Time Series Analysis for US's Inflation Rate

STAT 485 Applied Time Series Analysis

Team #3

Jiayi Wang (301417338)

Yuxuan Zhang (301390258)

Yilun Zhou ()

Ziying Peng (301417046)

1. Introduction

Inflation, which is defined as a decrease in the purchasing power of money, represents a large proportion of the economy. A too-high or too-low inflation rate will most likely bring negative consequences. To stabilize inflation, for example, the government usually has to interpose politically, as well as the central bank needs to adjust the interest rate. However, before any moves are made, fluctuations in the rate of inflation are inevitable, and we assume the current external contributions to the inflation rate are likely to happen in the future. In our project, we will use the inflation data of the U.S. from 2000 to 2022 to analyze by using time series models. To select models, the Box-Jenkins method and the Dynamic method are used. The goal of the project is to predict how the U.S. inflation rate is going to change in a given period.

2.Time Series Analysis

2.1 Model Identification

STEP 1: Plot the time series plot and check stationary

The first step of doing model specification is to check the stationary. If the series is stationary, then move to the next step; otherwise, we need to do some transformations. Here we plotted the time series for the original data.

Figure 2.1.1 Time series plot of the Inflation rate in the US from 2000 to 2022

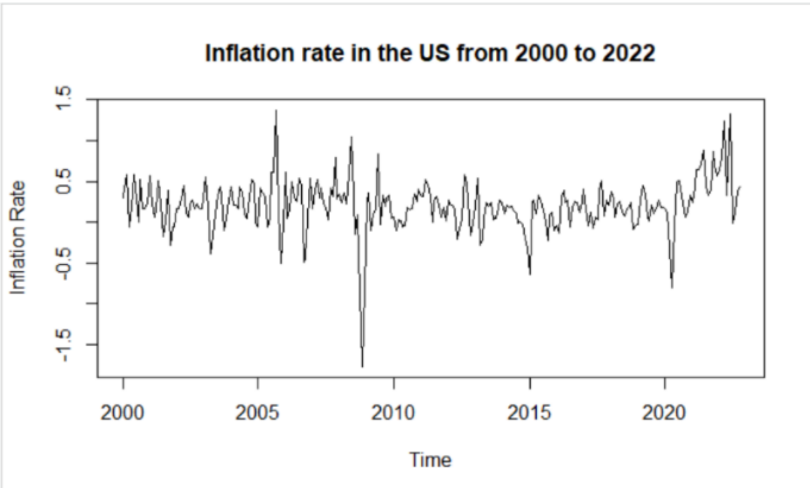


Figure 2.1.1 shows the Time Series Plot of the Inflation rate in the US from 2000 to 2022. The x-axis is the month of the year, and the y-axis is the inflation rate. As can be seen from the plot, the maximum inflation rate happened in October 2005; the minimum inflation rate happened in November 2008. However, the change down in November 2008 is significantly "larger" than all other changes, therefore, that may be an outlier. There is no general trend and significant seasonality. Besides, this plot shows a constant mean (0.2082), and the variance (0.1009) of this model is almost equalized, therefore this model may be stationary.

To further verify this conclusion, we can check the stationary by using the Dickey-Fuller test. Appendix Table 2.1.0 shows the output of the Dickey-Fuller Test. As the result shows p-value = 0.01, which is significantly less than 0.05. Therefore, we have evidence that the time series tends to be stationary.

STEP 2: Identify the order of p and q and interpret ACF, PACF and EACF

In this part, the observations are assumed to be generated by an ARMA(p,q) model. We compared the autocorrelation function (ACF), partial autocorrelation function (PACF), and extended autocorrelation function (EACF) using the Box-Jenkins method to identify the order p and q of the model.

Appendix Figure 2.1.2 displays a graph of the sample partial autocorrelation out to lag 30 for the Inflation rate time series. From Figure 2.1.2, we can observe that the sample ACF value at lag 1 is highly significant. The autocorrelation at lag 10, 11, 15, and 28 is barely significant. To conclude that the sample ACF becomes "small" and stays small for lags larger than 1. The information given in this plot leads us to consider an MA(1) model for this series.

The PACF plot (see Appendix Figure 2.1.3) displays a graph of the sample partial autocorrelation out to lag 30 for the Inflation rate time series, and we were recommended to choose either AR(1) or AR(2).

Then the EACF table (see Appendix Figure 2.1.4) displays a table of the extended autocorrelation for the Inflation rate time series, where the symbol X means the sample correlation is significantly different from 0. The triangular region of zeros shown in the EACF table indicates clearly that a mixed model ARMA(1,2) and MA(1) would be more appropriate.

Alternatively, we also use the dynamic method, which does not assume that the data come from a particular ARMA(p, q), and which focuses on explaining the dependencies in the data by increasing the order p of the ARMA(p, p-1) model to make it sufficiently tight. The value of p was chosen when the model's performance of explaining the dependence had no more significant improvement.

Based on the information available, we pick the model starting with an ARMA(3, 2) and then subtracting a parameter to see if it will change much, picking ARMA(2, 2) up to ARMA(1, 2).

2.2 Model Estimation

Information criterion: A method for estimating the best model among two or more suspect models based on, for example, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The AIC suggests that select the model that minimizes $AIC = -2\log(maximum\ likelihood) + 2k$, where $k = p + q + 1$ if the model contains an intercept or constant term.

Table 2.2.1 AIC of MA(1), AR(1), AR(2), ARMA(3,2), ARMA(2,2), ARMA(2,1) and ARMA(1,2) models

	MA(1)	AR(1)	AR(2)	ARMA(3,2)	ARMA(2,2)	ARMA(2,1)	ARMA(1,2)
AIC	74.15	80.26	74.1	71.75	74.13	74.03	72.54

From the table above, we can notice that except for the AR(1), all other models' ACI are relatively similar. For the ARMA(3,2), ARMA(2,2), ARMA(2,1) and ARMA(1,2), according to the principle of parsimony, the model with the least variables is chosen, and box-Jenkins selected ARMA(1,2) as the best, therefore, we may only consider ARMA(1,2) model instead of considering all three ARMA model.

