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A Model of Mortgage Default

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ABSTRACT

In this paper, we solve a dynamic model of households' mortgage decisions incorporating labor income, house price, inflation, and interest rate risk. Using a zero-profit condition for mortgage lenders, we solve for equilibrium mortgage rates given borrower characteristics and optimal decisions. The model quantifies the effects of adjustable versus fixed mortgage rates, loan-to-value ratios, and mortgage affordability measures on mortgage premia and default. Mortgage selection by heterogeneous borrowers helps explain the higher default rates on adjustable-rate mortgages during the recent U.S. housing downturn, and the variation in mortgage premia with the level of interest rates.

THE EARLY YEARS OF THE 21ST CENTURY were characterized by unprecedented instability in house prices and mortgage market conditions, both in the United States and globally. After the housing credit boom in the mid-2000s, the housing downturn of the late 2000s saw dramatic increases in mortgage defaults. Foreclosures appear to have had negative feedback effects on the values of neighboring properties, worsening the decline in house prices (Campbell, Giglio, and Pathak (2011)). Losses to mortgage lenders stressed the financial system and contributed to the larger economic downturn. These events have underscored the importance of understanding household incentives to default on mortgages, and the way in which these incentives vary across different types of mortgage contracts.

In this paper, we study the mortgage default decision using a theoretical model of a rational utility-maximizing household. We solve a dynamic model of a household that finances the purchase of a house with a mortgage, and must in each period decide how much to consume and whether to exercise options to default, prepay, or refinance the loan. Several sources of risk affect household

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decisions and the value of the options on the mortgage, including house prices, labor income, inflation, and real interest rates. We use multiple data sources to parameterize these risks.

Importantly, we study household decisions for endogenously determined mortgage rates. We model the cash flows of mortgage providers, including a loss on the value of the house in the event the household defaults. We then use risk-adjusted discount rates and a zero-profit condition to determine the mortgage premia that in equilibrium should apply to each contract. Since household mortgage decisions depend on interest rates and mortgage premia, and these decisions affect the profits of banks, we must solve several iterations of our model for each mortgage contract to find a fixed point. Thus, our model is not only a model of mortgage default, but also a micro-founded model of the determination of mortgage premia.

The literature on mortgage default emphasizes the role of house prices and home equity accumulation for the default decision. Deng, Quigley, and Van Order (2000) estimate a model, based on option theory, in which a household's option to default is exercised if the option is in the money by some specific amount. Borrowers do not default as soon as home equity becomes negative; they prefer to wait since default is irreversible and house prices may increase. Earlier empirical papers by Vandell (1978) and Campbell and Dietrich (1983) also emphasize the importance of home equity for the default decision.

As in this literature, in our model mortgage default is triggered by negative home equity, which tends to occur for a particular combination of the shocks that the household faces: house price declines in a low-inflation environment with large nominal mortgage balances outstanding. Also as in previous literature, households do not default as soon as home equity becomes negative.

A novel prediction of our model is that the level of negative home equity that triggers default depends on the extent to which households are borrowing constrained. As house prices decline, households with tightly binding borrowing constraints will default sooner than unconstrained households, because they value the immediate budget relief from default more highly relative to the longer-term costs. The degree to which borrowing constraints bind depends on the realizations of income shocks, the endogenously chosen level of savings, the level of interest rates, and the terms of the mortgage contract. For example, adjustable-rate mortgages (ARMs) tend to default when interest rates increase, because high interest rates increase required mortgage payments on ARMs, tightening borrowing constraints and triggering defaults.

We use our model to illustrate these triggers for mortgage default and to explore several interesting questions about the effects of the mortgage system on defaults and mortgage premia.

First, we use our model to examine how the adjustability of mortgage rates affects default behavior, comparing default rates for ARMs and fixed-rate mortgages (FRMs). Not surprisingly, both ARMs and FRMs experience high default rates when there are large declines in house prices. However, for aggregate states with moderate declines in house prices, ARM defaults tend to occur when interest rates are high—because high rates increase the required payments on ARMs—whereas the reverse is true for FRMs.

