# The Mortgage Credit Channel of Macroeconomic Transmission\*

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#### **Abstract**

I investigate how the structure of the mortgage market influences macroeconomic dynamics, using a general equilibrium framework with prepayable debt and a limit on the ratio of mortgage payments to income — features that prove essential to reproducing observed debt dynamics. The resulting environment amplifies transmission from interest rates into debt, house prices, and economic activity. Monetary policy more easily stabilizes inflation, but contributes to larger fluctuations in credit growth. A relaxation of payment-to-income standards appears vital for explaining the recent boom. A cap on payment-to-income ratios, not loan-to-value ratios, is the more effective macroprudential policy for limiting boom-bust cycles.

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# 1 Introduction

Mortgage debt is central to the workings of the modern macroeconomy. The sharp rise in residential mortgage debt at the start of the twenty-first century in the US and countries around the world has been credited with fueling a dramatic boom in house prices and consumer spending. At the same time, high levels of mortgage debt and household leverage have been blamed for the severity of the subsequent bust. Since mortgage credit evolves endogenously in response to economic conditions, its critical position in the macroeconomy raises a number of important questions. How, if at all, does mortgage credit growth propagate and amplify macroeconomic fluctuations in general equilibrium? How does mortgage finance affect the ability of monetary policy to influence economic activity? Finally, what role did changing credit standards play in the boom, and how might regulation have limited the resulting bust?

These questions all center on what I will call the *mortgage credit channel* of macroeconomic transmission: the path from primitive shocks, through mortgage credit issuance, to the rest of the economy. Characterizing this channel requires confronting the institutional environment, which profoundly shapes the US mortgage landscape. The market is dominated by the Government Sponsored Enterprises — Fannie Mae and Freddie Mac — who wield an outsize influence on underwriting standards and the form of the typical mortgage contract. Consequently, the resulting system of mortgage finance exhibits specific and often complex functional forms that may not be well represented as the solution to an optimal contracting problem. Long-term prepayable fixed-rate mortgages are the predominant contract, while borrowers face multiple constraints at origination that depend mechanically on both individual and aggregate economic variables. Although the typical approach in general equilibrium macroeconomics has been to abstract from many of these institutional details, I will argue in this paper that they play a pivotal role in macroeconomic dynamics.

To this end, I develop a tractable modeling framework that embeds key institutional features in a New Keynesian dynamic stochastic general equilibrium (DSGE) environment. The framework centers on two components that shape the mortgage credit channel. First, the size of new loans is limited not only by the ratio of the loan's balance to the value of the underlying collateral ("loan-to-value" or "LTV"), but also by the ratio of the mortgage payment to the borrower's income ("payment-to-income" or "PTI"). While a

<sup>&</sup>lt;sup>1</sup>The payment-to-income ratio is also commonly known as the "debt-to-income" or "DTI" ratio. I use the term "payment-to-income" for clarity, since under either name the ratio measures the flow of payments

vast literature documents the impact of LTV constraints on debt dynamics, the influence of PTI limits on the macroeconomy remains relatively unstudied, despite their central role in underwriting in the US and abroad. Second, borrowers choose whether to prepay their existing loans and replace them with new loans, a process that incurs a transaction cost. This prepayment option allows the model to capture two empirical facts: only a small minority of borrowers obtain new loans in a given quarter, but the fraction that choose to do so is volatile and co-moves strongly with house prices and interest rates.<sup>2</sup>

These two features map to the two key links in the chain of transmission: PTI limits affect the amount of available credit, while endogenous prepayment determines how much of this potential debt is actually issued. Applied jointly, they deliver an excellent fit of aggregate US debt dynamics, which existing specifications are unable to reproduce. Since a realistic implementation of both features involves accounting for population heterogeneity — with endogenous and time-varying fractions of the population limited by each constraint, and choosing to prepay their loans, respectively — I develop aggregation procedures to capture these phenomena, and calibrate them to US mortgage data at the aggregate, household, and loan levels.

Using this framework, I present two main sets of findings. First, I find that these novel features of the model greatly amplify transmission from nominal interest rates into debt, house prices, and economic activity. The initial step of transmission is that PTI limits are highly sensitive, allowing 8% more borrowing in response to a 1% fall in nominal rates. However, because only a minority of borrowers are constrained by PTI at equilibrium, this direct impact on PTI constraints has only moderate quantitative importance.

Instead, the key to strong transmission is the *constraint switching effect*, a novel propagation mechanism through which changes in which of the two constraints is binding for borrowers translate into large movements in house prices. As PTI limits loosen following a fall in interest rates, more borrowers find themselves constrained by LTV. Since LTV-constrained households can relax their borrowing limits with additional housing collateral, but PTI-constrained households cannot, this switch boosts housing demand, raising house prices. This force causes price-to-rent ratios to rise by 3% in response to a 1% fall in nominal rates alone, compared to a response near zero in traditional models. Rising house prices in turn loosen borrowing constraints for the LTV-constrained majority of the population, leading to nearly twice as much credit growth as under an alternative economy with an LTV constraint alone.

relative to a borrower's income, not the stock of debt relative to a borrower's income.

<sup>&</sup>lt;sup>2</sup>See Section A.7 in the Appendix for empirical evidence of this pattern.

For transmission into output, borrowers' option to prepay their loans turns out to be critical, due to its influence on the timing of credit growth. When borrowers hold this option, a fall in rates leads to a wave of prepayments, new issuance, and new spending on impact, generating a large output response — a phenomenon that I call the *frontloading effect*. Quantitatively, this effect amplifies the impact of a 1% fall in the term premium on output more than three-fold (0.14% to 0.50%). Alternative economies without endogenous prepayment generate much slower issuance of credit with little effect on output, despite similar increases in debt limits. These results have important consequences for monetary policy, which is more effective at stabilizing inflation due to these forces, but contributes to larger swings in credit growth, posing a potential trade-off for central bankers concerned with stabilizing both markets.

My second set of findings concern credit standards and the sources of the recent boom and bust, where I argue that a relaxation of PTI limits was essential to the events that unfolded. Although a substantial body of work has looked to credit liberalization to explain the boom in house prices and lending, the macroeconomic literature has typically focused on changes in LTV limits, while overlooking PTI limits. However, analysis of loan-level data reveals a massive loosening of PTI limits that far outstrips changes in LTV standards over the same period. An experiment conservatively implementing this relaxation of PTI in the model reveals that this change was a major contributor to the boom, by itself explaining more than one third of the observed increase in price-to-rent and loan-to-income ratios over the period. This strong response is once again due to the constraint switching effect, which is critical to obtaining a large rise in house prices, allowing for increased borrowing across the entire population.

Moreover, while a liberalization of PTI constraints is partially sufficient for explaining the boom, it also appears *necessary* for other factors to have played as large a role as they did. To show this, I first incorporate additional shocks — optimistic house price expectations, the observed fall in interest rates, and a small relaxation of LTV standards — to reproduce the full peak increases in price-to-rent and loan-to-income ratios found in the data. I find that compared to this baseline, a counterfactual experiment enforcing PTI limits at their historical levels would have reduced the size of the boom by nearly 60%, indicating that the contemporaneous relaxation of PTI standards increased the contribution of these remaining forces by more than half. These results have important implications for macroprudential regulation, implying that a cap on PTI ratios, not LTV ratios, is the more effective policy for limiting boom-bust cycles. As a final application, I study the 43% cap

on PTI ratios imposed by the Dodd-Frank legislation. Although this limit is looser than historical norms, I find that it could have dampened the boom by more than one third had it been in place, and is likely to be even more effective going forward.

**Literature Review.** This paper builds on several existing strands of the literature.<sup>3</sup> On the empirical side, it relates to a large and growing body of work demonstrating important links among mortgage credit, house prices, and economic activity, and documenting patterns of credit growth in the boom.<sup>4</sup> My study complements these works by analyzing the theoretical mechanisms behind these links in general equilibrium.

Turning to theoretical models, the literature can be broadly split into two camps. The first comprises heterogeneous agent models, which often include rich specifications of idiosyncratic risk, costly financial transactions, and long-term mortgage contracts, but cannot tractably incorporate inflation, monetary policy, and endogenous output in general equilibrium.<sup>5</sup> In contrast, a set of monetary DSGE models with housing and collateralized debt can easily handle these macroeconomic features, but use simplified loan structures that rule out important features of debt dynamics.<sup>6</sup> In this paper I seek to combine these two approaches, embedding a realistic mortgage structure in a tractable general equilibrium environment. The resulting framework can easily be merged with existing macroeconomic models used by central banks and regulators around the world, making this hybrid approach valuable for policy analysis.

Further, to my knowledge, Corbae and Quintin (2015) represents the only prior macroe-conomic model to incorporate a PTI constraint and use its relaxation as a proxy for the housing boom. These authors introduce the PTI constraint to explore the relationship between endogenously priced default risk and credit growth in a model with exogenous house prices. While their setup delivers important findings regarding default and fore-closure, both absent from my model, these authors do not study the implications of the PTI constraint for interest rate transmission, or, through its influence on house prices, on the LTV constraint — the key to the results of this paper.

<sup>&</sup>lt;sup>3</sup>See Davis and Van Nieuwerburgh (2014) for a survey of the recent literature on housing, mortgages, and the macroeconomy.

<sup>&</sup>lt;sup>4</sup>See, e.g., Aladangady (2014), Mian and Sufi (2014), Adelino, Schoar, and Severino (2015), Favara and Imbs (2015), Foote, Loewenstein, and Willen (2016), Mian and Sufi (2016), Di Maggio and Kermani (2017).

<sup>&</sup>lt;sup>5</sup>See, e.g., Chen, Michaux, and Roussanov (2013), Corbae and Quintin (2015), Khandani, Lo, and Merton (2013), Laufer (2013), Guler (2014), Beraja, Fuster, Hurst, and Vavra (2015), Campbell and Cocco (2015), Chatterjee and Eyigungor (2015), Gorea and Midrigan (2015), Landvoigt (2015), Wong (2015), Elenev, Landvoigt, and Van Nieuwerburgh (2016), Kaplan, Mitman, and Violante (2017).

<sup>&</sup>lt;sup>6</sup>See, e.g., Iacoviello (2005), Monacelli (2008), Iacoviello and Neri (2010), Ghent (2012), Liu, Wang, and Zha (2013), Rognlie, Shleifer, and Simsek (2014).

This work is also related to research connecting a relaxation of credit standards to the recent boom-bust. My findings largely support the importance of credit liberalization in the boom, with the specific twist that a relaxation of PTI constraints appears key. Of particular relevance is Justiniano, Primiceri, and Tambalotti (2015b), who find that the interaction of an LTV constraint with an exogenous lending limit can generate strong effects of movements in the non-LTV constraint on debt and house prices — a result echoed in many of the findings of this paper. By utilizing an endogenous PTI constraint in place of an exogenous fixed limit on lending, I am able to connect these dynamics to interest rate transmission, calibrate to observed relaxations of PTI standards in the data, and analyze the effects of a regulatory cap on PTI limits, such as the one imposed by Dodd-Frank.

Additionally, this paper parallels research on the redistribution channel of monetary policy. When borrowers hold adjustable-rate mortgages, changes in interest rates lead to changes in payments on the existing stock of debt, influencing borrower spending. This channel is separate from, and complementary to, the mortgage credit channel, which operates instead through the flow of *new* credit driven by changes in borrowing constraints. Interestingly, while allowing borrowers to prepay their loans does allow for substantial changes in payments when interest rates fall, and therefore large redistributions between borrowers and savers, the redistribution channel is nonetheless weak in my framework, leading to very small aggregate stimulus. The key difference is in the timing: under fixed-rate mortgages, while changes in interest payments eventually become large as borrowers refinance, they occur too slowly to influence output.

Finally, this work connects to an older literature on the effects of inflation on mortgages. As argued by e.g., Lessard and Modigliani (1975), when inflation is high, a fixed-rate nominal mortgage implies a more frontloaded path of real payments, leading to high payment-to-income ratios in the early years of the loan. These authors intuited that this heavy initial payment burden could lead to a contraction in housing demand and lending, a mechanism that I now derive and generalize in a full general equilibrium model.<sup>9</sup>

**Overview.** The remainder of the paper is organized as follows. Section 2 provides a simple example and presents facts from the data. Section 3 constructs the theoretical

<sup>&</sup>lt;sup>7</sup>See, e.g., Campbell and Hercowitz (2005), Kermani (2012), Iacoviello and Pavan (2013), Favilukis, Ludvigson, and Van Nieuwerburgh (2017).

<sup>&</sup>lt;sup>8</sup>See, e.g., Rubio (2011), Calza, Monacelli, and Stracca (2013), Auclert (2015), Garriga, Kydland, and Sustek (2015).

<sup>&</sup>lt;sup>9</sup>Also relevant is Boldin (1993), who finds econometric evidence that changes in mortgage affordability due to movements in interest rates have strong effects on housing demand.

model. Section 4 describes the calibration and evaluates the model through comparison with macroeconomic data. Section 5 presents the results on interest rate transmission, and the consequences for monetary policy. Section 6 discusses the role of credit standards in the boom-bust, and the implications for macroprudential policy. Section 7 concludes. Additional results, extensions, and data definitions can be found in the appendix.

## 2 Background: LTV and PTI Constraints

This section presents a simple numerical example, and demonstrates the empirical properties of LTV and PTI limits in the data.

### 2.1 Simple Numerical Example

To provide intuition for model's core mechanisms, I present a simplified example from an individual borrower's perspective. I describe the intuition below, and formalize the problem behind these results in Appendix A.3.

Consider a prospective home-buyer who prefers to pay as little as possible in cash today, perhaps because she must save for the down payment and delaying purchase is costly. This borrower's annual income is \$50k, and she faces a 28% PTI limit, meaning that she can put at most \$1.2k per month toward her mortgage payment. At an interest rate of 6%, this maximum payment is associated with a loan size of \$160k, which is therefore the most she can borrow subject to her PTI limit. Her maximum LTV ratio is 80% so that, including the minimum 20% down payment, she reaches her maximum loan size at at a house price of \$200k.

This \$200k house price represents the threshold at which the borrower switches from being LTV-constrained to PTI-constrained. This creates a kink in the borrower's required down payment as a function of house price, shown as the solid blue line in Figure 1. Below this threshold price, the borrower is constrained by the value of her collateral. In this region, increasing her house value by \$1 allows her to borrow an additional 80 cents, requiring her to pay only 20 cents more in down payment. But above the kink, she is constrained by her income. In this region she cannot obtain any additional debt no matter how valuable her collateral is, and must pay for any additional housing in cash. This discrete change around the kink implies that a "corner solution" price of exactly

 $<sup>^{10}</sup>$ For simplicity, I abstract in this example from property taxes, insurance, and non-mortgage debt payments, and round quantities to the nearest \$1k = \$1,000.

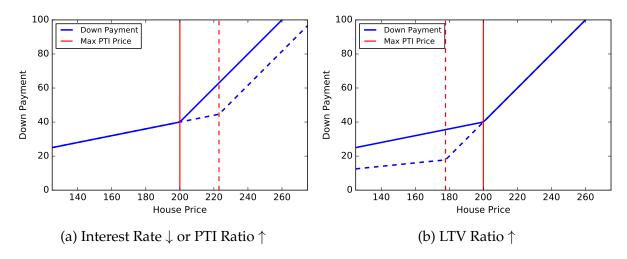


Figure 1: Simple Example: House Price vs. Down Payment

\$200k is a likely optimum for this borrower. For this example, let us assume that this is indeed her choice.

From this starting point, imagine that the mortgage interest rate now falls from 6% to 5%, displayed as the dashed lines in Figure 1a. While the borrower's maximum monthly payment has not changed, at a lower interest rate this \$1.2k payment is now associated with a larger loan of \$178k. But because of her LTV constraint, the borrower can only take advantage of this larger loan limit if she obtains a more valuable house as collateral. This shifts the kink in the down payment function to the right, with the threshold price now occurring at \$223k — an 11% increase. If the borrower once again chooses her threshold house size, the result is a substantial increase in demand, potentially contributing to a large rise in house prices if others do the same. Note that this result depends crucially on the interaction of the LTV and PTI constraints, and would not be present under either constraint in isolation.

This example can also be used to analyze changes in credit standards. First, consider an increase in allowed PTI ratios. Since this intervention increases the maximum PTI loan size, the impact on the down payment function is the same as if the interest rate had fallen. Specifically, a rise from a 28% to a 31% PTI ratio exactly replicates the change in Figure 1a, once again raising the threshold house price, and potentially boosting housing demand.

In contrast, an increase in the maximum LTV ratio from 80% to 90%, shown in Figure 1b, has a starkly different impact. In this case, the borrower's maximum loan size given her income is unchanged, at \$160k. But with only a 10% down payment, the house price

associated with this loan falls to \$178k, an 11% *decrease*. If the borrower once again follows her corner solution, the result is a fall in her housing demand, potentially contributing to a decline in house prices.

To understand this result, note that prior to the LTV loosening, moving from a \$200k house to a \$178k house would have let the borrower keep only \$4.4k in cash, since she would have been forced to cut her loan size. But after the relaxation, the borrower can keep the entire \$22k difference, dramatically increasing her cash savings from downsizing. Alternatively, consider that a relaxation of the LTV limit increases the effective supply of collateral, since each unit of housing can collateralize more debt, but does not increase the demand for collateral, since the borrower's overall loan size is still constrained by her PTI limit. An increase in supply holding demand fixed pushes down the price of collateral, depressing the value of housing. This result, again due to the interaction of the two limits, is not found in models in which borrowers face only an LTV constraint, where lower down payments typically increase housing demand and house prices.

#### 2.2 LTV and PTI in the Data

This section considers the empirical properties of the LTV and PTI constraints, providing evidence on the influence of PTI limits after the housing bust, as well as on the liberalization of PTI limits during the boom.

To begin, Figure 2 shows the distribution of combined LTV (CLTV) and PTI ratios on newly issued conventional fixed-rate mortgages securitized by Fannie Mae for two points in time: the height of the boom (2006 Q1) and a recent post-crash date (2014 Q3).<sup>11</sup> Beginning with the CLTV distributions, we can observe two patterns of interest. First, the influence of LTV limits is obvious, with the majority of borrowers grouped in large spikes at known institutional limits and cost discontinuities.<sup>12</sup> Second, the cross-sectional distribution of CLTV changes little between 2006 and 2014, and appears if anything looser after the bust, consistent with similar CLTV standards imposed in both the boom and post-crash environments.

Turning to the PTI plots, we observe markedly different patterns. While the distributions do not display large individual spikes as in the CLTV case, the clear influence of the institutional limit (45%) can be seen in the 2014 data, with the distributions building

<sup>&</sup>lt;sup>11</sup>Combined LTV is the ratio of total mortgage debt to the value of the house, summing if necessary over multiple mortgages against the same property. Identical plots using Freddie Mac data can be seen in Figure A.1 in the appendix.

<sup>&</sup>lt;sup>12</sup>The largest spikes occur at 80%, where borrowers must start paying for private mortgage insurance.

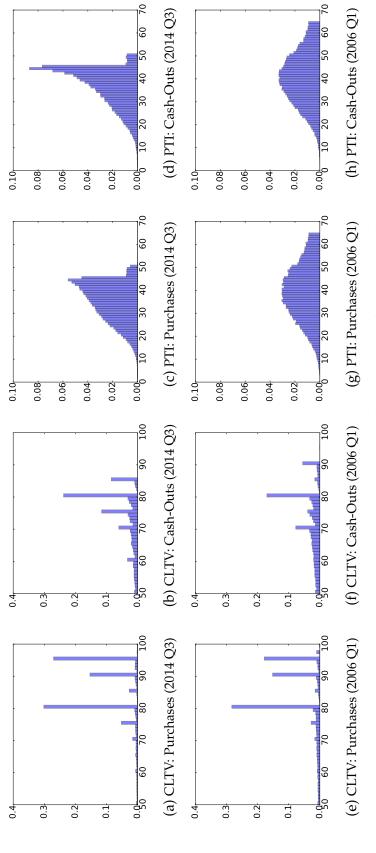


Figure 2: Fannie Mae Data, CLTV and PTI on Newly Originated Mortgages

Note: Histograms are weighted by Ioan balance. Source: Fannie Mae Single Family Dataset. PTI histograms for additional years can be found in the Appendix, Figures B.1 and B.2.

toward this limit before undergoing nearly complete truncation. The appearance of this smooth shape, rather than a single spike, likely stems from search frictions. Many borrowers may prefer the threshold price described in Section 2.1, but are unable to find a house at precisely this value. If borrowers are willing to buy a house below but not above the threshold price, the joint pattern of LTV spikes and a truncated PTI distribution will emerge naturally.<sup>13</sup> The distribution of cash-out refinances — where borrowers remain in their existing homes and do not search — bolsters this argument, displaying much more PTI concentration near the institutional limit, but less bunching in CLTV.

Overall, the 2014 data indicate that a nontrivial minority of borrowers are influenced by PTI limits. Since the Dodd-Frank legislation imposes a 43% cap on PTI ratios that will eventually apply to most mortgages, this influence is likely to persist, and may strengthen further if interest rates rise from their current historic lows.<sup>14</sup>

In complete contrast, the 2006 data display no evidence of a PTI limit imposed at *any* level. Instead, the PTI histogram displays a smooth shape until 65% of pre-tax borrower income is committed to recurring debt payments, at which point the data are top-coded by the provider. In this sample, 55% of debt for home purchases went to loans violating the traditional PTI limit of 36%, while 19% of debt went to loans with PTI ratios exceeding 50%. As a whole, these data point to extremely loose PTI standards during the boom period, while comparison with the CLTV distribution indicates that PTI limits likely underwent the larger change over this span.

While the data used for Figure 2 is not available prior to 2000, at which point PTI limits already appear loose, Figure B.4 in the Appendix displays histograms from the Black Knight Mortgage Performance (McDash) dataset, covering a longer sample including the 1990s, as well as non-GSE loans. While the coverage within this population is not as complete as the Fannie Mae data in Figure 2, the Black Knight data reinforce the findings of extremely loose PTI limits during the boom, and display patterns strongly consistent with a liberalization of PTI limits between 1998 and 2000. Prior to 1999, these data

 $<sup>^{13}</sup>$ Bank preapproval letters often cap the price at which a buyer can make an offer to exactly this threshold price by default, potentially explaining this asymmetry.

<sup>&</sup>lt;sup>14</sup>To be more precise, the Dodd-Frank limit is not a hard cap, but is the limit for "Qualified Mortgages," which banks are strongly incentivized to issue. While this limit has already taken effect, GSE-insured loans — the vast majority of loans issued since the bust — are exempt from this limit until 2020, and instead follow the self-imposed GSE limit of 45%. See DeFusco, Johnson, and Mondragon (2017) for more details on this regulation and its influence on credit supply.

<sup>&</sup>lt;sup>15</sup>The corresponding numbers for cash-out refinances are 59% to loans exceeding 36% PTI, and 20% to loans exceeding 45% PTI.

<sup>&</sup>lt;sup>16</sup>The Black Knight data has a large number of missing values for the PTI field, which servicers often fail to report. See Foote, Gerardi, Goette, and Willen (2010) for further discussion of this phenomenon. It is

display many borrowers bunching in a single PTI bin, while few loans exhibit PTI ratios above 50%. After 1999, this pattern is reversed, with little bunching and many PTI ratios above 50%. This shift suggests that loose PTI limits were not a longstanding feature of US mortgage underwriting, but were the product of a massive relaxation in the years just prior to the boom.<sup>17</sup>

#### 3 Model

This section constructs the model and presents its key equilibrium conditions.