Appendix Figure 2.2.2: Maximum Likelihood Estimates for a Simulated MA($\mathbf{1}$) Model shows that $\hat{\theta}_1 = 0.5119$, we also see that the estimated noise variance is $\hat{\sigma}_e^2 = 0.07555$. The estimated model would be written as

$$\widehat{Y}_t - 0.2088 = e_t - 0.5119e_{t-1} \text{ or } \widehat{Y}_t = 0.2088 + e_t - 0.5119e_{t-1}$$

Appendix Figure 2.3.3 Maximum Likelihood Estimates for a Simulated AR(2) Model shows that $\widehat{\emptyset}_1 = 0.5625, \widehat{\emptyset}_2 = -0.1710$, we also see that the estimated noise variance is $\hat{\sigma}_e^2 = 0.07498$. The estimated model would be written as

$$\widehat{Y}_t - 0.2088 = e_t + 0.5625(Y_{t-1} - 0.2088) - 0.171(Y_{t-2} - 0.2088)$$

$$\text{or } \widehat{Y}_t = 0.1271 + e_t + 0.5625Y_{t-1} - 0.171Y_{t-2}$$

Appendix Figure 2.3.4 Maximum Likelihood Estimates for a Simulated ARMA(1, 2) Model shows that $\widehat{\emptyset}_1 = -0.4148$ $\hat{\theta}_1 = 0.9998$, $\hat{\theta}_2 = 0.3492$ we also see that the estimated noise variance is $\hat{\sigma}_e^2 = 0.074$. The estimated model would be written as

$$\widehat{Y}_t - 0.2088 = -0.4148(Y_{t-1} - 0.2088) + e_t - 0.9998\, e_{t-1} - 0.3492e_{t-2}$$

$$\text{or } \widehat{Y}_t = -0.4148Y_{t-1} + e_t - 0.9998\, e_{t-1} - 0.3492e_{t-2} + 0.2954$$

In the next step, we will extract the MA(1) and ARMA(1,2) for further diagnostics.

2.3 Model Diagnostics

Testing the goodness-of-fit of the models, the first step is to analyze the predicted residuals et-hat. If the model was proven "correct", we will then check the White noise hypothesis in three aspects: normality, correlogram, and plot of residuals.

Here we first look at the time series plot of the residuals over time. If the model is adequate, the graph should indicate that the normalized residuals are around the zero horizontal level and there is no trend pattern.

Left: Figure 2.3.1 Residuals from ARMA(1,2) Model
Right: Figure 2.3.2 Residuals from MA(1) Model

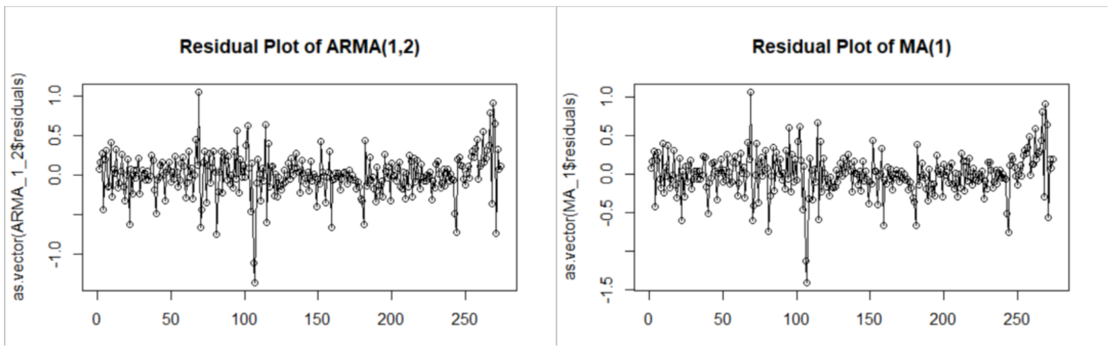


Figure 2.3.1 shows the plot of the residuals from the ARMA(1,2) model fitted to the time series of the inflation rate. Figure 2.3.2 shows the plot of the residuals from the MA(1) model. It is easy to observe that the difference between the two diagrams is negligible. If we zoom in and compare these two plots, the observations of figure 2.3.1 hang together more than figure 2.3.2. However, both two plots adequately support the ARMA(1,2) model and MA(1) model as there are no clear trends of variation present. Both models were fitted using maximum likelihood estimation and there are no obvious anomalous residuals in the series that show a pattern associated with the fitted trend. Therefore, they should be a standard normal distribution.

The second step is to check the gross non-normality, which can be checked by plotting a histogram of the residuals and the quantile-quantile plot. We first look at the histogram of the residuals for the ARMA(1,2) model and MA(1) model.

Left: Figure 2.3.3 Histogram of ARMA(1,2) Model
Right: Figure 2.3.4 Histogram of MA(1) Model

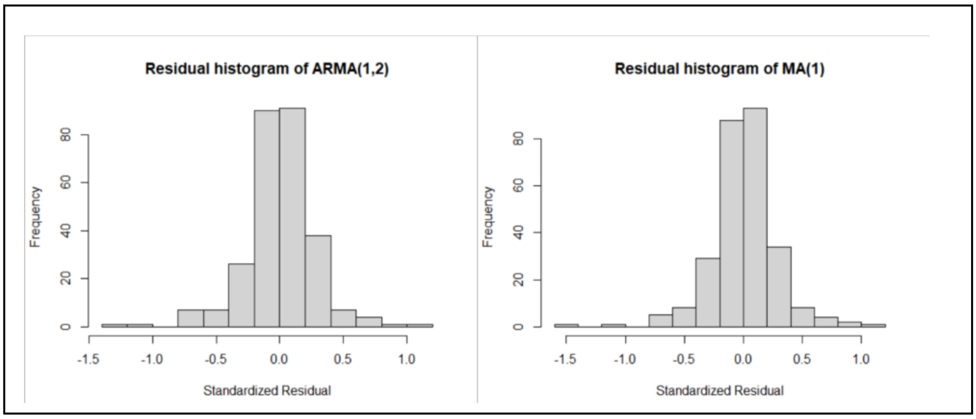


Figure 2.3.3 Histogram shows the sampling distribution of the ARMA (1,2) model and Figure 2.3.4 Histogram of the MA(1) Model. For both plots, the time series of the inflation rate is almost normal. Modes are at 0 and approximately symmetric, and have tails at both the high and low ends like a normal distribution. The corresponding approximation to the normal distribution may therefore be suitable for the sampling distribution of the parameter estimator. However, If we zoom in on two histograms, compared with the ARMA(1,2) model, there is a wider tail in the low end for MA(1) model. Besides, the histogram of MA(1) looks more symmetric than the histogram of ARMA(1,2).

The normalization cannot be determined accurately only by the histogram; we should see further analysis of the quantile-quantile plot. Quantile-quantile plot is an effective tool for checking normality. Here we apply the Q-Q plot to residuals.

