ELSEWEDY UNIVERSITY OF TECHNOLOGY
- POLYTECHNIC OF EGYPT -

Physics for Electrical Engineering

Lecture 5: Wave and Oscillations

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Simple harmonic motion (SHM)

Introduction

- When the force on a body **is proportional to its displacement** from an equilibrium position, and if this force **acts always toward the equilibrium position** of the body, there is a repetitive **back-and-forth** motion about this position.
- **Such motion is called periodic or oscillatory motion.**

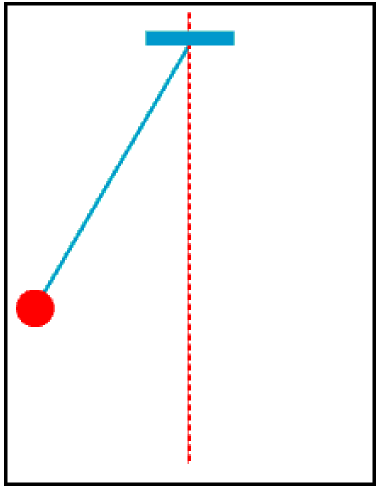
Simple harmonic motion (SHM)

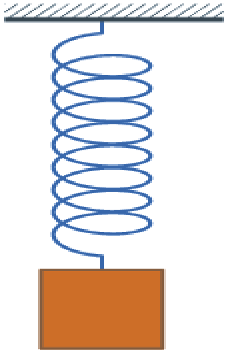
- **Aperiodic motion**

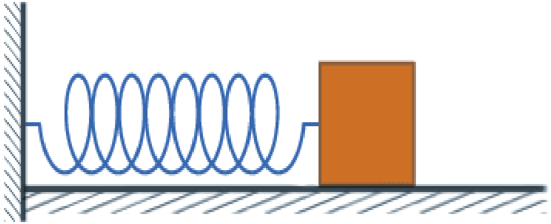
It is a motion of an object that regularly returns to a given position after a fixed time interval.

- **Simple Harmonic Motion (SHM)**

It is a special kind of periodic motion occurs in mechanical system when the force acting on an object is proportional to the position of the object relative to some equilibrium position







Mass-Spring System

When the mass is at the position x=0,

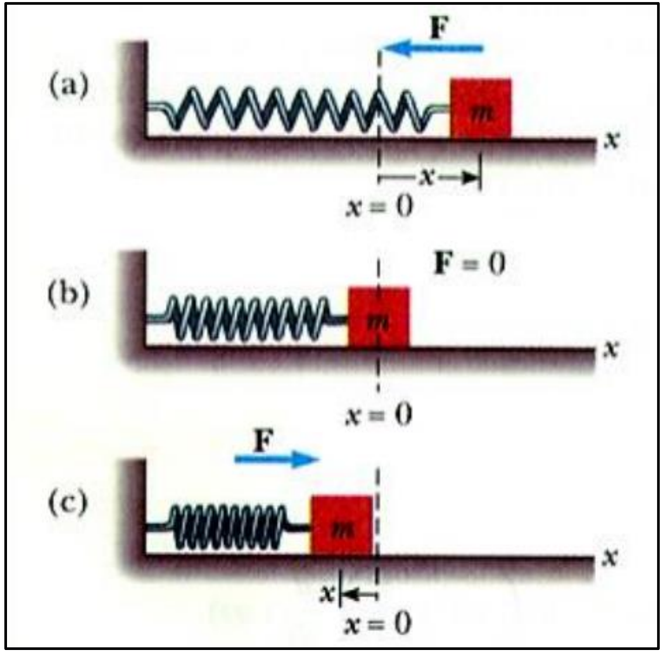
this position is called:

The equilibrium position

The system will oscillate back and forth

if disturbed from its equilibrium position

 and will describe a SHM



Mass-Spring System

- **Motion of a mass attached to a spring**

When the mass is displaced a small distance x from equilibrium, the spring exerts a force on m given by:

$$F = -kx$$

The force F is a linear restoring force

because it is linearly proportional to the displacement and is always directed toward the equilibrium position and therefore opposite the displacement.

