ELSEWEDY UNIVERSITY OF TECHNOLOGY
- POLYTECHNIC OF EGYPT -

Physics for Electrical Engineering

Lecture 3: Elasticity

© Spring 2024 – **Dr. Rania Gamal**

References

- University Physics Volume 1, 2016
  - https://cnx.org/content/col12031/1.10
- University Physics Volume 2, 2016
  - https://cnx.org/content/col12074/1.9
- D. C. Giancoli, Physics: Principles with Applications, 6th ed. Pearson.

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Classification of material

Solids have a definite size and shape.

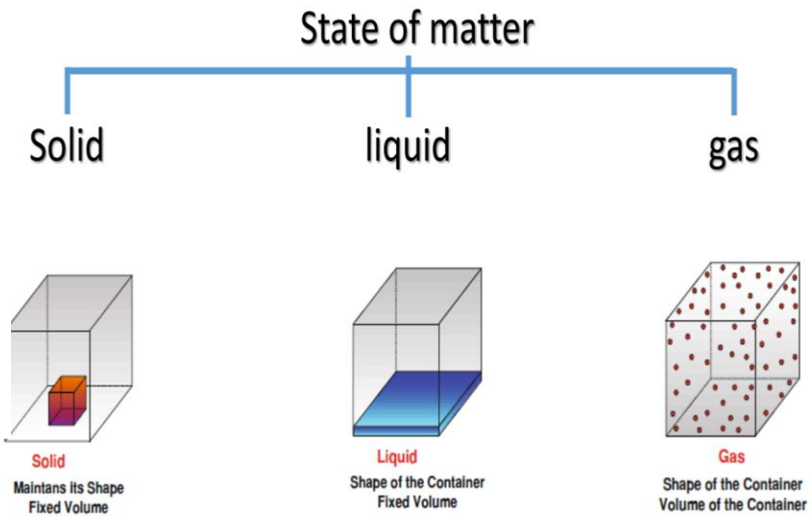
المواد الصلبة لها شكل وحجم محدد.

Liquids have a definite size but take on the shape of the container.

السوائل لها حجم محدد ، ولكن تأخذ شكل الإناء الذي يحويها.

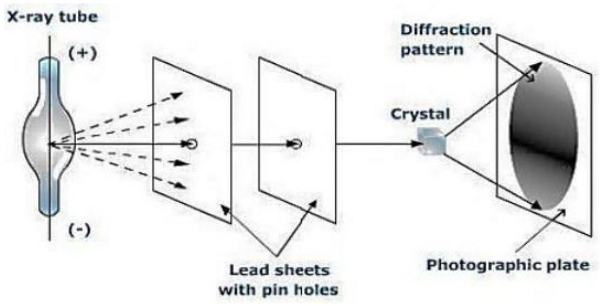
Gases have no definite size or shape and move into all available space

الغازات ليس لها شكل او حجم محددين بل تنتقل فى أي مسحاة متاحة لها.



Atomic structure of solids

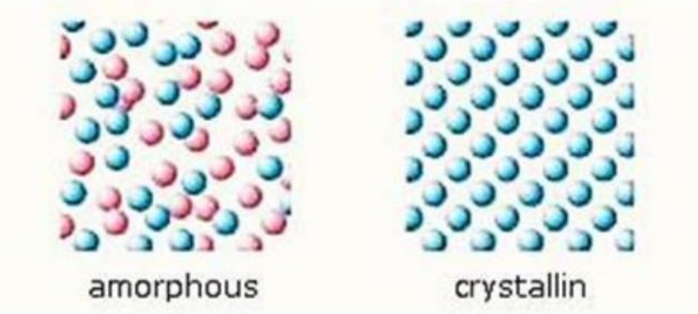
Atomic structure of solids can by determined using **X-ray**



Classification of solids

(a) crystalline , like $NaCl$

(b) Non-crystalline (amorphous), like rubber, glass and plastic



Deformation of Solids تشوه المواد الصلبه

When an object is subjected to external forces, it undergoes changes in size or shape or both. The changes depend on the arrangement and bonding of the atoms in the material.