Left: Figure 2.3.5 Q-Q Normal plot of ARMA(1,2) Model
Right: Figure 2.3.6 Q-Q Normal plot of MA(1) Model

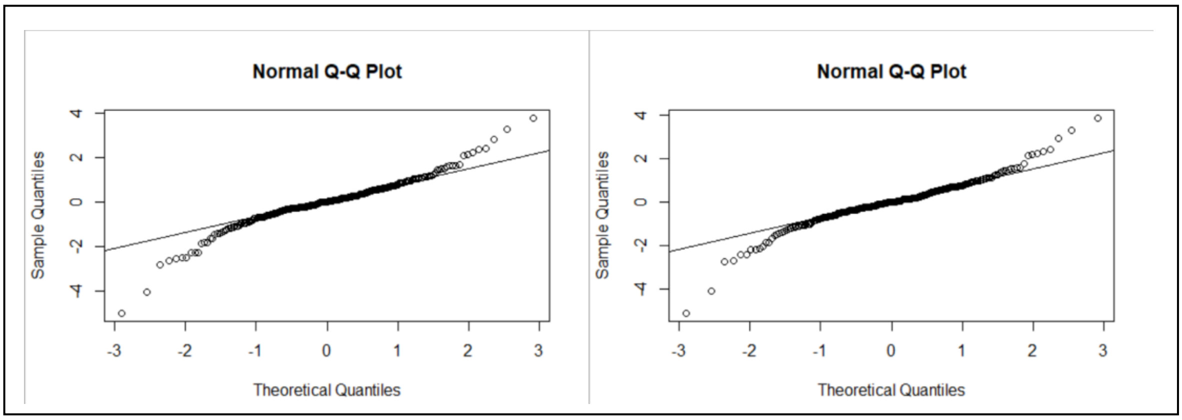


Figure 2.3.5 shows the normal Q-Q plot of the residuals from the ARMA(1,2) model fitted to the time series of the inflation rate, and Figure 2.3.6 is for MA(1) model. For both two plots, most of the points seem to follow the straight line fairly closely, however, they also show that the outliers are quite prominent.

Left: Figure 2.3.7 ACF from ARMA(1,2) Model

Right: Figure 2.3.8 ACF from MA(1) Model

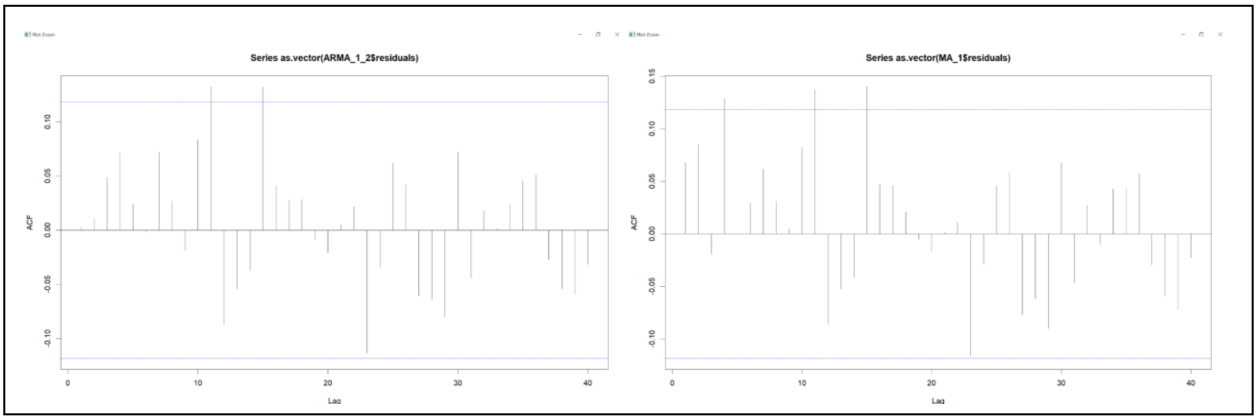


Figure 2.3.7 and Figure 2.3.8 separately display the sample ACF of residuals of the ARMA(1,2) model and MA(1) model for the inflation rate times series. The dashed horizontal lines plotted are based on the standard error of $\pm\frac{2}{\sqrt{n}}$ which is $\pm\frac{2}{\sqrt{274}} = \pm 0.1208$ in this model. In Figure 2.3.7, the lag 11 autocorrelation and the lag 15 autocorrelation exceed the standard error of 0.1208, while in Figure 2.3.8, the lag 4, lag 11 autocorrelation and the lag 15 autocorrelation exceed the standard error of 0.1208.

Although the graphs from the ACF show that both models have values outside of the 2 standard deviation corridors. we cannot reject the null hypothesis that the residuals are independent. But relatively speaking, ARMA (1, 2) is better than MA (1), so we conclude that we choose the ARMA (1, 2) model to analyze and forecast the inflation rate in the US.

2.4 Forecasting

One of the main objectives of building a model for the inflation rate is to be able to predict how the inflation rate in the US will behave at further times. Here we predict the future values by using the mean squared error (MSE) forecast as the measurement of accuracy. Based on the observations from time 2000 to 2022, the minimum mean square error forecast at ℓ time units in the future were expressed as: $\widehat{Y_t} = E(Y_{t+\ell})$, where t denotes the forecast origin and "ℓ" is the lead time for the forecast.

Based on the above analysis, we have our ARMA(1,2) model:

$Y_t = \emptyset_1 Y_{t-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$ ----- equation 1

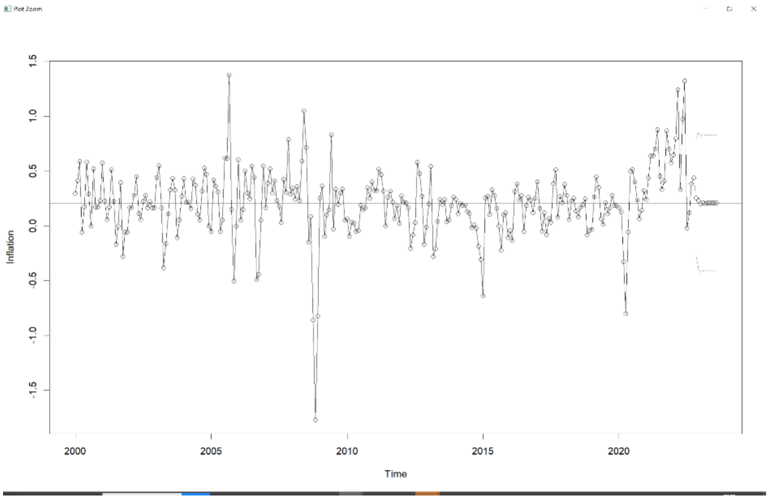
and the forecasting:

$$\widehat{Y_t}(1) = \mu + \emptyset_1(Y_t - \mu) - \theta_1 E(e_t|Y_1, Y_2, \dots, Y_t) - \theta_2 E(e_{t-1}|Y_1, Y_2, \dots, Y_t)$$
$$= \emptyset_1(Y_t - \mu) - \theta_1 e_t - \theta_2 e_{t-1}$$
$$\widehat{Y_t}(2) = \mu + \emptyset_1\left(\widehat{Y_t}(1) - \mu\right) - \theta_1 E(e_{t+1}|Y_1, Y_2, \dots, Y_t) - \theta_2 E(e_t|Y_1, Y_2, \dots, Y_t)$$
$$= \emptyset_1\left(\widehat{Y_t}(1) - \mu\right) - \theta_2 e_t$$
$$\dots$$

$\widehat{Y_t}(\ell) = \mu + \emptyset_1\left(\widehat{Y_t}(\ell - 1) - \mu\right) \, for \; \ell \geq 3$ ----- equation 2

From equation 1 we can see the noise terms e_t, e_{t-1}, e_{t-2} appear directly in the computation of the forecasts for leads $\ell = 1,2$. However, for $\ell \geq 3$ from equation 2 we can see the nature of the forecasts for long lead times will be determined by the autoregressive parameter $\emptyset_1$.