**Demographics and Preferences.** The economy consists of two families, each populated by a continuum of infinitely-lived households. The households in each family differ in their preferences: one family contains relatively impatient households named "borrowers," denoted with subscript b, while the other family contains relatively patient households named "savers," denoted with subscript s. The measures of the two populations are  $\chi_b$  and  $\chi_s = 1 - \chi_b$ , respectively. Households trade a complete set of contracts for consumption and housing services within their own family, providing perfect insurance against idiosyncratic risk, but cannot trade these securities with members of the other family. Both types supply perfectly substitutable labor.

Each agent of type  $j \in \{b, s\}$  maximizes expected lifetime utility over nondurable consumption  $c_{j,t}$ , housing services  $h_{j,t}$ , and labor supply  $n_{j,t}$ 

$$\mathbb{E}_{t} \sum_{k=0}^{\infty} \beta_{j}^{k} u(c_{j,t+k}, h_{j,t+k}, n_{j,t+k})$$
(1)

where utility takes the separable form

$$u(c, n, h) = \log(c) + \xi \log(h) - \eta_j \frac{n^{1+\varphi}}{1+\varphi}.$$
 (2)

Preference parameters are identical across types with the exceptions that  $\beta_b < \beta_s$ , so that borrowers are less patient than savers, and that the  $\eta_i$  are allowed to differ, so that the

also worth noting that Black Knight typically reports "front-end" PTI ratios, excluding non-mortgage debt payments, while Figure 2 reports "back-end" ratios including these payments.

<sup>&</sup>lt;sup>17</sup>Acharya, Richardson, Van Nieuwerburgh, and White (2011) describe how political pressure on the GSEs, combined with the entry of private label securitizers, contributed to the relaxation of credit standards at this time.

two types provide supply the same amount of labor in steady state. For notation, I define the marginal utility and stochastic discount factor for each type by

$$u_{j,t}^c = \frac{\partial u(c_{j,t}, n_{j,t}, h_{j,t})}{\partial c_{j,t}} \qquad \qquad \Lambda_{j,t+1} = \beta_j \frac{u_{j,t+1}^c}{u_{j,t}^c}$$

with analogous expressions for  $u_{j,t}^n$  and  $u_{j,t}^h$ .

**Asset Technology.** For notation, stars (e.g.,  $q_t^*$ ) differentiate values for newly originated loans from the corresponding values for existing loans in the economy — a distinction necessary under long-term fixed-rate debt. The symbol "\$" before a quantity indicates that it is measured in nominal terms.

The essential financial asset in the paper, and the only source of borrowing in the model economy, is the mortgage contract, whose balances (long for the saver, short for the borrower) are denoted m. The mortgage is a nominal perpetuity with geometrically declining payments, as in Chatterjee and Eyigungor (2015). I consider a fixed-rate mortgage contract, which is the predominant contract in the US, but extend the model for the case of adjustable-rate mortgages in Appendix A.6.

To allow for changes in the real interest rate similar to movements in term premia or mortgage spreads, I introduce a proportional tax  $\Delta_{q,t}$  on all future mortgage payments associated with a given loan, that is assumed to follow the stochastic process

$$\Delta_{q,t} = (1 - \phi_q)\mu_q + \phi_q \Delta_{q,t-1} + \varepsilon_{q,t} \tag{3}$$

where  $\varepsilon_{q,t}$  is a white noise process that I will call a term premium shock. This tax does not map to any existing policy, but is instead used to introduce a time-varying wedge that can exogenously move the real cost of borrowing, and is rebated lump-sum to savers.

Putting these pieces together, under the fixed-rate mortgage contract the saver gives the borrower \$1 at origination. In exchange, the saver receives  $(1 - \nu)^k (1 - \Delta_{q,t}) q_t^*$  at time t + k, for all k > 0 until prepayment, where  $q_t^*$  is the equilibrium coupon rate at origination, and  $\nu$  is the fraction of principal paid each period.

As is standard in the US, mortgage debt is prepayable, meaning that the borrower can choose to repay the principal balance on a loan at any time, thereby canceling all future payments of the loan. If a borrower chooses to prepay her loan, she may choose a new loan size  $m_{i,t}^*$  subject to her credit limits (defined below). Obtaining a new loan

incurs a transaction cost  $\kappa_{i,t} m_{i,t}^*$ , where  $\kappa_{i,t}$  is drawn i.i.d. across individual members of the family and across time from a distribution with c.d.f.  $\Gamma_{\kappa}$ . This heterogeneity is needed to match the data, as otherwise identical model borrowers must make different prepayment decisions so that only an endogenous fraction prepay in each period. The borrower's optimal policy is to prepay the loan if her cost draw  $\kappa_{i,t}$  falls below a threshold value.

To allow for aggregation, I make a simplifying assumption: as part of the mortgage contract, borrowers must precommit to a threshold cost policy  $\bar{\kappa}_t$  that can depend arbitrarily on any aggregate states, but cannot depend on the positions of their individual loans within the cross-section. As a result, while the model prepayment rate will endogenously respond to key macroeconomic conditions, such as the average interest rate on new vs. existing loans, the total amount of home equity available to be extracted, and forward looking expectations of all aggregate state variables, it loses the ability to react to shifts in the shapes of the individual loan distributions relative to their means. In return, this abstraction yields a major gain in tractability, since the probability of prepayment (prior to the draws of  $\kappa_{i,t}$ ) becomes constant across borrowers at any single point in time — a key property for my aggregation result.

Turning to credit limits, a new loan for borrower *i* must satisfy both an LTV and a PTI constraint, defined by

$$\frac{m_{i,t}^*}{p_t^h h_{i,t}^*} \le \theta^{LTV} \qquad \qquad \frac{(q_t^* + \alpha) m_{i,t}^*}{w_t n_{i,t} e_{i,t}} \le \theta^{PTI} - \omega$$

where  $m_{i,t}^*$  is the balance on the new loan, and  $\theta^{LTV}$  and  $\theta^{PTI}$  are the maximum LTV and PTI ratios, respectively. These constraints are treated as institutional, and are not the outcome of any formal lender optimization problem.<sup>19</sup> The LTV ratio divides the loan balance by the borrower's house value, given by the product of house price  $p_t^h$  and the quantity of housing purchased  $h_{i,t}^*$ . The key property of the LTV limit is that it moves proportionally with  $p_t^h$ , so that a rise in house prices loosens this constraint.

For the PTI ratio, the numerator is the borrower's initial payment, while the denominator is the borrower's labor income, equal to the product of the wage  $w_t$ , labor supply  $n_{i,t}$ , and an idiosyncratic labor efficiency shock  $e_{i,t}$ , drawn i.i.d. across borrowers and time with mean equal to unity and c.d.f.  $\Gamma_e$ . This income shock serves to generate variation

<sup>&</sup>lt;sup>18</sup>I calibrate the transaction cost parameters in Section 4.2 to match the average prepayment rate and prepayment sensitivity implied by the data so as to remove any bias due to this assumption on average.

<sup>&</sup>lt;sup>19</sup>This choice is motivated by the observation that industry standards for these ratios can persist for decades, despite large changes in economic conditions.

among borrowers, so that an endogenous fraction is limited by each constraint at equilibrium. The term  $\alpha$  is used to account for taxes and insurance (included in typical PTI calculations) as well as to ensure that the different amortization schemes in the model and data do not distort the tightness of the constraint (see Section 4). Finally, the offsetting term  $\omega$  adjusts for the underwriting convention that the numerator of PTI typically includes payments on all recurring debt (e.g., car loans, student loans, etc.) by assuming that these payments require a fixed fraction of borrower income. The presence of  $q_t^*$  in the PTI ratio makes the PTI limit extremely sensitive to movements in interest rates — as already seen in the simple example of Section 2.1 — a property that will be crucial in the results to follow.

These expressions imply the maximum debt balances

$$ar{m}_{i,t}^{LTV} = heta^{LTV} p_t^h h_{i,t}^* \qquad \qquad ar{m}_{i,t}^{PTI} = rac{( heta^{PTI} - \omega) w_t n_{i,t} e_{i,t}}{q_t^* + lpha}$$

consistent with each of the two limits. Since the borrower must satisfy both constraints, her overall debt limit is  $m_{i,t}^* \leq \bar{m}_{i,t} = \min(\bar{m}_{i,t}^{LTV}, \bar{m}_{i,t}^{PTI})$ . This constraint is applied at origination of the loan only, so that borrowers are not forced to delever if they violate these constraints later on. At equilibrium, this constraint will bind for all newly issued loans, consistent with Figure 2, which shows few unconstrained borrowers at origination. However, households usually wait years between prepayments in the model, during which time they are typically away from their borrowing constraints and accumulating home equity.

In addition to mortgages, households can trade a one-period nominal bond, whose balances are denoted  $b_t$ . One unit of this bond costs \$1 at time t and pays  $R_t$  with certainty at time t + 1. This bond is in zero net supply, and is used by the monetary authority as a policy instrument. Since the focus of the paper is on mortgage debt, I assume that positions in the one-period bond must be non-negative, so that it is traded by savers only at equilibrium.

The final asset in the economy is housing, which produces a service flow each period equal to its stock, and can be owned by both types. A constant fraction  $\delta$  of house value must be paid as a maintenance cost at the start of each period. Borrower and saver hold-

 $<sup>^{20}</sup>$ While I model  $e_{i,t}$  as an income shock, it could stand in for any shock that varies the ratio of house price to income in the population. Without variation in this ratio, all borrowers would be limited by the same constraint in a given period.

<sup>&</sup>lt;sup>21</sup>Since the dynamics of non-mortgage debt are beyond the scope of this paper, I assume this debt is owed to other borrowers, so that it has no other influence beyond this constraint.

ings of housing are denoted  $h_{b,t}$  and  $h_{s,t}$ , respectively. To simplify the analysis, I fix the total housing stock to be  $\bar{H}$ , which implies that the price of housing fully characterizes the state of the housing market.<sup>22</sup> Additionally, to focus on the use of housing as a collateral asset, I assume that saver demand is fixed at  $h_{s,t} = \bar{H}_s$ , so that a borrower is always the marginal buyer of housing.<sup>23</sup> Saver demand is fixed for both owned housing and housing services, so that borrowers do not rent from savers at equilibrium.<sup>24</sup> Finally, as is standard in the US, each loan is linked to a specific house, so that only prepaying households can adjust their housing holdings.

**Taxation.** Both types are subject to proportional taxation of labor income at rate  $\tau_y$ . All taxes are returned in lump sum transfers equal to the amount paid by that type. Borrower interest payments, defined as  $(q_{i,t-1} - \nu)m_{i,t-1}$ , are tax deductible.

Representative Borrower's Problem. As demonstrated in Appendix ??, the borrower's problem conveniently aggregates to that of a single representative borrower. The endogenous state variables for the representative borrower's problem are: total start-of-period debt balances  $m_{t-1}$ , total promised payments on existing debt  $x_{t-1} \equiv q_{t-1}m_{t-1}$ , and total start-of-period borrower housing  $h_{b,t-1}$ . If we define  $\rho_t = \Gamma_\kappa(\bar{\kappa}_t)$  to be the fraction of loans prepaid, then the laws of motion for these state variables are given by

$$m_t = \rho_t m_t^* + (1 - \rho_t)(1 - \nu)\pi_t^{-1} m_{t-1}$$
(4)

$$x_{b,t} = \rho_t q_t^* m_t^* + (1 - \rho_t)(1 - \nu) \pi_t^{-1} x_{b,t-1}$$
(5)

$$h_{b,t} = \rho_t h_{b,t}^* + (1 - \rho_t) h_{b,t-1} \tag{6}$$

<sup>&</sup>lt;sup>22</sup>Modeling a fixed housing stock precludes the dampening effect of supply on prices. However, from perspective of credit growth, the key variable is total collateral value: the product of price and quantity. Under a flexible housing supply, smaller movements in price are compensated by larger movements in quantity, leading to similar overall effects. Moreover, my numerical results focus on price-to-rent ratios. These should not be strongly affected by supply responses, which typically move prices and rents in parallel. For results on spending and output, borrowing used for nondurable consumption in this model would be instead spent on residential investment in a flexible supply specification.

<sup>&</sup>lt;sup>23</sup>This assumption is useful under divisible housing to prevent excessive flows of housing between the two groups, which would otherwise occur unrealistically along the intensive margin of house size.

<sup>&</sup>lt;sup>24</sup>The existence of a perfect rental market with an unconstrained representative landlord, as in Kaplan et al. (2017), would imply that shifts in credit constraints cannot directly influence house prices. In reality, heterogeneity in the suitability of properties as rental units, and the widespread use of mortgages by landlords, imply that house prices should still be sensitive to credit conditions. Establishing quantitatively the degree to which rental markets can dampen house price responses to changes in credit availability is an important area for future research.

The representative borrower chooses consumption  $c_{b,t}$ , labor supply  $n_{b,t}$ , the size of newly purchased houses  $h_{b,t}^*$ , the face value of newly issued mortgages  $m_t^*$ , and the fraction of loans to prepay  $\rho_t$ , to maximize (1) using the aggregate utility function

$$u(c_{b,t}, h_{b,t-1}, n_{b,t}) = \log(c_{b,t}/\chi_b) + \xi \log(h_{b,t-1}/\chi_b) - \eta_b \frac{(n_{b,t}/\chi_b)^{1+\varphi}}{1+\varphi}$$

subject to the budget constraint

$$c_{b,t} \leq \underbrace{(1-\tau_y)w_t n_{b,t}}_{\text{labor income}} - \underbrace{\pi_t^{-1}\big((1-\tau_y)x_{b,t-1} + \tau_y \nu m_{t-1}\big)\big)}_{\text{payment net of deduction}} + \underbrace{\rho_t\big(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1}\big)}_{\text{new issuance}} - \underbrace{\delta p_t^h h_{b,t-1}}_{\text{housing purchases}} - \underbrace{\rho_t p_t^h \big(h_{b,t}^* - h_{b,t-1}\big)}_{\text{housing purchases}} - \underbrace{(\Psi(\rho_t) - \bar{\Psi}_t)\,m_t^*}_{\text{transaction costs}} + T_{b,t}$$

the debt constraint

$$m_t^* \le \bar{m}_t = \underbrace{\bar{m}_t^{PTI} \int_t^{\bar{e}_t} e_i \, d\Gamma_e(e_i)}_{\text{PTI Constrained}} + \underbrace{\bar{m}_t^{LTV} (1 - \Gamma_e(\bar{e}_t))}_{\text{LTV Constrained}}.$$
 (7)

and the laws of motion (4) - (6), where

$$\bar{m}_t^{LTV} = \theta^{LTV} p_t^h h_{b,t}^* \qquad \qquad \bar{m}_t^{PTI} = \frac{(\theta^{PTI} - \omega) w_t n_{b,t}}{q_t^* + \alpha} \tag{8}$$

are the population average LTV and PTI limits. The term  $\bar{e}_t \equiv \bar{m}_t^{LTV}/\bar{m}_t^{PTI}$  is the threshold value of the income shock  $e_{i,t}$  so that for  $e_{i,t} < \bar{e}_t$ , borrowers are constrained by PTI, while

$$\Psi(\rho_t) = \int^{\Gamma^{-1}(\rho_t)} \kappa d\Gamma_{\kappa}(\kappa)$$

is the average transaction cost per unit of issued debt,  $\bar{\Psi}_t$  is a proportional rebate that returns these transaction costs to the borrowers at equilibrium,  $T_{b,t}$  rebates borrower taxes. Note that because (7) aggregates smoothly over endogenous fractions limited by each constraint, there is no issue with occasionally binding constraints, allowing debt dynamics to be effectively captured by a perturbation solution.

<sup>&</sup>lt;sup>25</sup>I choose to rebate the transaction costs, as they likely stand in for non-monetary frictions such as inertia, matching evidence that borrowers often do not refinance even when financially advantageous (see, e.g., Andersen, Campbell, Nielsen, and Ramadorai (2014), Keys, Pope, and Pope (2014)).

**Representative Saver's Problem.** The individual saver's problem also aggregates to the problem of a representative saver, who chooses consumption  $c_{s,t}$ , labor supply  $n_{s,t}$ , and the face value of newly issued mortgages  $m_t^*$  to maximize (1) using the utility function

$$u(c_{s,t}, n_{s,t}) = \log(c_{s,t}/\chi_s) + \xi \log(\bar{H}_s/\chi_s) - \eta_s \frac{(n_{s,t}/\chi_s)^{1+\varphi}}{1+\varphi}$$

subject to the budget constraint

$$c_{s,t} \leq \underbrace{(1-\tau_y)w_t n_{s,t}}_{\text{labor income}} + \underbrace{\pi_t^{-1}x_{s,t-1}}_{\text{mortgage payments}} - \underbrace{\rho_t \left(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1}\right)}_{\text{new issuance}}$$

$$- \underbrace{\delta p_t^h \bar{H}_s}_{\text{maintenance}} - \underbrace{\left(R_t^{-1}b_t - b_{t-1}\right)}_{\text{net bond purchases}} + \underbrace{\Pi_t}_{\text{profits}} + T_{s,t},$$

the law of motion (4), and

$$x_{s,t} = (1 - \Delta_{q,t})\rho_t q_t^* m_t^* + (1 - \rho_t)(1 - \nu)\pi_t^{-1} x_{s,t-1}$$
(9)

where  $\Pi_t$  are intermediate firm profits, and  $T_{s,t}$  rebates saver taxes.

**Productive Technology.** The production side of the economy is populated by a competitive final good producer and a continuum of intermediate goods producers owned by the saver. The final good producer solves the static problem

$$\max_{y_t(i)} P_t \left[ \int y_t(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}} - \int P_t(i) y_t(i) di$$

where each input  $y_t(i)$  is purchased from an intermediate good producer at price  $P_t(i)$ , and  $P_t$  is the price of the final good.

The producer of intermediate good i chooses price  $P_t(i)$  and operates the linear production function

$$y_t(i) = a_t n_t(i)$$

to meet the final good producer's demand, where  $n_t(i)$  is labor hours and  $a_t$  is total factor productivity (TFP), which evolves according to

$$\log a_{t+1} = (1 - \phi_a)\mu_a + \phi_a \log a_t + \varepsilon_{a,t+1}$$

where  $\varepsilon_{a,t+1}$  is a white noise process that I will refer to as a productivity or TFP shock. Intermediate good producers are subject to price stickiness of the Calvo-Yun form with indexation. Specifically, a fraction  $1 - \zeta$  of firms are able to adjust their price each period, while the remaining fraction  $\zeta$  update their existing price by the rate of steady state inflation.

**Monetary Authority.** The monetary authority follows a Taylor rule, similar to that of Smets and Wouters (2007), of the form

$$\log R_{t} = \log \bar{\pi}_{t} + \phi_{r} (\log R_{t-1} - \log \bar{\pi}_{t-1}) + (1 - \phi_{r}) \left[ (\log R_{ss} - \log \pi_{ss}) + \psi_{\pi} (\log \pi_{t} - \log \bar{\pi}_{t}) \right]$$
(10)

where the subscript "ss" refers to steady state values, and  $\bar{\pi}_t$  is a time-varying inflation target defined by

$$\log \bar{\pi}_t = (1 - \psi_{\bar{\pi}}) \log \pi_{ss} + \psi_{\bar{\pi}} \log \bar{\pi}_{t-1} + \varepsilon_{\bar{\pi},t}$$

where  $\varepsilon_{\bar{\pi},t}$  is a white noise process that I will refer to as an inflation target shock. These shocks correspond to near-permanent changes in monetary policy that, as in Garriga et al. (2015), shift the entire term structure of nominal interest rates. In contrast to term premium shocks, inflation target shocks move nominal rates while influencing real rates very little — and in the opposite direction — making them convenient for analyzing the effect of changing nominal rates in isolation.

It will also be useful to define the special case  $\psi_{\pi} \to \infty$ , corresponding to the case of perfect inflation stabilization, in which case the policy rule (10) collapses to

$$\pi_t = \bar{\pi}_t \tag{11}$$

which implicitly defines the value of  $R_t$  needed to attain equality.

**Equilibrium.** A competitive equilibrium in this model is defined as a sequence of endogenous states  $(m_{t-1}, x_{t-1})$ , allocations  $(c_{j,t}, n_{j,t})$ , mortgage and housing market quantities  $(h_{b,t}^*, m_t^*, \rho_t)$ , and prices  $(\pi_t, w_t, p_t^h, R_t, q_t^*)$  that satisfy borrower, saver, and firm opti-

mality, and the following market clearing conditions:

Resources:  $c_{b,t} + c_{s,t} + \delta p_t^h \bar{H} = y_t$ 

Bonds:  $b_{s,t} = 0$ 

Housing:  $h_{b,t} + \bar{H}_s = \bar{H}$ 

Labor:  $n_{b,t} + n_{s,t} = n_t$ .

#### 3.1 Model Solution

In this section, I present two borrower optimality conditions that summarize the main innovations of the model: simultaneously imposed LTV and PTI constraints, and long-term debt with endogenous prepayment. The remaining optimality conditions, as well as those for the saver and intermediate producers, can be found in Appendix A.1.

The influence of the constraint structure appears most strongly in the borrower's first order condition for housing, which requires the equilibrium house price to satisfy

$$p_{t}^{h} = \frac{u_{b,t}^{h}/u_{b,t}^{c} + \mathbb{E}_{t}\left\{\Lambda_{b,t+1}p_{t+1}^{h}\left[1 - \delta - (1 - \rho_{t+1})C_{t+1}\right]\right\}}{1 - C_{t}}.$$

The term  $C_t = \mu_t F_t^{LTV} \theta^{LTV}$  represents the marginal collateral value of housing — the benefit the borrower would receive from an additional dollar of housing through its ability to relax her debt limit — where  $\mu_t$  is the multiplier on the constraint, and  $F_t^{LTV} = 1 - \Gamma_e(\bar{e}_t)$  is the fraction of new borrowers constrained by LTV. Division by  $1 - C_t$  reflects a collateral premium for housing, raising its price when collateral demand is high.<sup>26</sup>

In a model with an LTV constraint only,  $C_t$  would equal  $\mu_t \theta^{LTV}$ , the product of the amount by which the constraint is relaxed ( $\theta^{LTV}$ ) and the rate at which the borrower values the relaxation ( $\mu_t$ ). But when both constraints are imposed, the debt limits of PTI-constrained borrowers are not altered by an additional unit of housing, so that only LTV-constrained households actually receive this collateral benefit. As a result, the collateral value is scaled by  $F_t^{LTV}$ . Because of this scaling, any macroeconomic forces that shift the fraction of borrowers who are LTV-constrained will also influence collateral values. I call this mechanism — through which changes in which limit is binding for borrowers translate into movements in house prices — the *constraint switching effect*. This effect

 $<sup>^{26}</sup>$ In contrast, the appearance of  $C_{t+1}$  in the numerator of (3.1) occurs because, with probability  $1 - \rho_{t+1}$ , the borrower will not prepay her loan. In these states of the world, the borrower will not use her housing holdings to collateralize a new loan, and does not receive the collateral benefit of housing.

generalizes the dynamics of the simple example in Section 2.1 to an environment with heterogeneous borrowers.

