数字射频与标签技术-作业1

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1.1 给定两个矢量 $A=e_x2+e_y3-e_z4$和$B=e_x4-e_y5+e_z6$, 求:

(1) e_A

$e_A=\frac{e_x2+e_y3-e_z4}{\sqrt{2^2+3^2+(-4)^2}}=\frac{e_x2+e_y3-e_z4}{\sqrt{29}}$

(2) A·B

$A\cdot B=2\times4-3\times5-4\times6=-31$

(3) A与B的夹角

$cos\theta=\frac{A\cdot B}{|A||B|}=\frac{-31}{\sqrt{2^2+3^2+(-4)^2}\sqrt{4^2+(-5)^2+6^2}}=\frac{-31}{\sqrt{2233}}\approx-0.656$

$\theta_{AB}=arccos(-0.656)=131^\circ$

(4) A×B

$A\times B=\begin{vmatrix}e_x & e_y & e_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$

$=\begin{vmatrix}e_x & e_y & e_z\\ 2 & 3 & -4\\ 4 & -5 & 6\end{vmatrix}$

$=e_x(3\times6-(-4)\times(-5))+e_y((-4)\times4-2\times6)+e_z(2\times(-5)-3\times4)$

$=-2e_x-28e_y-22e_z$

1.2 给定3个矢量: $A=e_x+e_y2-e_z3$、$B=-e_y4+e_z$和$C=e_x5-e_z2$, 求:

(1) A·(B×C)和(A×B)·C

$B\times C=\begin{vmatrix}e_x & e_y & e_z\\ 0 & -4 & 1\\ 5 & 0 & -2\end{vmatrix}$

$=e_x(-4\times-2)+e_y(1\times5)+e_z(-(-20))$

$=e_x8+e_y5+e_z20$

$A\cdot(B\times C)=(e_x+e_y2-e_z3)\cdot(e_x8+e_y5+e_z20)$

$=1\times8+2\times5+20\times(-3)=-42$

$$A \times B = \begin{vmatrix} e_x & e_y & e_z \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix}$$

$$= e_x(1 \times 2 - (-3) \times (-4)) - e_y + e_z$$

$$= -10e_x - e_y - 4e_z$$

$$(A \times B) \cdot C = (-10e_x - e_y - 4e_z) \cdot (e_x 5 - e_z 2) = (-10) \times 5 - (-4) \times (-2) = -42$$

(2) A×(B×C)和(A×B)×C

$$A \times (B \times C) = (e_x + e_y 2 - e_z 3) \times (e_x 8 + e_y 5 + e_z 20)$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ 1 & 2 & -3 \\ 8 & 5 & 20 \end{vmatrix}$$

$$= e_x 55 - e_y 44 - e_z 11$$

$$(A \times B) \times C = (-10e_x - e_y - 4e_z) \times (e_x 5 - e_z 2)$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ -10 & -1 & -4 \\ 5 & 0 & -2 \end{vmatrix}$$

$$= e_x 2 - e_y 40 + e_z 5$$

1.3 证明两个矢量$A = e_x 9 + e_y - e_z 6$和$B = e_x 4 - e_y 6 + e_z 5$是互相垂直的; 两个矢量$A = e_x 2 + e_y 5 + e_z 3$和$B = e_x 4 + e_y 10 + e_z 6$是互相平行的。

$A \cdot B = 9 \times 4 - 6 \times 1 - 6 \times 5 = 0$, 故矢量A和B是互相垂直的。

两矢量各项对应系数之比$\frac{2}{4} = \frac{5}{10} = \frac{3}{6} = \frac{1}{2}$, 故矢量A和B是互相平行的。

1.4 已知直角坐标系中的点$P_1(-3, 1, 2)$和$P_2(2, -3, 4)$, 求:

(1) 直角坐标系中, 点P_1和P_2的空间位置矢量r_1和r_2

(2) 点P_1和P_2距离矢量的大小和方向

(1)$r_1 = -e_x 3 + e_y 1 + e_z 2$

$r_2 = e_x 2 - e_y 3 + e_z 4$

(2)$R_{12} = r_2 - r_1 = e_x 5 - e_y 4 + e_z 2$

1.8 用球坐标表示的场$E = e_r(25/r^2)$, 求:

(1) 在点(-3, 4, -5)处的|E|和E_x

(2) E与矢量$B = e_x 2 - e_y 2 + e_z$构成的夹角

(1)