Mass-Spring System

- We apply **Newton's second law** to the motion of the mass in the x direction:

$F = -kx$

$F = ma \qquad (1)$

$-kx = ma$

$ma + kx = 0 \qquad (2)$

$$a + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad (3)$$

Mass-Spring System

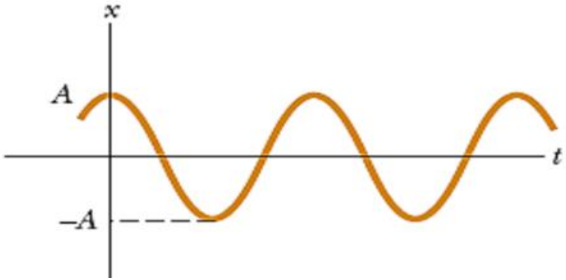
$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

The solution of this differential equation must be that of simple harmonic motion:

$$x(t) = A\cos(\omega t + \varphi)$$

where,

A is called the amplitude of the motion, which is the maximum displacement of the particle in either the positive or the negative x direction



The Periodic time, The frequency and The angular frequency

- φ is called the phase constant (*phase* angle)
- A and φ are determined by the initial conditions of the particle and tell us what was the displacement at $t = 0$.
- ω is called angular frequency (*radians/second*) $\omega = \sqrt{\frac{k}{m}}$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- The quantity $(\omega t + \varphi)$ is called the phase of the motion.

The Periodic time, The frequency and The angular frequency

- The period T of the motion is the time it takes the particle to go through one full cycle

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

- The inverse of the period is called the frequency of the motion (f) (Hz)

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Motion of a mass attached to a spring

- The equation of **displacement is:**

$$\boldsymbol{x(t) = Acos(\omega t + \varphi)}$$

- We obtained **the velocity** of the particle by differentiating the equation of **displacement** with respect to time:

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \varphi)$$

- We obtained **the acceleration** of the particle by differentiating the equation of **velocity** with respect to time:

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \varphi)$$

Motion of a mass attached to a spring

Since $x = A\cos(\omega t + \varphi)$

then: $a = -\omega^2 x$

Since the sine and cosine functions oscillate between ± 1, the extreme values of v $are\ \pm A\omega$ and the extreme value of the acceleration are $\pm A\omega^2$.

Therefore, the maximum values of the speed and acceleration are:

$$v_{max} = \pm A\omega$$
$$a_{max} = \pm A\omega^2$$

Summary

- The equation of displacement

$$\boldsymbol{x(t) = Acos(\omega t + \varphi)}$$

The maximum displacement is: $x_{max} = A$

- The equation of velocity

$$v(t) = -A\omega \sin(\omega t + \varphi)$$

The maximum velocity is: $v_{max} = \omega A$

- The equation of acceleration

$$a(t) = -A\omega^2 \cos(\omega t + \varphi)$$

$$a(t) = -\omega^2 x$$

The maximum acceleration is: $a_{max} = \omega^2 A$

Problems

- **A simple harmonic oscillator takes 12 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency (ω) in radian per second?**

Solution:

(a) the period of its motion

$$T = \frac{12}{5} = 2.4\ s$$

(b) the frequency in hertz

$$f = \frac{1}{T} = \frac{1}{2.4} = 0.42\ Hz$$

(c) the angular frequency (ω) in radian per second

$$\omega = 2\pi f = 2 \times 3.14 \times 0.42 = 2.6\ rad/sec$$

Problems

- **A piston in a gasoline engine is in simple harmonic motion. Taking the extremes of its position relative to its center point as ± 5 cm , Find magnitude of (a) the maximum velocity of the piston when the angular frequency is $120\,\pi\, rad/sec$. and (b) the maximum acceleration of the piston.**