Next, the influence of long-term prepayable debt can be seen in the borrower's optimality condition for prepayment, which sets the fraction of prepaid loans to

$$\rho_{t} = \Gamma_{\kappa} \left\{ \underbrace{\left(1 - \Omega_{b,t}^{m} - \Omega_{b,t}^{x} q_{t-1}\right) \left(1 - \frac{(1 - \nu)\pi_{t}^{-1} m_{t-1}}{m_{t}^{*}}\right)}_{\text{new debt incentive}} - \underbrace{\Omega_{b,t}^{x} \left(q_{t}^{*} - q_{t-1}\right)}_{\text{interest rate incentive}} \right\}$$

$$(12)$$

where  $\Omega_{b,t}^m$  and  $\Omega_{b,t}^x$  are the marginal continuation costs to the borrower of an additional unit of face-value debt, and of promised payment, respectively (see Appendix A.1 for details), and where  $q_{t-1}$  is the average coupon rate on existing time t-1 loans. The term inside the c.d.f.  $\Gamma_{\kappa}$  represents the marginal benefit to prepaying an additional unit of debt, which can be decomposed into two terms reflecting borrowers' distinct motivations to prepay.

The first term represents the hypothetical benefit from taking on new debt at the average interest rate on *existing* debt: the product of the net benefit of an additional dollar of debt (\$1 today minus continuation costs of additional principal and promised payments) and the net increase in debt per dollar of face value, after deducting the portion of the new loan used to prepay existing debt. The second term reflects the borrower's interest rate incentive: under fixed-rate debt, prepayment is more beneficial when the coupon rate on new debt  $(q_t^*)$  is low relative to the rate on existing debt  $(q_{t-1})$ . These forces will drive the frontloading effect in Section 5.2 that is key to transmission into output.

## 4 Calibration and Model Evaluation

This section describes the calibration procedure, and tests the model's fit of the macroe-conomic data, showing that the model delivers impulse responses in line with the data. This calibration succeeds in matching the dynamics of aggregate US mortgage leverage, generating a substantially improved fit of the data relative to existing models.

#### 4.1 Calibration

The calibrated parameter values are presented in Table 1. While some parameters can be set to standard values, a number of others relate to features new to the literature, and are

calibrated directly to mortgage data.

For the income shock distribution  $\Gamma_e$ , I choose the log-normal specification  $\log e_{i,t} \sim N\left(-\sigma_e^2/2, \sigma_e^2\right)$ . This parameterization implies

$$\int^{\bar{e}_t} e_i d\Gamma_e(e_i) = \Phi\left(\frac{\log \bar{e}_t - \sigma_e^2/2}{\sigma_e}\right)$$

where  $\Phi$  is the standard normal c.d.f., facilitating the computation of (7). In reality, unlike in the model, borrowers may differ both in their incomes and in the size of the house that they purchase. As a result, to capture dispersion in which constraint is binding, I set  $\sigma_e$  to match the standard deviation of  $\log(PTI_{i,t}) - \log(CLTV_{i,t})$  in the data. This term is the difference of individual borrowers' log PTI and CLTV ratios at origination, which is equal to  $\log e_{i,t}$  in the model, up to the offset term  $\omega$ . I compute this standard deviation for purchase loans in the Fannie Mae data for each quarter from 2000 to 2014, and set  $\sigma_e = 0.411$  to be the average of this series.<sup>27</sup> This procedure has the additional benefit of allowing  $e_{i,t}$  to account for borrower variation in non-mortgage debt service (i.e.,  $\omega_{i,t}$ ), which appear in the data measure of  $PTI_{i,t}$ .

Next, the parametric form for the transaction cost distribution,  $\Gamma_{\kappa}$ , is inspired by the observation that in the data, the fraction of loans prepaid in a single quarter varies from a minimum of 1.0% to a maximum of 20.8%, despite a wide range of interest rate and housing market conditions.<sup>28</sup> With an upper bound so far below unity, the fit is improved by choosing  $\Gamma_{\kappa}$  to be a mixture, such that with 1/4 probability,  $\kappa$  is drawn from a logistic distribution, and with 3/4 probability,  $\kappa = \infty$ , in which case borrowers never prepay, delivering

$$\Gamma_{\kappa}(\kappa) = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(-\frac{\kappa - \mu_{\kappa}}{s_{\kappa}}\right)}.$$

This functional form is parameterized by a location parameter  $\mu_{\kappa}$  and a scale parameter  $s_{\kappa}$ , which are calibrated to fit aggregate leverage dynamics in Section 4.2 below.

I calibrate the fraction of borrowers  $\chi_b$  to match the 1998 Survey of Consumer Finances. Consistent with the model, I classify borrower households in the data to be those with a house and mortgage, but less than two months' income in liquid assets, yielding  $\chi_b = 0.319$ . For the remaining preference parameters, I calibrate the housing preference

<sup>&</sup>lt;sup>27</sup>Results using analogous data from Freddie Mac are very similar.

<sup>&</sup>lt;sup>28</sup>Source: eMBS, Fannie Mae 30-Year MBS (code: FNM30).

<sup>&</sup>lt;sup>29</sup>Although 45.3% of those households that hold more than two months' liquid assets also hold a mortgage in the data, I still categorize them as savers as they do not appear to be liquidity-constrained, implying

Table 1: Parameter Values: Baseline Calibration

| Parameter                | Name              | Value     | Internal     | Target/Source                             |
|--------------------------|-------------------|-----------|--------------|---|
|                          | L                 | Demograph | ics and Prej | ferences                                  |
| Fraction of borrowers    | $\chi_b$          | 0.319     | N            | 1998 Survey of Consumer Finances          |
| Income dispersion        | $\sigma_{e}$      | 0.411     | N            | Fannie Mae Loan Performance Data          |
| Borr. discount factor    | $eta_b$           | 0.965     | Y            | Value-to-income ratio (1998 SCF)          |
| Saver discount factor    | $eta_s$           | 0.987     | N            | Avg. 10Y rate, 1993-1997                  |
| Housing preference       | ξ                 | 0.25      | N            | Davis and Ortalo-Magné (2011)             |
| Borr. labor disutility   | $\eta_b$          | 8.190     | Y            | $n_{b,ss}/\chi_b = 1/3$                   |
| Saver labor disutility   | $\eta_s$          | 5.662     | Y            | $n_{s,ss}/\chi_s=1/3$                     |
| Inv. Frisch elasticity   | $\varphi$         | 1.0       | N            | Standard                                  |
| Housing and Mortgages    |                   |           |              |   |
| Mortgage amortization    | ν                 | 0.435%    | N            | See text                                  |
| Income tax rate          | $	au_{y}$         | 0.204     | N            | Elenev et al. (2016)                      |
| Max PTI ratio            | $	heta^{ec{PT}I}$ | 0.36      | N            | See text                                  |
| Max LTV ratio            | $	heta^{LTV}$     | 0.85      | N            | See text                                  |
| Issuance cost mean       | $\mu_{\kappa}$    | 0.348     | Y            | Nonlinear LS (see Section 4.2)            |
| Issuance cost scale      | $S_{\mathcal{K}}$ | 0.152     | Y            | Nonlinear LS (see Section 4.2)            |
| PTI offset (taxes, etc.) | α                 | 0.285%    | Y            | $q_{ss}^* + \alpha = 10.6\%$ (annualized) |
| PTI offset (other debt)  | $\omega$          | 0.08      | N            | See text                                  |
| Term premium (mean)      | $\mu_q$           | 0.320%    | Y            | Avg. mortgage rate, 1993 - 1997           |
| Term premium (pers.)     | $\phi_q$          | 0.852     | N            | Autocorr. of (mort. rate - 1Y rate)       |
| Log housing stock        | log Ħ             | 2.178     | Y            | $p_{ss}^{h} = 1$                          |
| Log saver housing stock  | $\log \bar{H}_s$  | 1.867     | Y            | 1998 Survey of Consumer Finances          |
| Housing depreciation     | δ                 | 0.005     | N            | Standard                                  |
| Productive Technology    |                   |           |              |   |
| Productivity (mean)      | $\mu_a$           | 1.099     | Y            | $y_{ss}=1$                                |
| Productivity (pers.)     | $\phi_a$          | 0.964     | N            | Garriga et al. (2015)                     |
| Variety elasticity       | $\lambda$         | 6.0       | N            | Standard                                  |
| Price stickiness         | ζ                 | 0.75      | N            | Standard                                  |
|                          |                   | Mon       | etary Policy | /   |
| Steady state inflation   | $\pi_{ss}$        | 1.008     | N            | Avg. infl. expectations, 1993 - 1997      |
| Taylor rule (inflation)  | $\psi_\pi$        | 1.5       | N            | Standard                                  |
| Taylor rule (smoothing)  | $\phi_r$          | 0.89      | N            | Campbell, Pflueger, and Viceira (2014)    |
| Infl. target (pers.)     | $\phi_{ar{\pi}}$  | 0.994     | N            | Garriga et al. (2015)                     |

*Note:* The model is calibrated at quarterly frequency. Parameters denoted "Y" in the "Internal" column are internally calibrated, meaning that they are not set explicitly in closed form, but are instead chosen implicitly to match a particular moment at steady state.

weight  $\xi$  to 0.25, to target a housing expenditure share of 20%, equal to the 24% share estimated by Davis and Ortalo-Magné (2011), net of 4% to account for utilities. I choose the borrower discount factor to match the steady state ratio of borrower house value to income ( $p_t^h h_{b,t}/w_t n_{b,t}$ ) in the 1998 SCF (8.89 quarterly), which yields  $\beta_b = 0.965$ .

Next, I calibrate the interest rate and inflation parameters. Since the key rates in the model concern long-term mortgage debt, I calibrate the saver discount factor  $\beta_s$ , average inflation  $\pi_{ss}$ , and average term premium  $\mu_q$  to match the 1993 - 1997 average of 10-year interest rates (6.46%), 10-year inflation expectations (3.25%), and mortgage rates (7.81%), respectively. I set the persistence of the term premium shock  $\phi_q$  to match the average quarterly autocorrelation of the spread between mortgage rates and two-year treasuries.

For the debt limit parameters, I set  $\theta^{LTV}=0.85$  as a compromise between the mass bunching at 80%, and the masses constrained at higher institutional limits such as 90% or 95%. Because of the presence of the PTI limit, the average LTV ratio across newly originated mortgages is 80.5% at steady state, in line with the data. For the PTI limit, I choose  $\theta^{PTI}=0.36$  to match the pre-boom underwriting standard and  $\omega=0.08$  to match the traditional PTI limit excluding other debt (28%). It is worth noting, however, that since the recent housing crash, the maximum PTI ratio on new loans appears to be not 36% but 45%, while in the future, the relevant ratio is likely to be the Dodd-Frank limit of 43%. Results using this value are similar, and can be found in Section A.6 in the Appendix.

Turning to the other mortgage contract parameters, I set  $\nu=0.435\%$  to match the average share of principal paid on existing loans.<sup>31</sup> This low value, which implies an average duration of more than 57 years, adjusts for the fact that, because of prepayment, the loan distribution is biased toward younger loans, whose payments contain a lower share of principal due to their earlier position in the amortization schedule. Since even with this adjustment, the simpler geometrically decaying coupons in the model might apply too much principal repayment at the start of the loan, I calibrate the offset term  $\alpha$  to ensure that this does not imply unrealistically tight PTI limits. Specifically, I set  $\alpha$ 

that their consumption should not be sensitive to changes in their debt limits or transitory changes to income. In the model, savers can trade mortgages (and any other financial contracts) within the saver family. Defining all mortgagors to be borrowers would further amplify transmission. A small fraction of borrowers have home equity lines of credit and may not be effectively liquidity constrained; excluding these households would yield a similar borrower share of 0.286.

<sup>&</sup>lt;sup>30</sup>See Figure B.3 in the Appendix.

<sup>&</sup>lt;sup>31</sup>Specifically, for each month in 2000:01 - 2015:01, I compute the average loan age and interest rate for existing loans in Fannie Mae 30-Year MBS (FNM30), weighted by loan balance. Given the age and rate, the fraction of the loan balance paid off as principal  $v_t$  can be computed from a standard amortization schedule. I calibrate v so that (1 - v) is the geometric average of  $(1 - v_t)$  over all months in the sample.

so that  $q_t^* + \alpha$  is equal to 10.47% (annualized) at steady state, which is the interest and principal payment on a loan with the steady state mortgage interest rate (7.81%) under the *exact* amortization scheme for a fixed-rate mortgage, plus 1.75% annually for taxes and insurance.

For the remaining parameters, I calibrate the housing stock  $\bar{H}$  and saver housing demand  $\bar{H}_s$  so that the price of housing is unity at steady state, and the ratio of saver house value to income is the same as in the 1998 SCF (11.40 quarterly). I set the tax rate  $\tau_y$  following Elenev et al. (2016) to the national average prior to mortgage interest deductions. To calibrate the exogenous processes for productivity  $a_t$  and the inflation target  $\bar{\pi}_t$ , I follow Garriga et al. (2015), who also study the impact of these shocks on long-term mortgage rates.

## 4.2 Matching Aggregate Leverage Dynamics

In this section, I calibrate the parameters  $\mu_{\kappa}$ , and  $s_{\kappa}$  to match the observed dynamics of aggregate leverage. In the process, I demonstrate that these dynamics cannot be explained by standard models, but can be reproduced by jointly accounting for PTI constraints, a liberalization of PTI during the boom, and endogenous prepayment by borrowers.

**Methodology.** To compare the ability of different models to fit the data, my approach is to derive a general law of motion for aggregate household leverage that nests a wide set of specifications. By using actual data in place of model variables, I can directly evaluate this block in isolation, without making any assumptions about the remainder of the model. The specifications can then be compared on their respective forecast errors to evaluate their ability to match observed debt dynamics.

To begin, divide through equation (4) by the value of residential housing  $v_t \equiv p_t^h \bar{H}$  to obtain

$$LTV_t = \rho_{t-k}LTV_{t-k}^* + (1 - \rho_{t-k})(1 - \nu)G_t^{-1}LTV_{t-1}.$$
 (13)

where  $LTV_t$  is the aggregate loan-to-value ratio  $(m_t/v_t)$ ,  $LTV_t^*$  is the LTV on newly originated loans  $(m_t^*/v_t)$ , and  $G_t$  is house value growth  $(v_t/v_{t-1})$ . While the model specifies the lag k=0, generalizing to k>0 is useful for matching the data, as it allows for a delay between when the decision to take on a loan is made and the terms are set, and when the new debt is issued and shows up in the national accounts.<sup>32</sup> For all specifications below,

 $<sup>\</sup>overline{^{32}}$ Since  $m_t$  is end-of-quarter debt, an additional lag may also be needed to accommodate data measures

I use k = 2, which provides the best empirical fit, although results with k = 0 and k = 1 are similar.

While this paper's framework implies specific parametric forms for  $LTV_t^*$  and  $\rho_t$ , the general law of motion (13) nests many existing macro-housing models. We can therefore compare this paper's benchmark model with existing specifications from the literature according to how each version of (13) fits the observed data. To give each model the best possible chance of matching the data, I estimate each model's specific parameters, denoted  $\gamma$ , using nonlinear least squares:

$$\hat{\gamma} = \arg\min_{\gamma} \frac{1}{T} \sum_{t=1}^{T} \left( LTV_{t} - \rho_{t-k} LTV_{t-k}^{*} - (1 - \rho_{t-k})(1 - \nu)G_{t}^{-1} LTV_{t-1} \right)^{2}.$$

Each model contains a formula for computing  $LTV_{t-k}^*$  and  $\rho_{t-k}$  as direct functions of the time t-k data and parameters  $\gamma$ , while the remaining variables  $LTV_t$  and  $G_t$  are taken directly from the data. The estimation sample is 1980 Q1 - 2015 Q4, which is the longest overlapping span for the full set of series needed for the exercise. Parameter estimates, including standard errors, can be found in Table B.1 in the Appendix.

While minimizing the one-quarter forecast errors is useful for estimating the parameters, the more relevant metric for policymakers is likely the ability of the model to produce accurate long-term forecasts of credit growth given assumed paths for house prices and other relevant macro variables. To test performance on this front, I compute an implied "forecast" series  $\widehat{LTV}_t$  for each model given the true paths for the other variables. Specifically, I initialize  $\widehat{LTV}_0$  at its true value  $LTV_0$ , and repeatedly apply the law of motion

$$\widehat{LTV}_t = \hat{\rho}_{t-k} LTV_{t-k}^* + (1 - \hat{\rho}_{t-k})(1 - \nu)G_t^{-1} \widehat{LTV}_{t-1}$$
(14)

using the fitted parameter values  $\gamma = \hat{\gamma}$ . While I still take  $G_t$  directly from the data, I update (14) using the previous forecast value  $\widehat{LTV}_{t-1}$ , and compute the implied prepayment rate  $\hat{\rho}_{t-k}$  (when needed) using the implied value  $\widehat{LTV}_{t-k}$ .<sup>33</sup> Finally, the implied loan-to-income ratio  $\widehat{LTI}_t$  can be computed by multiplying  $\widehat{LTV}_t$  by the ratio of value to household disposable income.

that do not follow this timing convention.

<sup>&</sup>lt;sup>33</sup>None of the models considered imply that  $LTV_t^*$  depends on  $LTV_t$ , so there is no difference between using the actual and implied path of  $LTV_t$  on  $LTV_t^*$ .

Existing Models. Figure 3 displays the resulting paths for  $\widehat{LTV}_t$  and  $\widehat{LTI}_t$  from this paper's framework, along with those from three popular specifications from the literature, and compares them to their counterparts in the data. To start, consider the existing specifications shown in Figures 3a and 3c, which follow the standard assumption in the literature of constant  $LTV^*$ . First, the path titled "One-Period," follows, e.g., Iacoviello (2005) in imposing one-period debt ( $\rho_t = 1$ ) so that  $LTV_t = LTV^*$  for all t. After estimating  $\gamma = LTV^*$ , this specification is able to capture the flat LTV ratios and rising LTI ratios observed during the boom — a period of rapid turnover (high  $\rho_t$ ) — but exaggerates leverage at the start of the sample, and implies that households delever far too quickly in the bust.

Next, the path titled "Ratchet" follows, e.g., Justiniano et al. (2015b) in specifying

$$\rho_t = \begin{cases} 1 & \text{for } LTV^* > (1 - \nu_t)G_t^{-1}LTV_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

so that borrowers renew all their loans each period, unless this would require them to delever, in which case they keep their existing balances. This mechanism is designed to avoid the unrealistically fast deleveraging found in the bust under the One-Period specification. Since this model is specially designed to capture the boom-bust period, I estimate  $\gamma = LTV^*$  on a shorter sample from 1998 Q1 onward.<sup>34</sup> While this version performs better than the One-Period model over the bust period, it offers little insight into debt dynamics in the pre-boom period, where it still seriously overstates leverage.

For the final existing model, the path titled "Exog. Prepay" follows, e.g., Midrigan and Philippon (2016), in specifying that a fixed fraction of loans are renewed each period ( $\rho_t = \bar{\rho} < 1$ ). For this application, I set  $LTV^*$  to a scaled version of the baseline calibration  $\theta^{LTV} = 0.85$  that adjusts for the difference between the aggregate and borrower populations due to, e.g., outright owners, <sup>35</sup> and estimate  $\gamma = \bar{\rho}$ . While this specification performs much better than the one-period debt models in the early period, and captures the persistent rise in LTV ratios during a slow post-crash deleveraging, it seriously un-

 $<sup>^{34}</sup>$ The ratchet specification fitted over the full sample performs poorly — the nonlinear least squares criterion is minimized by setting  $LTV^*$  so low that  $\rho_t=0$  over the entire sample.  $^{35}$  Specifically, I use the limit  $LTV^*=0.747\cdot\theta^{LTV}$ , yielding a value of 0.635. This scaling is chosen so

<sup>&</sup>lt;sup>35</sup> Specifically, I use the limit  $LTV^* = 0.747 \cdot \theta^{LTV}$ , yielding a value of 0.635. This scaling is chosen so that the ratio of  $LTV_{1998}$  (0.420) to  $LTV^*$  is the same as the ratio of median LTV among mortgage holders in the 1998 SCF (0.562) to the baseline LTV limit (0.85). This ensures that the effective fraction of extractable equity is the same as for the typical mortgagor in 1998.

<sup>&</sup>lt;sup>36</sup>This procedure estimates a reasonable average annual prepayment rate of 13.0%, validating the scaling procedure for  $\theta^{LTV}$  described in the previous footnote.

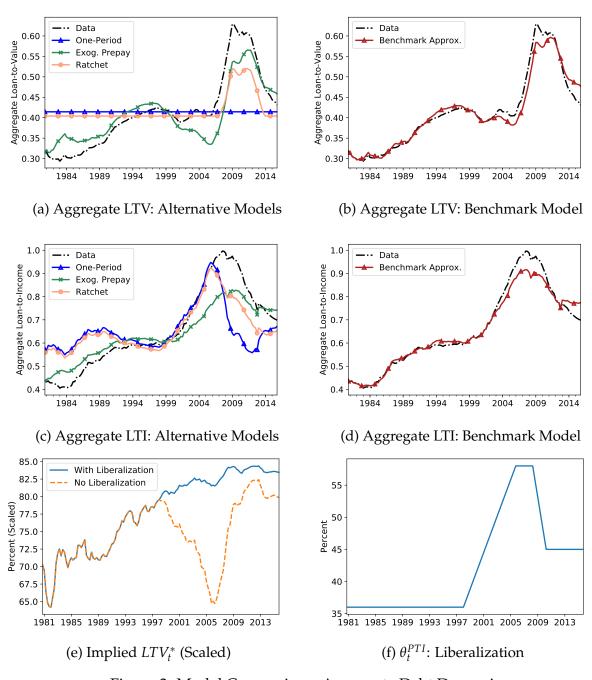


Figure 3: Model Comparison, Aggregate Debt Dynamics

Note: See Table A.1 in the Appendix for data sources. Aggregate Loan-to-Value and Aggregate Loan-to-Income are computed as the ratios of household mortgage debt to the value of household residential real estate and household disposable income, respectively. Panel (e) shows the scaled value  $LTV_{\tau}^*/0.747$ , which adjusts for the presence of outright owners for easier comparison with the baseline value  $\theta^{LTV}=0.747$ . The paths "One-Period" and "Ratchet" estimate  $\gamma=LTV^*$ , while "Benchmark Approx" estimates  $\gamma'=(\mu_{\kappa},s_{\kappa})$ , and all other specifications estimate  $\gamma=\bar{\rho}$ . The sample spans 1980 Q1 - 2015 Q4.

derstates debt accumulation during the boom, missing nearly half the rise in LTI ratios. Overall, this exercise shows that none of the existing models is able to match the path of aggregate leverage over the full sample.

**Benchmark Model.** To improve upon the performance of these specifications, a close fit of the data can be obtained by incorporating this paper's two main modeling innovations (PTI limits and endogenous prepayment) alongside its primary finding from the microdata (loose PTI limits during the boom).

As a first step, we can endogenize the debt limit to incorporate the PTI limit. To do this, I use the overall constraint (7) to compute  $LTV_t^*$ , using actual data on aggregate house values and pre-tax income, and average interest rates on new mortgages.<sup>37</sup> But, perhaps surprisingly, it turns out that uniformly imposing this debt limit throughout the sample would deliver a *worse* fit relative to the version with constant limit  $LTV^*$ .<sup>38</sup> The reason is simple: a uniform PTI limit would bind heavily during the housing boom. This would imply low values of  $LTV_t^*$ , shown in Figure 3e under the label "No Liberalization," that would have dramatically limited debt accumulation over this period.