Figure 2.4.1 Forecasting for the ARMA(1,2) model



As the graph indicates, we can predict the future 3 months' inflation rate with the ARMA(1,2) model. However, the prediction results of the ARMA(1,2) model will gradually approach the mean value and lose the prediction power. (There is a zoom-in version provided in appendix figure 2.4.3)

Furthermore, we can compute the plot of fitted values and the original time series to see the performance of our model.

Figure 2.4.2 Fitted Values and Original Time Series

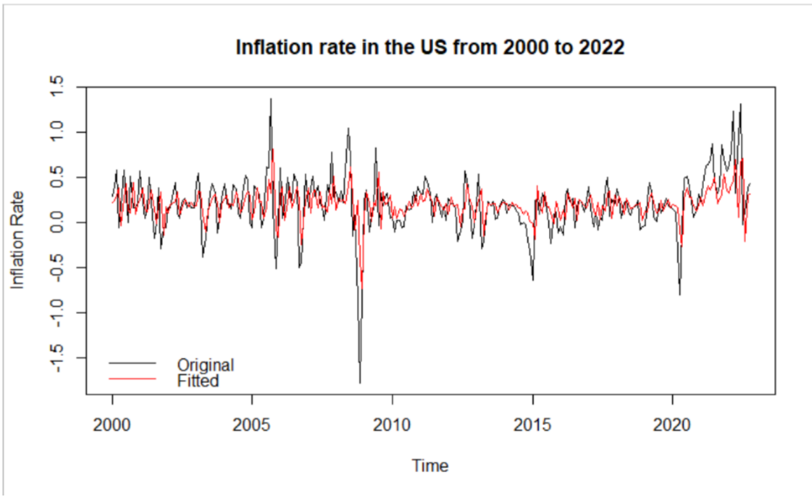


Figure 2.4.2 displays the actual inflation rates from 2000 to 2022 versus the fitted values by ARMA(1,2) model. Overall, the plot shows that this model fits the actual data quite well. However, we can still observe from the graph that for the larger shocks in the original data, our predictions are not very accurate.

3. Conclusion

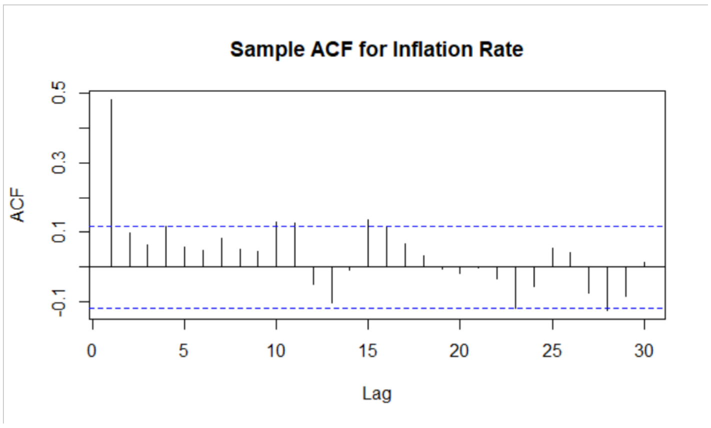
In conclusion, after fitting different types of time series models we learned from the course, we find that ARMA(1,2) is the best model. However, we are unable to predict future inflation changes due to the limitation of model options. We only obtain a result that the inflation rate is likely going to float by 0.3167599 per month. To predict the fluctuations of future inflation rates, more high-class models are needed to successfully predict future inflation.

Appendix

Table 2.1.0 Dickey-Fuller Test Output

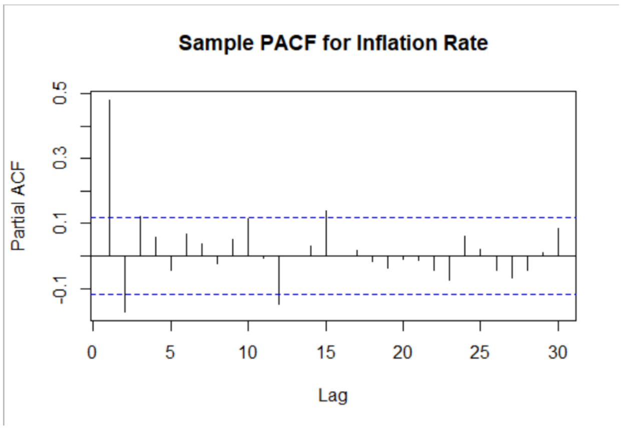
```
> adf.test(data.ts)
                    Augmented Dickey-Fuller Test
data:   data.ts
Dickey-Fuller = -4.8772, Lag order = 6, p-value = 0.01
alternative hypothesis: stationary
```

Figure 2.1.2 Sample ACF for the Inflation rate




```
> acf(as.vector(data.ts), main="Sample ACF for Inflation Rate",lag.max=30)
```

Figure 2.1.3 Sample PACF for the Inflation rate




```
> pacf(as.vector(data.ts), main="Sample PACF for Inflation Rate",lag.max=30)
```

Figure 2.1.4 Sample EACF for the Inflation rate

```
> eacf(data.ts)
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 x o o o o o o o o x x  o  o  o
1 x x o o o o o o o o o  o  o  o
2 x x x o o o o o o o o  o  o  o
3 x x x x o o o o o o o  o  o  o
4 x o x x o o o o o o o  o  o  o
5 x x x x o o o o o o o  o  o  o
6 x x x o x o o o o o o  o  o  o
7 x x x x x o o o o o o  o  o  o
```

> eacf(data.ts)

Figure 2.2.2 Maximum Likelihood Estimates for a Simulated MA(**1**) Model

```
Call:
arima(x = data.ts, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.5119     0.2088
s.e.  0.0456     0.0251

sigma^2 estimated as 0.07555:  log likelihood = -35.07,  aic = 74.15
```

> MA_1 = arima(data.ts, order=c(0, 0, 1))
> MA_1

Figure 2.2.3 Maximum Likelihood Estimates for a Simulated AR(2) Model

```
Call:
arima(x = data.ts, order = c(2, 0, 0))

Coefficients:
         ar1      ar2  intercept
      0.5625  -0.1710     0.2086
s.e.  0.0594   0.0594     0.0271

sigma^2 estimated as 0.07498:  log likelihood = -34.05,  aic = 74.1
```

