ELSEWEDY UNIVERSITY OF TECHNOLOGY
- POLYTECHNIC OF EGYPT -

Applied Discrete Mathematics

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ELSEWEDY UNIVERSITY OF TECHNOLOGY
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Applied Discrete Mathematics
Lecture 1

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Goals of a Discrete Mathematics Course

- *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments.

- *Combinatorial Analysis:* An important problem-solving skill is the ability to count or enumerate objects.

- *Discrete Structures:* A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects

- *Algorithmic Thinking:* Certain classes of problems are solved by the specification of an algorithm.

- *Applications and Modeling:* Discrete mathematics has applications to almost every conceivable area of study.

Applied Discrete Mathematics

- Topics in discrete mathematics will be important in many courses that you will take in the future:

- **Computer Science:** Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ...

- **Mathematics:** Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization, ...

- **Other Disciplines:** You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.

Applied Discrete Mathematics

References

- Kenneth Rosen, (2019) Discrete Mathematics and Its Applications, McGraw-Hill Higher.
- https://www.youtube.com/watch?v=eFDzhn1Inc4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz
- https://www.youtube.com/watch?v=Ho7BD7Yqqtk&list=PLZyQU-WOzZF1rmALoJZthmDKPsqxCV4mW

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- Relations
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The Foundations: Logic and Proofs

Propositional Logic

A **proposition** is a **declarative sentence** (that is, a sentence that declares a fact) **that is either true or false,** but not both**.**

Not [Questions - Command - Opinion - probabilities – variables]

EXAMPLE 1 All the following declarative sentences are propositions.

1. Washington, D.C., is the capital of the United States of America.
2. Toronto is the capital of Canada.
3. $1 + 1 = 2$.
4. $2 + 2 = 3$.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

The Foundations: Logic and Proofs

Some sentences that are not propositions

EXAMPLE 2 Consider the following sentences.

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.
4. $x + y = z$.

- Sentences 1 and 2 are not propositions because they are not declarative sentences.
- Sentences 3 and 4 are not propositions because they are neither true nor false.

- Note that each of sentences 3 and 4 can be turned into a proposition if we assign values to the variables.

The Foundations: Logic and Proofs

We use letters to denote **propositional variables** (or **statement variables**), that is, variables that **represent propositions**, just as letters are used to denote numerical variables.

The conventional letters used for propositional variables are p, q, r, s, The truth value of a proposition is true, denoted by **T**, if it is a true proposition, and the truth value of a proposition is false, denoted by **F**, if it is a false proposition.

The Foundations: Logic and Proofs

Simple propositions contain one sentence true or false.

Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Compound propositions combines between more than one propositions and need to connect between them.

It is raining today, and I am happy.

Negation:

DEFINITION 1

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

EXAMPLE Find the negation of the proposition

"Vandana's smartphone has at least 32GB of memory"

and express this in simple English.

Solution: The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

or even more simply as

"Vandana's smartphone has less than 32GB of memory."

The Foundations: Logic and Proofs

The truth table for the Negation of the propositions.	
P	$\neg p$
T	F
F	T

This table has a row for each of the two possible truth values of a proposition p. Each row shows the truth value of $\neg p$ corresponding to the truth value of p for this row.

The Foundations: Logic and Proofs

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

DEFINITION 3

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

It is raining today, and I am happy.

TABLE 2 **The Truth Table for the Conjunction of Two Propositions.**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 **The Truth Table for the Disjunction of Two Propositions.**

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The Foundations: Logic and Proofs

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

The Foundations: Logic and Proofs

TABLE 4 **The Truth Table for the Exclusive Or of Two Propositions.**

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The Foundations: Logic and Proofs

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

The Foundations: Logic and Proofs

Precedence of logical operators.

Brackets first ()

Precedence	Logical operator
1	$\neg$
2	$\wedge$
3	$\vee$
4	$\rightarrow$
5	$\leftrightarrow$

The Foundations: Logic and Proofs

Truth Tables of Compound Propositions

EXAMPLE 11 Construct the truth table of the compound proposition

$$(p \vee \neg q) \rightarrow (p \wedge q).$$

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Logic and Bit Operations

Computers represent information using bits. A **bit** is a symbol with two possible values, namely, 0 (zero) and 1 (one). This meaning of the word bit comes from *b*inary dig*it*, because zeros and ones are the digits used in binary representations of numbers

A bit can be used to represent a truth value, because there are two truth values, namely, *true* and *false*. As is customarily done, we will use a 1 bit to represent true and a 0 bit to represent false. That is, 1 represents T (true), 0 represents F (false). A variable is called a **Boolean variable** if its value is either true or false. Consequently, a Boolean variable can be represented using a bit.

Truth Value	*Bit*
T	1
F	0

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

The Foundations: Logic and Proofs

DEFINITION 7 A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

Ex: 101010011 is a bit string of length nine.

EXAMPLE Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

EXAMPLE 13 Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

Solution: The bitwise *OR*, bitwise *AND*, and bitwise *XOR* of these strings are obtained by taking the *OR*, *AND*, and *XOR* of the corresponding bits, respectively. This gives us

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

The Foundations: Logic and Proofs

Precedence of logical operators.

Precedence	Logical operator
1	$\neg$
2	$\wedge$
3	$\vee$
4	$\rightarrow$
5	$\leftrightarrow$

Example: Prove that

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

This is the distributive law of disjunction over conjunction.

No of propositions =3
p, q, and r

Solution :

We construct the truth table for these compound propositions

The Foundations: Logic and Proofs

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	**T**	T	T	**T**
T	T	F	F	**T**	T	T	**T**
T	F	T	F	**T**	T	T	**T**
T	F	F	F	**T**	T	T	**T**
F	T	T	T	**T**	T	T	**T**
F	T	F	F	**F**	T	F	**F**
F	F	T	F	**F**	F	T	**F**
F	F	F	F	**F**	F	F	**F**

The columns corresponding to $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are identical, then these two compound propositions are logically equivalent.

