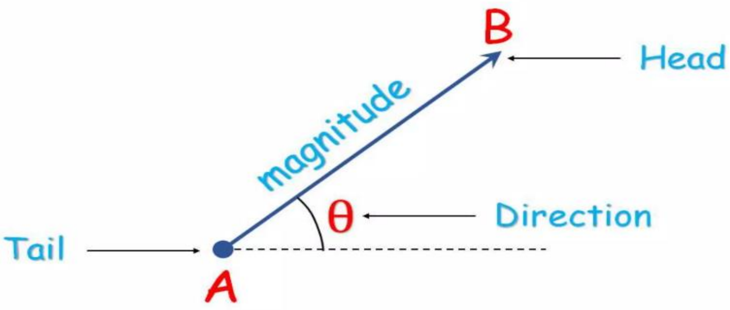
Vectors and scalars definition

Vectors and scalars definition

- **A scalar quantity** has only magnitude.
- **A vector quantity** has both magnitude and direction.

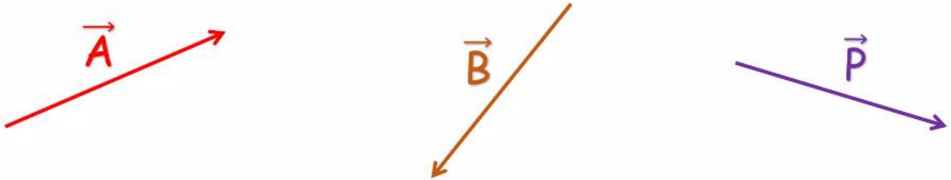
Scalar Quantity	Vector Quantity
Length, volume, speed, mass, pressure, energy, power, density ….	Displacement, velocity, acceleration, force ……

Representation of A vector



Symbolically it is represented as $\overrightarrow{AB}$

They are also represented by a single capital letter with an arrow above it.



Types of Vectors

- **Parallel Vectors**

Two vectors are said to be parallel vectors, if they have same direction.

- **Anti-parallel Vectors**

Two vectors are said to be anti-parallel vectors, if they are in opposite directions.

Types of Vectors

- **Equal Vectors**

Two parallel vectors are said to be equal vectors, if they have same magnitude.

$\vec{A} = \vec{B}$ $\quad\quad$ $\vec{P} = \vec{Q}$

- **Negative Vectors**

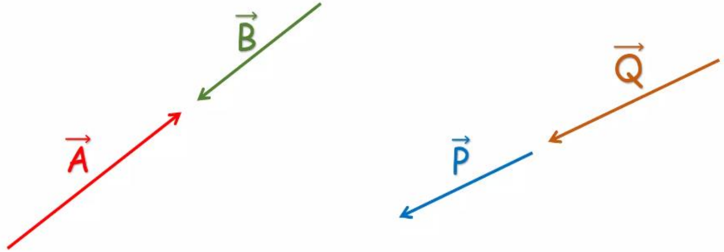
Two anti-parallel vectors are said to be negative vectors, if they have same magnitude.

$\vec{A} = -\vec{B}$ $\quad\quad$ $\vec{P} = -\vec{Q}$

Types of Vectors

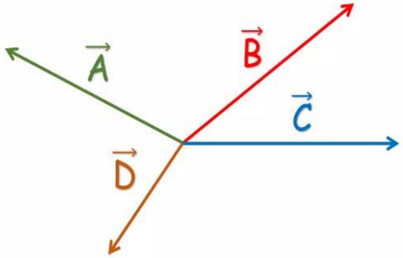
- **Collinear Vectors**

Two vectors are said to be collinear vectors, if they act along a same line.



- **Co-initial Vectors**

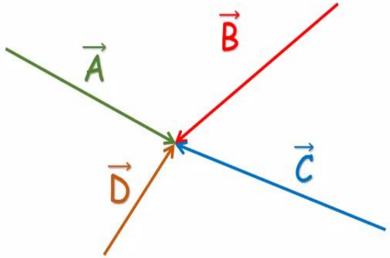
Two or more vectors are said to be co-initial vectors, if they have common initial point.



Types of Vectors

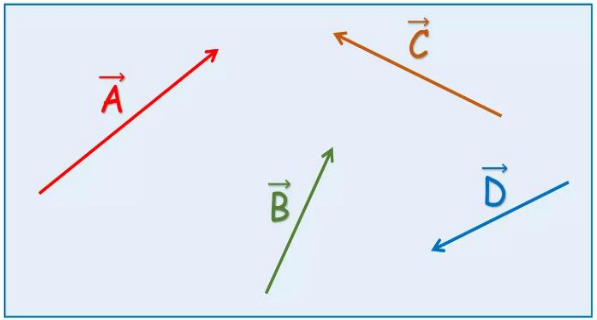
- **Co-terminus Vectors**

Two or more vectors are said to be co-terminus vectors, if they have common terminal point.