عندما يتعرض جسم إلي قوة خارجية ، فهذا يسبب تغير الجسم فى الحجم أو الشكل او كليهما وهذه التغيرات تعتمد علي ترتيب وترابط ذرات المادة

Deformation التشوه	
Elastic المرن	**Inelastic (Plastic) غير مرن (لدن)**
When the external force is removed the object returns to its original shape and size	**When the external force is removed the object doesn't return to its original shape and size**
عندما تزول القوة الخارجية يستعيد الجسم شكله الأصلي مرة أخري	**عندما تزول القوة الخارجية لا يستعيد الجسم شكله الأصلي مرة أخري بل يحدث له تشوه دائم**

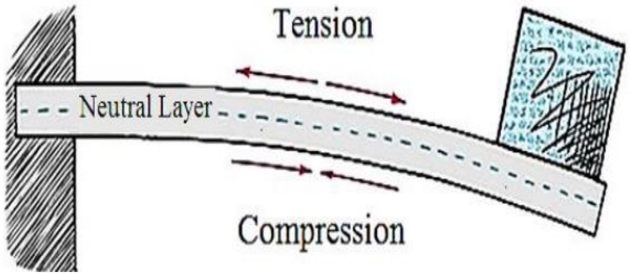
Examples for the deformation of solids

1- A cantilever beam (كمرة معلقة) الكابولي

Top part is under tension السطح العلوي يكون تحت تأثير شد

Bottom part is under compression السطح السفلي تحت تأثير إنضغاط

Middle part is a neutral layer, in which there is neither tension nor compression الجزء الأوسط يكون طبقة المتعادلة ، حيث لا يوجد أي قوى شد أو انضغاط

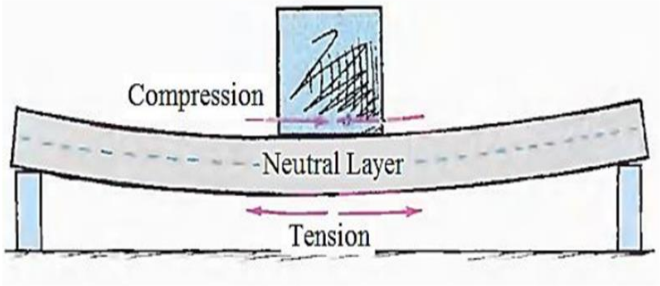


2- A horizontal beam كمرة أفقية

Top part is under compression السطح العلوي يكون تحت تأثير إنضغاط

Bottom part is under tension السطح السفلي تحت تأثير شد

Middle part is a neutral layer, in which there is neither tension nor compression الجزء الأوسط يكون طبقة المتعادلة ، حيث لا يوجد أي قوى شد أو انضغاط

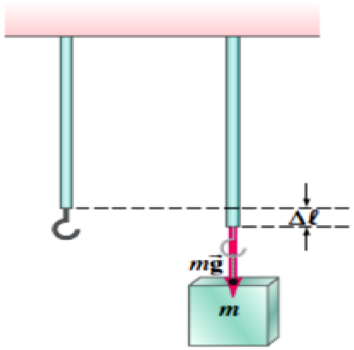


Hooke's Law

When an object is distorted, the amount of distortion stretch or compression is directly proportional to the applied distorting force.

عند تعرض جسم ما للتشوه، فإن ذلك التشوه يتناسب مع القوة المسببة لذلك التشوه

When a stretching force F is applied to the end of a rod of length, then the amount of stretch ΔL in the rod is proportional to the stretching force, F this region is so called elastic region.



عندما نؤثر بقوة شد F على نهاية قضيب طوله L فإن مقدار التمدد ΔL الحادث في القضيب يتناسب مع قوة الشد F ، وتسمى هذه المنطقة بمنطقة المرونة

$$F = k\Delta L$$

k ... *proportionality constant*   ثابت التناسب

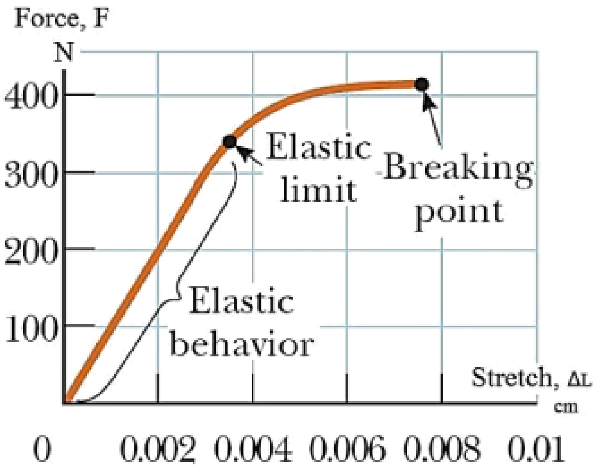
If an elastic material is stretched or compressed beyond a certain amount, it will not return to its original state and will remain distorted, and the region in this called is called plastic region

إذا تمددت (أو انكمشت) المادة المرنة بعد حد معين ، فمن الممكن ان لا تستعيد المادة أبعادها الأصلية بل تظل مشوهه، وتسمى هذه المنطقة بمنطقة اللدونة

Hooke’s Law

Hooke's law holds only as long as the force does not stretch or compress the material beyond its elastic limit.