Second, we determine mortgage premia in the model and compare the results to the data. For most parameterizations and household characteristics the model predicts that mortgage premia should increase with the level of interest rates. In U.S. data, this appears to be the case for FRMs, but not for ARMs. The model is able to generate ARM premia that decrease with interest rates when we assume that ARM borrowers have labor income that is not only riskier on average, but also correlated with the level of interest rates. Such a correlation arises naturally if interest rates tend to be lower in recessions. We use our model to perform welfare calculations and show that households with this type of income risk benefit more from ARMs relative to FRMs, supporting the hypothesis that such households disproportionately borrow at adjustable rates.

Even though our model can generate the qualitative patterns of mortgage premia observed in the data, it is harder to match those patterns quantitatively. Our model does not easily explain the large ARM premia observed in U.S. data when interest rates are low. Furthermore, our model generally predicts a larger positive effect of interest rates on FRM mortgage premia than that observed in U.S. data. Our model can deliver FRM mortgage premia that better match the data if there is refinancing inertia (Miles (2004), Campbell (2006)), so that households do not refinance their FRMs as soon as it is optimal to do so.[1]

Third, we use our model to investigate how ratios at mortgage origination such as loan-to-value (LTV), loan-to-income (LTI), and mortgage-payment-to-income (MTI) affect default probabilities. The LTV ratio measures the household's initial equity stake, while LTI and MTI are measures of initial mortgage affordability. A clear understanding of the relation between these ratios and mortgage defaults is particularly important in light of the recent U.S. experience. Figure 1 plots aggregate ratios for newly originated U.S. mortgages over the last couple of decades, using data from the Monthly Interest Rate Survey (MIRS) of mortgage lenders conducted by the Federal Housing Finance Agency (FHFA).[2] This figure shows that there was an increase in the average LTV in the years before the crisis, but to a level that does not seem high by historical standards. A caveat is that the survey omits information on second mortgages, which became far more common during the 2000s.[3] Even looking only at first mortgages, however, there is a striking increase in the average LTI ratio, from an average of 3.3 during the 1980s and 1990s to as high as 4.5 in the mid-2000s. This pattern in the LTI ratio is not confined to the United States; in the United Kingdom the average LTI ratio increased from roughly two in the 1970s and 1980s to above 3.5 in the years leading up to the credit crunch (Financial Services Authority (2009)). Interestingly, as can be seen from Figure 1, the

[1] Guiso and Sodini (2013) provide a survey of the household finance literature.

[2] The LTV series is taken directly from the survey, and the LTI series is calculated as the ratio of the average loan amount obtained from the same survey to the median U.S. household income obtained from census data. The survey is available at www.fhfa.gov.

[3] In addition, the figure shows the average LTV, not the right tail of the distribution of LTVs, which may be relevant for mortgage default.

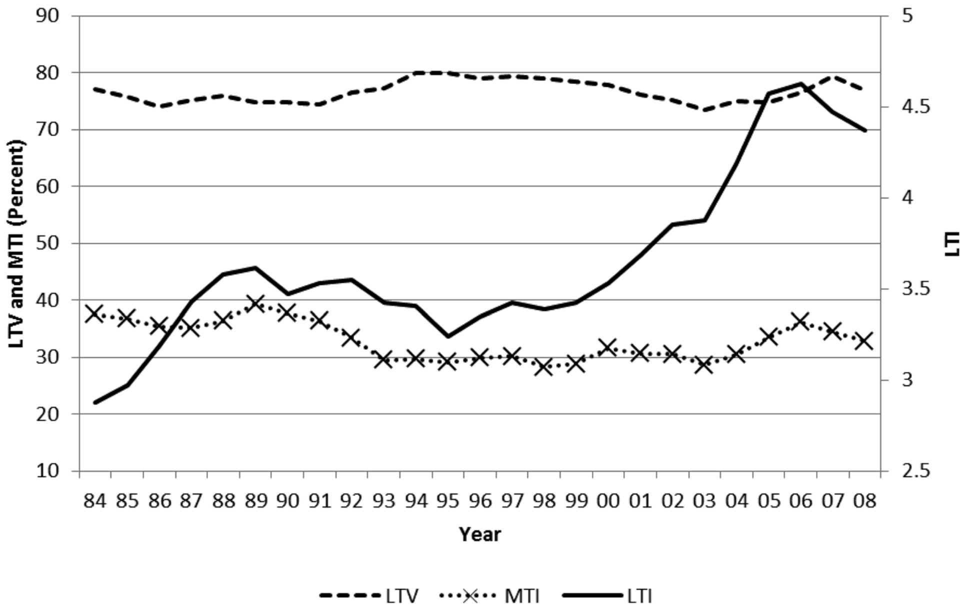


Figure 1. LTV, MTI, and LTI over time for the United States. LTV data are from the MIRS of the FHFA, LTI are calculated as the ratio of the average loan amount obtained from the same survey to the median U.S. household income obtained from Census data, MTI are calculated using the same income measure and the loan amount, maturity, and mortgage interest rate data from the MIRS.