> AR_2 = arima(data.ts, order=c(2, 0, 0))
> AR_2

Figure 2.2.4 Maximum Likelihood Estimates for a Simulated ARMA(1, 2) Model

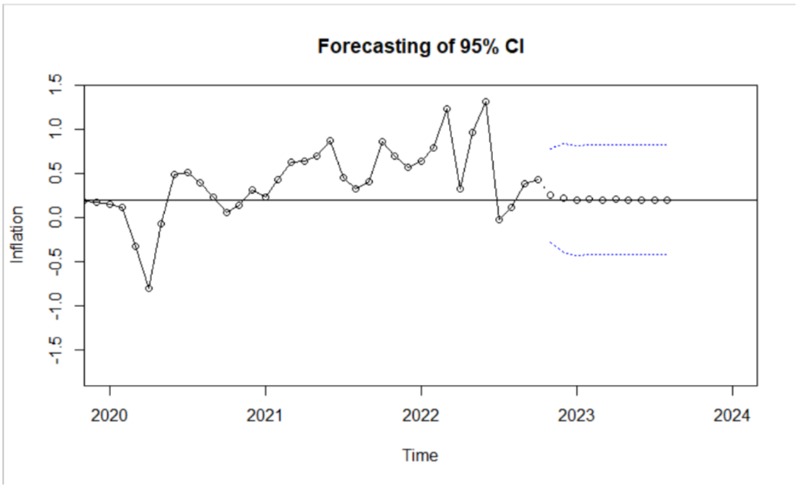
```
Call:
arima(x = data.ts, order = c(1, 0, 2))

Coefficients:
          ar1     ma1     ma2  intercept
      -0.4148  0.9998  0.3492     0.2085
s.e.   0.3122  0.2989  0.1512     0.0272

sigma^2 estimated as 0.074:  log likelihood = -32.27,  aic = 72.54
```

```
> ARMA_1_2 = arima(data.ts, order=c(1, 0, 2))
> ARMA_1_2
```

Figure 2.4.3 zoom in on the version of Forecasting for the ARMA(1,2) model



Code of Residuals from ARMA(1,2) Model and MA(1) Model

```
> plot(as.vector(ARMA_1_2$residuals), type="o", main='Residual Plot of ARMA(1,2)'
> plot(as.vector(MA_1$residuals), type="o", main='Residual Plot of MA(1)')
>abline(h=0)
```

Code of Histogram of ARMA(1,2) Model and MA(1) Model

```
> hist(ARMA_1_2$residuals, xlab='Standardized Residual',main='Residual histogram of ARMA(1,2)')
> hist(MA_1$residuals, xlab='Standardized Residual',main='Residual histogram of MA(1)')
```

Code of Q-Q Normal plot of ARMA(1,2) Model and MA(1) Model

```
> qqnorm(rstandard(ARMA_1_2)) > qqline(rstandard(ARMA_1_2))
> qqnorm(rstandard(MA_1)) > qqline(rstandard(MA_1))
```

Code of ACF from ARMA(1,2) Model and MA(1) Model

```
> acf(as.vector(ARMA_1_2$residuals), lag.max = 40)
> acf(as.vector(MA_1$residuals), lag.max = 40)
```

Code of forecasting of ARMA(1,2)

```
> plot(as.vector(ARMA_1_2),n.ahead=10 ,xlab='Time', ylab='Inflation',main='Forecasting of 95% CI')
> abline(h=coef(ARMA_1_2)[names(coef(ARMA_1_2))=='intercept'])
```

Code of Fitted Values and Original Time Series

```
> plot(window(data.ts, start=c(2000,1)), main="Inflation rate in the US from 2000 to 2022",
ylab = "Inflation Rate")
> lines(fitted(ARMA_1_2), col="red", lty=1)
> legend("bottomleft", c("Original", "Fitted"),   col=c("black", "red"), lty=1, bty="n")
```

Code of Time series plot of the Inflation rate in the US from 2000 to 2022

```
> plot(window(data.ts, start=c(2000,1)),
   main="Inflation rate in the US from 2000 to 2022", ylab = "Inflation Rate")
```

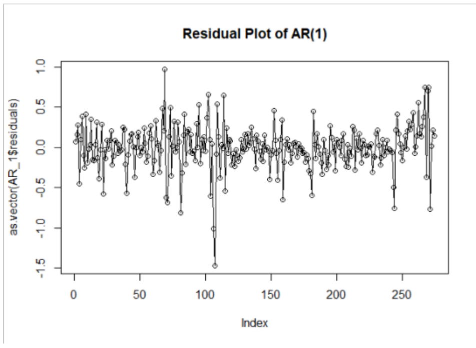


Figure A1: Residuals from AR(1) Model

```
> plot(as.vector(AR_1$residuals), type="o",
main='Residual Plot of AR(1)')
```

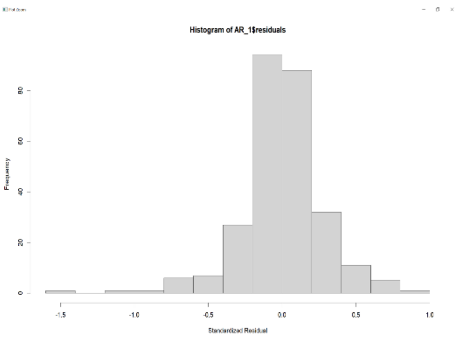


Figure A2: Histogram of AR(1) Model

```
> hist(AR_1$residuals, xlab='Standardized
Residual')
```

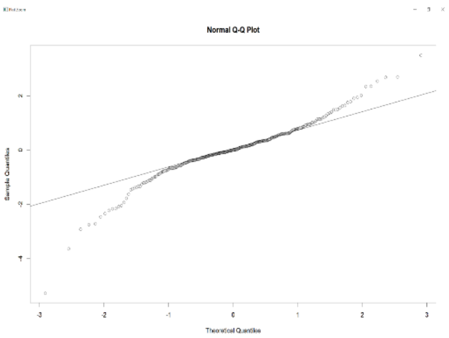


Figure A3: Q-Q Noraml plot of AR(1) Model

```
> qqnorm(rstandard(AR_1))
> qqline(rstandard(AR_1))
```