This failure occurs because a constant PTI limit is at odds with the data presented in Figure 2, which instead show extremely loose PTI standards during the boom. To correct this, I impose a time-varying path for the maximum PTI ratio  $\theta_t^{PTI}$ , shown in Figure 3f, that is inspired by the observed distributions over this period.<sup>39</sup> This limit takes on the baseline value of 36% in the pre-boom era, then increases over the first years of the boom to 58%, before falling to 45% as PTI limits are restored following the bust.<sup>40</sup> Once this liberalization is included, PTI limits substantially improve the model's fit, allowing for heavy debt accumulation under loose PTI limits in the boom, restricting leverage in the early sample, when high interest rates induced tight PTI constraints.<sup>41</sup>.

 $<sup>\</sup>overline{\phantom{a}}^{37}$ As with house values, aggregate income must be scaled to adjust for outright owners and also for non-owning renters. Specifically, I scale the credit limit parameters, using the scaled values  $\tilde{\theta}^{LTV}=0.747\cdot \theta^{LTV}$  and  $\tilde{\theta}_t^{PTI}=0.555\cdot \theta_t^{PTI}$ . The scaling for LTV is identical to that of the Exog. Prepay specification, described above. For the PTI scaling, recall that the threshold income shock at which the PTI limit binds  $(\bar{e}_t)$  is proportional to the aggregate ratio of income to house value. This ratio is different in the overall and mortgagor populations, due to the presence of outright owners as well as renters who earn income but own no housing. The scaling for  $\theta_t^{PTI}$  corrects for this discrepancy, as 0.555 times the overall income-value ratio (0.81) is equal to the median income-value ratio for mortgagors in the 1998 SCF (0.45).

<sup>&</sup>lt;sup>38</sup>See path "Exog. Prepay + PTI" in Appendix Figures B.6a and B.6b.

<sup>&</sup>lt;sup>39</sup>See Appendix, Figures B.1 - B.3 for the timing of changes in PTI, and Section 6 for an explanation of the specific value (58%) applied in the boom.

 $<sup>^{40}</sup>$ More precisely, from the start of the boom in 1998 Q1, through 2004 Q4,  $\theta_t^{PTI}$  increases linearly from 36% to 58%. It remains there until 2008 Q2, then declines linearly to its final value of 45% in 2009 Q4.

<sup>&</sup>lt;sup>41</sup>See path "Exog. Prepay + PTI" in Appendix Figures B.6a and B.6b.

Finally, to move to the full benchmark model, we can endogenize the prepayment rate  $\rho_t$ . While imposing (12) directly would require a complex nonlinear filtering exercise, we can instead approximate the optimal  $\rho_t$  by replacing  $\Omega_{b,t}^m$  and  $\Omega_{b,t}^x$  with their steady state values from the model. Imposing this approximation and re-expressing some variables in terms of LTV instead of the debt level m yields

$$\rho_{t} = \Gamma_{\kappa} \left\{ \left( 1 - \Omega_{b,ss}^{m} - \Omega_{b,ss}^{x} q_{t-1} \right) \left( 1 - \frac{(1 - \nu)G_{t}^{-1}LTV_{t-1}}{LTV_{t}^{*}} \right) - \Omega_{b,ss}^{x} \left( q_{t}^{*} - q_{t-1} \right) \right\}. \tag{15}$$

Under this approximation,  $\rho_t$  can be directly computed given data on the average coupon rate on existing debt  $(q_{t-1})$ , the coupon rate on new debt  $(q_t^*)$ , and leverage  $(LTV_{t-1})$ , making it straightforward to estimate  $\gamma = (\mu_{\kappa}, s_{\kappa})'$  through nonlinear least squares. <sup>42</sup>

The resulting series, labeled "Benchmark Approx" in Figures 3b and 3d, provides a superior fit of the data, matching leverage in three widely different settings: the early 1980s, when rising interest rates created an unfavorable refinancing environment; the mid-2000s, when soaring house prices offered unprecedented opportunities to extract equity; and the post-bust period, when low levels of home equity encouraged borrower inaction. This close fit of the data throughout the sample, unmatched by existing models, is not due to one force alone, but depends on the full combination of PTI limits, their liberalization, and the endogenous prepayment option.<sup>43</sup> To ensure that the model inherits these realistic dynamics, the fitted values  $\mu_{\kappa} = 0.348$  and  $s_{\kappa} = 0.152$  are used in the baseline calibration, yielding a steady state annualized prepayment rate of 14.2%.<sup>44</sup>

It is worth noting that, while the "borrower" population as defined in the model makes up only a subset of all mortgagors — a distinction important for generating realistic consumption responses to debt issuance<sup>45</sup> — the prepayment sensitivity is calibrated to match the dynamics of *total* mortgage debt. Although excluding "non-borrower" mortgages from the model causes the level of mortgage debt to be too low (equal to 36.2% of

The data equivalent of q (payment per unit of face value) is obtained by dividing the household mortgage debt service ratio by ratio of disposable income to total mortgage debt. Since the terms  $\Omega_{b,ss}^m$  and  $\Omega_{b,ss}^x$  depend on the values of  $(\mu_{\kappa}, s_{\kappa})$ , I iteratively fit  $(\mu_{\kappa}, s_{\kappa})$  and re-solve the model to update  $(\Omega_{b,ss}^m, \Omega_{b,ss}^x)$ . This procedure converges rapidly to a fixed point.

<sup>&</sup>lt;sup>43</sup>Figure B.6 in the Appendix shows that removing any of these features from the Benchmark path would substantially compromise the fit.

<sup>&</sup>lt;sup>44</sup>The corresponding value for Fannie Mae 30-Year MBS (FNM30), which includes rate refinances that do not affect debt issuance and are therefore ignored in the computations above, is 17.8% over the sample Jan 1994 - Jan 2015. Source: eMBS.

<sup>&</sup>lt;sup>45</sup>Mortgagors with low liquid wealth should be much more likely to spend out of new borrowing than mortgagors with substantial liquid saving, following the theory of, e.g., Kaplan and Violante (2014).

annual pre-tax income in the model vs. 51.7% in the 1998 data), this calibration approach implies that the *proportional* response of total debt should be roughly correct. As a result, percent changes in debt from impulse responses and boom-bust experiments can be interpreted as paths for total debt, not only "borrower" debt.<sup>46</sup>

#### 4.3 Response to Identified Productivity Shocks

To check that the model generates reasonable dynamics, and does not exaggerate transmission into house prices, I compare the responses of macroeconomic variables to a TFP shock in the model and the data. I choose a TFP shock for this exercise for three reasons: (i) several data measures of these shocks exist and have been extensively studied (see, e.g., Ramey (2016)); (ii) it straightforward to implement analogous TFP shocks in both model and data; (iii) TFP shocks interact with the key distinguishing features of the PTI constraint by pushing nominal interest rates down (through their deflationary influence) while increasing labor income.

For the model version, I compute impulse responses from the linearized solution around the deterministic steady state. For the data version, I follow Ramey (2016) in applying the local projection method of Jordà (2005). Specifically, for each forecast horizon  $h \ge 0$ , and each variable of interest y, I run the regression

$$y_{t+h} = \beta_h + \beta_{1,h} \varepsilon_{a,t} + \beta'_{2,h} X_{t-1} + u_{t,t+h}$$
(16)

where the notation in (16) is unrelated to the model notation aside from the productivity shock  $\varepsilon_{a,t}$ . Controls  $X_{t-1}$  include the lagged variable  $y_{t-1}$ , two lags of the shock  $\varepsilon_{a,t-1}, \varepsilon_{a,t-2}$ , and additional variables chosen for each y variable as likely forecasters of  $y_{t+h}$  given time t-1 information. In this specification, the fitted coefficient  $\hat{\beta}_{1,h}$  represents the estimated response of the y variable to a 1% productivity shock h quarters after impact. For the data measure of  $\varepsilon_{a,t}$ , I use the technology shock series from Francis, Owyang, Roush, and DiCecio (2014). Further details, as well as similar results using differences in utilization-adjusted TFP from Fernald (2014), can be found in Appendix A.5.

Figure 4 displays model and data impulse responses for six macroeconomic variables, along with their 90% confidence bands. Overall, despite the model's relative parsimony, the model and data responses match up well, generating paths in the same direction and

<sup>&</sup>lt;sup>46</sup>This approach also conservatively assumes that the model "borrowers" prepay their loans at the same rate as the overall population — assuming that liquidity-constrained borrowers extract equity at a higher rate than other mortgagors would generate larger spending responses to credit issuance.

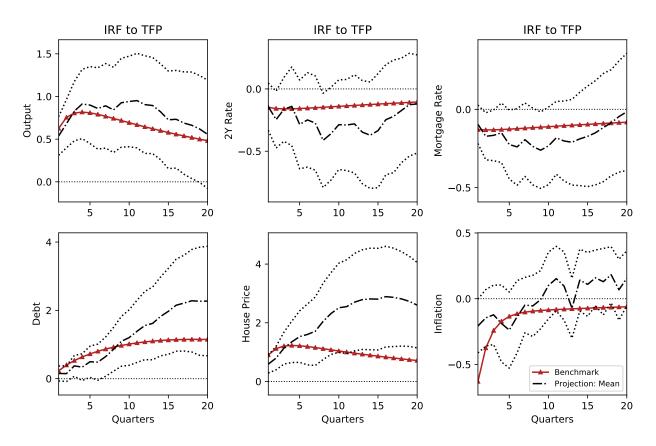


Figure 4: Response to 1% Productivity Shock, Model vs. Data Projections

*Note:* A value of 1 represents a 1% increase relative to the initial value (data) or steady state (model), except for 2Y Rate and Mort. Rate, which are measured in percentage points. The full data definitions, sources, and lists of controls can be found in the Appendix. The 2Y rate in the model is computed as the implied yield on a geometrically decaying nominal perpetuity with average duration of 8Q. Standard errors for each horizon h are corrected for serial correlation due to overlapping data using the Newey-West procedure with h lags.

of similar magnitudes for all variables. The main point of difference is that the model has no mechanism capable of generating the sluggish house price adjustment observed in the data. But reassuringly, the model does not appear to overstate the strength of the transmission mechanism. If anything, the responses of debt and house prices appear larger in the data than in the model, despite similar or smaller movements in output and interest rates. These results therefore imply that the simplifying assumptions fixing saver housing demand and the size of the housing stock do not appear to be inflating house price responses relative to the data.

#### **5** Results: Interest Rate Transmission

This section illustrates how the novel features of the model amplify transmission from nominal interest rates into debt, house prices, and economic activity, and demonstrates the implications for monetary policy. These quantitative results are obtained by linearizing the model around the deterministic steady state and computing impulse responses to the model's fundamental shocks  $(\varepsilon_{\bar{\pi}}, \varepsilon_{q}, \varepsilon_{q})$ .

## 5.1 The Constraint Switching Effect

For the first main result, I find that the addition of the PTI constraint alongside the LTV constraint generates powerful transmission from interest rates into debt and house prices. To isolate the effects of the credit limit structure, I compare the model as described to this point — hereafter the *Benchmark economy* — with two alternatives: the *PTI economy* which imposes only the PTI constraint  $\bar{m}_t = \bar{m}_t^{PTI}$ , and the *LTV economy* which imposes only the LTV constraint  $\bar{m}_t = \bar{m}_t^{LTV}$ . These economies are otherwise identical in their specification and parameter values, with the exception that the credit limit parameters  $\theta^{LTV}$  and  $\theta^{PTI}$  are recalibrated in the PTI and LTV economies so that their steady state debt limits match those of the Benchmark economy.<sup>47</sup>

To demonstrate how this channel can work through movements in nominal rates only, Figure 5 displays the response to a near-permanent -1% (annualized) shock to the inflation target. This shock induces a near 1% fall in nominal mortgage rates while causing a slight rise in real rates. The first panel shows that the three economies differ widely in their debt responses to the shock. To begin, the PTI economy displays a much larger increase of debt than the LTV economy, with 2.5 times the increase after 20Q (8.08% vs. 3.19%). This occurs because PTI limits are strongly affected by interest rates, which directly shift PTI constraints with an elasticity near 8, potently increasing the size of new loans in the PTI economy. In contrast, debt limits in the LTV economy are only indirectly affected by interest rates through house prices, and remain largely unchanged. As a result, the LTV economy's modest debt response is driven by a combination of lower inflation and an increase in the share of borrowers prepaying to lock in lower fixed rates on their mortgages, rather than by an increase in loan size.

Turning to the Benchmark economy, we observe a substantial increase in debt (5.94% after 20Q) that, perhaps surprisingly, is closer to that of the PTI economy than that of the

 $<sup>\</sup>overline{^{47}}$ The required values are  $\theta^{LTV}=0.731$  and  $\theta^{PTI}=0.272$ , respectively.

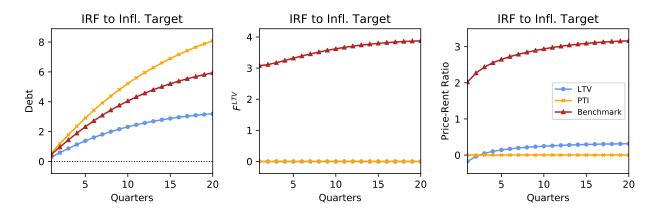


Figure 5: Response to -1% (Ann.) Inflation Target Shock

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is reported in real terms. The price-to-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. The responses of additional variables can be found in the Appendix, Figure B.13.

LTV economy. This occurs despite the fact that in the model, the majority of borrowers are constrained by LTV at the moment of origination (74% at steady state), consistent with the pattern observed the data (e.g., Figure 2). This makes clear that the Benchmark economy is not simply a convex combination of the LTV and PTI economies, but displays qualitatively different behavior due to the constraint switching effect. As PTI limits loosen in the Benchmark economy, many borrowers formerly constrained by PTI now find LTV to be more restrictive, driving  $F^{LTV}$  up by more than three percentage points. These borrowers can now increase their borrowing limit with additional housing collateral, boosting housing demand. As a result, the implied price-to-rent ratio, defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , rises up to 3% in the Benchmark economy, compared to a small or zero change in the LTV and PTI economies.<sup>48</sup>

The constraint switching effect not only provides a novel transmission mechanism into house prices, but is also key to the Benchmark economy's amplified debt response. While debt limits are directly increased for PTI-constrained households, there are too few of these households to generate the observed impact from this response alone. But because higher house prices increase collateral values, LTV constraints are relaxed to a much greater extent in the Benchmark economy than in the LTV economy. It is in fact this strong debt response of the LTV-constrained households — the majority of the borrower

<sup>&</sup>lt;sup>48</sup>The slight rise in the price-to-rent ratio in the LTV economy is due to the "tilt" effect noted by e.g., Lessard and Modigliani (1975). Lower inflation implies a more backloaded real payment schedule for a mortgage with fixed nominal payments. This benefits impatient borrowers who prefer to postpone repayment, increasing the collateral value of housing through  $\mu_t$ .

population — that causes the LTV and Benchmark economy paths to diverge so widely. <sup>49</sup> The interaction of the two constraints therefore creates a transmission chain from interest rates, through PTI limits, into house prices, and finally into LTV limits.

This analysis can be generalized to an arbitrary set of shocks. Since the constraint switching effect operates through movements in  $F_t^{LTV}$ , the influence of the constraint structure (i.e., moving from the LTV economy to the Benchmark economy) depends on the relative responses of the credit limits  $\bar{m}_t^{PTI}$  and  $\bar{m}_t^{LTV}$ . For shocks that, all else equal, would shift PTI limits without a strong direct effect on house prices, such as the inflation target shock, we will see house prices and debt move much more in the Benchmark economy than in the LTV economy. Next, shocks that would move  $\bar{m}_t^{PTI}$  and  $\bar{m}_t^{LTV}$  in parallel will induce more similar responses across the Benchmark and LTV economies. The term premium shock, which directly influences both both PTI limits (by moving interest rates) and house prices (by changing the real cost of borrowing), falls in this category, but still delivers stronger responses in the Benchmark economy for  $\rho_q$  not too close to unity. Finally, shocks that impact housing markets without directly affecting  $\bar{m}^{PTI}$  — such as a shock to expected housing utility — will be dampened in the Benchmark economy, as  $F_t^{LTV}$  moves against the initial impulse to house prices.

### 5.2 The Frontloading Effect

While the interaction of LTV and PTI limits is sufficient to generate transmission from interest rates into debt and house prices, it turns out that endogenous prepayment by borrowers is crucial for transmission into output. In this class of New Keynesian model, an increase in borrowing and consumer spending can increase output, but only if it occurs in the short run, before most intermediate firms have an opportunity to reset their prices.<sup>52</sup> But although a fall in interest rates raises debt limits immediately, under long-

 $<sup>\</sup>overline{\ }^{49}$ Figure B.7 in the Appendix shows a counterfactual impulse response that shuts down the constraint switching effect by holding  $F^{LTV}$  fixed. In this case, the debt and price-to-rent responses of the Benchmark economy are smaller, and close to that of the LTV economy.

<sup>&</sup>lt;sup>50</sup>A full set of impulse responses to term premium and productivity shocks in the Benchmark, LTV, and PTI economies can be found in the Appendix, Figures B.14 and B.15.

 $<sup>^{51}</sup>$ Quantitatively, term premium shocks for  $\rho_q$  close to unity move house prices and interest rates by similar magnitudes and therefore display closely matching responses across the Benchmark and LTV economies. However, a less persistent  $\Delta_{q,t}$  process still delivers substantial amplification in the Benchmark economy, since expected reversion to the mean weakens the initial impact on house prices, allowing the rise in  $\bar{m}_t^{PTI}$  to outpace that of  $\bar{m}_t^{LTV}$ .

<sup>&</sup>lt;sup>52</sup>While nominal rigidities are important for transmission into output, the results on transmission into house prices and debt in Section 5.1 and in the boom-bust experiments of Section 6 would be similar in a flexible price model (see Figure B.8 in the Appendix).

term mortgages this will not translate into an increase in debt balances or spending until borrowers prepay their existing loans and take on new ones.

If borrowers always prepaid at the average rate — 3.8% of loans per quarter — most new credit issuance and spending would occur too far in the future to influence output. But when borrowers can choose when to prepay, a fall in rates can induce a wave of new debt issuance, as many borrowers choose to both lock in lower fixed rates and make use of their newly higher debt limits, which have been raised due to the mechanisms of the previous section.

This immediate increase in credit growth leads to a large increase in spending on impact, amplifying the economy's output response, a phenomenon that I call the *frontloading effect*. To see this mechanism in action, we can once again compare alternative economies, this time contrasting the Benchmark economy, where prepayment rates are endogenously determined by (12), with "exogenous prepayment" versions of the Benchmark and LTV economies, where  $\rho_t$  is fixed to equal its steady state value  $\rho_{ss}$  at all times.

To demonstrate how the frontloading effect can amplify shocks at business cycle frequencies, Figure 6 shows the response to a -1% term premium shock. This induces a decline in the the real mortgage rate that is close to 1% on impact, before gradually decaying. Due to the constraint switching effect, this fall in rates generates much larger increases in debt limits in both versions of the Benchmark economy relative to the LTV economy. But despite a similar rise in debt limits, the paths of credit issuance across the variations of the Benchmark economy are sharply different. The endogenous prepayment version delivers a much more frontloaded path of issuance that begins far above, and eventually falls below, the smaller but more persistent issuance of the exogenous prepayment variety.

This pattern leads to highly disparate effects on output, whose response is more than three times larger on impact in the endogenous prepayment Benchmark economy (0.50%) relative to its exogenous prepayment counterpart (0.14%), which is instead close to that of the exogenous prepayment LTV economy (0.06%). Overall, these results suggest that borrower prepayment is of primary importance for the transmission from long-term interest rates into output.<sup>53</sup>

A natural question in light of this finding is whether it is the reduction in interest payments or the issuance of new credit that causes prepayment to influence demand so strongly. Despite potentially large redistributions between borrowers and savers as interest rates change following prepayment, and an extreme difference in marginal propen-

<sup>&</sup>lt;sup>53</sup>These findings complement those of Wong (2015), who obtains a similar result in a partial equilibrium heterogeneous agent setting.

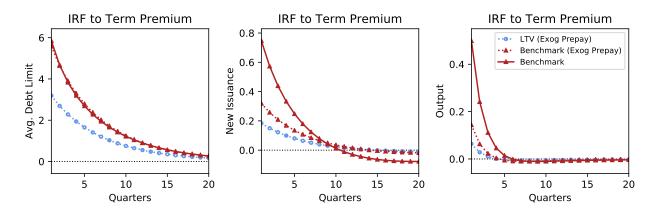


Figure 6: Response to 1% Term Premium Shock

*Note:* A value of 1 represents a 1% increase relative to steady state, except for New Issuance,  $\rho_t(m_t^* - (1 - \nu)\pi_t^{-1}m_{t-1})$ , which is measured as a percentage of steady state output (both quarterly). All variables are reported in real terms. The responses of additional variables can be found in the Appendix, Figure B.16.

sities to consume between the two types, it turns out that the change in payments contributes almost nothing to the output response, which is instead driven entirely by credit growth.<sup>54</sup> The cause is a variation on the frontloading effect: while borrowers' interest savings may be large in present value, most of the lower payments occur far in the future, where they have little influence on output.<sup>55</sup> In contrast, newly issued credit can be spent immediately upon receipt, with much larger stimulatory effects.

# 5.3 Monetary Policy

These results on interest rate transmission have important implications for monetary policy. Regarding unconventional monetary policy, the findings above show directly how the mortgage credit channel can produce strong macroeconomic responses to changes in mortgage rates. This channel therefore provides theoretical backbone for one important pathway — mortgage issuance — through which policies targeting long rates, such as Quantitative Easing, can act. Moreover, the results above connect to recent proposals — such as in Blanco (2015) — to raise the inflation target in order to provide policymak-

<sup>&</sup>lt;sup>54</sup>Figure B.9 shows that a counterfactual impulse response removing the effect of prepayment on interest rates delivers identical output responses.

<sup>&</sup>lt;sup>55</sup>When borrowers are expected to keep their loans for many years before prepaying — such as when they have locked in extremely low interest rates, or when mortgages have been specially modified under the Home Affordable Refinance Program — there is an additional dampening effect as the change in payments is close to a permanent income shock, inducing a large offsetting consumption response by the saver.

<sup>&</sup>lt;sup>56</sup>This pathway through mortgage issuance complements others previously considered in the literature, such as through financial intermediaries in e.g., Gertler and Karadi (2011).

ers with more room to cut rates before reaching the zero lower bound. Specifically, the responses in Figure 5 indicate that one important consequence of such a policy could a substantial contraction in house values and mortgage credit.

Turning now to conventional monetary policy, I find that stabilizing inflation is easier due to the mortgage credit channel, but contributes to larger swings in credit markets, posing a potential trade-off for policymakers. To demonstrate this, I present results using the alternative policy rule (11), under which the central bank moves the policy rate as much as needed to perfectly stabilize inflation, which in this simple framework also stabilizes output (the "divine coincidence"). While not as empirically realistic as (10), this rule provides a natural benchmark for evaluating the strength of the monetary authority: the less the policy rate must move to keep inflation at target following a shock, the more effective is monetary policy.