- **Coplanar Vectors**

Three or more vectors are said to be coplanar vectors, if they lie in the same plane.



Example

- **Find out the scalar and vector quantity from the given list?**

Force, Speed, Electric field, Angular Momentum, Magnetic Moment, Temperature, Linear Momentum, Average Velocity.

Solution:

From the given list,

- Scalar Quantities : Speed, Temperature.
- Vector Quantities : Force, Electric field, Angular Momentum, Magnetic Moment, Linear Momentum, Average Velocity.

- **Find the magnitude of v = i + 4j?**

Solution:

$$|V| = \sqrt{a^2 + b^2}$$

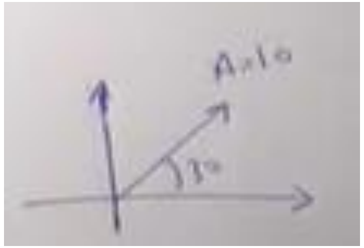
$$a = 1, b = 4$$

$$|V| = \sqrt{1^2 + 4^2}$$

$$|V| = \sqrt{17}$$

Example

- In figure find the component?



Solution:

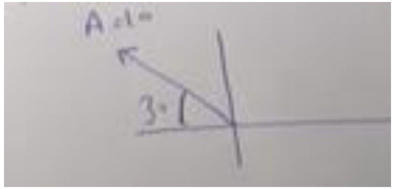
$A_x = 10\cos(30) = 8.66$

$A_y = 10\sin(30) = 5$

$\vec{A} = 8.66\hat{\imath} + 5\hat{\jmath}$

- In figure find the component?

Solution:



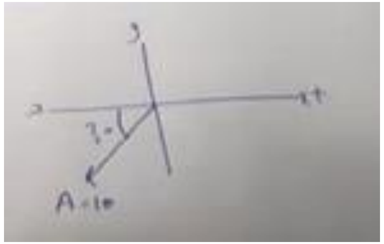
$A_x = 10\cos(150) = -8.66$

$A_y = 10\sin(150) = 5$

$\vec{A} = -8.66\hat{\imath} + 5\hat{\jmath}$

Example

- In figure find the component?



Solution:

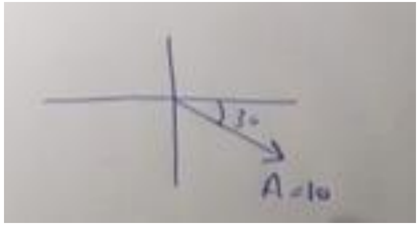
$A_x = 10\cos(210) = -8.66$

$A_y = 10\sin(210) = -5$

$\vec{A} = -8.66\hat{\imath} - 5\hat{\jmath}$

- In figure find the component?

Solution:



$A_x = 10\cos(330) = 8.66$

$A_y = 10\sin(330) = -5$

$\vec{A} = 8.66\hat{\imath} - 5\hat{\jmath}$

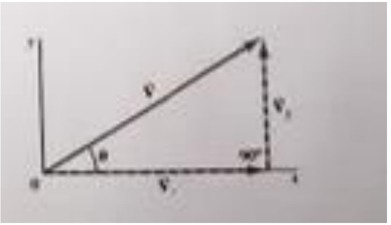
Example

- If $\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}$ find the magnitude and the direction?

Solution:

$$|V| = \sqrt{{V_x}^2 + {V_y}^2}$$

$$\theta = tan^{-1}(\frac{V_y}{V_x})$$



- If $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ find the magnitude and the direction?

Solution:

$$|V| = \sqrt{3^2 + 4^2} = 5$$

$$\theta = tan^{-1}\left(\frac{4}{3}\right) = 53.1°$$

- If $\vec{V} = -3\hat{\imath} + 4\hat{\jmath}$ find the magnitude and the direction?

Solution:

$$|V| = \sqrt{(-3)^2 + (4)^2} = 5$$

$$\theta = tan^{-1}\left(\frac{4}{-3}\right) = -53.1°$$

$$direction = 180 - 53.1 = 126.9°$$

Addition of Vectors

Addition of Vectors

- Example 1: Find $\vec{A} + \vec{B}$, where $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = 5\hat{\imath} + 10\hat{\jmath}$?

