Physics for Electrical Engineering

Lecture 7: Photoelectric Effect

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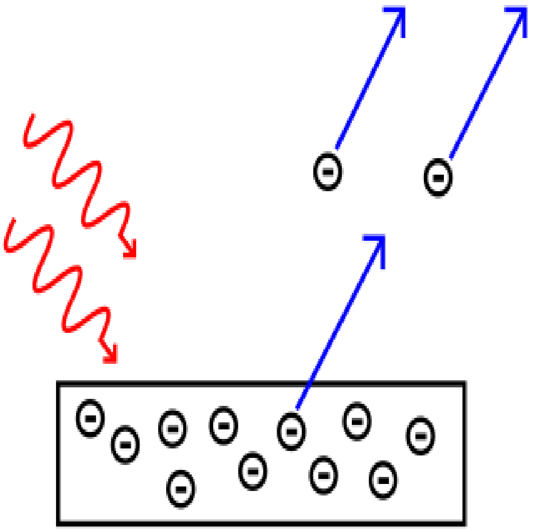
Introduction

What is the photoelectric effect?

- When light shines on a metal, electrons can be ejected from the surface of the metal in a phenomenon known as the *photoelectric effect*. This process is also often referred to as *photoemission*, and the electrons that are ejected from the metal are called *photoelectrons*. In terms of their behavior and their properties, photoelectrons are no different from other electrons. The prefix, *photo*-, simply tells us that the electrons have been ejected from a metal surface by incident light.



Albert Einstein Model

Albert Einstein Model

- That model was developed by Albert Einstein, who proposed that light sometimes behaved as particles of electromagnetic energy which we now call *photons*. The energy of a photon could be calculated using Planck's equation:

$$\boldsymbol{E = hv}$$

- where $\boldsymbol{E}$ is the energy of a photon in joules (J), $\boldsymbol{h}$ is Planck's constant $6.626 \times 10^{-34} J \cdot s$, and $\boldsymbol{v}$ is the frequency of the light in Hz. According to Planck's equation, the energy of a photon is proportional to the frequency of the light, *v*. The amplitude of the light is then proportional to the number of photons with a given frequency.

Albert Einstein Model

- **As the wavelength of a photon increases, what happens to the photon's energy?**

According to Planck's equation, the energy of a photon is proportional to the light frequency, *v*:

$$\boldsymbol{E = h\nu}$$

The light frequency ν is inversely proportional to wavelength λ:

$$\boldsymbol{c = \lambda\nu}$$

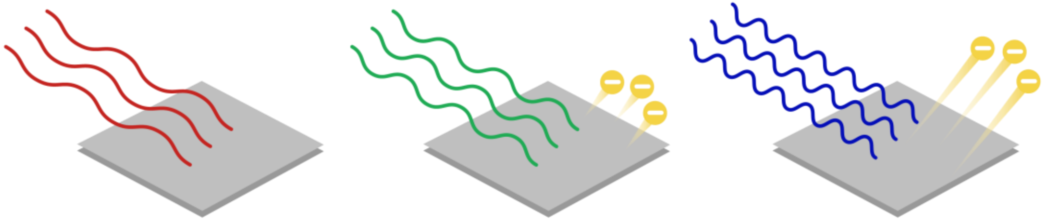
where $\boldsymbol{c}$ is the speed of light. That means that increasing the wavelength decreases the light's frequency. Therefore, as the wavelength of a photon increases, its energy decreases.

Light frequency and the cut off frequency

Light frequency and the cut off frequency

- **Light frequency and the cut off frequency**

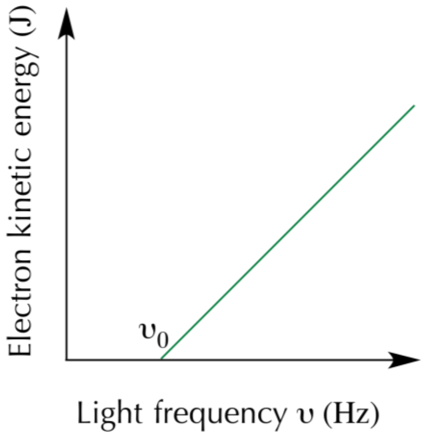
We can think of the incident light as a stream of photons with an energy determined by the light frequency. When a photon hits the metal surface, the photon's energy is absorbed by an electron in the metal. The graphic below illustrates the relationship between light frequency and the kinetic energy of ejected electrons.