يتم تطبيق قانون هوك طالما القوة المسببة للتمدد (أو الإنضغاط) لا تتخطي حد المرونة



Stress and Strain الإجهاد والإنفعال

Stress: is the external force acting on an object per unit cross sectional area.

الإجهاد : هو القوة المؤثرة على الجسم لكل وحدة عبر مساحة المقطع

$$Stress = \frac{Force}{Area} = \frac{F}{A} \qquad (N/m^2 \text{ or } Pa)$$

Strain: is a measure of the degree of deformation and the type of strain produced depends on the way in which the stress is applied

الإنفعال : هو مقياس لدرجة التشوه الحادثة فى المادة ، ونوع الانفعال يتوقف على الطريقة التي يتم بها الإجهاد

$$Strain = \frac{amount\ of\ deformation}{undestorted\ shape}$$

Type of Stress and Strain

- **Type of Stress**

1. Tensile Stress $= F/A$
2. Volume Stress $= F/A$
3. Shear Stress $= F/A$

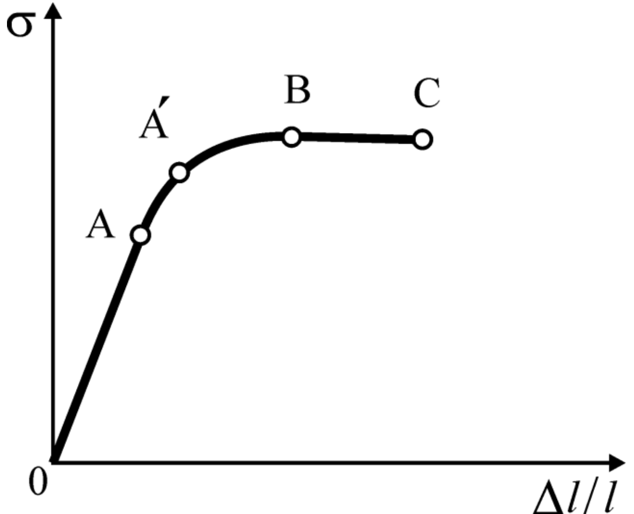
- **Type of Strain**

1. Tensile Strain $= \Delta L/L$
2. Volume Strain $= \Delta V/V$
3. Shear Strain $= \emptyset = x/h$

Relation between Stress and Strain

Discuss Stress-Strain Curve?

- 0A: Linear relation elastic region obeys hock’s law.
- AA’: Non-linear relation elastic region don’t obey hock’s law.
- A’: Elastic Limit: The maximum stress which the material can withstand before the crystal structure is destroyed.
- A’B: Permanent set region.
- BC: Yield region.
- C: Breaking point.

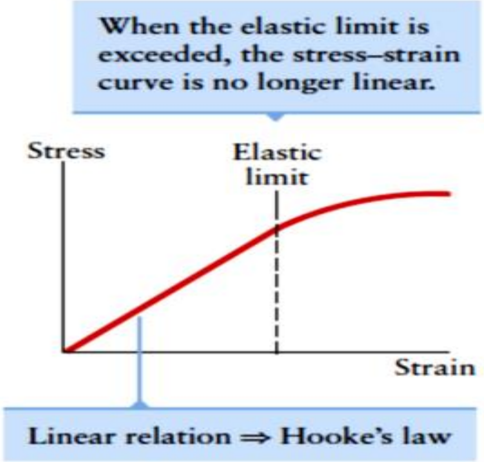


Relation between Stress and Strain

Stress is proportional to strain and the constant of proportionality depends on the material being deformed and on the nature of the deformation.

يتناسب الإجهاد مع الإنفعال وثابت التناسب يعتمد على نوع المادة المشوهة وعلى طبيعة القوة المسببة للتشوه

$$Stress = Constant \times Strain$$



We call this proportionality constant **the elastic modulus**.