low interest rate environment in the 2000s prevented the increase in LTI from driving up MTI to any great extent.

Our model allows us to understand the channels through which LTV and initial mortgage affordability ratios affect mortgage default. A higher LTV ratio (equivalently, smaller down payment) increases the probability of negative home equity and mortgage default, an effect that is documented empirically by Schwartz and Torous (2003) and more recently by Mayer, Pence, and Sherlund (2009). The unconditional default probabilities predicted by our model become particularly large for LTV ratios in excess of 90%.

The LTI ratio affects default probabilities through a different channel. A higher initial LTI ratio does not increase the probability of negative equity; however, it reduces mortgage affordability, making borrowing constraints more likely to bind. The level of negative home equity that triggers default becomes less negative, and in turn default probabilities increase. Our model implies that mortgage providers and regulators should think about combinations of LTV and LTI rather than try to control these parameters in isolation.[4]

[4] Regulators in many countries, including Austria, Poland, China, and Hong Kong, ban high LTV ratios in an effort to control the incidence of mortgage default. Some countries, such as the Netherlands, China, and Hong Kong, have also imposed thresholds on the mortgage affordability

Fourth, we model heterogeneity in labor income growth, labor income risk, and other household characteristics such as intertemporal preferences and inherent reluctance to default. For instance, we consider two households that have the same current income, but differ in terms of the expected growth rate of their labor income. The higher the growth rate, the smaller are the incentives to save, which increases default probabilities. However, we find that this effect is slightly weaker than the direct effect of higher future income on mortgage affordability, as measured, for example, by the MTI ratio later in the life of the loan. Therefore, the mortgage default rate and the equilibrium mortgage premium decrease with the expected growth rate of labor income.

Finally, we use our model to simulate developments during a downturn like the one experienced by the United States in the late 2000s. One motivation for this exercise is that, during the downturn, U.S. default rates were considerably higher for ARMs than for FRMs even though interest rates were declining, which contradicts our model's prediction that ARMs default primarily when interest rates increase. To try to understand this fact, we simulate our model for a path of aggregate variables that matches the recent U.S. experience of declining house prices and low interest rates. We show that one explanation for the higher default rates of ARMs is that ARMs are particularly attractive to households that face higher labor income risk, particularly if their labor income is correlated with interest rates. In addition, we model ARMs with a teaser rate to capture the fact that interest-only and other alternative mortgage products have had higher delinquency and default rates than traditional principal-repayment mortgages (Mayer, Pence, and Sherlund (2009)).

Several recent empirical papers study mortgage default. Foote, Gerardi, and Willen (2008) examine homeowners in Massachusetts who had negative home equity during the early 1990s and find that fewer than 10% of these owners eventually lost their home to foreclosure, so that not all households with negative home equity default. Bajari, Chu, and Park (2008) study empirically the relative importance of the various drivers behind subprime borrowers' decision to default. They emphasize the nationwide decrease in home prices as the main driver of default, but also find that the increase in borrowers with high MTI ratios has contributed to increased default rates in the subprime market. Mian and Sufi (2009) further emphasize the importance of an increase in mortgage supply in the mid-2000s, driven by securitization that created moral hazard among mortgage originators.