The frequency of red light (left) is less than the cut off frequency of this metal $(v_{red} < v_0)$ *, so no electrons are ejected. The green (middle) and blue light (right) have* $(v > v_0)$ *, so both cause photoemission. The higher energy blue light ejects electrons with higher kinetic energy compared to the green light.*

Light frequency and the cut off frequency

- The scientists observed that if the incident light had a frequency less than a minimum frequency ν_0 , then no electrons were ejected regardless of the light amplitude. This minimum frequency is also called the *cut off frequency*, and the value of ν_0 depends on the metal. For frequencies greater than ν_0 , electrons would be ejected from the metal. Furthermore, the kinetic energy of the photoelectrons was proportional to the light frequency. The relationship between photoelectron kinetic energy and light frequency is shown in graph below.



Kinetic Energy of Electron

Relation between energy of photon and the kinetic energy of electron

- We can analyze the frequency relationship using the law of conservation of energy. The total energy of the incoming photon, E_{photon} , must be equal to the kinetic energy of the ejected electron, $KE_{electron}$, plus the energy required to eject the electron from the metal. The energy required to free the electron from a particular metal is also called the metal's *work function*, which is represented by the symbol W_0 (in units of J):

$$\boldsymbol{E_{photon} = KE_{electron} + W_0}$$

The photoelectron velocity

- Like the cut off frequency ν_0 , the value of W_0 also changes depending on the metal. We can now write the energy of the photon in terms of the light frequency using Planck's equation:

$$\boldsymbol{E_{photon} = h\nu = KE_{electron} + W_0}$$

- Rearranging this equation in terms of the electron's kinetic energy, we get:

$$\boldsymbol{KE_{electron} = h\nu - W_0}$$

- We can also use this equation to find the photoelectron velocity V, which is related to $\boldsymbol{KE_{electron}}$ as follows:

$$\boldsymbol{KE_{electron} = h\nu - W_0 = \frac{1}{2} m_e V^2}$$

where $\boldsymbol{m_e}$ is the rest mass of an electron $9.1 \times 10^{-31} kg$.

Metal's work function

Metal	Work function Φ (Joules, J)
Calcium, Ca	4.60×10^{-19}
Tin, Sn	7.08×10^{-19}
Sodium, Na	3.78×10^{-19}
Hafnium, Hf	6.25×10^{-19}
Samarium, Sm	4.33×10^{-19}

Example

- The work function only depends on the type of ………………

material	frequency	energy	intensity

- The kinetic energy of photoelectrons depends on the ………… Of the incident photons.

material	frequency	energy	intensity

- No emission of electrons if v of incident photon … … … … v_0.

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Example

- **The work function of copper metal is $W_0 = 7.53 \times 10^{-19} J$. If we shine light with a frequency of $3.0 \times 10^{16} Hz$ on copper metal and plank constant is $6.626 \times 10^{-34} J.s$, will the photoelectric effect be observed? What is the kinetic energy of the photoelectrons ejected from the copper by the light with a frequency of ?**

Solution

In order to eject electrons, we need the energy of the photons to be greater than the work function of copper. We can use Planck's equation to calculate the energy of the photon, E_{photon} :

$$E_{photon} = hv$$

$$E_{photon} = 6.626 \times 10^{-34} \times 3.0 \times 10^{16} = 2.0 \times 10^{-17} J$$

If we compare our calculated photon energy, E_{photon}, to copper's work function, we see that the photon energy is greater than W_0 :

$$E_{photon} > W_0$$

$$2.0 \times 10^{-17} > 7.53 \times 10^{-19}$$

Thus, we would expect to see photoelectrons ejected from the copper.

Cont. Example

$$E_{photon} = KE_{electron} + W_0$$

$$KE_{electron} = E_{photon} - W_0$$

$$KE_{electron} = 2.0 \times 10^{-17} - 7.53 \times 10^{-19} = 1.9 \times 10^{-17} J$$

Therefore, each photoelectron has a kinetic energy of $1.9 \times 10^{-17} J$

Wave-Particle Duality and The De Broglie Wavelength

Wave-particle duality and the de Broglie wavelength

- Another major development in quantum mechanics was pioneered by French physicist Louis de Broglie. Based on work by Planck and Einstein that showed how light waves could exhibit particle-like properties, de Broglie hypothesized that particles could also have wavelike properties.
- De Broglie derived the following equation for the wavelength of a particle of mass m (in kilograms kg), traveling at velocity V (in m/s), where λ is the de Broglie wavelength of the particle in meters and h is Planck's constant, $6.626 \times 10^{-34} kg \cdot m^2/s$:

