# DIAGNOSTIC TESTS

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

## A  DIAGNOSTIC TEST: ALGEBRA

1. Evaluate each expression without using a calculator.
   (a) $(-3)^4$
   (b) $-3^4$
   (c) $3^{-4}$
   (d) $\dfrac{5^{23}}{5^{21}}$
   (e) $\left(\dfrac{2}{3}\right)^{-2}$
   (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.
   (a) $\sqrt{200} - \sqrt{32}$
   (b) $(3a^3b^3)(4ab^2)^2$
   (c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplfy.
   (a) $3(x+6) + 4(2x-5)$
   (b) $(x+3)(4x-5)$
   (c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$
   (d) $(2x+3)^2$
   (e) $(x+2)^3$

4. Factor each expression.
   (a) $4x^2 - 25$
   (b) $2x^2 + 5x - 12$
   (c) $x^3 - 3x^2 - 4x + 12$
   (d) $x^4 + 27x$
   (e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$
   (f) $x^3y - 4xy$

5. Simplify the rational expression.
   (a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$
   (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x+3}{2x+1}$
   (c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x+1}{x+2}$
   (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

**6.** Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5} - 2}$

(b) $\dfrac{\sqrt{4 + h} - 2}{h}$

**7.** Rewrite by completing the square.

(a) $x^2 + x + 1$

(b) $2x^2 - 12x + 11$

**8.** Solve the equation. (Find only the real solutions.)

(a) $x + 5 = 14 - \tfrac{1}{2}x$

(b) $\dfrac{2x}{x + 1} = \dfrac{2x - 1}{x}$

(c) $x^2 - x - 12 = 0$

(d) $2x^2 + 4x + 1 = 0$

(e) $x^4 - 3x^2 + 2 = 0$

(f) $3|x - 4| = 10$

(g) $2x(4 - x)^{-1/2} - 3\sqrt{4 - x} = 0$

**9.** Solve each inequality. Write your answer using interval notation.

(a) $-4 < 5 - 3x \leqslant 17$

(b) $x^2 < 2x + 8$

(c) $x(x - 1)(x + 2) > 0$

(d) $|x - 4| < 3$

(e) $\dfrac{2x - 3}{x + 1} \leqslant 1$

**10.** State whether each equation is true or false.

(a) $(p + q)^2 = p^2 + q^2$

(b) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

(c) $\sqrt{a^2 + b^2} = a + b$

(d) $\dfrac{1 + TC}{C} = 1 + T$

(e) $\dfrac{1}{x - y} = \dfrac{1}{x} - \dfrac{1}{y}$

(f) $\dfrac{1/x}{a/x - b/x} = \dfrac{1}{a - b}$

## ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

**1.** (a) 81 (b) $-81$ (c) $\frac{1}{81}$
(d) 25 (e) $\frac{9}{4}$ (f) $\frac{1}{8}$

**2.** (a) $6\sqrt{2}$ (b) $48a^5b^7$ (c) $\dfrac{x}{9y^7}$

**3.** (a) $11x - 2$ (b) $4x^2 + 7x - 15$
(c) $a - b$ (d) $4x^2 + 12x + 9$
(e) $x^3 + 6x^2 + 12x + 8$

**4.** (a) $(2x - 5)(2x + 5)$ (b) $(2x - 3)(x + 4)$
(c) $(x - 3)(x - 2)(x + 2)$ (d) $x(x + 3)(x^2 - 3x + 9)$
(e) $3x^{-1/2}(x - 1)(x - 2)$ (f) $xy(x - 2)(x + 2)$

**5.** (a) $\dfrac{x + 2}{x - 2}$ (b) $\dfrac{x - 1}{x - 3}$
(c) $\dfrac{1}{x - 2}$ (d) $-(x + y)$

**6.** (a) $5\sqrt{2} + 2\sqrt{10}$ (b) $\dfrac{1}{\sqrt{4 + h} + 2}$

**7.** (a) $\left(x + \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$ (b) $2(x - 3)^2 - 7$

**8.** (a) 6 (b) 1 (c) $-3, 4$
(d) $-1 \pm \tfrac{1}{2}\sqrt{2}$ (e) $\pm 1, \pm \sqrt{2}$ (f) $\tfrac{2}{3}, \tfrac{22}{3}$
(g) $\tfrac{12}{5}$