The contribution of our paper is to propose a dynamic and unified microeconomic model of rational consumption and mortgage default in the presence of house price, labor income, and interest rate risk. Our goal is not to try to derive

ratios LTI and MTI, in the form of either guidelines or strict limits. For instance, in Hong Kong, in 1999 the maximum LTV of 70% was increased to 90% provided that borrowers satisfied a set of eligibility criteria based on a maximum debt-to-income ratio, a maximum loan amount, and a maximum loan maturity at mortgage origination.

the optimal mortgage contract (as in Piskorski and Tchistyi (2010, 2011)), but instead to study the determinants of the default decision within an empirically parameterized model, and to compare outcomes across different types of mortgages. In this respect, our paper is related to the literature on mortgage choice (see, for example, Brueckner (1994), Stanton and Wallace (1998, 1999), Campbell and Cocco (2003), Koijen, Van Hermert, and Van Nieuwerburgh (2009), and Ghent (2013)). Our work is also related to the literature on the benefits of homeownership, since default is a decision to abandon homeownership and move to rental housing. For example, we find that the ability of homeownership to hedge fluctuations in housing costs (Sinai and Souleles (2005)) plays an important role in deterring default. Similarly, the tax deductibility of mortgage interest not only creates an incentive to buy housing (Glaeser and Shapiro (2003), Poterba and Sinai, 2011), but also reduces the incentive to default on a mortgage. Relative to Campbell and Cocco (2003), in addition to characterizing default decisions, we make two main contributions. First, we assume that household permanent income shocks are only imperfectly correlated with house price shocks. This assumption is important since it allows us to assess, for each contract, the relative contributions of idiosyncratic and aggregate shocks to the default decision. Second, we use the profits of mortgage providers together with a zero-profit condition to solve for the mortgage premium that should apply to each contract.

Our paper is related to interesting recent research by Corbae and Quintin (2015). They solve an equilibrium model to evaluate the extent to which low down payment mortgages were responsible for the increase in foreclosures in the late 2000s, and find that mortgages with these features account for 60% of the observed foreclosure increase. Garriga and Schlagenhauf (2009) also solve an equilibrium model of long-term mortgage choice to examine how leverage affects the default decision, while Corradin (2014) solves a continuous-time model of household leverage and default in which the agent optimally chooses the down payment on an FRM, abstracting from inflation and real interest rate risk. Our paper does not attempt to solve for the housing market equilibrium, and therefore can examine household risks and mortgage terms in more realistic detail, distinguishing the contributions of short- and long-term risks, and idiosyncratic and aggregate shocks, to both the default decision and mortgage premia. We emphasize the influence of realized and expected inflation on the default decision, a phenomenon that is absent in real models of mortgage default. In this respect, our work complements the research of Piazzesi and Schneider (2012) on inflation and asset prices.

The paper is organized as follows. In Section I, we set up the model, building on Campbell and Cocco (2003). This section also describes our solution method and the calibration of model parameters. In Section II, we study unconditional average default rates for standard principal-repayment mortgages, that is FRMs and ARMs, for different human capital characteristics and household preference parameters. We also study ARMs with a teaser rate.

Section III looks at default rates conditional on specific realizations of aggregate state variables, thereby clarifying the relative contributions of aggregate and idiosyncratic shocks to the default decision. A particular path that we study is one of declining house prices and low interest rates, which matches the recent U.S. experience. Section IV concludes. An Internet Appendix provides additional analysis.[5]

I. The Model

A. Setup

A.1. Time Parameters and Preferences

We model the consumption and default choices of a household i with a T-period horizon that uses a mortgage to finance the purchase of a house of fixed size H_i. We assume that household preferences are separable in housing and nondurable consumption, and are given by

$$\max E_1 \sum_{t=1}^{T} \beta_i^{t-1} \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \beta_i^T b_i \frac{W_{i,T+1}^{1-\gamma_i}}{1-\gamma_i}, \tag{1}$$

where T is the terminal age, β_i is the time discount factor, C_{it} is nondurable consumption, and γ_i is the coefficient of relative risk aversion. The household derives utility from both consumption and terminal real wealth, $W_{i,T+1}$, which can be interpreted as the remaining lifetime utility from reaching age $T+1$ with wealth $W_{i,T+1}$. Terminal wealth includes both financial and housing wealth. The parameter b_i measures the relative importance of the utility derived from terminal wealth. Households are heterogeneous and our notation uses the subscript i to take this into account. We solve the model for different household characteristics.