$r^2 = (-3)^2 + 4^2 + 5^2 = 50$

$|E| = |e_r \frac{25}{r^2}| = \frac{1}{2}$

$E_x = e_x E = |Ecos\theta_{rx}| = \frac{1}{2} \times \frac{-3}{5\sqrt{2}} = -\frac{3\sqrt{2}}{20}$

(2)

在点(-3,4,-5)处, $r = -e_x 3 + e_y 4 - e_z 5$

所以 $E = \frac{25}{r^2} = \frac{25r}{r^3} = \frac{-e_x 3 + e_y 4 - e_z 5}{10\sqrt{2}}$

$\theta_{EB} = arccos(\frac{E \cdot B}{|E||B|}) = arccos\frac{-19/10\sqrt{2}}{3/2} = 153.6^\circ$

1.11 已知矢量$A = e_x x^2 + e_y (xy)^2 + e_z 24x^2y^2z^3$, 求:

(1) ∇·A

$\nabla \cdot A = \frac{\partial(x^2)}{\partial x} + \frac{\partial x^2y^2}{\partial y} + \frac{\partial 24x^2y^2z^3}{\partial}$

$= 2x + 2x^2y + 72x^2y^2z^2$

(2) A对中心在原点的一个单位立方体表面的积分

$\int_S \nabla A dS = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} (\frac{1}{2})^2 dydz - \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} (-\frac{1}{2})^2 dydz + \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} 2x^2 (\frac{1}{2})^2 dxdz -$

$\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} 2x^2 (-\frac{1}{2})^2 dxdz + \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} 24x^2y^2 (\frac{1}{2})^3 dxdy - \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} 24x^2y^2 (\frac{1}{2})^3 dxdy = \frac{1}{24}$

(3) 验证高斯散度定理

$\int_\tau \nabla A d\tau = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} (2x + 2x^2y + 72x^2y^2z^2) dxdydz = \frac{1}{24} = \int_S \nabla A dS$

1.15 求下列矢量的旋度

(1) $A = e_x yz_+ e_y zx + e_z xy$

$$rotA = \nabla \times A = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$= e_x(\frac{\partial xy}{\partial y} - \frac{\partial zx}{\partial z}) + e_y(\frac{\partial yz}{\partial y} - \frac{\partial xy}{\partial x}) + e_z(\frac{\partial zx}{\partial x} - \frac{\partial yz}{\partial y})$

$= 0$

(2) $A = e_x(y^2 + z^2) + e_y(z^2 + x^2) + e_z(x^2 + y^2)$

$$= rotA = \nabla \times A = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2+z^2 & z^2+x^2 & x^2+y^2 \end{vmatrix}$$

$$= e_x(\frac{\partial x^2+y^2}{\partial y} - \frac{\partial z^2+x^2}{\partial z}) + e_y(\frac{\partial y^2+z^2}{\partial y} - \frac{\partial x^2+y^2}{\partial x}) + e_z(\frac{\partial z^2+x^2}{\partial x} - \frac{\partial y^2+z^2}{\partial y})$$

$$= e_x(2y-2x) + e_y(2z-2x) + e_z(2x-2y)$$

1.16 求下列标量的梯度:

(1) $u = 4x^2y + y^2z - 4xz$

$$\nabla u = e_x\frac{\partial u}{\partial x} + e_y\frac{\partial u}{\partial y} + e_z\frac{\partial u}{\partial z}$$

$$= e_x(8xy-4) + e_y(4x^2+2yz) + e_z(y^2-4x)$$

(2) $u = xyz - x^2 + y^2$

$$\nabla u = e_x\frac{\partial u}{\partial x} + e_y\frac{\partial u}{\partial y} + e_z\frac{\partial u}{\partial z}$$

$$= e_x(yz-2x) + e_y(xz+2y) + e_z xy$$

1.17 已知矢量$A = e_x x + e_y xy^2$, 求:

(1) $\nabla \times A$

(2) 矢量A沿圆周$x^2+y^2=a^2$的闭合线积分

(3) 验证斯托克斯定理

(1)

$$\nabla \times A = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & xy^2 & 0 \end{vmatrix}$$

$$= e_z y^2$$

(2)

$$\int_C Adl = \int_C xdx + xy^2dy = \int_0^{2\pi}(-a^2cos\phi sin\phi + a^4cos^2\phi sin^2\phi)d\phi = \frac{\pi a^4}{4}$$

(3)

$$\int_S \nabla \times AdS = \int_S e_z(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})e_z dS = \int_S y^2 dS = \int_0^a\int_0^{2\pi} r^2 sin^2\phi r d\phi dr = \frac{\pi a^4}{4} = \int_C Adl$$