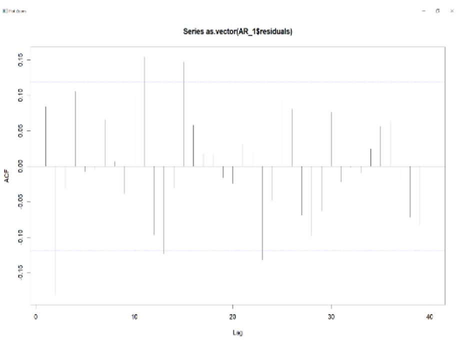
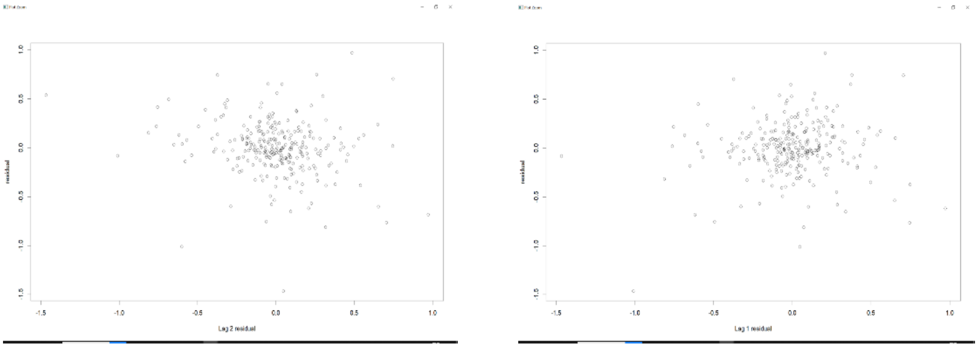


Figure A4: ACF from AR(1) Model

```
> acf(as.vector(AR_2$residuals), lag.max = 40)
```

Figure A5: Residuals VS lag 1 residual and lag 2 residual from AR(1) Model




```
## Residual vs lag 1 residual
> plot(y=AR_1$residuals, x=zlag(AR_1$residuals, d=1),
    xlab = 'Lag 1 residual', ylab='residual')
## Residual vs lag 2 residual
> plot(y=AR_1$residuals, x=zlag(AR_1$residuals, d=2),
    xlab = 'Lag 2 residual', ylab='residual')
```

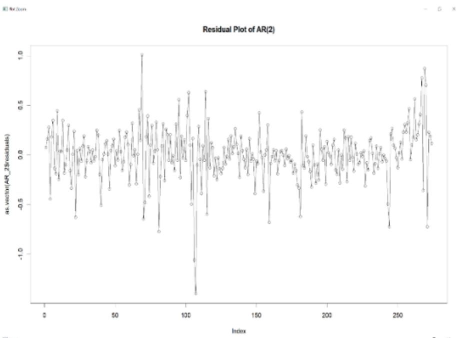


Figure B1: Residuals from AR(2) Model

```
> plot(as.vector(AR_2$residuals), type="o",
main='Residual Plot of AR(2)')
```

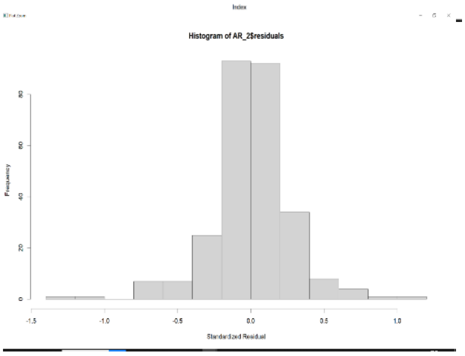


Figure B2: Histogram of AR(2) Model

```
> hist(AR_2$residuals, xlab='Standardized
Residual')
```

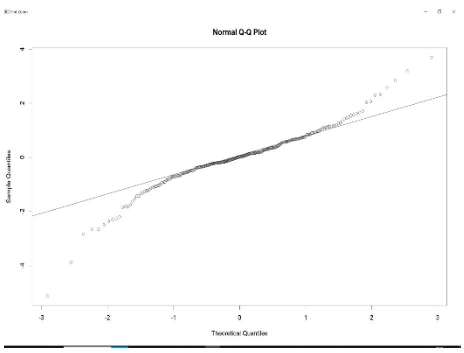


Figure B3: Q-Q Noraml plot of AR(2) Model

```
> qqnorm(rstandard(AR_2))
> qqline(rstandard(AR_2))
```

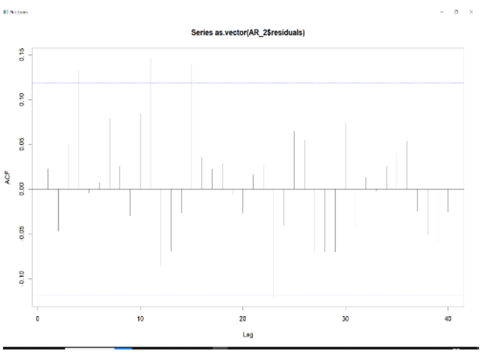
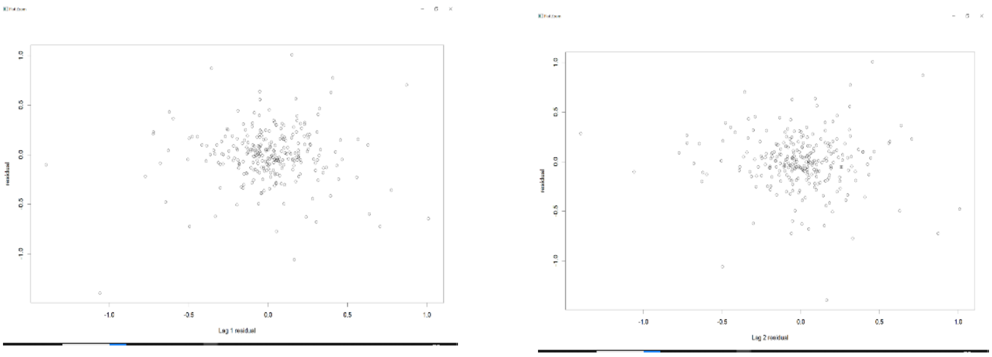


Figure B4: ACF from AR(2) Model

> acf(as.vector(AR_2$residuals), lag.max = 40)

Figure B5: Residuals VS lag 1 residual and lag 2 residual from AR(2) Model



Residual vs lag 1 residual
> plot(y=AR_2$residuals, x=zlag(AR_2$residuals, d=1),
    xlab = 'Lag 1 residual', ylab='residual')
Residual vs lag 2 residual
> plot(y=AR_2$residuals, x=zlag(AR_2$residuals, d=2),
    xlab = 'Lag 2 residual', ylab='residual')

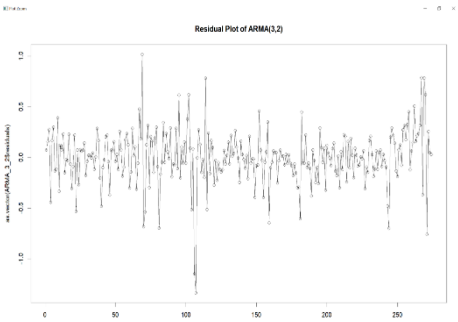


Figure C1: Residuals from ARMA(3,2) Model

plot(as.vector(ARMA_3_2$residuals), type="o", main='Residual Plot of ARMA(3,2)')