Since we assume that housing and nondurable consumption are separable and that H_i is fixed, we do not need to include housing explicitly in household preferences. However, the above preferences are consistent with

$$\max E_1 \sum_{t=1}^{T} \beta_i^{t-1} \left[\frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \theta_i \frac{H_{it}^{1-\gamma_i}}{1-\gamma_i} \right] + \beta_i^T b_i \frac{W_{i,T+1}^{1-\gamma_i}}{1-\gamma_i}, \tag{2}$$

for $H_{it} = H_i$ fixed, where the parameter θ_i measures the importance of housing relative to nondurable consumption.

In reality H_i is not fixed and depends on household preferences and income, among other factors. We simplify the analysis here by abstracting from housing choice, but we do study mortgage default for different values of H_i. In the Internet Appendix, we consider a simple model of housing choice to assure that our main results are robust to this consideration.

[5] The latest Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

A.2. Interest and Inflation Rates

Nominal interest rates are variable over time. This variability comes from movements in both the expected inflation rate and the ex ante real interest rate. We use a simple model that captures variability in both these components of the short-term nominal interest rate.

We write the nominal price level at time t as P_t, and normalize the initial price level to P_1=1. We adopt the convention that lowercase letters denote log variables, and thus $p_t \equiv \log(P_t)$ and the log inflation rate is $\pi_t = p_{t+1} - p_t$. To simplify the model, we abstract from one-period uncertainty in realized inflation; thus, expected inflation at time t is the same as inflation realized from t to $t+1$. While clearly counterfactual, this assumption should have little effect on our results since short-term inflation uncertainty is quite modest. We assume that expected inflation follows an AR(1) process. That is,

$$\pi_t = \mu_\pi(1 - \phi_\pi) + \phi_\pi \pi_{t-1} + \epsilon_t, \tag{3}$$

where ϵ_t is a normally distributed white noise shock with mean zero and variance σ_ϵ^2. The ex ante real interest rate also follows an AR(1) process. The expected log real return on a one-period bond, $r_{1t} = \log(1 + R_{1t})$, is given by

$$r_{1t} = \mu_r(1 - \phi_r) + \phi_r r_{1,t-1} + \varepsilon_t, \tag{4}$$

where ε_t is a normally distributed white noise shock with mean zero and variance σ_ε^2.

The log nominal yield on a one-period nominal bond, $y_{1t} = \log(1 + Y_{1t})$, is equal to the log real return on a one-period bond plus expected inflation:

$$y_{1t} = r_{1t} + \pi_t. \tag{5}$$

We let expected inflation be correlated with the ex ante real interest rate and denote the coefficient of correlation by $\rho_{\pi,r}$.

A.3. Labor Income and Taxation

Household i is endowed with stochastic gross real labor income in each period t, L_{it}, which cannot be traded or used as collateral for a loan. As usual we use a lowercase letter to denote the natural log of the variable, so $l_{it} \equiv \log(L_{it})$. The household's log real labor income is exogenous and given by

$$l_{it} = f_i(t, Z_{it}) + v_{it} + \omega_{it}, \tag{6}$$

where $f_i(t, Z_{it})$ is a deterministic function of age t and other individual characteristics Z_{it}, and v_{it} and ω_{it} are random shocks. In particular, v_{it} is a permanent shock assumed to follow a random walk,

$$v_{it} = v_{i,t-1} + \eta_{it}, \tag{7}$$

where η_{it} is an i.i.d. normally distributed random variable with mean zero and variance $\sigma_{\eta_i}^2$. The other shock represented by ω_{it} is transitory and follows an i.i.d. normal distribution with mean zero and variance $\sigma^2_{\omega_i}$. Thus, log income is the sum of a deterministic component and two random components, one transitory and one persistent.

We let real transitory labor income shocks, ω_{it}, be correlated with innovations to the stochastic process for expected inflation, ϵ_t, and denote the corresponding coefficient of correlation $\rho_{\omega_i,\epsilon}$. In a world where wages are set in real terms, this correlation is likely to be zero. If wages are set in nominal terms, however, the correlation between real labor income and inflation may be negative. As before, we use the subscript i throughout to model the fact that households are heterogenous in the characteristics of their labor income, including the variance of the income shocks they face.

We model the tax code in the simplest possible way, assuming a linear taxation rule. Gross labor income, L_{it}, and nominal interest earned are taxed at the constant tax rate τ. We allow for deductibility of nominal mortgage interest at the same rate.