**9.** (a) $[-4, 3)$ (b) $(-2, 4)$
(c) $(-2, 0) \cup (1, \infty)$ (d) $(1, 7)$
(e) $(-1, 4]$

**10.** (a) False (b) True (c) False
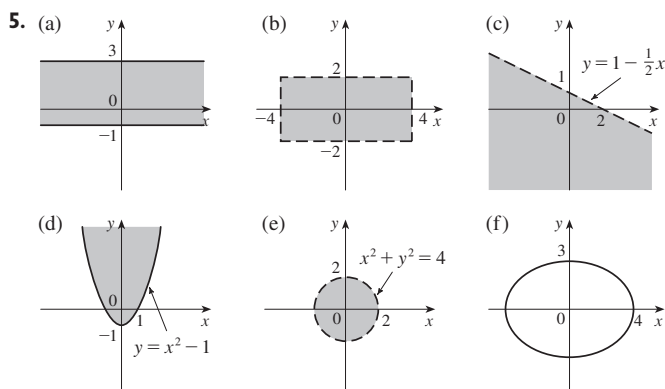(d) False (e) False (f) True

If you have had difficulty with these problems, you may wish to consult the Review of Algebra on the website **www.stewartcalculus.com**.

## B    DIAGNOSTIC TEST: ANALYTIC GEOMETRY

**1.** Find an equation for the line that passes through the point $(2, -5)$ and
    (a) has slope $-3$
    (b) is parallel to the $x$-axis
    (c) is parallel to the $y$-axis
    (d) is parallel to the line $2x - 4y = 3$

**2.** Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

**3.** Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

**4.** Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
    (a) Find the slope of the line that contains $A$ and $B$.
    (b) Find an equation of the line that passes through $A$ and $B$. What are the intercepts?
    (c) Find the midpoint of the segment $AB$.
    (d) Find the length of the segment $AB$.
    (e) Find an equation of the perpendicular bisector of $AB$.
    (f) Find an equation of the circle for which $AB$ is a diameter.

**5.** Sketch the region in the $xy$-plane defined by the equation or inequalities.
    (a) $-1 \le y \le 3$
    (b) $|x| < 4$ and $|y| < 2$
    (c) $y < 1 - \frac{1}{2}x$
    (d) $y \ge x^2 - 1$
    (e) $x^2 + y^2 < 4$
    (f) $9x^2 + 16y^2 = 144$

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### ANSWERS TO DIAGNOSTIC TEST B: ANALYTIC GEOMETRY

**1.** (a) $y = -3x + 1$      (b) $y = -5$
    (c) $x = 2$      (d) $y = \frac{1}{2}x - 6$

**2.** $(x + 1)^2 + (y - 4)^2 = 52$

**3.** Center $(3, -5)$, radius $5$

**4.** (a) $-\frac{4}{3}$
    (b) $4x + 3y + 16 = 0$; $x$-intercept $-4$, $y$-intercept $-\frac{16}{3}$
    (c) $(-1, -4)$
    (d) $20$
    (e) $3x - 4y = 13$
    (f) $(x + 1)^2 + (y + 4)^2 = 100$

**5.** (a), (b), (c), (d), (e), (f)

## C  DIAGNOSTIC TEST: FUNCTIONS



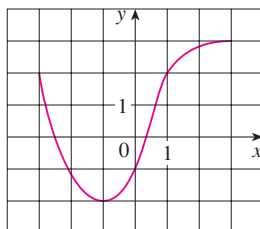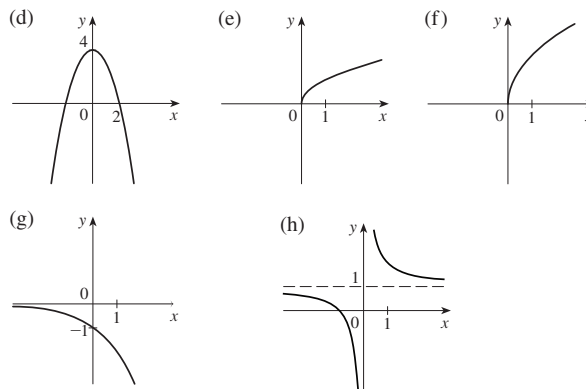**FIGURE FOR PROBLEM 1**

**1.** The graph of a function $f$ is given at the left.
   (a) State the value of $f(-1)$.
   (b) Estimate the value of $f(2)$.
   (c) For what values of $x$ is $f(x) = 2$?
   (d) Estimate the values of $x$ such that $f(x) = 0$.
   (e) State the domain and range of $f$.