Figure 7 compares the response to a 1% productivity shock under the Benchmark economy, and a "control" economy — the exogenous prepayment LTV economy — to demonstrate the combined contribution of the model's novel features. This shock is deflationary and persistent, so the central bank in both economies must persistently cut rates to return inflation to target. However, the initial required fall in the policy rate is more than 25% larger in the control economy relative to the Benchmark (132bp vs. 105bp). In the Benchmark case, as long rates fall due to expectations of low future short rates, a wave of new borrowing takes place. The increase in demand as newly borrowed funds are spent puts upward pressure on prices, thus requiring less monetary stimulus to correct the deflationary shock relative to the control economy.

Overall, these results indicate that monetary policy is stronger due to the mortgage credit channel, requiring smaller movements in the policy rate to stabilize inflation. But importantly, these smaller changes in the policy rate are associated with *larger* shifts in mortgage issuance, with debt rising by over 66% more in the Benchmark economy (0.70% vs. 0.42%) after 20Q. If policymakers are concerned with the stability of credit growth as well as inflation, these dynamics may present a difficult dilemma.

For an important example, consider the position of the Federal Reserve in the early 2000s, which chose to cut rates during a massive expansion of mortgage credit. Taylor (2007) has blamed this decision for the ensuing housing boom and bust, while Bernanke (2010) has argued that this action was appropriate given deflationary concerns. The preceding analysis suggests that this debate may be impossible to fully resolve, as there may have been no way to use interest rates to stabilize inflation without further contributing

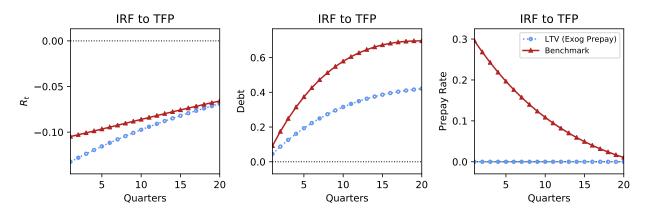


Figure 7: Response to 1% Productivity Shock, Full Inflation Stabilization

*Note:* A value of 1 represents a 1% increase relative to steady state, except for Prepay Rate ( $\rho_t$ ), which is measured in percentage points (annualized). Debt ( $m_t$ ) is measured in real terms. The interest rate  $R_t$  is annualized. The responses of additional variables can be found in the Appendix, Figure B.17.

to the credit boom. These results therefore provide a potential rationale for imperfect inflation stabilization, or for the use of instruments other than monetary policy to influence credit markets.

### 6 Results: Credit Standards and the Boom

The analysis until this point has focused on model dynamics under a single credit regime, with  $\theta^{LTV}$  and  $\theta^{PTI}$  fixed, as these maximum ratios are typically stable at business cycle frequencies. But credit standards can change over time, and did so dramatically during the recent boom-bust episode, as evidenced in Section 2. To better understand the role of credit changes in driving this cycle, and the type of policy that might have limited its severity, I present several experiments varying credit conditions. In particular, I compute three sets of responses: to changes in credit parameters alone, to a broader set of shocks that can collectively explain the entire boom, and to these same shocks under alternative macroprudential policies.

To simulate each hypothetical boom-bust cycle, I trace out nonlinear transition paths in a deterministic version of the Benchmark economy, applying the "L-B-J" solution technique described in Juillard, Laxton, McAdam, and Pioro (1998). The transition begins from steady state with a surprise announcement that certain parameters — e.g.,  $\theta^{LTV}$  or  $\theta^{PTI}$  — have changed permanently, followed later by a second surprise announcement that credit parameters have permanently reverted to their baseline values. For each

experiment, I report the resulting rise in price-to-rent ratios  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$  and loan-to-disposable-income (LTI) ratios  $m_t/(1-\tau_y)y_t$  over the model boom period, compared to their peak increases in the data (60% and 67%, respectively). For timing, I assume that the first announcement arrives in 1998 Q1 (the start of the sustained rise in price-to-rent ratios) and that the time gap between the announcements is 36Q. This choice implies a boom through 2006 Q4, selected as a compromise between the peaks of price-to-rent ratios (2006 Q1) and LTI ratios (2007 Q3), respectively. The results of these experiments are reported in Table 2, and are further analyzed below.

Before proceeding, note that while I treat changes in these parameters as exogenous, shifts in credit standards were surely influenced by prevailing economic conditions and expectations. Since lenders only take losses in default when the property is not valuable enough to recover the principal balance, beliefs that house prices will continue to increase at a rapid pace can rationally induce a relaxation of debt limits. While analyzing this endogenous formation of credit standards is an important topic for future research, the exogenous credit liberalizations considered below are the correct ones to address two critical policy questions: could restrictions on credit standards preventing them from loosening have dampened the boom-bust cycle, and if so, which standards should be targeted?

**Credit Liberalization Experiments.** For the first set of experiments, I present the responses to changes in the credit standard parameters in Figure 8. To begin, the LTV Liberalized experiment increases  $\theta^{LTV}$  from 0.85 to 0.99, followed by a reversal. While the exact amount by which LTV limits were relaxed over this period is unclear, this near-complete relaxation is designed to give LTV liberalization the best possible chance to make a quantitatively important contribution to the boom. Although a liberalization of LTV standards is often proposed as a candidate cause of the boom, the responses, labeled "LTV Liberalized" fail to generate a large boom when PTI limits are held at their baseline values. Instead, we observe only a small rise in debt, while price-to-rent ratios actually fall. This result is entirely due to the presence of the PTI limit, as a similar liberalization in the LTV economy would indeed produce a large increase in prices relative to rents. <sup>57</sup>

The presence of PTI limits dampens the response to LTV liberalization for two reasons. First, there is a direct effect, since PTI-constrained borrowers cannot increase their credit balances in response to this change. But, more importantly, there is a general equilibrium response due to the constraint switching effect. As LTV limits loosen, many previously

<sup>&</sup>lt;sup>57</sup>See Figure B.10 in the Appendix.

Table 2: Results: Boom Experiments

| Experiment  | Price-Rent | (Of Actual) | LTI | (Of Actual) |  |  |  |
|---|------------|-------------|-----|-------------|--|--|--|
| Data  | 60%        |             | 67% |             |  |  |  |
| Figure 8: Credit Liberalization Experiments       |            |             |     |             |  |  |  |
| LTV Liberalized                                   | -1%        | (-2%)       | 10% | (15%)       |  |  |  |
| PTI Liberalized                                   | 21%        | (35%)       | 22% | (33%)       |  |  |  |
| Both Liberalized                                  | 29%        | (48%)       | 47% | (69%)       |  |  |  |
| Figure 9: Decomposing the Boom                    |            |             |     |             |  |  |  |
| PTI Lib. + Low Rates                              | 35%        | (58%)       | 42% | (62%)       |  |  |  |
| Complete Boom                                     | 60%        | (100%)      | 67% | (100%)      |  |  |  |
| Figure 10: Macroprudential Policy Counterfactuals |            |             |     |             |  |  |  |
| No PTI  | 25%        | (42%)       | 29% | (43%)       |  |  |  |
| Dodd-Frank  | 39%        | (65%)       | 44% | (65%)       |  |  |  |
| Additional Experiments (Not Shown)                |            |             |     |             |  |  |  |
| Low Rates Only (LTV Economy)                      | 7%         | (12%)       | 11% | (17%)       |  |  |  |
| Low Rates Only                                    | 12%        | (20%)       | 17% | (26%)       |  |  |  |
| Complete Boom, Exog. Prepay                       | 57%        | (95%)       | 55% | (82%)       |  |  |  |

*Note:* Table corresponds to the paths in Figures 8, 9, and 10. For each experiment, "Price-Rent" and "LTI" (loan-to-disposable-income) columns denote the rise from the start of the experiment to the peak of the boom, 36Q later, for price-to-rent and debt-disposable income ratios, respectively. The columns "(Of Actual)" denote the fraction of the observed increase in each variable from 1998 Q1 to its peak (2006 Q1 for price-to-rent, 2007 Q3 for LTI) explained by this experiment.

LTV-constrained borrowers now find their PTI limits to be more restrictive. The resulting fall in  $F_t^{LTV}$  of 14 percentage points depresses collateral demand and price-to-rent ratios. The failure of house prices to boom in turn limits the ability of LTV-constrained house-holds to borrow, dampening the increase in debt.

Next, the PTI Liberalized experiment computes the response to an increase in  $\theta^{PTI}$  from 36% to 58%, chosen to approximate the 90th percentile of the PTI distribution during the boom (see Figure B.3 in the Appendix) — a conservative calibration in practice since fewer than 10% of model borrowers are constrained by PTI during the key boom experiments below. Returning to Figure 8, we observe that this PTI liberalization generates a much larger boom than its LTV counterpart, explaining more than one third of the

<sup>&</sup>lt;sup>58</sup>The relaxation of PTI was likely further exacerbated by the rise of exotic mortgage products and low-documentation loans — products that are excluded from the Fannie Mae data in Figure 2. Adjustable-rate and low-amortization/interest-only mortgages offered lower initial payments during the boom, while low-documentation loans allowed borrowers to inflate their stated income, in both cases lowering the effective PTI ratios on a given loan.

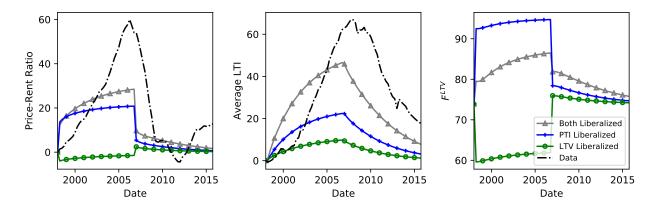


Figure 8: Credit Liberalization Experiments

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. The price-to-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . See Figure B.18 for the responses of additional variables.

observed rise in both price-to-rent and LTI ratios. While these results clearly leave room for other factors, they point to an important role for changing PTI standards in propelling the boom-bust cycle.

That the PTI-driven boom vastly exceeds the LTV-driven boom, despite the fact that only a minority of borrowers are PTI-constrained, is once again due to the constraint switching effect. As PTI limits have loosened, more borrowers find themselves constrained by LTV, pushing up the demand for collateral, which in turn drives up house prices and relaxes debt limits for the LTV-constrained majority. Importantly, this pathway provides a new perspective on recent empirical research showing that debt increased evenly across the income spectrum during the boom, and that credit growth was closely linked to increases in house values.<sup>59</sup> While this simulated boom is *initiated* by the relaxation of income-based constraints, new borrowing in the experiment is largely undertaken by LTV-constrained households responding to the rise in house prices, consistent with these empirical findings.<sup>60</sup>

While the above results consider each liberalization in isolation, we can also investigate whether a relaxation of LTV limits fits the data well once PTI limits have already been loosened. To this end, the series "Both Liberalized" shows the results of simultaneously relaxing  $(\theta^{LTV}, \theta^{PTI})$  from (0.85, 0.36) to (0.99, 0.58). The simultaneous liberalization of

<sup>&</sup>lt;sup>59</sup>See, e.g., Adelino et al. (2015) and Foote et al. (2016).

<sup>&</sup>lt;sup>60</sup>It is also worth noting that high income households can nonetheless become PTI constrained if they buy a sufficiently expensive house.

PTI does indeed boost the impact of the LTV liberalization, allowing for a positive net impact on price-to-rent ratios, and a much larger net increase in aggregate LTI. However, the constraint switching effect still ensures that the accumulation of debt under an LTV liberalization is vastly larger than the rise in price-to-rent ratios — a pattern inconsistent with the data, where the two ratios rose essentially in parallel. This result, useful for the decomposition exercise below, implies that a relaxation of LTV limits played a limited role in explaining the remainder of the boom.

**Decomposing the Boom.** The results above imply that a complete explanation of the boom requires looking to alternative forces beyond credit standards. A natural starting point is the observed decline in mortgage rates, with 30-year fixed mortgage rates falling from an average of 7.81% over the years 1993-2007 to an average of 6.06% for the period 2003-2007. To accommodate this phenomenon, at the start of the boom period I impose a permanent fall in average inflation ( $\pi_{ss}$ ) of 0.82% (annualized) to match the drop in average 10-year inflation expectations from 1993-1997 to 2003-2007, as well as a permanent fall in the average term premium of 1.09% (annualized) to match an interest rate of 6.06% over the final five years of the boom era. The resulting paths, labeled "PTI Lib + Low Rates" in Figure 9, show that the fall in rates was indeed quantitatively important, explaining an additional 23% of the observed rise in price-to-rent ratios and 29% of the observed rise in LTI ratios, while capturing a majority of the boom in combination with loosened PTI limits.

That interest rates have such a large effect is due the presence, and liberalization, of the PTI constraint. Specifically, these increases are more than 2.5 times larger than would be observed after an identical drop in interest rates, in isolation, applied to the LTV economy (see "Additional Experiments" in Table 2). This occurs for two reasons. First, due to the constraint switching effect, the response to a fall in rates in isolation would already be stronger in the Benchmark economy relative to the LTV economy. Second, because collateral value  $C_t$  varies with the product of  $F_t^{LTV}$  and the multiplier  $\mu_t$ , the impact of a fall in the real cost of borrowing on  $\mu_t$  is further amplified when  $F_t^{LTV}$  has already been

 $<sup>^{61}</sup>$ The choice of a permanent shift is motivated by the fact that mortgage rates have not returned to their previous levels, instead falling even lower since the bust. Explaining the entire fall in rates using movements in term premia (real rates) instead of inflation expectations would strengthen the responses further. For consistency, I choose the size of the change in  $\mu_q$  to match the fall in rates in the "Complete Boom" experiment below, which better explains how much the observed fall in rates contributed to the boom.

<sup>&</sup>lt;sup>62</sup>This amplification is mostly due to the change in average inflation, similar to an inflation target shock, while permanent changes in term premia have similar effects in the two economies.

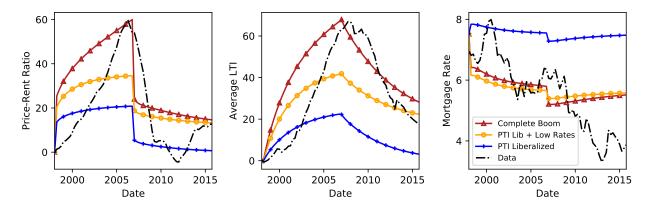


Figure 9: Decomposing the Boom

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. The price-to-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . For the "Complete Boom" path, in addition to the changes in parameters, agents learn at time 0 (1997 Q4) that in 36Q, the housing preference parameter  $\xi$  will increase from 0.250 to 0.312. After 36Q, however, the agents are surprised to learn that the parameter will instead remain at its initial value. See Figure B.19 for the responses of additional variables.

raised by the liberalization of PTI limits.

To account for the remainder of the boom, I impose two additional shocks. First, I incorporate an increase in expected house price expectations, emphasized as important by, e.g., Kaplan et al. (2017). Specifically, I impose that agents learn in 1998 Q1 that after 36Q, the housing preference parameter  $\xi$  will increase to a higher value  $\xi^H$ . After 36Q, however, the agents are surprised to learn that the parameter will instead remain at its initial value. For the second shock, I add a small liberalization of LTV limits.

The exact mixture of these two shocks to hit both the price-to-rent and loan-income targets is pinned down by the fact that the house price expectations shock moves house prices more than debt, while relaxing the LTV limit increases debt much more than house prices. The resulting fit implies an expected increase in  $\xi$  from 0.250 to 0.312, which explains most of the remaining boom (bringing the totals to 97% and 89% of observed price-to-rent and LTI increases, respectively),<sup>63</sup> while a modest increase in  $\theta^{LTV}$  from 85% to 89.1% captures the residual.

Overall, this exercise characterizes a realistic boom that is not dominated by a single cause, but where credit liberalization, interest rates, and expected appreciation all play important roles. The model's main shortcomings relative to the data are a lack of sluggishness in the response of house prices in the boom (similar to the local projection

<sup>&</sup>lt;sup>63</sup>These numbers follow from an experiment (not shown) that applies PTI liberalization, low rates, and optimistic house price expectations, but does not relax LTV limits.

results of Section 4.3), and a less severe house price crash, likely driven in reality by housing market and financial frictions that lie beyond the scope of this paper. However, the model does predict a return to higher price-to-rent ratios in the recovery due to a combination of lower interest rates and looser PTI limits.<sup>64</sup> Finally, endogenous prepayment plays an important role in the background, explaining an additional 18% of debt accumulation relative to an identical set of shocks applied under exogenous prepayment, in addition to capturing the asymmetry between the rapid rise of debt in the boom and the slow deleveraging in the bust.<sup>65</sup>

Macroprudential Policy Counterfactuals. This experiment fully accounting for the boom is also useful as a laboratory for evaluating the effects of macroprudential policies, whose effects are shown in Figure 10. First, the path labeled "No PTI Lib" plots the response to the all the shocks applied in the Complete Boom experiment except for the PTI liberalization. Notably, while relaxing PTI limits in isolation was able to generate at most 35% of the boom, *removing* PTI liberalization from the full set of shocks reduces the size of the boom by at least 57% for both ratios, implying that the net effect of the remaining forces is more than 1.5 times larger with PTI liberalization than without it. This is largely due to a sharp reduction in the influence of the house price expectations shock, once again due to the constraint switching effect. Since the expected increase in housing utility increases house prices today, it endogenously relaxes borrowers' LTV constraints. Just as in the exogenous LTV liberalization case, this force puts downward pressure on collateral demand in the presence of a tight PTI limit, dampening the resulting boom.<sup>66</sup>

These results yield implications for macroprudential regulation. As noted by Jácome and Mitra (2015), while caps on both LTV and PTI limits are common regulatory measures around the world, these is little theoretical guidance indicating how each limit should be used. To this end, the experiments above clearly indicate that a cap on PTI limits is the more effective tool for limiting the size of boom-bust cycles. Specifically, restrictions on PTI limits can both prevent booms driven by lenders' relaxation of those very limits, as well as seriously dampen the influence of additional forces that would otherwise boost

<sup>&</sup>lt;sup>64</sup>Price-to-rent ratios may rise higher still if the post crash interest environment of extremely low interest rates persists. If the observed fall in rates is permanent, the model predicts that price-to-rent ratios should plateau 29% above their pre-boom levels. See Appendix, Figure B.11 for more details.

<sup>&</sup>lt;sup>65</sup>While the accumulation of debt is too rapid in the model, this is a symptom of an excessively fast rise in house prices. In both model and data, house prices and debt move nearly in tandem during the boom, while debt declines more slowly than house prices in the bust.

<sup>&</sup>lt;sup>66</sup>To isolate this effect, Figure B.12 in the Appendix shows that this house price expectations shock in isolation has a vastly larger impact applied to the LTV economy relative to the Benchmark economy.

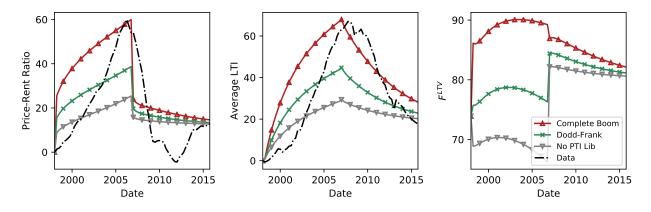


Figure 10: Macroprudential Policy Counterfactuals

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. The price-to-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . For each path, in addition to the changes in parameters, agents learn at time 0 (1997 Q4) that in 36Q, the housing preference parameter  $\xi$  will increase from 0.250 to 0.312. After 36Q, however, the agents are surprised to learn that the parameter will instead remain at its initial value. See Figure B.20 for the responses of additional variables.

house prices.<sup>67</sup> In contrast, restricting LTV limits is much less effective at limiting credit growth when house values are rising, and in some cases may even put further upward pressure on prices.

Of particular policy relevance is the Dodd-Frank legislation, which for the first time imposed a regulatory cap of 43% on PTI ratios for US mortgages, set to apply to nearly all loans by 2020. While this limit was framed as a microprudential tool to combat predatory lending, the results above indicate that it could also have important macroprudential consequences. To evaluate these, the path labeled "Dodd-Frank" applies the full set of shocks, but allows  $\theta^{PTI}$  to rise only to this 43% limit. Despite still allowing for a partial PTI liberalization, the resulting boom would have been more than one-third smaller had this regulation been active at the time. Since lenders' PTI standards now appear to be at or above the 43% limit, there should be much less room for PTI ratios to rise going forward relative to the experiment in Figure 10. As a result, this regulation is likely to be even more effective at dampening future boom-bust cycles if it remains in effect.

<sup>&</sup>lt;sup>67</sup>While I focus on PTI limits because they are a standard part of US underwriting, alternative limits that do not co-move positively with house prices, such as caps on LTI ratios, would inherit these same macroprudential benefits through the constraint switching effect.

### 7 Conclusion

In this paper, I developed a general equilibrium framework centered on two novel features: the combination of LTV and PTI limits, and the endogenous prepayment of long-term debt. When calibrated to US mortgage data, these features greatly amplify transmission from interest rates into debt, house prices, and economic activity. The effects on credit and house prices occur largely by the constraint switching effect, through which changes in which of the two constraints is binding for borrowers translate into movements in house prices. The effects on economic activity are due mainly to the frontloading effect, through which the prepayment decisions of borrowers generate waves of new borrowing and spending. This transmission channel implies that monetary policy can more potently stabilize inflation, but contributes to larger movements in credit growth. Finally, I found that a PTI liberalization appears essential to explaining the boom-bust, both through its direct contribution and through its amplification of other forces, and that restricting PTI ratios rather than LTV ratios is the more effective macroprudential policy.

Looking ahead, the macro-housing literature has now produced a number of well-crafted frameworks that, nonetheless, deliver starkly different explanations for the housing boom due to differences in modeling assumptions. For example, the house price effects driven by a relaxation of PTI limits found in this paper — in which all house-holds are effectively owners — would be completely ruled out under the perfect rental market/deep-pocketed landlord assumptions of Kaplan et al. (2017). Similarly, the important role for improved risk sharing in driving house prices in Favilukis et al. (2017), which assumes frictionless extraction of equity each period in a stochastic setting, are precluded in this paper due to my aggregation approach and deterministic transition experiments. Clearly rental markets are neither perfect nor completely absent. Similarly, the ability to use home equity to smooth consumption in the face of income risk is neither frictionless nor completely unvalued. Further work to understand *quantitatively* where on these spectra the true economy lies, and for which research questions each set of assumptions is appropriate, is a crucial step toward the unification of these competing approaches.

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# A Appendix

The appendix is structured as follows. Section A.1 completes the derivation of the equilibrium conditions for the model. Section A.2 demonstrates the aggregation result. Section A.3 formalizes the simple example of Section 2.1. Section A.4 describes the data used in the calibration and plots. Section A.5 describes the variables and controls used for the local projections, and displays results using an alternative set of TFP shocks. Section A.6 presents extensions of the baseline model. Supplementary tables and figures can be found in Appendix B.

#### A.1 Model Solution

This section supplements Section 3.1 by providing the set of optimality conditions for the model.