A.4. House Prices and Other Housing Parameters

We model house price variation as an aggregate process. Let P^H_t denote the date t real price of housing, and let $p^H_t \equiv \log(P^H_t)$. We normalize $P^H_1 = 1$ so that H also denotes the value of the house that the household purchases at the initial date. The real price of housing is a random walk with drift, so real house price growth can be written as

$$\Delta p^H_t = g + \delta_t, \tag{8}$$

where g is a constant and δ is an i.i.d. normally distributed random shock with mean zero and variance σ^2_δ . We assume that the shock δ_t is uncorrelated with inflation, so in our model housing is a real asset and an inflation hedge. It would be straightforward to relax this assumption.

We assume that innovations to the permanent component of the household's real labor income, η_{it}, are correlated with innovations to real house prices, δ_t, and denote by $\rho_{\eta_i,\delta}$ the corresponding coefficient of correlation. When this correlation is positive, states of the world with high house prices are also likely to have high permanent labor income. We let innovations to the real interest rate be correlated with house price shocks, and denote this correlation by $\rho_{\varepsilon,\delta}$.

We assume that in each period homeowners must pay property taxes, at rate τ_p, proportional to house value, and that property tax costs are income-tax deductible. In addition, homeowners must pay a maintenance cost, m_p, proportional to the value of the property. This can be interpreted as the maintenance cost of offsetting property depreciation. The maintenance cost is not income-tax deductible.

A.5. Mortgage Contracts

The household is not allowed to borrow against future labor income. Furthermore, the maximum loan amount is equal to the value of the house less a down payment. The initial loan amount (D_{i1}) is thus

$$D_{i1} \leq (1 - d_i)P_1 P_1^H H_i, \tag{9}$$

where d_i is the required down payment. We use a subscript i on the required down payment to allow for the possibility that it differs across households. We simplify the model by assuming that the household finances the initial purchase of the house of size H_i with previously accumulated savings and a nominal mortgage loan equal to the maximum allowed, $(1 - d_i)H_i$. (Recall that we normalize P_1^H and P_1 to one.) The LTV and LTI ratios at mortgage origination are therefore given by

$$LTV_i = (1 - d_i) \tag{10}$$

$$LTI_i = \frac{(1 - d_i)H_i}{L_{i1}}, \tag{11}$$

where L_{i1} denotes the level of household labor income at the initial date.

Required mortgage payments depend on the type of mortgage. We consider several types, including FRMs, ARMs, and ARMs with a teaser rate.

Let $Y_T^{i,FRM}$ be the interest rate that household i pays on an FRM with maturity T. It is equal to the expected interest rate over the life of the loan, or the yield on a long-term bond, plus an interest rate premium that depends on loan and borrower characteristics. The date t real mortgage payment, M_{it}^{FRM}, is given by the standard annuity formula:

$$M_{it}^{FRM} = \frac{(1 - d_i)H_i\left[\left(Y_T^{i,FRM}\right)^{-1} - \left(Y_T^{i,FRM}(1 + Y_T^{i,FRM})^T\right)^{-1}\right]^{-1}}{P_t}. \tag{12}$$

A distinctive feature of the U.S. mortgage market is that FRMs come with a refinancing option, which we model. In particular, if households take out FRMs when interest rates are high and rates subsequently decline, then households that have the required level of positive home equity, d_i, may refinance the loan to take advantage of the lower interest rates. We assume that refinancing costs are equal to a proportion c_r of the loan amount. We also assume that households refinance into an FRM with remaining maturity $T - t_r + 1$, where t_r denotes the refinancing period. More specifically, we assume that households refinance into the contract and the borrowing position that they would have been in period t_r had the interest rates at the time that the loan began been lower.[6]

[6] This simplifies the numerical solution of the problem since we only need to solve the model for the different possible levels of initial interest rates, sequentially, starting with the lowest, and

Let $Y_{1t}^{i,ARM}$ be the one-period nominal interest rate on an ARM taken out by household i, and let D_{it}^{ARM} be the nominal principal amount outstanding at date t. The date t real mortgage payment, M_{it}^{ARM}, is given by

$$M_{it}^{ARM} = \frac{Y_{1t}^{i,ARM} D_{it}^{ARM} + \Delta D_{i,t+1}^{ARM}}{P_t}, \tag{13}$$

where $\Delta D_{i,t+1}^{ARM}$ is the component of the mortgage payment at date t that goes to pay down principal rather than pay interest. We assume that, for the ARM, the principal loan repayments, $\Delta D_{i,t+1}^{ARM}$, equal those for the FRM. This assumption simplifies the solution of the model since the outstanding mortgage balance is not a state variable.