نسمي ثابت التناسب **بمعامل المرونة**

The elastic modulus is defined as the ratio of the stress to the resulting strain

معامل المرونة : هو النسبة بين الإجهاد والأنفعال الناتج من ذلك الإجهاد

$$Elastic\ Modulus = \frac{Stress}{Strain} \qquad (Hooke'sLaw)$$

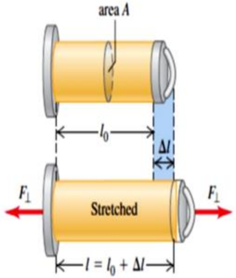
Modulus of Elasticity

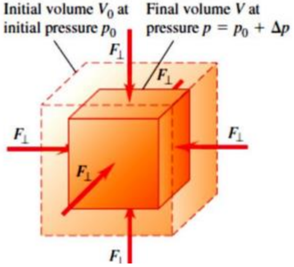
Modulus of Elasticity= $\frac{Sress}{Strain}$

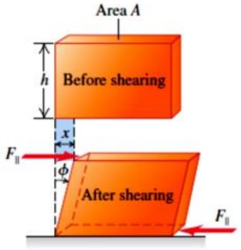
Young Modulus $Y = \frac{F/A}{\Delta L/L} = \frac{F\ L}{A\ \Delta L}$

Bulk Modulus $B = \frac{-P}{\Delta V/V} = \frac{-P\ V}{\Delta V}$

Shear Modulus $S = \frac{F/A}{\emptyset}$







Example

- A wire 3.0 m long and *0.8 mm* in diameter is hanging vertically. A load with a mass of *5 kg* is suspended from the free end of the wire. The length of the wire increased by *0.6 mm*. Determine the stress, the tensile strain, and Young's modulus.

Solution

$$\sigma = \frac{mg}{\pi r^2} = \frac{5 \times 9.8}{3.14 \times (0.4 \times 10^{-3})^2} = 97 \times 10^6\ N/m^2$$

$$\varepsilon_0 = \frac{\Delta L}{L} = \frac{0.6 \times 10^{-3}}{3} = 2 \times 10^{-4}$$

$$Y = \frac{\sigma}{\varepsilon_0} = \frac{97 \times 10^6}{2 \times 10^{-4}} = 48.5 \times 10^{10} N/m^2$$

Example

- A load of 7 $tons$ weight is hung from a vertical bar of rectangular section 2.5 cm x 1.5 cm. Find the tensile stress.

Solution

$$\sigma = \frac{F}{A} = \frac{mg}{A}$$

$$\sigma = \frac{9.8 \times 7 \times 10^3}{1.5 \times 10^{-2} \times 2.5 \times 10^{-2}} = \frac{68.6 \times 10^3}{3.75 \times 10^{-4}} = 18.3 \times 10^7 \; N/m^2$$

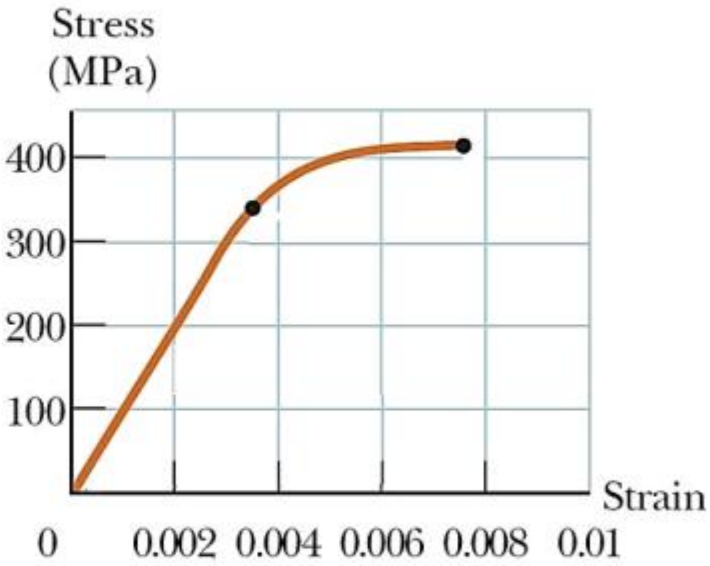
Example

- Evaluate Young's modulus for the material whose stress-strain curve is shown in Figure?