Solution:

$$\vec{A} + \vec{B} = 7\hat{\imath} + 13\hat{\jmath}$$

- Example 2: Find $\vec{C} + \vec{D}$ and the magnitude, where $\vec{C} = \hat{\imath} - \hat{\jmath} + 4\hat{K}$ and $\vec{D} = 2\hat{\imath} + 6\hat{K}$?

Solution:

$$\vec{C} + \vec{D} = 3\hat{\imath} - \hat{\jmath} + 10\hat{K}$$

$$\left|\vec{C} + \vec{D}\right| = \sqrt{(3)^2 + (-1)^2 + (10)^2} = 10.5$$

Subtraction of Vectors

Subtraction of Vectors

- Find $\vec{A} - \vec{B}$ and the magnitude, where $\vec{A} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{B} = 3\hat{\imath} - 2\hat{\jmath}$?

Solution:

$\vec{A} - \vec{B} = -\hat{\imath} + 3\hat{\jmath}$

$|\vec{A} + \vec{B}| = \sqrt{(-1)^2 + (3)^2} = 3.16$

Example

- **If $\boldsymbol{A = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}}$ and $\boldsymbol{B = 4\hat{\imath} - 8\hat{\jmath} - \hat{k}}$.**

Find, $\boldsymbol{A + B, A - B, 2A - 3B}$ and $\boldsymbol{A - 4B}$?

Solution:

- $\boldsymbol{A + B} = 6\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$
- $\boldsymbol{A - B} = -2\hat{\imath} + 12\hat{\jmath} - 2\hat{k}$
- $\boldsymbol{2A - 3B} = 2\left(2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}\right) - 3(4\hat{\imath} - 8\hat{\jmath} - \hat{k})$

  $2A - 3B = \left(4\hat{\imath} + 8\hat{\jmath} - 6\hat{k}\right) - \left(12\hat{\imath} - 24\hat{\jmath} - 3\hat{k}\right) = -8\hat{\imath} + 32\hat{\jmath} - 3\hat{k}$
- $\boldsymbol{A - 4B} = \left(2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}\right) - 4(4\hat{\imath} - 8\hat{\jmath} - \hat{k})$

  $A - 4B = \left(2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}\right) - \left(16\hat{\imath} - 32\hat{\jmath} - 4\hat{k}\right) = -14\hat{\imath} + 36\hat{\jmath} + \hat{k}$

Example

- A vector $\vec{A}$ has a component of (10) in the $+x$ direction, a component of (10) in the +y direction, and a component of (5) in the +z direction. The magnitude of this vector is :

Zero	**15**	**25**	**225**

Solution:

$$\vec{A} = 10\hat{\imath} + 10\hat{\jmath} + 5\hat{k}$$

$$\left|\vec{A}\right| = \sqrt{(10)^2 + (10)^2 + (5)^2} = 15$$

Multiplying Vectors

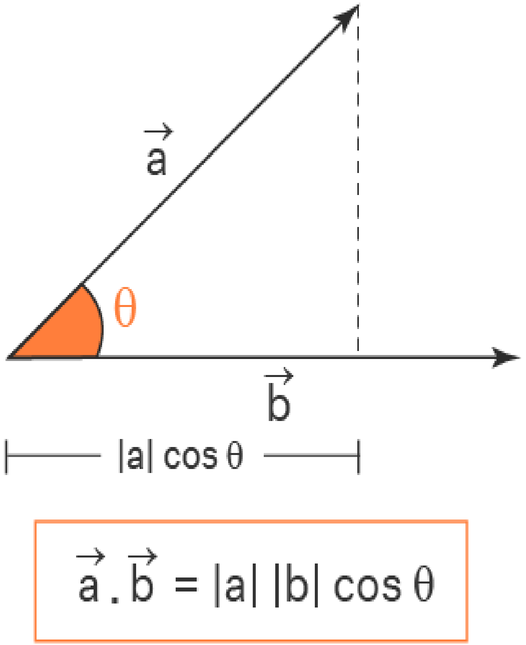
Dot Product

- The dot product of vectors is also called the scalar product of vectors. The resultant of the dot product of the vectors is a scalar value. Dot Product of vectors is equal to the product of the magnitudes of the two vectors, and the cosine of the angle between the two vectors. The resultant of the dot product of two vectors lie in the same plane of the two vectors. The dot product may be a positive real number or a negative real number.

- *Let a and b be two non-zero vectors, and θ be the included angle of the vectors. Then the scalar product or dot product is denoted by a.b, which is defined as:*

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|cos\theta$$

Here, $|\vec{a}|$ is the magnitude of $\vec{a}$, $|\vec{b}|$ is the magnitude of $\vec{b}$, and θ is the angle between them.