#### A.1.1 Borrower Optimality

Optimality of labor supply,  $n_{b,t}$ , implies the intratemporal condition

$$-\frac{u_{b,t}^n}{u_{b,t}^c} = (1 - \tau_y)w_t + \mu_t \rho_t \left(\frac{(\theta^{PTI} - \omega)w_t}{q_t^* + \alpha}\right) \int_{-\epsilon_t}^{\bar{e}_t} e_i d\Gamma_e(e_i). \tag{17}$$

where the second term on the right accounts for the borrower's incentive to relax the PTI constraint by working more.<sup>68</sup> Optimality of new debt,  $m_{i,t}^*$ , requires

$$1 = \Omega_{h\,t}^{m} + \Omega_{h\,t}^{x} q_{t}^{*} + \mu_{t} \tag{18}$$

where  $\mu_t$  is the multiplier on the borrower's aggregate credit limit, and  $\Omega_{b,t}^m$  and  $\Omega_{b,t}^x$  are the marginal continuation costs to the borrower of taking on an additional dollar of face value debt, and of promising an additional dollar of initial payments, defined by

$$\Omega_{b,t}^{m} = \mathbb{E}_{t} \left\{ \Lambda_{b,t+1} \pi_{t+1}^{-1} \left[ \nu \tau_{y} + (1-\nu)\rho_{t+1} + (1-\nu)(1-\rho_{t+1})\Omega_{b,t+1}^{m} \right] \right\}$$
(19)

$$\Omega_{b,t}^{x} = \mathbb{E}_{t} \left\{ \Lambda_{b,t+1} \pi_{t+1}^{-1} \Big[ (1 - \tau_{y}) + (1 - \nu)(1 - \rho_{t+1}) \Omega_{b,t+1}^{x} \Big] \right\}$$
 (20)

<sup>&</sup>lt;sup>68</sup>Because I assume that the borrower chooses her labor supply before deciding whether to prepay, this has a very small effect on labor supply, equivalent to a 2.0% increase in wages in steady state Results assuming that borrowers do not internalize the effect of their labor supply decision on their credit availability, which sets this term to zero, are virtually identical.

respectively.

### A.1.2 Saver Optimality

The saver optimality conditions are similar to those of the borrower, and are defined by

$$-\frac{u_{s,t}^{n}}{u_{s,t}^{c}} = (1 - \tau_{y})w_{t}$$

$$1 = R_{t}\mathbb{E}_{t} \left[ \Lambda_{s,t+1} \pi_{t+1}^{-1} \right]$$

$$1 = \Omega_{s,t}^{m} + \Omega_{s,t}^{x} (q_{t}^{*} - \Delta_{q,t}).$$

where  $\Omega_{s,t}^m$  and  $\Omega_{s,t}^x$  are the marginal continuation *benefits* to the saver of an additional unit of face value and an additional dollar of promised initial payments, respectively. These values are defined by

$$\Omega_{s,t}^{m} = \mathbb{E}_{t} \left\{ \Lambda_{s,t+1} \pi_{t+1}^{-1} \Big[ (1-\nu)\rho_{t} + (1-\nu)(1-\rho_{t+1}) \Omega_{s,t+1}^{m} \Big] \right\} 
\Omega_{s,t}^{x} = \mathbb{E}_{t} \left\{ \Lambda_{s,t+1} \pi_{t+1}^{-1} \Big[ 1 + (1-\nu)(1-\rho_{t+1}) \Omega_{s,t+1}^{x} \Big] \right\}.$$

Note that  $\Omega_{s,t}^m$  and  $\Omega_{s,t}^x$  capture forward looking expectations about marginal utility in the states in which the borrower will prepay, which can in turn influence the equilibrium coupon rate  $q_t^*$ .

Overall, the saver's optimality conditions are equivalent to the terms in the borrower's problem, with the following exceptions: savers are unconstrained ( $\mu = 0$ ), use a different stochastic discount factor, do not optimize over housing, face a proportional tax (wedge) on their mortgage payment receipts, and have an additional optimality condition from trade in the one-period bond.

### A.1.3 Intermediate and Final Good Producer Optimality

The solution to the intermediate and final good producers' problems is standard and can be summarized by the following system of equations

$$z_{1,t} = y_t \left(\frac{mc_t}{mc_{ss}}\right) + \zeta \mathbb{E}_t \left[\Lambda_{s,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}}\right)^{\lambda} z_{1,t+1}\right]$$
$$z_{2,t} = y_t + \zeta \mathbb{E}_t \left[\Lambda_{s,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}}\right)^{\lambda-1} z_{2,t+1}\right]$$

$$\tilde{p}_t = \frac{z_{1,t}}{z_{2,t}}$$

$$\pi_t = \pi_{ss} \left[ \frac{1 - (1 - \zeta)\tilde{p}_t^{1-\lambda}}{\zeta} \right]^{\frac{1}{\lambda - 1}}$$

$$\mathcal{D}_t = (1 - \zeta)\tilde{p}_t^{-\lambda} + \zeta(\pi_t/\pi_{ss})^{\lambda}\mathcal{D}_{t-1}$$

$$y_t = \frac{a_t n_t}{\Delta_t}$$

where  $y_t$  is total output,  $mc_t = w_t/a_t$  is the firm's marginal cost of production,  $z_{1,t}$  and  $z_{2,t}$  are auxiliary variables,  $\tilde{p}_t$  is the ratio of the optimal price for resetting firms relative to the average price, and  $\mathcal{D}_t$  is price dispersion.

## A.2 Aggregation

This section demonstrates the equivalence of the representative borrower's problem with the individual borrower's problem. The proof of the equivalence of problems of the individual saver and representative saver is symmetric.

In the individual's problem I assume that each borrower owns housing, but can also freely buy and sell housing services on an intra-borrower rental market. The individual borrower chooses consumption of nondurables  $c_{i,t}$ , rental of housing services  $h_{i,t}^{rent}$ , labor supply  $n_{i,t}$ , an indicator for the choice to prepay  $\mathbb{I}_t \in \{0,1\}$ , her target owned house size  $h_{i,t}^*$  and mortgage size  $m_{i,t}^*$  conditional on prepayment, and a vector of Arrow securities  $a_{i,t}(s_{t+1})$  traded among borrowers to maximize (1) subject to the budget constraint

$$\begin{split} c_{i,t} &\leq (1 - \tau_y) w_t n_{i,t} - \pi_t^{-1} x_{i,t-1} + \tau_y \pi_t^{-1} (x_{i,t-1} - \nu m_{t-1}) \\ &\quad + \operatorname{rent}_t (h_{i,t} - h_{i,t}^{rent}) - \delta p_t^h h_{i,t-1} \\ &\quad - \mathbb{I}_t (\kappa_{i,t}) \left[ \left( m_{i,t}^* - (1 - \nu) \pi_t^{-1} m_{i,t-1} \right) - p_t^h \left( h_{i,t}^* - h_{i,t-1} \right) \right. \\ &\quad - \left. (\kappa_{i,t} - \bar{\Psi}_t / \chi_b) \, m_{i,t}^* \right] \\ &\quad + a_{i,t-1} (s_t) + \sum_{s_{t+1} \mid s^t} p_t^a (s_{t+1}) a_{i,t} (s_{t+1}) + T_{b,t} \end{split}$$

the debt constraint

$$m_{i,t}^* \leq \min(\bar{m}_{i,t}^{LTV}, \bar{m}_{i,t}^{PTI})$$

and the laws of motion

$$m_{i,t} = \mathbb{I}_t(\kappa_{i,t})m_{i,t}^* + (1 - \mathbb{I}_t(\kappa_{i,t}))(1 - \nu)\pi_t^{-1}m_{i,t-1}$$
(21)

$$h_{i,t} = \mathbb{I}_t(\kappa_{i,t})h_{i,t}^* + (1 - \mathbb{I}_t(\kappa_{i,t}))h_{i,t-1}$$
(22)

$$x_{i,t} = \mathbb{I}_t(\kappa_{i,t})q_t^* m_{i,t}^* + (1 - \mathbb{I}_t(\kappa_{i,t}))(1 - \nu)\pi_t^{-1} x_{i,t-1}.$$
(23)

The assumption that prepayment can be chosen based only on aggregate and not individual conditions, other than the draw of the transaction cost  $\kappa_{i,t}$  is expressed by the lack of a subscript i on  $\mathbb{I}_t$ . This policy is chosen before time 0. The exact timing for the other controls is as follows:

- 1. Borrowers choose labor supply  $n_{i,t}$ .
- Borrowers choose how much housing they will purchase conditional on prepayment.
- 3. Borrowers draw  $\kappa_{i,t}$  and determine whether to prepay based on the pre-time-0 choice of  $\mathbb{I}_t(\kappa_{i,t})$ .
- 4. Borrowers draw  $e_{i,t}$ .
- 5. Prepaying borrowers choose their new loan size  $m_{i,t}^*$  subject to their credit limits.
- 6. Borrowers realize insurance claims, buy new Arrow securities, and choose consumption and rental housing.

The Lagrangian is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^{t}} \beta_{b}^{t} \int_{e_{i}^{t}} \int_{\kappa_{i}^{t}} \int \int \left\{ u(c_{i,t}, h_{i,t}^{rent}, n_{i,t}) + \lambda_{i,t} \left[ + (1 - \tau_{y}) w_{t} n_{i,t} - \pi_{t}^{-1} x_{i,t-1} + \tau_{y} \pi_{t}^{-1} (x_{i,t-1} - \nu m_{t-1}) \right] \right.$$

$$rent_{t} (h_{i,t} - h_{i,t}^{rent}) - \delta p_{t}^{h} h_{i,t-1}$$

$$- \mathbb{I}_{t} (\kappa_{i,t}) \left( \left( m_{i,t}^{*} - (1 - \nu) \pi_{t}^{-1} m_{i,t-1} \right) - p_{t}^{h} (h_{i,t}^{*} - h_{i,t-1}) \right.$$

$$- (\kappa_{i,t} - \bar{\Psi}_{t} / \chi_{b}) m_{i,t}^{*} \right)$$

$$+ a_{i,t-1} (s_{t}) + \sum_{s_{t+1} | s^{t}} p_{t}^{a} (s_{t+1}) a_{i,t} (s_{t+1}) - c_{i,t}$$

$$+ \mu_{i,t} \mathbb{I}_t(\kappa_{i,t}) \Big( \min(\bar{m}_{i,t}^{LTV}, \bar{m}_{i,t}^{PTI}) - m_{i,t}^* \Big) \Big] \Bigg\} d\Gamma_e(e_i^t) d\Gamma_{\kappa}(\kappa_i^t) di.$$

where superscript t implies the history from time 0 to t. The optimality conditions are

$$\begin{aligned} (c_{i,t}): & \quad u_{i,t}^{c} = \lambda_{i,t} \\ (a_{i,t}(s_{t+1})): & \quad p_{t}^{a}\lambda_{i,t} = \beta_{b}\mathbb{E}_{t}\lambda_{i,t+1} \\ (n_{i,t}): & \quad u_{i,t}^{n} + \lambda_{i,t}(1 - \tau_{y})w_{t} \int e_{i,t} \, d\Gamma_{e}(e_{i,t}) \\ & \quad + \lambda_{i,t}\mu_{i,t} \int \int I_{t}(\kappa_{i,t}) \frac{\partial \bar{m}_{i,t}^{PTI}}{\partial n_{i,t}} \mathbf{1}_{\left\{\bar{m}_{t}^{PTI} < \bar{m}_{t}^{LTV}\right\}} \, d\Gamma_{e}(e_{i,t}) \, d\Gamma_{\kappa}(\kappa_{i,t}) = 0 \\ (h_{i,t}^{rent}): & \quad u_{i,t}^{h} = \lambda_{i,t} \text{rent}_{t} \\ (h_{i,t}^{*}): & \quad \int \left[\Omega_{i,t}^{h} - p_{t}^{h} + \mu_{i,t} \mathbf{1}_{\left\{e_{i,t} \geq \bar{e}_{t}\right\}} \theta_{t}^{LTV} p_{t}^{h}\right] \, d\Gamma_{e}(e_{i,t}) = 0 \\ (m_{i,t}^{*}): & \quad \Omega_{i,t}^{m} + \Omega_{i,t}^{x} q_{t}^{*} - 1 + \mu_{i,t} = 0 \\ (\mathbb{I}_{t}(\kappa_{i,t})): & \quad \kappa_{t}^{*} = \int_{e_{i}^{t}} \int_{\kappa_{i}^{t-1}} \left\{ (1 - \Omega_{i,t}^{m})(m_{i,t}^{*} - (1 - \nu)\pi_{t}^{-1}m_{i,t-1}) - \Omega_{i,t}^{x}(q_{t}^{*}m_{i,t}^{*} - (1 - \nu)\pi_{t}^{-1}x_{i,t-1}) - (p_{t}^{h} - \Omega_{i,t}^{h})(h_{i,t}^{*} - h_{i,t-1}) \right\} d\Gamma_{e}(e_{i}^{t}) \, d\Gamma_{\kappa}(\kappa_{i}^{t-1}) \end{aligned}$$

where

$$\begin{split} &\Omega_{i,t}^{h} = \mathbb{E}_{t} \left\{ \Lambda_{i,t+1} \Big[ (\text{rent}_{t+1} - \delta) + \rho_{t+1} p_{t+1}^{h} + (1 - \rho_{t+1}) \Omega_{i,t+1}^{h} \Big] \right\} \\ &\Omega_{i,t}^{m} = \mathbb{E}_{t} \left\{ \Lambda_{i,t+1} \pi_{t+1}^{-1} \Big[ \nu \tau_{y} + (1 - \nu) \rho_{t+1} + (1 - \nu) (1 - \rho_{t+1}) \Omega_{i,t+1}^{m} \Big] \right\} \\ &\Omega_{i,t}^{x} = \mathbb{E}_{t} \left\{ \Lambda_{i,t+1} \pi_{t+1}^{-1} \Big[ (1 - \tau_{y}) + (1 - \nu) (1 - \rho_{t+1}) \Omega_{i,t+1}^{x} \Big] \right\} \end{split}$$

and where  $\Lambda_{i,t+1} = \beta \lambda_{i,t+1}/\lambda_{i,t}$ . Note that the  $\mathbb{I}(\kappa_{i,t})$  optimality condition follows from the threshold prepayer's indifference toward prepaying and not prepaying. Given the assumption that the prepayment decision cannot condition on individual states, the probability of prepayment in the next period  $\rho_{t+1}$  does not depend on i or on other time t controls.

I now demonstrate that these optimality conditions are equivalent to those derived from the representative borrower's problem. I seek a symmetric equilibrium, in which all borrowers have equal lifetime wealth at time 0. From the  $a_{i,t}(s_{t+1})$  optimality condition it follows that  $\Lambda_{i,t+1}$  takes the identical value  $\Lambda_{b,t+1}$  for all i. In the symmetric equilibrium,

this implies that  $\lambda_{i,t}$  is identical across all agents, and so  $c_{i,t}$  is identically equal to  $c_{b,t}/\chi_b$ . As a result, we immediately obtain  $h_{i,t}^{rent}$  identically equal to  $h_{b,t-1}/\chi_b$  across agents.

Since all of the components of the  $\Omega$  equations are identical, the  $\Omega_{i,t}^h$ ,  $\Omega_{i,t}^m$  and  $\Omega_{i,t}^x$  terms are identical across all agents i, and the  $\Omega_t^m$  and  $\Omega_t^x$  terms satisfy (19) and (20). Applying this result to the  $m_{i,t}^*$  condition, we find that the value of  $\mu_{i,t}$  is identical across borrowers, yielding (18). Substituting into the  $h_{i,t}^*$  equation we obtain

$$\Omega_t^h = (1 - \mu_t F_t^{LTV} \theta_t^{LTV}) p_t^h$$

which combined with the  $\Omega_t^h$  and  $h_{i,t}^{rent}$  conditions yields (3.1). Applying the results above, and the equilibrium condition  $h_{i,t}^* = h_{i,t} = h_{b,t}$  yields (12). We can also integrate the  $n_{i,t}$  condition over  $e_{i,t}$  and  $\kappa_{i,t}$  to yield

$$-\frac{u_{i,t}^n}{u_{i,t}^c} = (1 - \tau_y)w_t + \mu_t \rho_t \left(\frac{\theta_t^{PTI}w_t}{q_t^* + \alpha}\right) \int_{-\bar{q}_t}^{\bar{q}_t} e_i d\Gamma_e(e_{i,t})$$

which implies  $n_{i,t} = n_{b,t}/\chi_b$  for all i, and delivers (17). Finally, integrating (21) - (23) yields (4) - (5).

### A.3 Simple Example: Quantitative Version

This section provides a quantitative version of the simple example of Section 2.1, and closely follows the modeling exercise of Justiniano, Primiceri, and Tambalotti (2015a). The agent's problem is defined by

$$V(b_{t-1}, h_{t-1}, m_{t-1}) = \max_{b_t, h_t, m_t} u(c_t, h_t) + \beta V(b_t, h_t, m_t)$$

subject to the constraints

$$c_{t} \leq y_{t} + Rb_{t-1} - b_{t} + m_{t} - (1 + r_{m,t-1})m_{t-1} - p_{t}(h_{t} - h_{t-1})$$

$$m_{t} \leq \theta^{LTV}p_{t}h_{t}$$

$$m_{t} \leq \theta^{PTI}y_{t}/q(r_{m,t})$$

where q is a function that turns a raw mortgage rate into a coupon rate using the standard annuity formula. This is a simplified version of an individual borrower's problem in the benchmark model, but where the borrower automatically and costlessly prepays each

period for simplicity. For further parsimony, I follow Justiniano et al. (2015a) in assuming quasi-linear utility:  $u(c_t, h_t) = c_t + v(h_t)$ . The optimality condition for  $m_t$  implies

$$1 - \beta(1 + r_{m,t}) = \mu_t^{LTV} + \mu_t^{PTI} \equiv \bar{\mu}_t$$

where I define  $\mu_t^{LTV}$  and  $\mu_t^{PTI}$  to be the multipliers on the LTV and PTI constraints, respectively, and  $\bar{\mu}_t$  to be the sum of the multipliers. For the housing condition, I normalize the house price to  $p_t = 1$ , and assume a known growth rate, so that  $p_{t+1} = (1+g)$ . Next, differentiating the objective function with respect to  $h_t$  implies that the net marginal benefit from purchasing an additional unit of housing is given by

$$v'(h_t) + \beta(1+g) - (1 - \mu_t^{LTV}\theta^{LTV}).$$

The value of  $\mu_t^{LTV}$  depends on which of the two borrowing constraints is binding, which in turn depends on  $h_t$ . Define

$$\bar{h}_t = rac{ heta^{PTI} y_t}{ heta^{LTV} q(r_{m,t})}$$

For  $h_t < \bar{h}_t$ , the LTV constraint is strictly tighter, so the PTI constraint is slack, yielding  $\mu_t^{LTV} = \bar{\mu}_t$ . For  $h_t > \bar{h}_t$ , the PTI constraint is strictly tighter, so the PTI constraint is slack, yielding  $\mu_t^{LTV} = 0$ . This introduces a corner solution at  $h_t = \bar{h}_t$ , where the net marginal benefit from an additional unit of housing jumps discontinuously downwards. In particular, the borrower will choose precisely  $h_t = \bar{h}_t$  whenever

$$v'(\bar{h}_t) + \beta(1+g) - (1 - \bar{\mu}\theta^{LTV}) > 0 > v'(\bar{h}_t) + \beta(1+g) - 1.$$
 (24)

We can calibrate this example at monthly frequency as in the example by setting  $\beta = 0.85^{1/12}$ ,  $1 + r_{m,t} = 1.06^{1/12}$ ,  $y_t = 50/12$ ,  $\theta^{LTV} = 0.8$ ,  $\theta^{PTI} = 0.28$ ,  $1 + g = 1.02^{1/12}$ ,  $v(h) = 0.0015 \cdot \log(h)$ , and any  $R < \beta^{-1}$ . It is easily checked that condition (24) holds for all the experiments of Section 2.1, verifying that the borrower indeed follows the corner solution as pictured.

### A.4 Data Description

This section describes the various data used in the paper, and provides additional histograms and moments to support the empirical claims of the paper.

#### A.4.1 Macroeconomic Data

Sources for the various macroeconomic data used in the paper can be found in Table A.1 below.

#### A.4.2 Fannie Mae Loan-Level Data

This set is taken from Fannie Mae's Single Family Loan Performance Data.<sup>69</sup> From the Fannie Mae data description:

The population includes a subset of Fannie Mae's 30-year, fully amortizing, full documentation, single-family, conventional fixed-rate mortgages. This dataset does not include data on adjustable-rate mortgage loans, balloon mortgage loans, interest-only mortgage loans, mortgage loans with prepayment penalties, government-insured mortgage loans, Home Affordable Refinance Program (HARP) mortgage loans, Refi Plus mortgage loans, and non-standard mortgage loans. Certain types of mortgage loans (e.g., mortgage loans with LTVs greater than 97 percent, Alt-A, other mortgage loans with reduced documentation and/or streamlined processing, and programs or variances that are ineligible today) have been excluded in order to make the dataset more reflective of current underwriting guidelines. Also excluded are mortgage loans originated prior to 1999, sold with lender recourse or subject to other third-party risk-sharing arrangements, or were acquired by Fannie Mae on a negotiated bulk basis.

The sample contains over 21 million loans acquired from Jan, 2000 to March 2012. Additional histograms and quantiles from this dataset are displayed in Figures B.1 - B.3 below.

#### A.4.3 Freddie Mac Loan-Level Data

This set is taken from Freddie Mac's Single Family Loan-Level Dataset.<sup>70</sup> The data set contains approximately 17 million 30-year, fixed-rate mortgages originated between January 1, 1999, and September 30, 2013. Data plots corresponding to those for Fannie Mae data in the main text can be found in Figure A.1.

### A.4.4 Pool-Level Agency MBS Data

This data set from eMBS<sup>71</sup> contains pool-level MBS data on all Fannie Mae, Freddie Mac, and Ginnie Mae products. The data are available at monthly frequency and are disaggre-

<sup>&</sup>lt;sup>69</sup>http://www.fanniemae.com/portal/funding-the-market/data/loan-performance-data.html

 $<sup>^{70} \</sup>verb|http://www.freddiemac.com/news/finance/sf_loanlevel_dataset.html|$ 

<sup>71</sup>http://www.embs.com

Table A.1: Data Definitions

| Name                   | Definition                                   | Source | Code            |
|------------------------|--|--------|-----------------|
|                        | Main Text + Boom-Bust Experiments            |        |                 |
| Income                 | Household + Nonprofit Disposable Income      | FoF    | FA156012005.Q   |
| Value                  | Household + Nonprofit Res. Real Estate       | FoF    | FL155035005.Q   |
| Debt                   | Household + Nonprofit Home Mort. (Liability) | FoF    | FL153165105.Q   |
| Gross Income           | Household + Nonprofit Personal Income        | FoF    | FA156010001.Q   |
| Mortgage Rate          | 30Y Conventional Mortgage Rate               | BoG    | MORTG           |
| Mortgage Service       | Mortgage Debt Service as % of Disp. Income   | BoG    | MDSP            |
| Housing Services       | PCE: Services, Housing and Utilities         | BEA    | DHUTRC1Q027SBEA |
| Inflation Expectations | 10Y Inflation Expectations                   | CleFed |                 |
| Prepayment Rate        | Fannie Mae 30Y MBS CPR                       | eMBS   | FNM30           |
|                        | Local Projections                            |        |                 |
| Output                 | Real GDP                                     | BEA    | GDPC1           |
| 2Y Rate                | 2Y Treasury Constant Maturity Rate           | BoG    | GS2             |
| House Price            | All-Transactions House Price Index           | FHFA   | USSTHPI         |
| GDP Deflator           | GDP: Implicit Price Deflator                 | BEA    | GDPDEF          |
| Population             | Civilian Noninstitutional Population         | BLS    | CNP160V         |
| 10Y Rate               | 10Y Treas. Constant Mat. Rate                | BoG    | GS10            |
| EBP                    | Excess Bond Premium                          | ZS     |                 |
| Hours                  | NFB Sector: Hours of All Persons             | BLS    | HOANBS          |
| Stock Wealth           | Household Corporate Equities                 | FoF    | LM153064105.Q   |

Note: Data sources are Bureau of Economic Analysis (BEA), Bureau of Labor Statistics (BLS), Federal Reserve Bank of Cleveland (CleFed), Federal Reserve Board of Governors (BoG), Flow of Funds (FoF), Federal Housing Finance Administration (FHFA), Gilchrist and Zakrajšek (2012) (GZ). Codes are FRED mnemonics with the exception of data from FoF and eMBS. The stock wealth series has been put in seasonally adjusted terms by combining the unadjusted levels and seasonally adjusted flows.