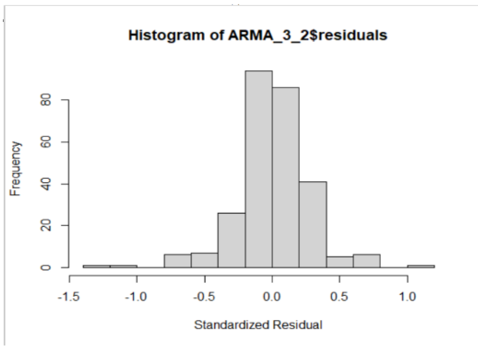


Figure C2: Histogram of ARMA(3,2) Model

> hist(ARMA_3_2$residuals, xlab='Standardized Residual')

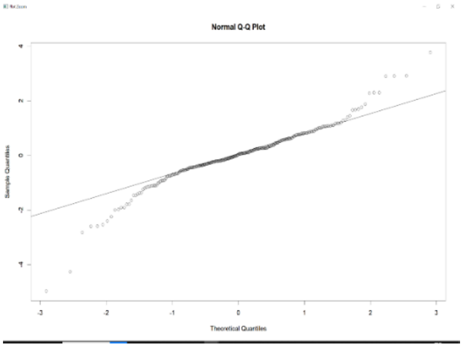


Figure C3: Q-Q Noraml plot of ARMA(3,2) Model

> qqnorm(rstandard(ARMA_3_2))
> qqline(rstandard(ARMA_3_2))

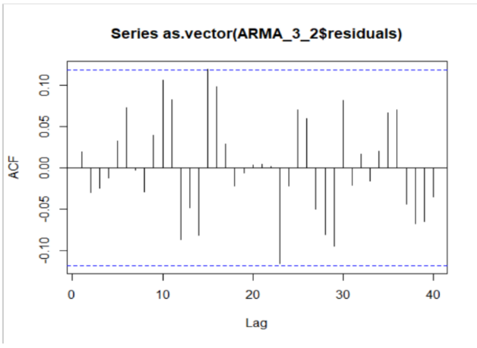
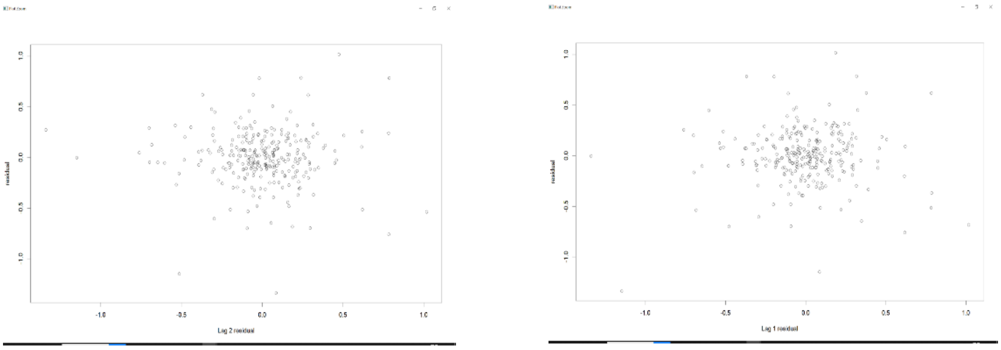


Figure C4: ACF from ARMA(3,2) Model

> acf(as.vector(ARMA_3_2$residuals), lag.max = 40)

Figure C5: Residuals VS lag 1 residual and lag 2 residual from ARMA(3,2) Model



Residual vs lag 1 residual

> plot(y=ARMA_3_2$residuals, x=zlag(ARMA_3_2$residuals, d=1), xlab = 'Lag 1 residual', ylab='residual')

Residual vs lag 2 residual
> plot(y=ARMA_3_2$residuals, x=zlag(ARMA_3_2$residuals, d=2),
    xlab = 'Lag 2 residual', ylab='residual')

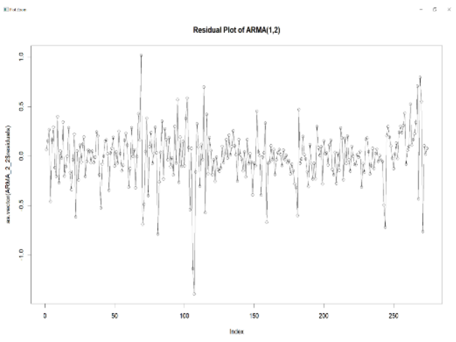


Figure D1: Residuals from ARMA(2,2) Model

> plot(as.vector(ARMA_2_2$residuals), type="o", main='Residual Plot of ARMA(2,2)')

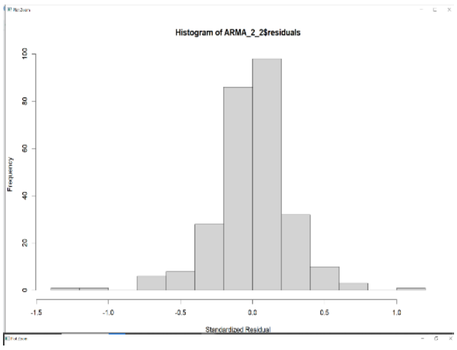


Figure D2: Histogram of ARMA(2,2) Model

> hist(ARMA_2_2$residuals, xlab='Standardized Residual')

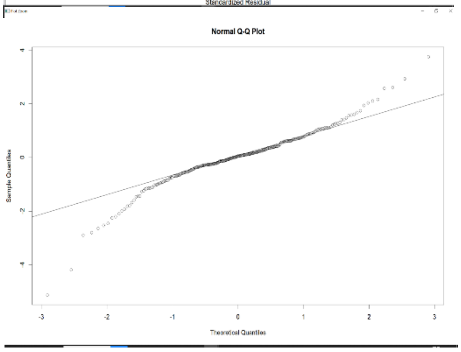


Figure D3: Q-Q Noraml plot of ARMA(2,2) Model

> qqnorm(rstandard(ARMA_2_2))
> qqline(rstandard(ARMA_2_2))

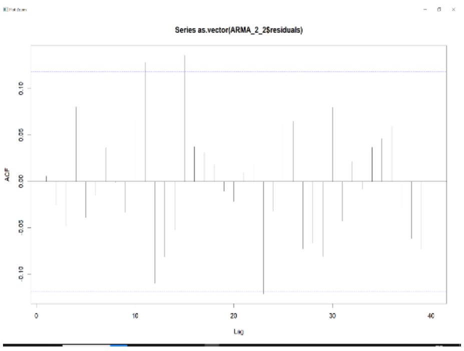
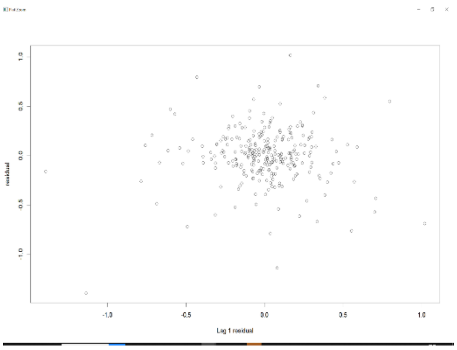
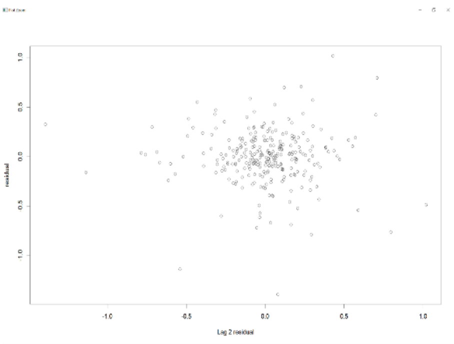