**2.** If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2 + h) - f(2)}{h}$ and simplify your answer.

**3.** Find the domain of the function.

   (a) $f(x) = \dfrac{2x + 1}{x^2 + x - 2}$      (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$      (c) $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

**4.** How are graphs of the functions obtained from the graph of $f$?
   (a) $y = -f(x)$      (b) $y = 2f(x) - 1$      (c) $y = f(x - 3) + 2$

**5.** Without using a calculator, make a rough sketch of the graph.
   (a) $y = x^3$      (b) $y = (x + 1)^3$      (c) $y = (x - 2)^3 + 3$
   (d) $y = 4 - x^2$      (e) $y = \sqrt{x}$      (f) $y = 2\sqrt{x}$
   (g) $y = -2^x$      (h) $y = 1 + x^{-1}$

**6.** Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

   (a) Evaluate $f(-2)$ and $f(1)$.    (b) Sketch the graph of $f$.

**7.** If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
   (a) $f \circ g$      (b) $g \circ f$      (c) $g \circ g \circ g$

### ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

**1.** (a) $-2$      (b) 2.8
   (c) $-3, 1$      (d) $-2.5, 0.3$
   (e) $[-3, 3], [-2, 3]$

**2.** $12 + 6h + h^2$

**3.** (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
   (b) $(-\infty, \infty)$
   (c) $(-\infty, -1] \cup [1, 4]$

**4.** (a) Reflect about the $x$-axis
   (b) Stretch vertically by a factor of 2, then shift 1 unit downward
   (c) Shift 3 units to the right and 2 units upward

**5.** (a)    (b)    (c) 

(d)    (e)    (f) 

(g)    (h) 

**6.** (a) $-3, 3$
   (b) 

**7.** (a) $(f \circ g)(x) = 4x^2 - 8x + 2$
   (b) $(g \circ f)(x) = 2x^2 + 4x - 5$
   (c) $(g \circ g \circ g)(x) = 8x - 21$

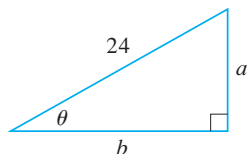If you have had difficulty with these problems, you should look at Sections 1.1–1.3 of this book.

## D | DIAGNOSTIC TEST: TRIGONOMETRY

**I.** Convert from degrees to radians.
  (a) $300°$       (b) $-18°$

**2.** Convert from radians to degrees.
  (a) $5\pi/6$       (b) $2$

**3.** Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30°$.

**4.** Find the exact values.
  (a) $\tan(\pi/3)$       (b) $\sin(7\pi/6)$       (c) $\sec(5\pi/3)$

**5.** Express the lengths $a$ and $b$ in the figure in terms of $\theta$.

**6.** If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where $x$ and $y$ lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

**7.** Prove the identities.
  (a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$

  (b) $\dfrac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

**8.** Find all values of $x$ such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

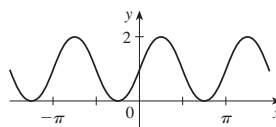**9.** Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

24

$a$

$\theta$

$b$

**FIGURE FOR PROBLEM 5**

ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

**I.** (a) $5\pi/3$       (b) $-\pi/10$

**2.** (a) $150°$       (b) $360/\pi \approx 114.6°$

**3.** $2\pi$ cm

**4.** (a) $\sqrt{3}$       (b) $-\frac{1}{2}$       (c) $2$

**5.** (a) $24 \sin \theta$       (b) $24 \cos \theta$

**6.** $\frac{1}{15}\left(4 + 6\sqrt{2}\right)$

**8.** $0, \pi/3, \pi, 5\pi/3, 2\pi$

**9.**



If you have had difficulty with these problems, you should look at Appendix D of this book.