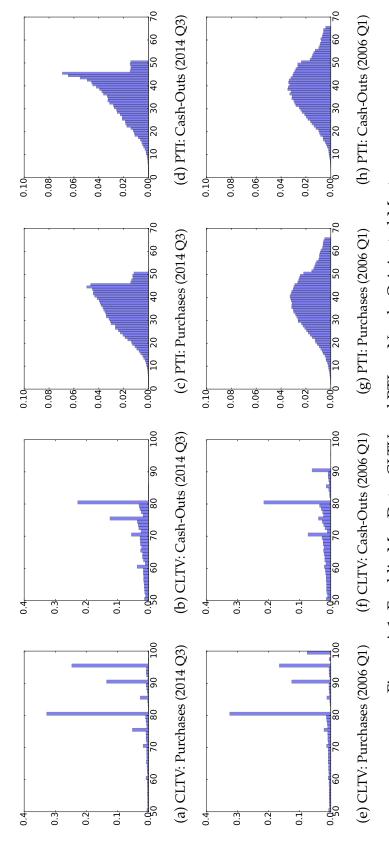


Figure A.1: Freddie Mac Data: CLTV and PTI on Newly Originated Mortgages

Note: Histograms are weighted by loan balance. Source: Freddie Mac Single Family Loan-Level Dataset.

gated by product type (e.g., 30-Year Fixed Rate), by coupon bin (in increments of 0.25% or 0.5%), and by either production year or state. Available variables include principal balance, conditional prepayment rate, level of issuance, weighted average coupon, and weighted average time to maturity.

#### A.4.5 Black Knight Loan Performance Data

Black Knight (also known as McDash) data contains servicer-provided information on a wide range of loans including loans guaranteed by Fannie Mae, Freddie Mac, Ginnie Mae, and private label securitization, as well as portfolio loans. The total sample contains 173 million loans.

### A.5 Local Projections: Details and Robustness

This section contains details on the implementation of the local projections used to compute the data responses to TFP shocks, as well as additional results for robustness. Data definitions can be found in Table A.2.

Table A.2: Data Definitions: Projections

| Name                              | Definition                                | Source      | Code          | Log | Def | Pop |
|-----------------------------------|---|-------------|---------------|-----|-----|-----|
| Output                            | Real GDP                                  | BEA         | GDPC1         | Y   | N   | Y   |
| 2Y Rate                           | 2Y Treas. Constant Mat. Rate              | BoG         | GS2           | N   | N   | N   |
| Mort Rate                         | 30Y Conventional Mortgage Rate            | BoG         | MORTG         | N   | N   | N   |
| Debt                              | Household Home Mortgages                  | FoF         | FL153165105.Q | Y   | Y   | Y   |
| House Price                       | All-Trans. House Price Index              | <b>FHFA</b> | USSTHPI       | Y   | Y   | N   |
| Inflation                         | ( $\Delta$ ) GDP: Implicit Price Deflator | BEA         | GDPDEF        | Y   | N   | N   |
| Additional Variables and Controls |   |             |               |     |     |     |
| Population                        | Civilian Noninstitutional Pop.            | BLS         | CNP16OV       | N   | N   | N   |
| 10Y Rate                          | 10Y Treas. Constant Mat. Rate             | BoG         | GS10          | N   | N   | N   |
| House Values                      | Household Real Estate Values              | FoF         | LM155035015.Q | Y   | Y   | Y   |
| EBP                               | Excess Bond Premium                       | GZ          |               | N   | N   | N   |
| Hours                             | NFB Sector: Hours of All Persons          | BLS         | HOANBS        | Y   | N   | Y   |
| Stock Wealth                      | Household Corp. Equities                  | FoF         |               | Y   | Y   | Y   |

*Note:* Data sources can be found in Table A.1.

Since the projection is intended to identify the change in the conditional expectation due to the time t shock, control variables should be chosen to provide a good fit of the expectation of the variable conditional on time t-1 data. With this in mind, I chose the

controls for each variable as follows. Output, Inflation: labor productivity (log of GDP divided by hours), stock wealth, and the excess bond premium. 2Y Rate: slope of term structure (10Y rate minus 2Y rate), excess bond premium. Mort Rate: 4Q log house price growth, mortgage spread (mortgage rate minus 10Y rate). Debt: output, aggregate log LTV (debt / value), relative mortgage rate (mortgage rate minus its 5Y moving average). House Price: output, 4Q log house price growth, mortgage rate. Prepay rate: aggregate LTV, one-year house price growth, rate incentive (weighted average coupon on FNM30 loans minus average new rate on FNM30 loans).

Projections using the log differences in the utilization-adjusted TFP series of Fernald (2014) (dtfp\_util) are plotted in Figure A.2, below. While the bands are slightly wider, the overall fit is similar to that of Figure 4.

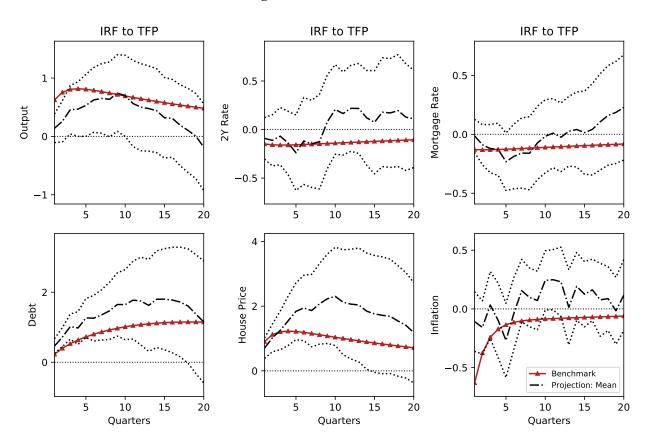


Figure A.2: Response to 1% Productivity Shock: Model vs. Data Projections (Fernald)

*Note:* A value of 1 represents a 1% increase relative to the initial value (data) or steady state (model), except for 2Y Rate and Mort. Rate, which are measured in percentage points. The full data definitions, sources, and lists of controls can be found in the appendix. The 2Y rate in the model is computed as the implied yield on a geometrically decaying nominal perpetuity with average duration of 8Q. Standard errors for each horizon h are corrected for serial correlation due to overlapping data using the Newey-West procedure with h lags.

### A.6 Extensions

This section contains two extensions to the baseline model: a specification with adjustable-rate mortgages, and a calibration with a higher PTI limit (43%) corresponding to the new limits under the Dodd-Frank Act.

#### A.6.1 Adjustable-Rate Mortgages

This section considers a version of the model using adjustable-rate mortgages (ARMs) instead of fixed-rate mortgages (FRMs). Under an ARM contract, the saver gives the borrower \$1 at origination. In exchange, the saver receives  $(1 - \nu)^k q_{t+k-1}^*$  at time t + k, for all k > 0 until prepayment, where  $q_{t+k-1}^* = (R_{t+k-1} - 1) + \nu$ . This coupon rate is obtained from arbitrage considerations, since a saver must be indifferent between holding an adjustable-rate mortgage for one period and the one-period bond, since both are short-term risk-free assets.

Under ARM contracts, promised payment is no longer an endogenous state variable, but is instead defined period-by-period using

$$x_t = q_t^* m_t$$
.

Correspondingly,  $\Omega_{j,t}^x$  and  $\Omega_{j,t}^m$  can be combined into a single term  $\Omega_{j,t}$ , that represents the total continuation cost of an additional unit of debt. As a result, the borrower's optimality conditions in the ARM case become

$$\rho_t = \Gamma_{\kappa} \left\{ (1 - \Omega_{b,t}) \left( 1 - \frac{(1 - \nu)\pi_t^{-1} m_{t-1}}{m_t^*} \right) \right\}$$

$$\Omega_{b,t} = 1 - \mu_t$$

for

$$\Omega_{b,t} = \mathbb{E}_t \left\{ \Lambda_{b,t+1}^{\$} \Big[ (1 - \tau_y) q_t^* + \tau_y \nu + (1 - \nu) \rho_{t+1} + (1 - \nu) (1 - \rho_{t+1}) \Omega_{b,t+1} \Big] \right\}.$$

The saver's optimality conditions for  $m_t^*$  in the ARM case becomes

$$\Omega_{s,t}=1$$

where

$$\Omega_{s,t} = \mathbb{E}_t \left\{ \Lambda_{s,t+1}^{\$} \Big[ (1 - \tau_q) q_t^* + (1 - \nu) \rho_{t+1} + (1 - \nu) (1 - \rho_{t+1}) \Omega_{s,t+1} \Big] \right\}.$$

To see the impact of the type of mortgage contract on the dynamics, we can compare the Benchmark economy with an *ARM Economy* in which contracts are defined as in this section. The difference between responses across economies depends substantially on the type of the shock. For near-permanent shocks to interest rates, the impulse responses are largely identical, as seen in the responses to an inflation target shock in Figure A.3.

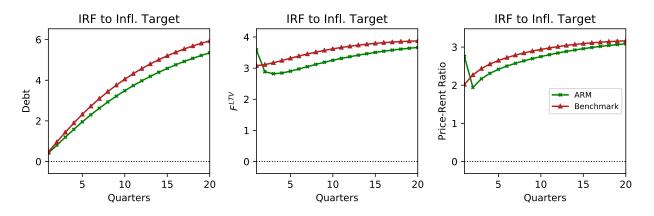


Figure A.3: Response to -1% (Ann.) Inflation Target Shock, Benchmark vs. ARM

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is measured in real terms. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services.

However, when shocks impose a temporary shift in mortgage rates, the effect on debt and prices is much stronger in the Benchmark setting, where borrowers rush to lock in lower rates before this temporary advantage expires, seen in the responses to a term premium shock plotted in Figure A.4. Note that, despite the name, the term premium shock also shifts adjustable rate mortgage payments (in this case it is better thought of as a mortgage spread shock) so the result is not hard-wired — the difference in responses is due to whether the change in payments will *continue* to be applied to new mortgages after the shock reverts.

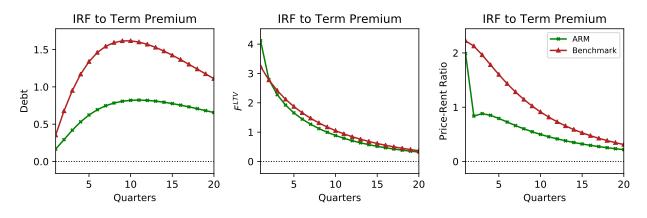


Figure A.4: Response to -1% (Ann.) Term Premium Shock, Benchmark vs. ARM

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is measured in real terms. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services.

For the final possibility, shocks not included in this model that would move the short end of the yield curve while leaving the long end unchanged would likely have a much larger effect in the ARM Economy, where they would lower initial payments and relax PTI limits, relative to the Benchmark, which should see little impact.

#### A.6.2 Alternative PTI Calibration

In this section, I present results using a higher calibration for the PTI limit of 43%, corresponding to the maximum for Qualified Mortgages under the Dodd-Frank Act. Impulse responses, shown in Figure A.5, demonstrate strong effects of incorporating PTI limits alongside LTV limits, although an even smaller minority of borrowers (16%) are constrained by PTI at equilibrium. The key is that the constraint switching effect occurs at the margin. Although a smaller number of borrowers are PTI-constrained to begin with a similar number *switch* to being LTV-constrained under the shock as in the baseline calibration. This allows the alternative calibration to deliver a similar rise in house prices, leading to comparable overall effects on debt.

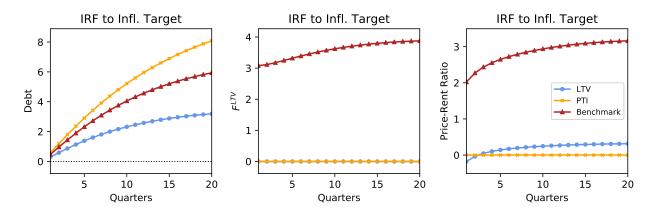


Figure A.5: Response to -1% (Ann.) Inflation Target Shock, 43% (Dodd-Frank) PTI Limit

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is measured in real terms. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services.

### A.7 Empirical Prepayment Behavior

This section displays the empirical behavior of the prepayment rate, showing that borrowers prepay only infrequently, but that the rate at which they do so is highly sensitive to interest rate incentives, and the availability of extractable equity. To this end, Figure A.6 shows the prepayment rate on Fannie Mae 30-Year FRM (FNM30) MBS, alongside the fitted values from a logistic regression of the prepayment rate on 4Q house price growth and the interest rate incentive, measured as the difference between interest rates on existing and new mortgages. Details and regression coefficients can be found in Table A.3.

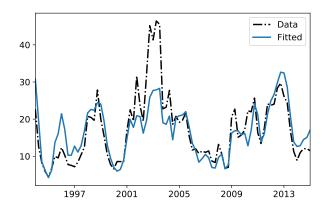


Figure A.6: Prepayment Rate vs. Regression Fit

*Note:* This figure plots the fitted values from the regression in Table A.3. A value of 1 on the y axis corresponds to a change of 1%.

Table A.3: Logistic Prepayment Regression, Fannie Mae 30-Year Fixed Rate Mortgages

|       | 4Q HP Growth | Rate Incentive | Const   | Adj. R <sup>2</sup> |
|-------|--------------|----------------|---------|---------------------|
| Value | 5.919        | 1.102          | -8.092  | 0.727               |
| (SE)  | (0.773)      | (0.068)        | (0.799) |                     |

*Note:* The left-hand-side variable is a logistic transform of the conditional prepayment rate of Fannie Mae 30-Year Fixed Rate Mortgages (source: eMBS), defined as the annualized fraction of loans that would be prepaid if the monthly prepayment rate continued for an entire year. "4Q HP Growth" is the 4Q log difference in the FHFA index, while "Rate Incentive" is difference in the average coupon rates on existing vs. newly issued FNM30 MBS. The house price growth measure is lagged by 1Q to allow for a delay between when the loan terms are set and when the loan is issued. Both right hand side variables are measured in percent, so a value of 1 implies 1% higher house price growth

# **B** Supplementary Tables and Figures

Table B.1: Nonlinear Least Squares Estimation

| Specification            | $LTV^*$ | $ar{ ho}$ | $\mu_{\kappa}$ | $S_{\mathcal{K}}$ | $100 \times RMSE$ |
|--------------------------|---------|-----------|----------------|-------------------|-------------------|
| One-Period               | 0.414   |           |                |                   | 8.926             |
|                          | (0.015) |           |                |                   |                   |
| Exog. Prepay             |         | 0.034     |                |                   | 0.402             |
|                          |         | (0.003)   |                |                   |                   |
| Ratchet                  | 0.404   |           |                |                   | 0.750             |
|                          | (0.004) |           |                |                   |                   |
| Exog. Prepay + PTI       |         | 0.048     |                |                   | 0.452             |
|                          |         | (0.004)   |                |                   |                   |
| Exog. Prepay + PTI + Lib |         | 0.046     |                |                   | 0.348             |
|                          |         | (0.003)   |                |                   |                   |
| Benchmark                |         |           | 0.348          | 0.152             | 0.318             |
|                          |         |           | (0.089)        | (0.061)           |                   |

*Note:* Standard errors, reported in parentheses, are corrected for heteroskedasticity. The value of RMSE has been scaled by 100 for easier reading. The sample spans 1980 Q1 - 2015 Q4, except for the "Ratchet" specification, whose estimation sample spans 1998 Q1 - 2015 Q4.

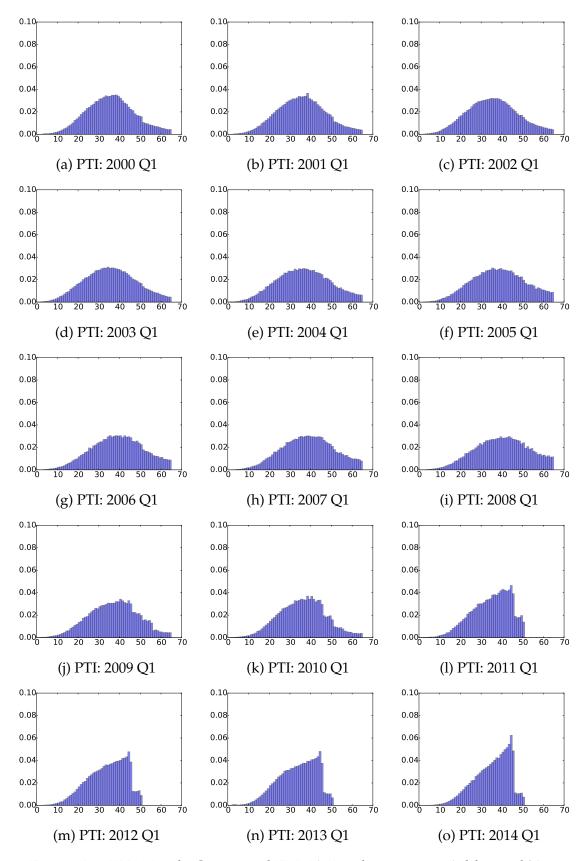


Figure B.1: PTI, Newly Originated FNMA Purchase Loans, Additional Years

Note: Histograms are weighted by loan balance. Source: Fannie Mae Single Family Dataset.

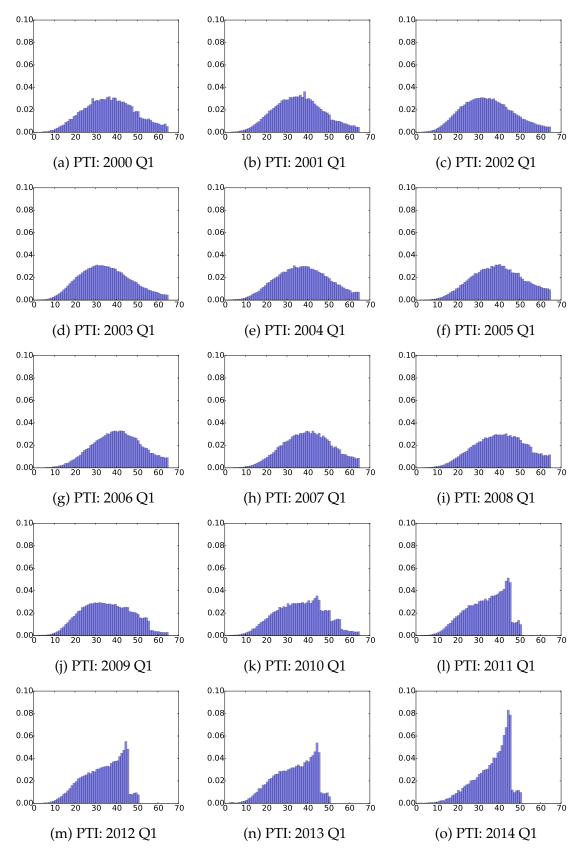


Figure B.2: PTI, Newly Originated FNMA Cash-Out Refi Loans, Additional Years *Note:* Histograms are weighted by loan balance. Source: Fannie Mae Single Family Dataset.

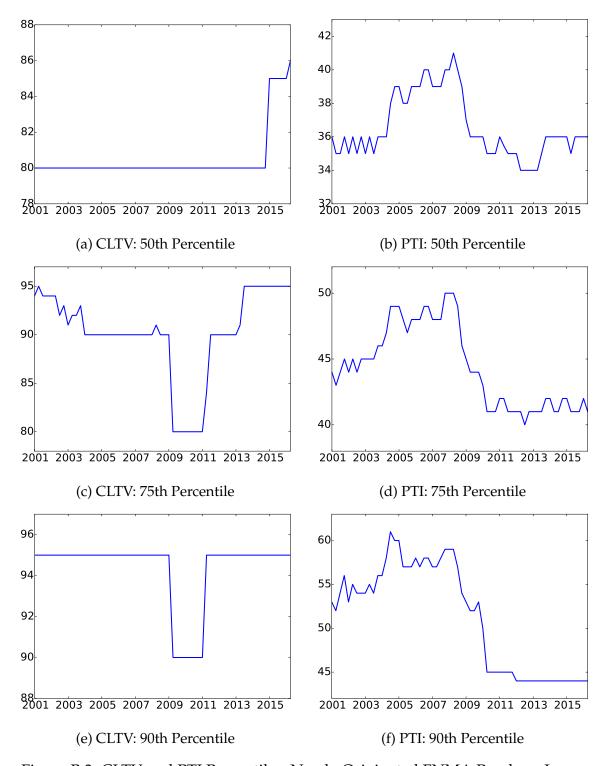


Figure B.3: CLTV and PTI Percentiles, Newly Originated FNMA Purchase Loans

Note: Plots report percentiles weighted by loan balance. Source: Fannie Mae Single Family Dataset.

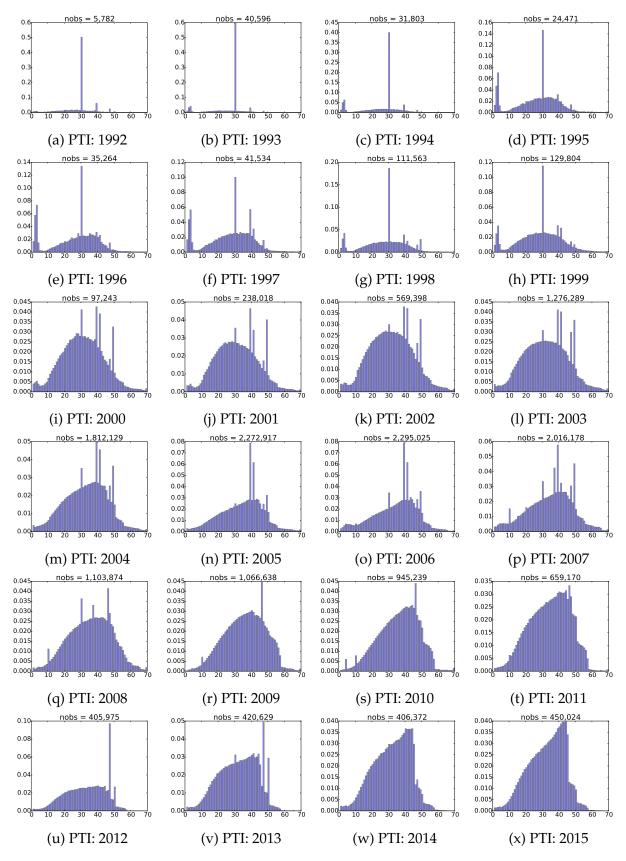


Figure B.4: PTI Ratios, Black Knight Data, Purchase Loans

Note: Plots display unweighted histograms of the front-end PTI ratio at origination by year of closing.

