Principles of Physics

Lecture 2: Units and Dimensions

References

- University Physics Volume 1, 2016
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- University Physics Volume 2, 2016
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Units

Base and Derived Units

- In any system of units, the units for some physical quantities must be defined through a measurement process. These are called the **base quantities** for that system and their units are the system's **base units**. All other physical quantities can then be expressed as algebraic combinations of the base quantities. Each of these physical quantities is then known as a **derived quantity** and each unit is called a **derived unit**.

Base quantity

Base Quantity	SI units
Length, l	metres, m
Mass, m	kilogram, kg
Time, t	second, s
Temperature, T	Kelvin, k
Electrical current, I	Ampere, A

Derived quantity

Derived Quantity	Units
Volume, V	m^3
Density, ρ	kgm^{-3}
Velocity, v	ms^{-1}
Force, F	N
Acceleration, a	ms^{-2}

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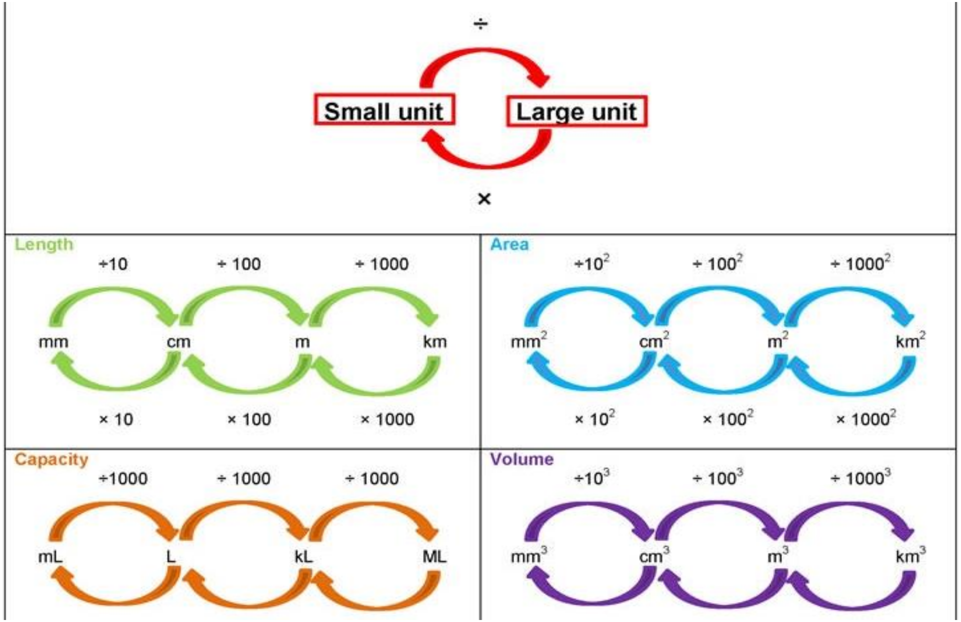
Units Conversion

Unit Conversion

- A conversion factor is an expression used to change from one unit to another.



Unit Conversion

Mass

$1\ \text{kg} = 10^3\ \text{g}$
$1\ \text{g} = 10^{-3}\ \text{kg}$
$1\ \text{u} = 1.66 \times 10^{-24}\ \text{g} = 1.66 \times 10^{-27}\ \text{kg}$
$1\ \text{slug} = 14.6\ \text{kg}$
$1\ \text{metric ton} = 1000\ \text{kg}$

Length

$1\ \text{Å} = 10^{-10}\ \text{m}$
$1\ \text{nm} = 10^{-9}\ \text{m}$
$1\ \text{cm} = 10^{-2}\ \text{m} = 0.394\ \text{in.}$
$1\ \text{yd} = 3\ \text{ft}$
$1\ \text{m} = 10^{-3}\ \text{km} = 3.281\ \text{ft} = 39.4\ \text{in.}$
$1\ \text{km} = 10^3\ \text{m} = 0.621\ \text{mi}$
$1\ \text{in.} = 2.54\ \text{cm} = 2.54 \times 10^{-2}\ \text{m}$
$1\ \text{ft} = 0.305\ \text{m} = 30.5\ \text{cm}$
$1\ \text{mi} = 5280\ \text{ft} = 1609\ \text{m} = 1.609\ \text{km}$
$1\ \text{ly (light year)} = 9.46 \times 10^{12}\ \text{km}$
$1\ \text{pc (parsec)} = 3.09 \times 10^{13}\ \text{km}$