The date t nominal interest rate for the ARM is equal to the short rate plus a constant premium

$$Y_{1t}^{i,ARM} = Y_{1t} + \psi^{i,ARM}, \tag{14}$$

where the mortgage premium $\psi^{i,ARM}$ compensates the lender for the prepayment and default risk of borrower i. In the case of an ARM with a teaser rate, the mortgage premium is set to zero for one initial period.

For an FRM the interest rate is fixed and equals the average interest rate over the loan maturity (the average zero-coupon bond yield for that maturity under the expectations hypothesis of the term structure) plus a premium $\psi^{i,FRM}$. In addition to prepayment and default risk, the FRM premium compensates the lender for the interest rate refinancing option that borrowers receive. At times when the one-year yield is low (high), the term structure is upward (downward) sloping, and long-term rates are higher (lower) than short-term rates. As previously noted, we assume that mortgage interest payments are deductible at the income tax rate τ.

A.6. Mortgage Default and Home Rental

In each period, the household decides whether to default on the mortgage loan. The household may be forced to default because it has insufficient cash to meet the mortgage payment. However, the household may also find it optimal to default even if it has the cash to meet the payment.

We assume that, in the case of default, a mortgage lender has no recourse to the household's financial savings or future labor income. The mortgage lender seizes the house, the household is excluded from credit markets, and since it cannot borrow the funds needed to buy another house it is forced into the rental market for the remainder of the time horizon. These assumptions simplify a complex reality. In the United States, the rules regarding recourse vary across states. In some states home mortgages are explicitly nonrecourse, whereas in others recourse is allowed but onerous restrictions on deficiency

using the value function as an input into the problem when initial interest rates are higher. We provide further details on the numerical solution in Section I.B.

judgments render many loans effectively nonrecourse.[7] In addition, defaulting households in the United States are excluded from credit markets for a period of time but not permanently. To understand the effect of recourse, in the Internet Appendix we consider a variation of our model in which lenders can seize borrowers' current financial assets in the event of default, but have no claim on their future labor income.

The rental cost of housing equals the user cost of housing times the value of the house (Poterba (1984), Diaz and Luengo-Prado (2008)). That is, the date t real rental cost U_{it} for a house of size H_i is given by

$$U_{it} = [Y_{1t} - \mathrm{E}_t[\exp(\Delta p^H_{t+1} + \pi_t) - 1] + \tau_p + m_p] P^H_t H_i, \tag{15}$$

where Y_{1t} is the one-period nominal interest rate, $\mathrm{E}_t[\exp(\Delta p^H_{t+1} + \pi_{1t}) - 1]$ is the expected one-period proportional nominal change in the house price, and τ_p and m_p are the property tax rate and maintenance costs, respectively.[8] This formula implies that in our model the rent-to-price ratio varies with the level of interest rates.[9]

Relative to owning, renting is costly for two main reasons. First, homeowners benefit from the income-tax deductibility of mortgage interest and property taxes, without having to pay income tax on the implicit rent they receive from their home occupancy. Second, owning provides insurance against future fluctuations in rents and house prices (Sinai and Souleles (2005)). When permanent income shocks are positively correlated with house price shocks, however, households have an economic hedge against rent and house price fluctuations even if they are not homeowners.

We assume that in the case of default the household is guaranteed a lower bound of $\underline{X}$ in per-period cash-on-hand, which can be viewed as a subsistence level. This assumption can be motivated by the existence of social welfare programs such as means-tested income support. In terms of our model, it implies that consumption and default decisions are not driven by the probability of extremely high marginal utility, which would be the case for power utility if there was a positive probability of extremely small consumption.

