ELSEWEDY UNIVERSITY OF TECHNOLOGY
- POLYTECHNIC OF EGYPT -

Applied Discrete Mathematics

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Applied Discrete Mathematics
Lecture 8

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Permutations

A **permutation** of a set of distinct objects is an ordered arrangement of these objects.

We also are interested in ordered arrangements of some of the elements of a set.

An ordered arrangement of r elements of a set is called an ***r*-permutation**.

Let $S = \{1, 2, 3\}$.
The ordered arrangement 3, 1, 2 is a permutation of S.
The ordered arrangement 3, 2 is a 2-permutation of S.

The number of r-permutations of a set with n elements is denoted by $P(n, r)$.
We can find $P(n, r)$ using the product rule.

Permutations and Combinations.

Permutations

The number of permutations of n different elements is $n!$ where $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

For instance, the number of permutations of the four letters a, b, c, and d will be $4! = 24$.

Permutations and Combinations.

Permutations

THEOREM 1 If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1)$$

r-permutations of a set with n distinct elements.

If n and r are integers with $0 \leq r \leq n$, then $P(n, r) = \dfrac{n!}{(n-r)!}$.

Permutations and Combinations.

Permutations

Let $S = \{a, b, c\}$. The 2-permutations of S are the ordered arrangements a, b; a, c; b, a; b, c; c, a; and c, b.

Consequently, there are six 2-permutations of this set with three elements. There are always six 2-permutations of a set with three elements

If n and r are integers with $0 \le r \le n$, then $P(n, r) = \dfrac{n!}{(n-r)!}$.

Permutations and Combinations.

Permutations

Let's say you have the set {A, B, C} and you want to find all possible permutations of this set.

1.The permutations of this set are:

1. ABC
2. ACB
3. BAC
4. BCA
5. CAB
6. CBA

Permutations and Combinations.

Permutations

1. **Problem:** How many different arrangements can be made using the letters of the word "APPLE"?**Solution:** The word "APPLE" has 5 letters. To find the number of permutations, we use the formula for permutations of n objects taken r at a time, which is $P(n, r) = \frac{n!}{(n-r)!}$.For "APPLE", $n = 5$ (number of letters). We want to arrange all 5 letters, so $r = 5$.$P(5, 5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.Therefore, there are 120 different arrangements of the letters in "APPLE".
2. **Problem:** How many different 4-digit numbers can be formed using the digits 1, 2, 3, and 4 without repetition?**Solution:** We need to find the number of permutations of 4 digits taken from the set {1, 2, 3, 4}.Using the formula for permutations, $P(n, r) = \frac{n!}{(n-r)!}$, where $n = 4$ (number of digits) and $r = 4$ (number of positions), we have:$P(4, 4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4! = 4 \times 3 \times 2 \times 1 = 24$.Therefore, there are 24 different 4-digit numbers that can be formed using the digits 1, 2, 3, and 4 without repetition.

Permutations and Combinations.

Problem: In how many ways can you arrange 3 books on a shelf out of a collection of 5 different books?

The formula for permutations of n objects taken r at a time is:

$$P(n, r) = \frac{n!}{(n-r)!}$$

where n is the total number of objects and r is the number of objects to be chosen.

In this problem:

- $n = 5$ (total number of books)
- $r = 3$ (number of books to be arranged on the shelf)

Plugging these values into the permutation formula, we get:

$$P(5, 3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\times4\times3\times2\times1}{2\times1} = 5 \times 4 \times 3 = 60$$

So, there are 60 different ways to arrange 3 books on a shelf out of a collection of 5 different books.

Permutations and Combinations.

Permutations

Example 1:

Consider a set of elements, such as $S = \{a, b, c, d, e\}$.

What is the number of permutation of subsets of **3** elements selected from S is?

Permutations and Combinations.

Permutations

Example 1:

Consider a set of elements, such as $S = \{a, b, c, d, e\}$.