Area

$1\ \text{cm}^2 = 10^{-4}\ \text{m}^2 = 0.1550\ \text{in.}^2 = 1.08 \times 10^{-3}\ \text{ft}^2$
$1\ \text{m}^2 = 10^4\ \text{cm}^2 = 10.76\ \text{ft}^2 = 1550\ \text{in.}^2$
$1\ \text{in.}^2 = 6.94 \times 10^{-3}\ \text{ft}^2 = 6.45\ \text{cm}^2 = 6.45 \times 10^{-4}\ \text{m}^2$
$1\ \text{ft}^2 = 144\ \text{in.}^2 = 9.29 \times 10^{-2}\ \text{m}^2 = 929\ \text{cm}^2$

Volume

$1\ \text{cm}^3 = 10^{-6}\ \text{m}^3 = 3.35 \times 10^{-5}\ \text{ft}^3 = 6.10 \times 10^{-2}\ \text{in.}^3$
$1\ \text{m}^3 = 10^6\ \text{cm}^3 = 10^3\ \text{L} = 35.3\ \text{ft}^3 = 6.10 \times 10^4\ \text{in.}^3 = 264\ \text{gal}$
$1\ \text{liter} = 10^3\ \text{cm}^3 = 10^{-3}\ \text{m}^3 = 1.056\ \text{qt} = 0.264\ \text{gal}$
$1\ \text{in.}^3 = 5.79 \times 10^{-4}\ \text{ft}^3 = 16.4\ \text{cm}^3 = 1.64 \times 10^{-5}\ \text{m}^3$
$1\ \text{ft}^3 = 1728\ \text{in.}^3 = 7.48\ \text{gal} = 0.0283\ \text{m}^3 = 28.3\ \text{L}$
$1\ \text{qt} = 2\ \text{pt} = 946\ \text{cm}^3 = 0.946\ \text{L}$
$1\ \text{gal} = 4\ \text{qt} = 231\ \text{in.}^3 = 0.134\ \text{ft}^3 = 3.785\ \text{L}$

Time

$1\ \text{h} = 60\ \text{min} = 3600\ \text{s}$
$1\ \text{day} = 24\ \text{h} = 1440\ \text{min} = 8.64 \times 10^4\ \text{s}$
$1\ \text{y} = 365\ \text{days} = 8.76 \times 10^3\ \text{h} = 5.26 \times 10^5\ \text{min} = 3.16 \times 10^7\ \text{s}$

Speed

$1\ \text{m/s} = 3.60\ \text{km/h} = 3.28\ \text{ft/s} = 2.24\ \text{mi/h}$
$1\ \text{km/h} = 0.278\ \text{m/s} = 0.621\ \text{mi/h} = 0.911\ \text{ft/s}$
$1\ \text{ft/s} = 0.682\ \text{mi/h} = 0.305\ \text{m/s} = 1.10\ \text{km/h}$
$1\ \text{mi}/h = 1.467\ \text{ft/s} = 1.609\ \text{km/h} = 0.447\ \text{m/s}$
$60\ \text{mi/h} = 88\ \text{ft/s}$

Force

$1\ \text{N} = 0.225\ \text{lb}$
$1\ \text{lb} = 4.45\ \text{N}$
Equivalent weight of a mass of 1 kg on Earth's surface $= 2.2\ \text{lb} = 9.8\ \text{N}$
$1\ \text{dyne} = 10^{-5}\ \text{N} = 2.25 \times 10^{-6}\ \text{lb}$

Pressure

$1\ \text{Pa} = 1\ \text{N/m}^2 = 1.45 \times 10^{-4}\ \text{lb/in.}^2 = 7.5 \times 10^{-3}\ \text{mm Hg}$
$1\ \text{mm Hg} = 133\ \text{Pa} = 0.02\ \text{lb/in.}^2 = 1\ \text{torr}$
$1\ \text{atm} = 14.7\ \text{lb/in.}^2 = 101.3\ \text{kPa} = 30\ \text{in. Hg} = 760\ \text{mm Hg}$
$1\ \text{lb/in.}^2 = 6.89\ \text{kPa}$
$1\ \text{bar} = 10^5\ \text{Pa} = 100\ \text{kPa}$
$1\ \text{millibar} = 10^2\ \text{Pa}$