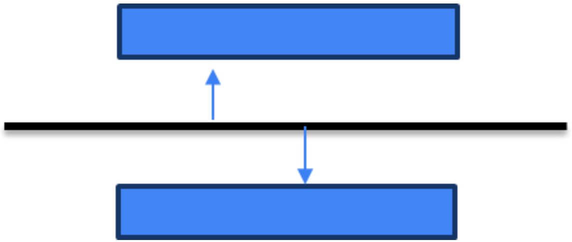
Solution:

$A = 5\ cm$

(a) the maximum velocity

$v_{max} = A\omega$

$v_{max} = 5 \times 120\,\pi = 1884\ cm/s = 18.84 m/s$



(b) the maximum acceleration

$a_{max} = A\omega^2$

$a_{max} = 5 \times (120\,\pi)^2 = 709891 cm/s^2 = 7099\ m/s^2$

Problems

- In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression $x = 5\cos(2t + \frac{180}{6})$, where x is in centimeters and t is in seconds, At t=0, find (a) the position of the particle, (b) it's velocity, and (c) its acceleration. Find (d) the period time and (e) the amplitude of the motion?

Solution

$A = 5\ cm$

$\omega = 2\ rad/sec$

At t=0

(a) $x = 5\cos\left(2t + \frac{180}{6}\right)$

$$x = 5\cos\left(0 + \frac{180}{6}\right) = 4.33\ \text{cm}$$

(b) $v = \frac{dx}{dt} = -10\sin\left(2t + \frac{180}{6}\right)$

$$v = -10\sin\left(0 + \frac{180}{6}\right) = -5\text{cm/s}$$

(c) $a = \frac{dv}{dt} = -20\cos\left(2t + \frac{180}{6}\right)$

$$a = -20\cos\left(0 + \frac{180}{6}\right) = -17.3\text{m/s}^2$$

(d) $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

$T = \pi$

(e) $A = 5\ cm$

Problems

- Body oscillates with SHM a long the x-axis. Its displacement varies with time according to the equation: $x = 4\cos(\pi t + \frac{180}{4})$. (a) Determine the amplitude, frequency and period of the motion. (b) Calculate the velocity and acceleration of the body at any time t. (c) Using the results to part (b), determine the position, velocity and acceleration of the body at t=0 s.

Solution

$x = 4\cos(\pi t + \frac{\pi}{4})$

(a) $A = 4\ m$

$f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} Hz$

$T = \frac{1}{f} = 2 sec$

(b) $x = 4\cos(\pi t + \frac{180}{4})$

$v = \frac{dx}{dt} = -4\pi \sin\left(\pi t + \frac{180}{4}\right)$

$a = \frac{dv}{dt} = -4\pi^2 \cos\left(\pi t + \frac{180}{4}\right)$

(c) At t=0 s

$x = 4\cos\left(0 + \frac{180}{4}\right) = 4\cos\left(\frac{180}{4}\right) = 2.8m$

$v = -4\pi \sin\left(0 + \frac{180}{4}\right) = -4\pi \sin\left(\frac{180}{4}\right) = -\ 8.89 m/s$

$a = -4\pi^2 \cos\left(0 + \frac{180}{4}\right) = -4\pi^2 \cos\left(\frac{180}{4}\right) = -27.91 m/s^2$

Problems

- An object attached to a spring vibrates with simple harmonic motion as described by figure. For this motion, find (a) the amplitude, (b) the period, (c) the angular frequency, the magnitude of (d) the maximum speed, and (e) the acceleration.

Solution:

$(a)\, A = 2\, m$

$(b) T = 4\, sec$

$(c)\, \omega = \frac{2\pi}{T} = \frac{2\pi}{4} = 1.57\, rad/sec$

$(d) v_{max} = A\omega = 2 \times 1.57 = 3.14\, m/s$

$(e)\, a_{max} = A\omega^2 = 2 \times (1.57)^2 = 4.9\, m/s^2$