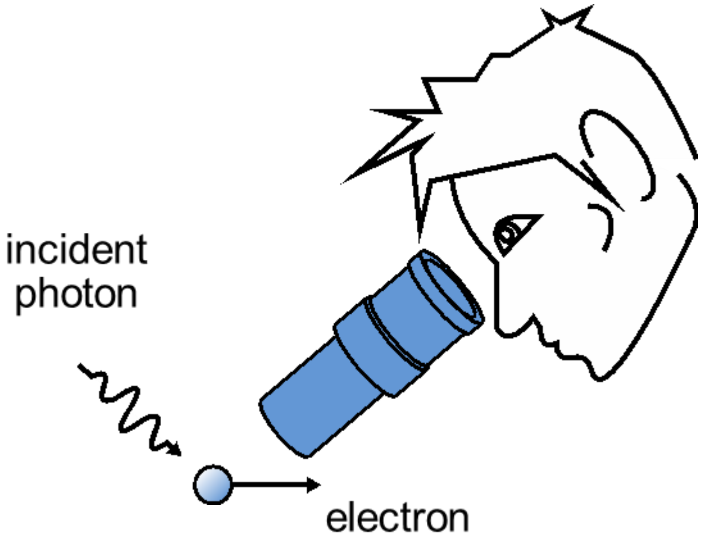
$$\lambda = \frac{h}{mV}$$

Note that the de Broglie wavelength and particle mass are inversely proportional.

Heisenberg Uncertainty Principle

Heisenberg Uncertainty Principle

- Formulated by the German physicist and Nobel laureate Werner Heisenberg in 1927, the uncertainty principle states that **we cannot know both the position and speed of a particle, such as a photon or electron, with perfect accuracy; the more we nail down the particle's position, the less we know about its speed and vice versa.**
- **Ex.** According to Heisenberg uncertainty principle we cannot know both …………. of a particle, such as a photon or electron, with perfect accuracy.



the position and speed	the intensity and speed	the intensity and position	the energy and frequency

Heisenberg Uncertainty Principle Equations

- If Δx is the error in position measurement and Δp is the error in the measurement of momentum, then
- $(\Delta P)(\Delta x) \geq \frac{h}{4\pi}$
- Since momentum, $p = mv$, Heisenberg's uncertainty principle formula can be alternatively written as,
- $(\Delta mV)(\Delta x) \geq \frac{h}{4\pi}$
- Or
- $(\Delta m)(\Delta V)(\Delta x) \geq \frac{h}{4\pi}$
- Where, ΔV is the error in the measurement of velocity, and assuming mass remains constant during the experiment,
- $(\Delta V)(\Delta x) \geq \frac{h}{4\pi m}$

Example 1

- If the position of the electron is measured within an accuracy of $+\ 0.002\ nm$ and Planck's constant is $6.626 \times 10^{-34} kg \cdot \frac{m^2}{s}$, (a) calculate the uncertainty in the momentum of the electron. (b) Suppose the momentum of the electron is $\frac{h}{4\times10^{-12}} \times 0.05nm$. Is there any problem in defining this value?

Solution:

a) Uncertainty in the momentum

$$\Delta x = 2 \times 10^{-12} m$$

$$(\Delta P)(\Delta x) \geq \frac{h}{4\pi}$$

$$(\Delta P)(\Delta x) \geq \frac{h}{4\pi}$$

$$(\Delta P) = \frac{h}{4\pi\Delta x} \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 2 \times 10^{-12}} = 2.64 \times 10^{-23} kg.m/s$$

b) Momentum

$$P = \frac{h}{4 \times 10^{-12}} \times 0.05nm$$

$$P = \frac{6.626 \times 10^{-34}}{4 \times 10^{-12}} \times 5 \times 10^{-11} = 28 \times 10^{-33} kg.m/s$$

Error in momentum measurement is 10^{10} times larger than the actual momentum. The given momentum will not be acceptable.

Example 2

- Position of a chloride ion on a material can be determined to a maximum error of 1μm. If the mass of the chloride ion is $5.86 \times 10^{-26} kg$ and Planck's constant is $6.626 \times 10^{-34} kg \cdot \frac{m^2}{s}$, what will be the error in its velocity measurement?

Solution:

$$\Delta x = 10^{-6} m$$

$$(\Delta m)(\Delta V)(\Delta x) \geq \frac{h}{4\pi}$$

$$\Delta V \geq \frac{h}{4\pi m \Delta x}$$

$$\Delta V \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 5.86 \times 10^{-26} \times 10^{-6}} = 9 \times 10^{-4} m/s$$

Thank You...