Solution:

$$Y = \frac{stress}{strain} = \frac{200 \times 10^6}{0.002}$$

$$Y = 100 \times 10^9\ Pa = 100\ GPa$$



Example

- A $200Kg$ load is hung on a wire of length $4.00\ m$, cross-sectional area $0.200 \times 10^{-4} m^2$, and Young's modulus $8.00 \times 10^{10} N/m^2$, what is its increase in length?

Solution:

$$\sigma = \frac{F}{A} = \frac{mg}{A}$$

$$\sigma = \frac{9.8 \times 200}{0.2 \times 10^{-4}} = 98 \times 10^{6}\ \ N/m^2$$

$$Y = \frac{\sigma}{\varepsilon_0}$$

$$\varepsilon_0 = \frac{\sigma}{Y} = \frac{98 \times 10^6}{8 \times 10^{10}} = 12.25 \times 10^{-4}$$

$$\varepsilon_0 = \frac{\Delta L}{L}$$

$$\Delta L = \varepsilon_0 L = 12.25 \times 10^{-4} \times 4 = 4.9 \times 10^{-3} m$$

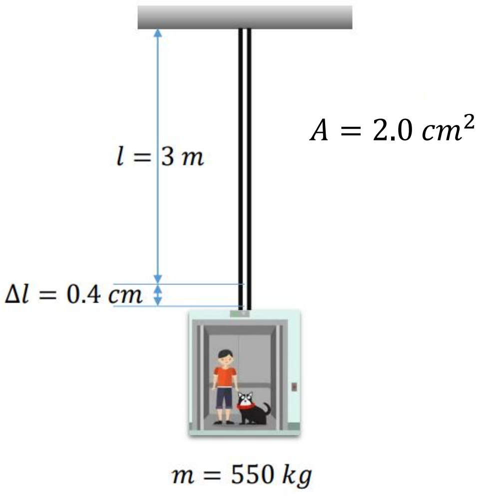
Example

- A small elevator with a mass of 550 kg hangs from a steel cable that is 3.0 m long when not loaded. The wires making up the cable have a total cross-sectional area of 2.0 cm^2 and with a 550 kg load, the cable stretches 0.40 cm beyond its unloaded length. Determine the cable's stress and strain? determine the value of Young's modulus for the cable's steel?

$$\text{stress} = \frac{F}{A} = \frac{550 \times 9.8}{2 \times 10^{-4}} = 26.95 \times 10^{6} Pa$$

$$\text{strain} = \frac{\Delta l}{l_o} = \frac{0.004}{3} = 1.33 \times 10^{-3}\text{m}$$

$$Y = \frac{stress}{strain} = \frac{26.95 \times 10^{6}}{1.33 \times 10^{-3}} = 2 \times 10^{10} Pa$$



Example

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.03 \times 10^5\ N/m^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is$2.03 \times 10^7\ N/m^2$. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?
[The bulk modulus of brass = $6.1 \times 10^{10}\ N/m^2$]

Solution

$$B = -\frac{\Delta P}{\Delta V/V_o} \qquad \rightarrow \qquad \Delta V = -\frac{V_o \Delta P}{B}$$

$$\Delta V = -\frac{(0.5)(2.03 \times 10^7 - 1.03 \times 10^5)}{6.1 \times 10^{10}}$$

$$\Delta V = -1.6 \times 10^{-4} \quad m^3$$

The negative sign indicates that the volume of the sphere decreases.

Example

The bulk modulus of water is $2.2 \times 10^9\ Pa$ .By how much does a cubic meter of water decrease in volume when it is taken from the surface of the ocean down to a depth of 1.0 km, where the pressure is $9.8 \times 10^6\ Pa$ greater than at the surface?

Solution:

$$B = -\frac{\Delta P}{\Delta V / V_o} = -\frac{V_o \times \Delta P}{\Delta V}$$

$$\Delta V = -\frac{V_o \times \Delta P}{B} = -\frac{(1)(9.8 \times 10^6)}{(2.2 \times 10^9)} = -4.45 \times 10^{-3} m^3$$

Example

When water freezes, it expands by about 9.00%. What pressure increase would occur inside your automobile engine block if the water in it froze? (The bulk modulus of ice is $2.00 \times 10^9\ N/m^2$)

Solution:

$$B = \frac{\sigma}{\varepsilon} = \frac{P \cdot V}{\Delta V}$$

$$2 \times 10^9 = \frac{P \cdot V}{0.09\ V}$$

$$P = 18 \times 10^7\ N/m^2$$

Thank You...