What is the number of permutation of subsets of **3** elements selected from S is?

$$P(n,r) = \frac{n!}{(n-r)!}$$

Solution:

$\boldsymbol{r = 3}$, $\quad \boldsymbol{n = 5}$

$$P(5,3) = \frac{5!}{(5-3)!}$$

$$= \frac{5!}{(2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$$

Permutations and Combinations.

Permutations

Example 2:

How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Permutations and Combinations.

Permutations

Example 2:

How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

$$P(n,r) = \frac{n!}{(n-r)!}$$

Solution:

$\boldsymbol{r = 3}, \qquad \boldsymbol{n = 100}$

$$P(100,3) = \frac{100!}{(100-3)!}$$

$$= \frac{100!}{(97)!} = 100 \times 99 \times 98 = 970{,}200$$

Permutations and Combinations.

Combinations:

The order does not matter

We now turn our attention to counting **unordered** selections of objects. To find the number of subsets of a particular size of a set with n elements, where n is a positive integer. An ***r*-combination** of elements of a set is an unordered selection of r elements from the set.

$$C(n,r) = C_r^n = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$C(n,r) = C(n,n-r)$$

Permutations and Combinations.

Combinations

- Let S be the set {1, 2, 3, 4}. Then {1, 3, 4} is a 3-combination from S.
- (Note that {4, 1, 3} is the same 3-combination as {1, 3, 4}, because the order in which the elements of a set are listed **does not matter.)**

- We see that C(4, 2) = 6, because the 2-combinations of {a, b, c, d} are the six subsets {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, and {c, d}.

Permutations and Combinations.

Combinations

Example 1:

How many possible selections of 3 balls from a box contains 10 colored balls ?

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

Solution:

$\boldsymbol{r = 3}, \qquad \boldsymbol{n = 10}$

$$C(10,3) = \frac{10!}{3!\,(10-3)!}$$

$$= \frac{10!}{3!\,7!} = 120$$

Permutations and Combinations.

Combinations

Example 2:

How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school ?

Permutations and Combinations.

Combinations

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How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school ?

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

Solution:

$\boldsymbol{r = 5}$, $\boldsymbol{n = 10}$

$$C(10,5) = \frac{10!}{5!\,(10-5)!}$$

$$= \frac{10!}{5!\,5!} = 252$$

Permutations and Combinations.

Combinations

EXAMPLE 3 A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

Solution: The number of ways to select a crew of six from the pool of 30 people is the number of 6-combinations of a set with 30 elements, because the order in which these people are chosen does not matter. By Theorem 2, the number of such combinations is

$$C(30, 6) = \frac{30!}{6!\,24!} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 593{,}775.$$

Permutations and Combinations.

Combinations

Example 3:

Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

Permutations and Combinations.

Combinations

Example 3:

Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

Solution:

$$C(9,3)\cdot C(11,4)=\frac{9!}{3!\,6!}\cdot\frac{11!}{4!\,7!}=84\cdot 330=27{,}720$$

Binomial Coefficients and Identities.

THE BINOMIAL THEOREM

Let x and y be variables and let n be a nonnegative integer.

Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n$$

The number of ***r*-combinations** from a set with n elements is often denoted by $\binom{n}{r}$

Binomial Coefficients

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

Binomial Coefficients and Identities.

Example 1:

What is the expansion of $(x+y)^4$?

Binomial Coefficients and Identities.

Example 1:

What is the expansion of $(x + y)^4$?

Solution:

From the binomial theorem it follows that

$$(x+y)^4 = \sum_{j=0}^{4} \binom{4}{j} x^{4-j} y^j$$

$$= \binom{4}{0} x^4 + \binom{4}{1} x^3 y + \binom{4}{2} x^2 y^2 + \binom{4}{3} x y^3 + \binom{4}{4} y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

Binomial Coefficients and Identities.

Example 2:

What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

Solution:

From the binomial theorem it follows that this coefficient is

$$\binom{25}{13} = \frac{25!}{13!\,12!} = 5{,}200{,}300.$$

Thank you !