Figure D4: ACF from ARMA(2,2) Model

> acf(as.vector(ARMA_2_2$residuals), lag.max = 40)

Figure D5: Residuals VS lag 1 residual and lag 2 residual from ARMA(2,2) Model






```
## Residual vs lag 1 residual
> plot(y=ARMA_2_2$residuals, x=zlag(ARMA_2_2$residuals, d=1),
    xlab = 'Lag 1 residual', ylab='residual')
## Residual vs lag 2 residual
> plot(y=ARMA_2_2$residuals, x=zlag(ARMA_2_2$residuals, d=2),
    xlab = 'Lag 2 residual', ylab='residual')
```

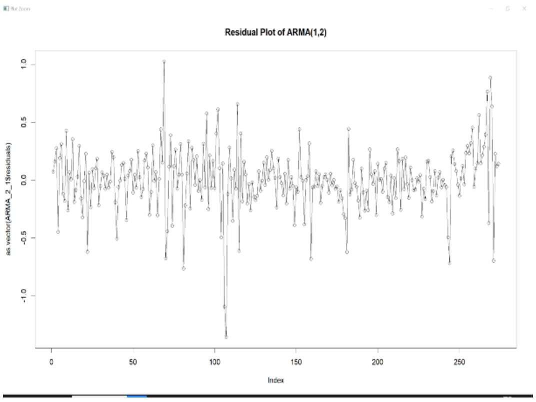


Figure E1: Residuals from ARMA(2,1) Model

```
>plot(as.vector(ARMA_2_1$residuals), type="o", main='Residual Plot of ARMA(2,1)')
```

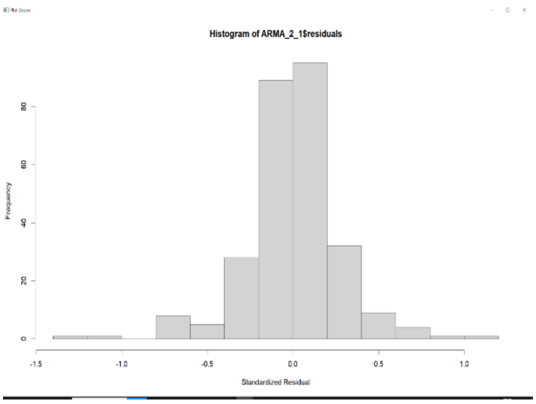


Figure E2: Histogram of ARMA(2,1) Model

```
> hist(ARMA_2_1$residuals, xlab='Standardized Residual')
```

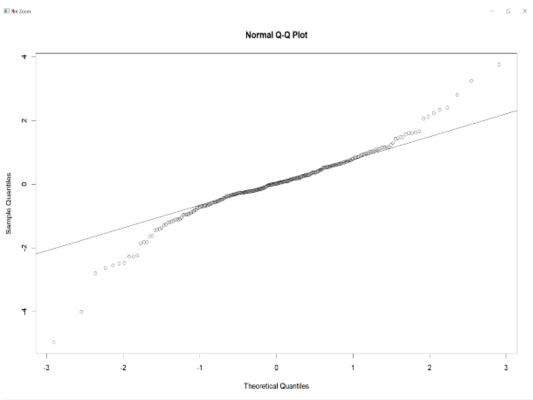


Figure E3: Q-Q Noraml plot of ARMA(2,1) Model

```
> qqnorm(rstandard(ARMA_2_1))
> qqline(rstandard(ARMA_2_1))
```

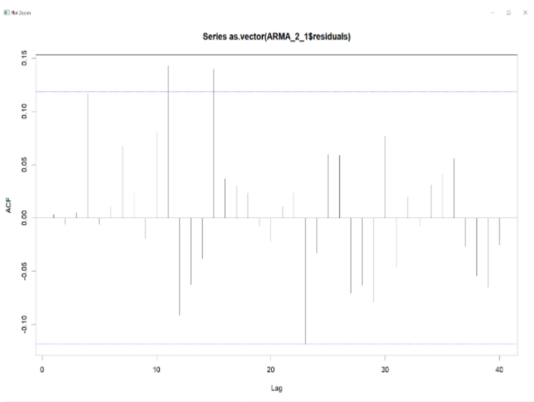
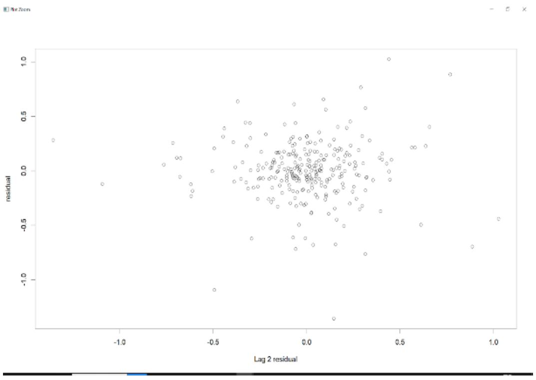
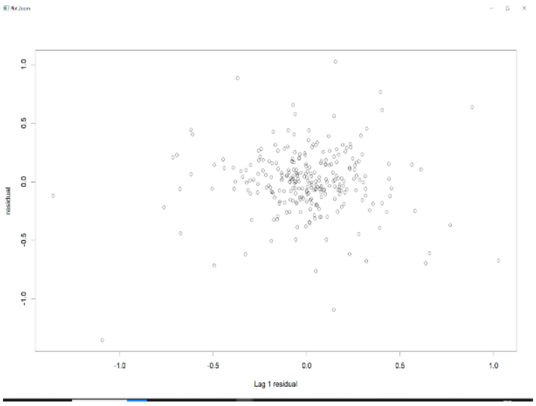


Figure E4: ACF from ARMA(2,1) Model

```
>acf(as.vector(ARMA_2_1$residuals), lag.max = 40)
```

Figure E5: Residuals VS lag 1 residual and lag 2 residual from ARMA(2,1) Model






```
## Residual vs lag 1 residual
> plot(y=ARMA_2_1$residuals, x=zlag(ARMA_2_1$residuals, d=1),
    xlab = 'Lag 1 residual', ylab='residual')
## Residual vs lag 2 residual
> plot(y=ARMA_2_1$residuals, x=zlag(ARMA_2_1$residuals, d=2),
    xlab = 'Lag 2 residual', ylab='residual')
```