Dot Product

- For the scalar multiplication of vectors, the two vectors are expressed in terms of unit vectors, i, j, k, along the x, y, z axes, then the scalar product is obtained as follows:
- If $\vec{A} = a_1\hat{\imath} + b_1\hat{\jmath} - c_1\hat{k}$ and $\vec{B} = a_2\hat{\imath} + b_2\hat{\jmath} - c_2\hat{k}$, then
- $\vec{A} \cdot \vec{B} = (a_1\hat{\imath} + b_1\hat{\jmath} - c_1\hat{k})(a_2\hat{\imath} + b_2\hat{\jmath} - c_2\hat{k})$
- $\vec{A} \cdot \vec{B} = (a_1a_2)(\hat{\imath} \cdot \hat{\imath}) + (a_1b_2)(\hat{\imath} \cdot \hat{\jmath}) + (a_1c_2)(\hat{\imath} \cdot \hat{k}) + (b_1a_2)(\hat{\jmath} \cdot \hat{\imath}) + (b_1b_2)(\hat{\jmath} \cdot \hat{\jmath}) + (b_1c_2)(\hat{\jmath} \cdot \hat{k}) + (c_1a_2)(\hat{k} \cdot \hat{\imath}) + (c_1b_2)(\hat{k} \cdot \hat{\jmath}) + (c_1c_2)(\hat{k} \cdot \hat{k})$
- $\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$.
- **Where :** $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = 0$$

$$\boldsymbol{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

Example 1

- Find $\vec{A} \cdot \vec{B}$, where $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} - 8\hat{\jmath} - \hat{k}$?

$$\vec{A} \cdot \vec{B} = 8 - 32 + 3 = -21$$

- Find $\vec{C} \cdot \vec{D}$, where $\vec{C} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{D} = 4\hat{\imath} + \hat{k}$?

$$\vec{C} \cdot \vec{D} = 8 + 0 + 1 = 9$$

Cross Product

- Cross Product is also called a Vector Product. Cross product is a form of vector multiplication, performed between two vectors of different nature or kinds. When two vectors are multiplied with each other and the multiplication is also a vector quantity, then the resultant vector is called the cross product of two vectors or the vector product. The resultant vector is perpendicular to the plane containing the two given vectors.

Cross Product

- Let us assume that $\vec{A}$ and $\vec{B}$ are two vectors, such that $\vec{A} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$ and $\vec{B} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ then by using determinations, we could find the cross-product multiplication of vectors, using the following matrix notation.
- $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & -\hat{\jmath} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$
- The cross-product multiplication of vectors is also represented using the cross-product formula as:

$$\vec{A} \times \vec{B} = \hat{\imath}(b_1c_2 - b_2c_1) - \hat{\jmath}(a_1c_2 - a_2c_1) + \hat{k}(a_1b_2 - a_2b_1)$$

Note: $\hat{\imath}, \hat{\jmath}$, and $\hat{k}$ are the unit vectors in the direction of x axis, y-axis, and z -axis, respectively.

$$\boldsymbol{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

Examples

Example 1

- Find $\vec{A} \cdot \vec{B}$, where $\vec{A} = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} - 8\hat{\jmath} - \hat{k}$?

$$\vec{A} \cdot \vec{B} = 8 - 32 + 3 = -21$$

- Find $\vec{C} \cdot \vec{D}$, where $\vec{C} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{D} = 4\hat{\imath} + \hat{k}$?

$$\vec{C} \cdot \vec{D} = 8 + 0 + 1 = 9$$

Example 4

- Find $\vec{A} \times \vec{B}$, where $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$?

Solution:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & -\hat{\jmath} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{\imath}(3 \times 3 - (-1 \times 2)) - \hat{\jmath}\big(2 \times 3 - (-1 \times 1)\big) + \hat{k}(2 \times 2 - (1 \times 3))$$

$$\vec{A} \times \vec{B} = \hat{\imath}(9 + 2) - \hat{\jmath}(6 + 1) + \hat{k}(4 - 3) = 11\hat{\imath} - 7\hat{\jmath} + \hat{k}$$

Example 5

- Find $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$, where $\vec{A} = 2\hat{\imath} + \hat{k}$ and $\vec{B} = 2\hat{\imath} + \hat{\jmath}$?

Solution:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & -\hat{\jmath} & \hat{k} \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix} = \hat{\imath}(0-1) - \hat{\jmath}(0-2) + \hat{k}(2-0) = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$$

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$$

$$\vec{B} \times \vec{A} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$$

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Thank You...