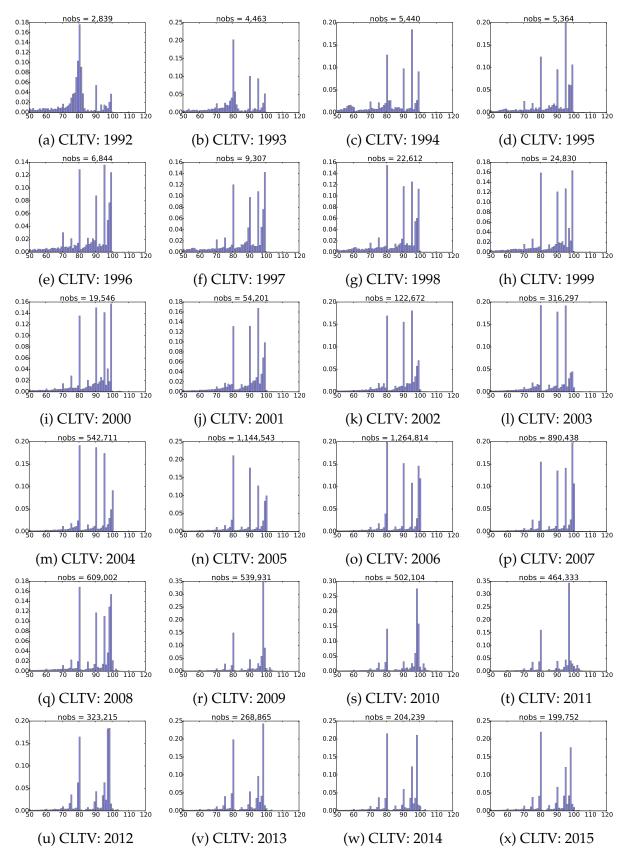


Figure B.5: CLTV Ratios, Black Knight Data, Purchase Loans

Note: Plots display unweighted histograms of the combined LTV ratio at origination by year of closing.

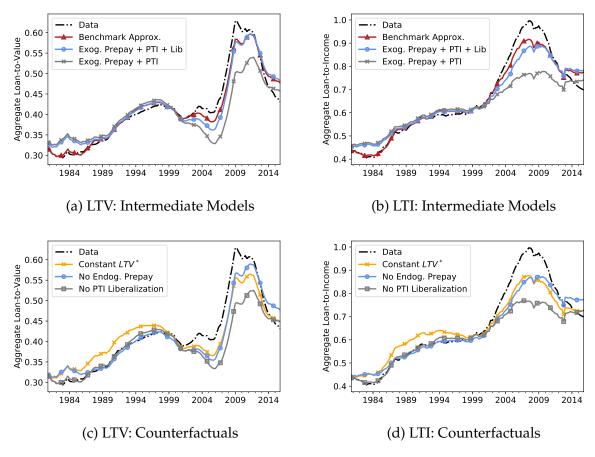


Figure B.6: Additional Paths, Aggregate LTV and LTI Distributions

*Note:* Counterfactual paths are generated by removing endogenous  $\rho_t$ , endogenous  $LTV_t^*$ , and the PTI liberalization from the Benchmark paths of Figure 3, without re-estimating the parameters. See Table A.1 in the appendix for full data sources and details. Aggregate Loan-to-Value and Aggregate Loan-to-Income are computed as the ratios of household debt to the value of household residential real estate and household disposable income. The sample spans 1980 Q1 - 2015 Q4.

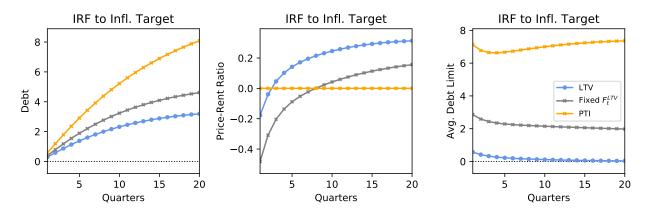


Figure B.7: Response to -1% (Ann.) Inflation Target Shock, Comparison of LTV, PTI, Fixed  $F_t^{LTV}$  Economies

A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is measured in real terms. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services.

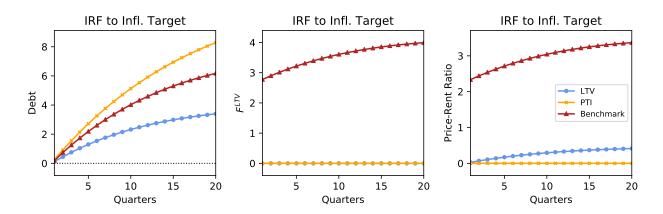


Figure B.8: Response to -1% (Ann.) Inflation Target Shock (Flexible Prices)

Note: Results are obtained in an alternative version of the model with  $\zeta=0$ , so that all intermediate goods prices are reset each period. A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. Debt  $(m_t)$  is measured in real terms. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services.

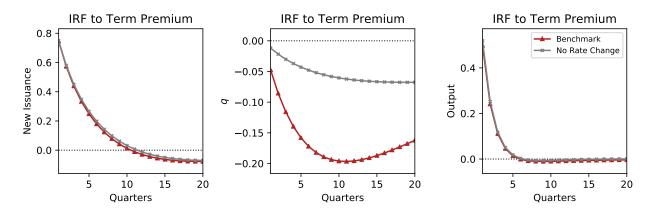


Figure B.9: Response to 1% (Ann.) Term Premium, Comparison of Benchmark, No Rate Change Economies

*Note:* The "No Rate Change" responses correspond to a counterfactual economy in which borrowers still prepay using the rule (12), but do not update the interest rate following prepayment, so that

$$x_t = q_t^* (m_t^* - (1 - \nu) \pi_t^{-1} m_{t-1}) + (1 - \nu) \pi_t^{-1} x_{t-1}.$$

A value of 1 represents a 1% increase relative to steady state, except for "New Issuance,"  $\rho_t(m_t^* - (1 - \nu)\pi_t^{-1}m_{t-1})$ , which is measured as a percentage of steady state output (both quarterly). All variables are reported in real terms.

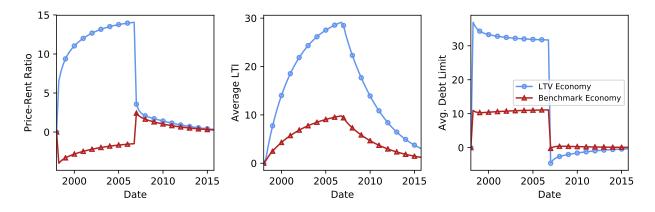


Figure B.10: Credit Liberalization Experiment: LTV Economy

Note: A value of 1 represents a 1% increase relative to steady state. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . Avg. Debt Limit  $\bar{m}_t$  is measured in real terms. For the LTV economy experiment, at time zero, the LTV limit  $\theta^{LTV}$  is unexpectedly loosened from 0.731 to 0.850, corresponding to the proportional loosening displayed in Figure 8, and after 36Q, is unexpectedly tightened back to 0.731.

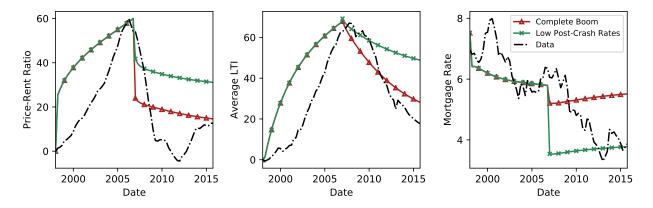


Figure B.11: Low Post-Crash Rates

*Note:* A value of 1 represents a 1% increase relative to steady state. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . For the "Post-Crash Rates" path, at the end of the boom, steady state inflation is permanently decreased by 0.659% (the average difference between 2003-2007 and 2013-2017) and the average term premium is permanently decreased by 1.13% to match an average mortgage interest rate over the period 2013-2017 of 3.92%.

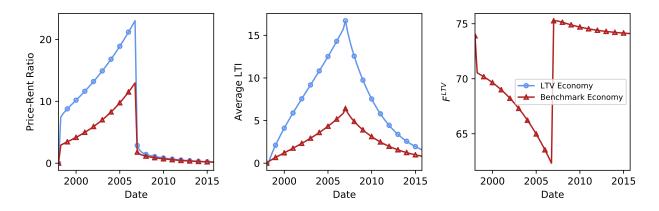


Figure B.12: House Price Expectations Experiments

*Note:* A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ , which is measured in percentage points. The price-rent ratio is defined as  $p_t^h/(u_{b,t}^h/u_{b,t}^c)$ , where the denominator is the implied price of rental services. Aggregate LTI is defined as  $m_t/(1-\tau_y)y_t$ . At time 0, agents learn that in 36Q, the housing preference parameter  $\xi$  will increase from 0.250 to 0.312. But after 36Q, the parameter unexpectedly is not increased.

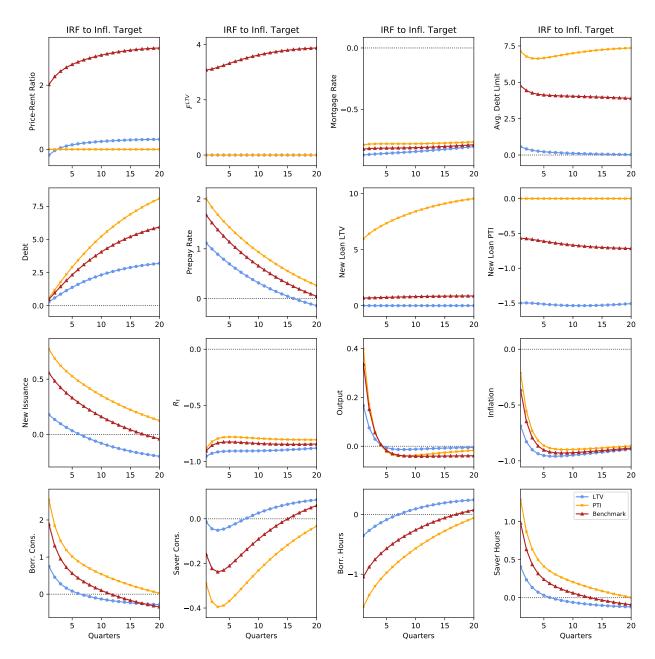


Figure B.13: Response to -1% (Ann.) Inflation Target Shock, Comparison of LTV, PTI, Benchmark Economies, Additional Variables

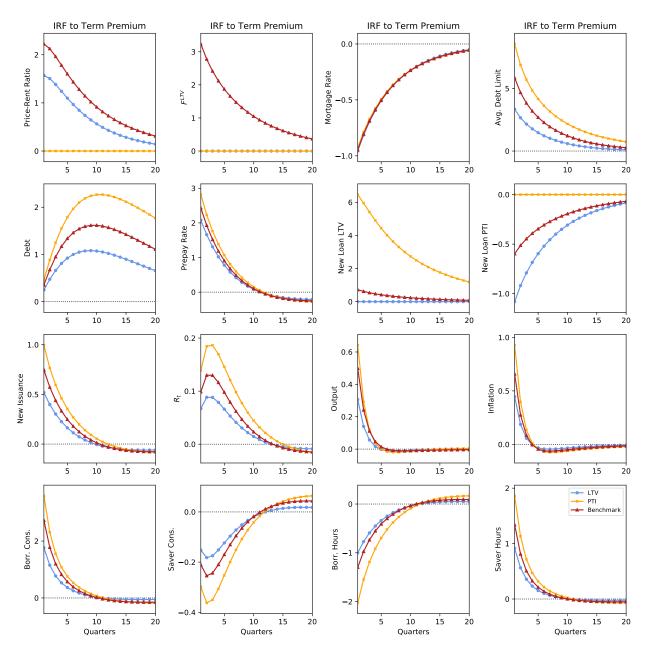


Figure B.14: Response to -1% (Ann.) Term Premium Shock, Comparison of LTV, PTI, Benchmark Economies, Additional Variables

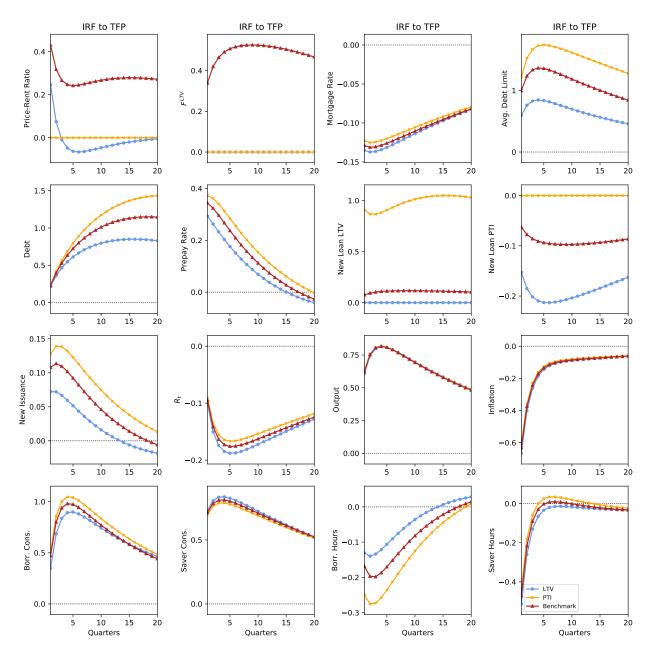


Figure B.15: Response to 1% Productivity Shock, Comparison of LTV, PTI, Benchmark Economies, Additional Variables

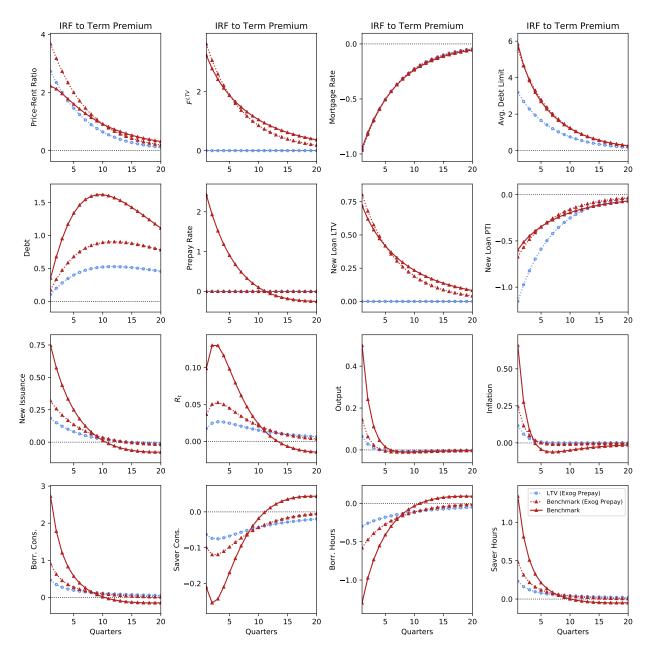


Figure B.16: Response to 1% Term Premium Shock, Comparison of LTV (Exog. Prepay), Benchmark (Exog. Prepay), and Benchmark (Endog. Prepay) Economies, Additional Variables

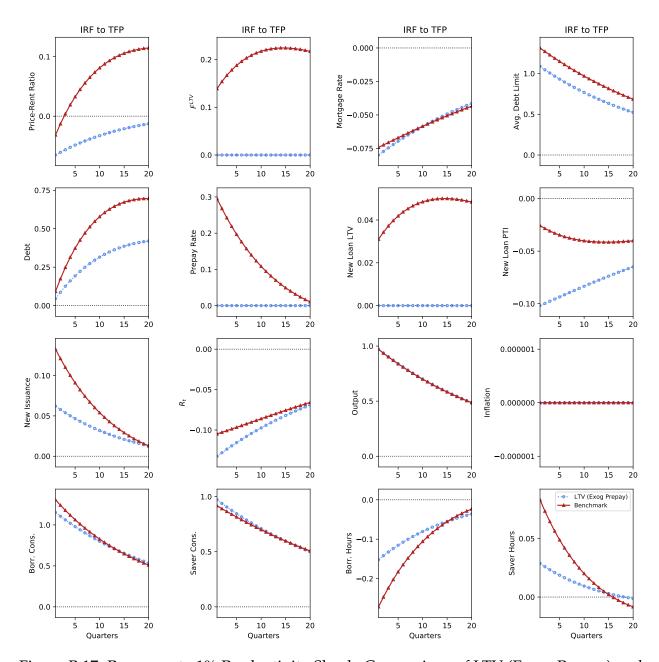


Figure B.17: Response to 1% Productivity Shock, Comparison of LTV (Exog. Prepay) and Benchmark (Endog. Prepay) Economies, Full Inflation Stabilization, Additional Variables

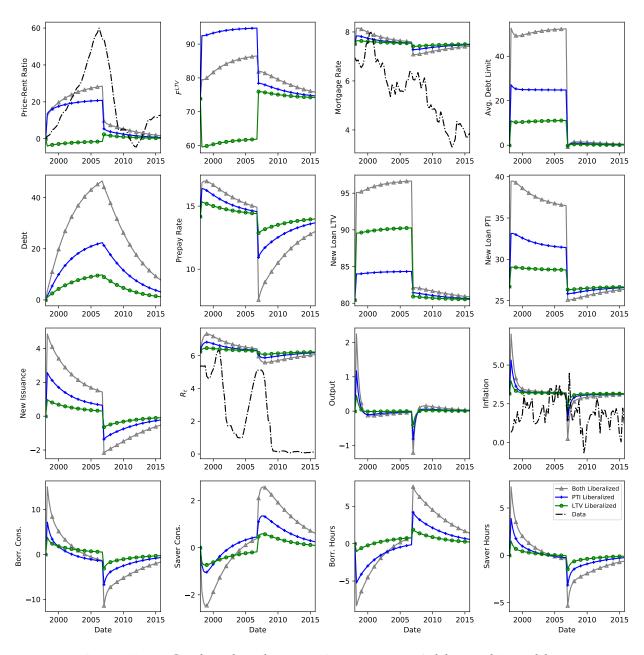


Figure B.18: Credit Liberalization Experiments, Additional Variables

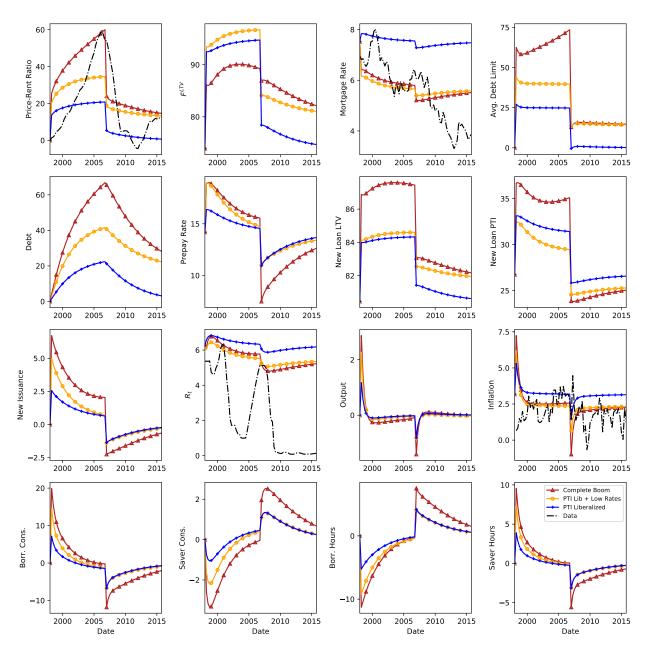


Figure B.19: Decomposing the Boom, Additional Variables

Note: For the "Complete Boom" path, in addition to the changes in parameters, agents learn at time 0 (1997 Q4) that in 36Q, the housing preference parameter  $\xi$  will increase from 0.250 to 0.312. After 36Q, however, the agents are surprised to learn that the parameter will instead remain at its initial value. Variable definitions are as follows. Price-Rent Ratio:  $p_t^h/(u_t^h/u_t^c)$ . Mortgage Rate:  $q_t^* - \nu$ . Avg. Debt Limit:  $\bar{m}_t$ , Debt:  $m_t$ . Prepay Rate:  $\rho_t$ . New Issuance:  $\rho_t(m_t^* - (1 - \nu)\pi_t^{-1}m_{t-1})$ . New Loan LTV:  $m_t^*/p_t^hh_{b,t}^*$ . New Loan PTI:  $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$ . Average LTV:  $m_t/p_t^hh_{b,t}$ . A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ ,  $q_t^*$ , Prepay Rate, New Loan LTV, and New Loan PTI, which are measured in percentage points, and New Issuance, which is measured as a fraction of steady state output. Avg. Debt Limit  $\bar{m}_t$ , Debt  $m_t$ , Output  $y_t$ , Borr. Cons.  $\hat{c}_{b,t}$ , and Saver Cons.  $\hat{c}_{s,t}$  are reported in real terms. Mortgage Rate, Prepay Rate,  $R_t$ , Output, and Inflation are annualized.

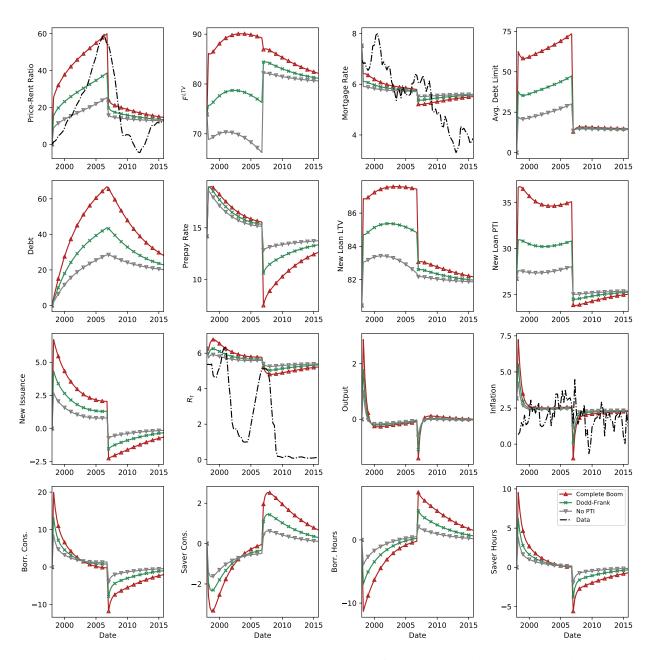


Figure B.20: Macroprudential Policy Counterfactuals, Additional Variables

Note: For each path, in addition to the changes in parameters, agents learn at time 0 (1997 Q4) that in 36Q, the housing preference parameter  $\xi$  will increase from 0.250 to 0.312. After 36Q, however, the agents are surprised to learn that the parameter will instead remain at its initial value. Variable definitions are as follows. Price-Rent Ratio:  $p_t^h/(u_t^h/u_t^c)$ . Mortgage Rate:  $q_t^* - v$ . Avg. Debt Limit:  $\bar{m}_t$ , Debt:  $m_t$ . Prepay Rate:  $\rho_t$ . New Issuance:  $\rho_t(m_t^* - (1-v)\pi_t^{-1}m_{t-1})$ . New Loan LTV:  $m_t^*/p_t^hh_{b,t}^*$ . New Loan PTI:  $(q_t^* + \alpha)m_t^*/w_tn_{b,t}$ . Average LTV:  $m_t/p_t^hh_{b,t}$ . A value of 1 represents a 1% increase relative to steady state, except for  $F^{LTV}$ ,  $q_t^*$ , Prepay Rate, New Loan LTV, and New Loan PTI, which are measured in percentage points, and New Issuance, which is measured as a fraction of steady state output. Avg. Debt Limit  $\bar{m}_t$ , Debt  $m_t$ , Output  $y_t$ , Borr. Cons.  $\hat{c}_{b,t}$ , and Saver Cons.  $\hat{c}_{s,t}$  are reported in real terms. Mortgage Rate, Prepay Rate,  $R_t$ , Output, and Inflation are annualized.