Energy

$1\ \text{J} = 0.738\ \text{ft}\cdot\text{lb} = 0.239\ \text{cal} = 9.48 \times 10^{-4}\ \text{Btu} = 6.24 \times 10^{18}\ \text{eV}$
$1\ \text{kcal} = 4186\ \text{J} = 3.968\ \text{Btu}$
$1\text{Btu} = 1055\ \text{J} = 778\ \text{ft}\cdot\text{lb} = 0.252\ \text{kcal}$
$1\ \text{cal} = 4.186\ \text{J} = 3.97 \times 10^{-3}\ \text{Btu} = 3.09\ \text{ft}\cdot\text{lb}$
$1\ \text{ft}\cdot\text{lb} = 1.36\ \text{J} = 1.29 \times 10^{-3}\ \text{Btu}$
$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$
$1\ \text{kWh} = 3.6 \times 10^6\ \text{J}$
$1\ \text{erg} = 10^{-7}\ \text{J} = 7.38 \times 10^{-6}\ \text{ft}\cdot\text{lb}$

Power

$1\ \text{W} = 1\ \text{J/s} = 0.738\ \text{ft}\cdot\text{lb/s} = 1.34 \times 10^{-3}\ \text{hp} = 3.41\ \text{Btu/h}$
$1\ \text{ft}\cdot\text{lb/s} = 1.36\ \text{W} = 1.82 \times 10^{-3}\ \text{hp}$
$1\ \text{hp} = 550\ \text{ft}\cdot\text{lb/s} = 745.7\ \text{W} = 2545\ \text{Btu/h}$

Mass–Energy Equivalents

$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} \leftrightarrow 931.5\ \text{MeV}$
$1\ \text{electron mass} = 9.11 \times 10^{-31}\ \text{kg} = 5.49 \times 10^{-4}\ \text{u} \leftrightarrow 0.511\ \text{MeV}$
$1\ \text{proton mass} = 1.673 \times 10^{-27}\ \text{kg} = 1.007\,267\ \text{u} \leftrightarrow 938.28\ \text{MeV}$
$1\ \text{neutron mass} = 1.675 \times 10^{-27}\ \text{kg} = 1.008\,665\ \text{u} \leftrightarrow 939.57\ \text{MeV}$

Temperature

$T_F = \frac{9}{5}T_C + 32$
$T_C = \frac{5}{9}(T_F - 32)$
$T_K = T_C + 273.15$

Angle

$1\ \text{rad} = 57.3°$
$1° = 0.0175\ \text{rad}$
$15° = \pi/12\ \text{rad}$
$30° = \pi/6\ \text{rad}$
$45° = \pi/4\ \text{rad}$
$60° = \pi/3\ \text{rad}$
$90° = \pi/2\ \text{rad}$
$180° = \pi\ \text{rad}$
$360° = 2\pi\ \text{rad}$
$1\ \text{rev/min} = (\pi/30)\ \text{rad/s} = 0.1047\ \text{rad/s}$

Dimensions

Dimensions

- The dimension of any physical quantity expresses its dependence on the base quantities as a product of symbols representing the base quantities. This table lists the base quantities and the symbols used for their dimension.

Base Quantity	Symbol for Dimension
Mass	M
Length	L
Time	T

Dimensions

Quantity	*Law*	*Unit*	*Dimension*
Area	$A = l^2$	m^2	L^2
Volume	$V = l^3$	m^3	L^3
Velocity	$v = l/t$	m/s	$L \cdot T^{-1}$
Acceleration	$a = v/t$	m/s^2	$L \cdot T^{-2}$
Force	$F = m\,a$	$kg \cdot m/s^2\ (N)$	$M \cdot L \cdot T^{-2}$
Energy	$E = F\,d$	$kg \cdot m^2/s^2\ (J)$	$M \cdot L^2 \cdot T^{-2}$
Denisty	$\mathrm{D} = m/V$	kg/m^3	$M \cdot L^{-3}$
Pressure	$P = F/A$	$kg/m \cdot s^2$	$M \cdot L^{-1} \cdot T^{-2}$

Important rules

قواعد هامة

١) يجب وضع اى كمية فيزيائية يطلب تحليل بعدى لها بدلالة الكميات الفيزيائية الثلاثة Mass (M) , Length (L) , Time (T).