The Foundations: Logic and Proofs

Applications of Propositional Logic:

Logic Circuits:

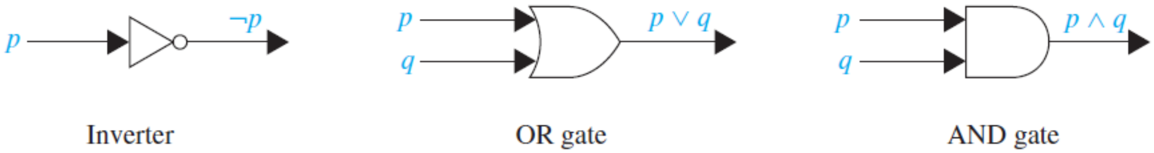


FIGURE 1 Basic logic gates.

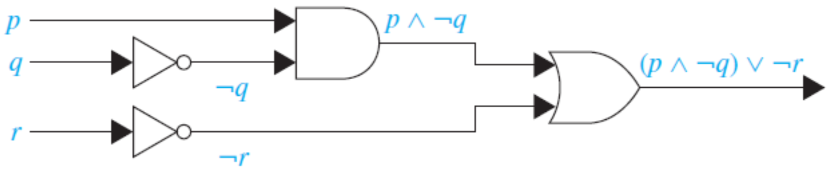
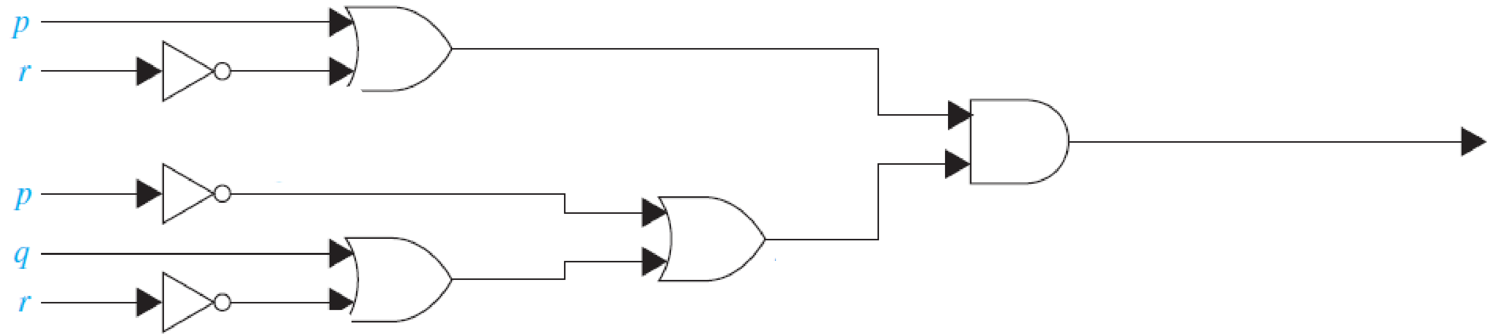


FIGURE 2 A combinatorial circuit.

The Foundations: Logic and Proofs

Applications of Propositional Logic:

Logic Circuits:



The Foundations: Logic and Proofs

Applications of Propositional Logic:

Logic Circuits:

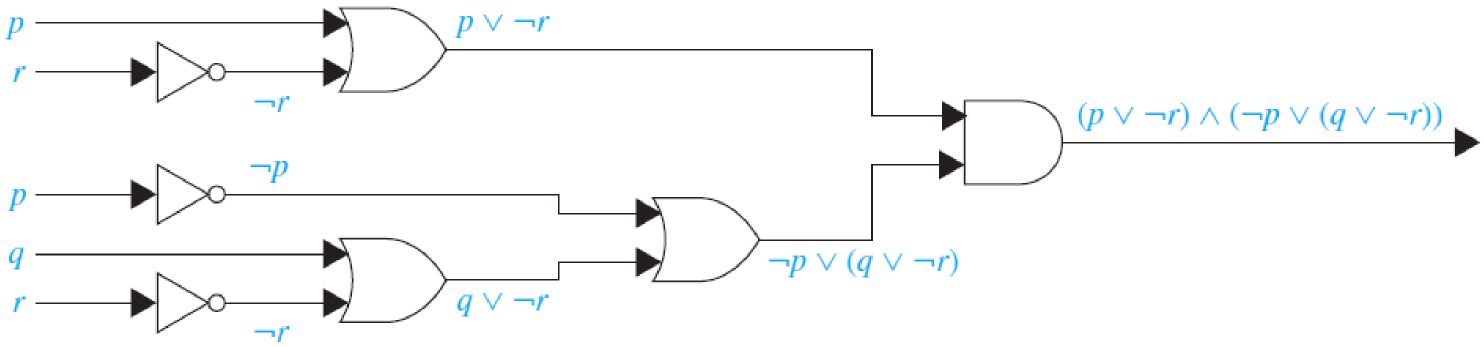


FIGURE 3 The circuit for $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$.

The Foundations: Logic and Proofs

36. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$ **b)** $(p \vee q) \wedge r$
c) $(p \wedge q) \vee r$ **d)** $(p \wedge q) \wedge r$
e) $(p \vee q) \wedge \neg r$ **f)** $(p \wedge q) \vee \neg r$

43. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001
b) 1111 0000, 1010 1010
c) 00 0111 0001, 10 0100 1000
d) 11 1111 1111, 00 0000 0000

Thank you !