٢) يفضل مراعاة الترتيب عند كتابة الـ Dimension لاى كمية فيزيائية بمعنى نكتب الـ Dimension كالاتى $(M \longrightarrow L \longrightarrow T)$.

٣) يمكن ضرب او قسمة الـ Dimension بمعنى $(LT^{-1} \times LT^{-1} = L^2T^{-2})$

٤) لا يمكن جمع او طرح الـ Dimension بمعنى $(LT^{-1} + LT^{-1} = LT^{-1})$ الجمع والطرح هنا ليس جبريا.

٥) اذا كان هناك كميتين A & B لهم وحدات قياس مختلفة فانه لايمكن جمعهم أو طرحهم (لا يمكن ان نقول A + B أو A – B).

٦) اذا كان هناك كميتين A & B لهم وحدات قياس متشابهة فانه يمكن جمعهم أو طرحهم (يمكن ان نقول A + B أو A – B).

٧) اذا كان هناك كميتين A & B لهم وحدات قياس مختلفة او متشابهة فانه يمكن ضربهم او قسمتهم (يمكن ان نقول A x B أو A / B).

Dimensionless quantity

- All the numbers
- Some constant (such as $\pi = {}^{22}/_{7} = 3.14$)
- Ratios (Proportions)
- Non-algebraic Functions

such as: *Logarithmic functions* $\log(x)$, $\ln(x)$ – *Exponential Functions* e^x, a^x – *Trigonometric Functions* $\sin(x)$, $\cos(x)$, $\tan(x)$.

Dimension of a dimensionless quantities is 1

Uses of the dimensions

Verify the validity of any equation التأكد من صحة أي معادلة

Verify the following equations:

- $\mathbf{x = \frac{1}{2}\, a\, t^2}$

$$L.H.S. = x = L$$

$$R.H.S. = \frac{1}{2}\, a\, t^2 = L\, T^{-2} \,.\, T^2 = L$$

$$\therefore L.H.S. = R.H.S.$$

The equation is correct

- $\mathbf{P = \rho\, g\, h}$

$$L.H.S. = P = \frac{F}{A} = \frac{M\, L\, T^{-2}}{L^2} = M\, L^{-1} T^{-2}$$

$$R.H.S. = \rho\, g\, h = ML^{-3} \cdot LT^{-2} \cdot L = ML^{-1}T^{-2}$$

$$\therefore L.H.S. = R.H.S.$$

The equation is correct

Deduce the law of any physical quantity إستنتاج القانون الخاص بأي كمية فيزيائية

Example : Prove that the period of oscillations of a simple pendulums is proportional to its length (L) , the acceleration of garvity (g) & the mass of pendulum (m)

<u>Solution</u>

هنفرض ان زمن الاهتزازات يساوى دالة فى المتغيرات (طول البندول – عجلة الجاذبية – كتلة البندول)

$$let\ that \quad T_p = f(l, g, m)$$

$$T_p = k(l^a g^b m^c) \qquad k:$$ ثابت ليس له وحدة

take the dimension of both sides

$$[T_p] = [l^a][g^b][m^c]$$

$$T = M^c * L^a * L^b.T^{-2b} = M^c\ L^{a+b}\ T^{-2b}$$

بمساواة الاسس فى الطرفين

$$w.r.t\ (T) \rightarrow -2b = 1 \Rightarrow b = -\frac{1}{2}$$

$$w.r.t\ (M) \rightarrow c = 0$$

$$w.r.t\ (L) \rightarrow a + b = 0 \Rightarrow a = \frac{1}{2}$$

$$T = k\, l^{0.5}\, g^{-0.5}\, m^0 \Rightarrow \boxed{T = k\sqrt{\frac{l}{g}}}$$

Problems

Example: Suppose we are told that the accelelration (a) of a particle moving with uniform speed (v) in a circle of radius (r) is proportional to some power of (r) , say (r^x) & some power of (v) , say (v^y). Determine the values of (x & y) and write the simplest form for the acceleration .

Solution

$a = k\,(r^x\, v^y)$ $\qquad$ k : ثابت ليس له وحدة

take the dimension of both sides

$[a] = [r^x]\,[v^y]$

$L\,.T^{-2} = L^x * (L.T^{-1})^y = L^{x+y}\,.T^{-y}$

بمساواة الاسس فى الطرفين

$$w.r.t\ (T) \quad \longrightarrow \quad -y = -2 \quad \Longrightarrow \quad y = 2$$
$$w.r.t\ (L) \quad \longrightarrow \quad x + y = 1 \quad \Longrightarrow \quad x = -1$$
$$a = k\, r^{-1}\, v^2$$

$$\boxed{a = k\frac{v^2}{r}}$$

Problems

- Assuming that the mass m of the largest stone that can be moved by a flowing river depends on the velocity V of the water, its density ρ, and the acceleration of gravity g. Show that m varies as the sixth power of the water velocity in the river.

Solution:

$$m \propto \rho\, V\, g$$

$$m = k\, V^a\, \rho^b\, g^c \qquad \Rightarrow (1)$$

$$M = (L\, T^{-1})^a\, (M\, L^{-3})^b\, (L\, T^{-2})^c$$

$$M = L^a\, T^{-a}\, M^b\, L^{-3b}\, L^c\, T^{-2c}$$

$$M = L^{a-3b+c}\, T^{-a-2c}\, M^b$$

$$\text{For M: } b = 1 \qquad \Rightarrow (2)$$

$$\text{For L: } a - 3b + c = 0$$

$$a + c = 3 \qquad \Rightarrow (3)$$

$$\text{For T: } -a - 2c = 0 \qquad \Rightarrow (4)$$

Add (3) and (4):

$$c = -3 \qquad \Rightarrow (5)$$

Substitute from (5) in (3):

$$a = 6 \qquad \Rightarrow (6)$$

Substitute from (2), (5), and (6) in (1):

$$\therefore \mathbf{m} = \frac{\mathbf{k\, V^6\, \rho}}{\mathbf{g^3}}$$

Problems

- Given that the time period T of oscillation of a gas bubble from an explosion under water depends upon P, d and E, where P is the static pressure, d the density of water and E is the total energy of explosion. Find dimensionally a relation for T.

Solution:

$$P = M\,.\,L^{-1}.\,T^{-2}$$

$$d = M.\,L^{-3}$$

$$E = M\,.\,L^{2}\cdot T^{-2}$$

$T \,\alpha\, P\, d\, E$

$$T = k\, P^{a}\, d^{b}\, E^{c} \qquad \Rightarrow (1)$$

$$T = (M\,.\,L^{-1}.\,T^{-2})^{a}\ (M.\,L^{-3})^{b}\ \ (M\,.\,L^{2}\cdot T^{-2})^{c}$$

$$T = M^{a}\,.\,L^{-a}.\,T^{-a} \qquad M^{b}.\,L^{-3b} \quad M^{c}.\,L^{2c}\cdot T^{-2c}$$

$$T = M^{a+b+c} \qquad L^{-a-3b+2c} \qquad T^{-a-2c}$$

Problems

For T : $-2a - 2c = 1 \Rightarrow a + c = \frac{-1}{2}$ $\Rightarrow (2)$

For M: $a + b + c = 0$ $\Rightarrow (3)$

For L: $-a - 3b + 2c = 0$ $\Rightarrow (4)$

Substitute from (2) in (3):

$-\frac{1}{2} + b = 0 \Rightarrow b = \frac{1}{2}$ $\Rightarrow (5)$

Substitute in (4):

$\therefore -a - 3\left(\frac{1}{2}\right) + 2c = 0 \Rightarrow -a + 2c = \frac{3}{2}$ $\Rightarrow (6)$

(2)+(6):

$3c = \frac{-1}{2} + \frac{3}{2} \Rightarrow 3c = \frac{2}{2} \Rightarrow 3c = 1 \Rightarrow c = \frac{1}{3}$ $\Rightarrow (7)$

$a + \frac{1}{3} = \frac{-1}{2} \rightarrow a = \frac{-5}{6}$ $\Rightarrow (8)$

Substitute in (1), (8), (7), (5):

$$\therefore T = k\ P^{\frac{-5}{6}}\ d^{\frac{1}{2}}\ E^{\frac{1}{3}} \Rightarrow T = \frac{k\ d^{\frac{1}{2}}\ E^{\frac{1}{3}}}{P^{\frac{5}{6}}}$$

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Thank You...