A.7. Early Mortgage Termination and Home Equity Extraction

We model several potential sources of early mortgage termination. As previously mentioned, we allow FRM borrowers to take advantage of a decrease in

[7] Ghent and Kudlyak (2011) use variation in state laws to empirically evaluate the impact of recourse on default decisions. Li, White, and Zhu (2011) argue that U.S. bankruptcy reform in 2005 affected mortgage default by making it harder for homeowners to use bankruptcy to reduce unsecured debt. See also the evidence in Demyanyk, Koijen, and Hemert (2010), and Chatterjee and Eyigungor (2009) and Mitman (2012), who solve equilibrium models of the macroeconomic effects of bankruptcy laws and foreclosure policies.

[8] To simplify we assume that maintenance costs are similar for homeowners and for rental properties. Alternatively, we could have reasonably assumed that homeowners take better care of the properties, thereby reducing maintenance expenses.

[9] Campbell, et al. (2009) provide an empirical variance decomposition for the rent-to-price ratio.

interest rates by refinancing their mortgage. In addition, we allow households that have accumulated positive home equity to sell their house, repay the outstanding debt, and move into rental accommodation. The house sale is subject to a realtor's commission, a fraction c of the current value of the property. In this way, at a cost c, households are able to access their accumulated housing equity and use it to finance nondurable consumption. We interpret this event as a cash-out prepayment.[10]

Ideally, in addition to a cash-out prepayment, we would like to explicitly model other ways in which households can draw down their accumulated home equity, for example, using second mortgages or home equity lines of credit. Home equity extraction can play an important role in consumption smoothing and can have macroeconomic implications (Chen, Michaux, and Roussanov (2013)). Unfortunately, this would increase the already large number of state variables in our model, so we leave this topic for future research.

In addition to the above endogenous sources of mortgage termination, we model exogenous random mortgage termination by assuming that, in each period, with probability φ_{it} borrowers are forced to move, in which case they sell the house, repay the principal outstanding, and move into the rental market. If a household is hit with a moving shock at a time of negative home equity, the household defaults on the loan. We allow for the possibility that negative home equity creates a "lock-in" effect by letting the probability of a forced move be a lower value $\varphi_{it}^{'}$ when home equity is negative.

A.8. Financial Institutions

We assume a competitive market for mortgage providers. In addition, we assume that financial institutions are able to screen borrowers and learn their characteristics. Therefore, the mortgage premium that they require for each contract will in equilibrium reflect the probability of prepayment, the default probability, and the expected losses given default of the specific borrower.[11]

Let $CF_{ijt}(S_t)$ denote the dollar nominal cash flow that the lender receives from household i on loan type j in period t when the state is S_t, for $j = ARM, FRM$. By state S_t we mean a given combination of values for the state variables in our model. Ex ante, many different values are possible; ex post, only one of them will be realized. The cash flow that lenders receive depends on whether household i chooses to default or prepay in period t, given state S_t, if he or she has not done so before. Given no default or prepayment, the lender receives the nominal mortgage payment

$$CF_{ijt} = P_t M_{it}^{j}, \text{ for } D_{def_{ijt}}^{C} = D_{prepay_{ijt}}^{C} = 0, \tag{16}$$

[10] In this paper, we use the terms cash-out, cash-out prepayment, and cash-out refinancing interchangeably.

[11] Since default and prepayment decisions depend on interest rates and mortgage premia, which also affect lenders' expected profits, this requires solving, for each borrower type and mortgage contract, several iterations of our model to find a fixed point. We provide further details in Section I.B.

where $D^C_{def_{ijt}}$ ($D^C_{prepay_{ijt}}$) is an indicator variable for default (prepayment) by household i in period t. When default occurs, the lender loses the outstanding mortgage principal but receives the house. We assume that foreclosure involves a deadweight cost equal to a proportion *loss* of the value of the house. The nominal cash flow received by the mortgage lender is given by

$$CF_{ijt} = (1 - loss)P_t P^H_t H_i, \text{ for } D^C_{def_{ijt}} = 1. \tag{17}$$

In the case of early mortgage termination due to a cash-out, the mortgage lender receives the outstanding loan principal,

$$CF_{ijt} = D^j_{it}, \text{ for } D^C_{prepay_{ijt}} = 1. \tag{18}$$

For the FRM there may also be early termination due to interest rate refinancing, in which case the lender receives the outstanding loan principal plus the refinancing cost paid by the borrower,

$$CF_{i,FRM,t} = D^{FRM}_{it} + c_r(1 - d_i)H_i, \text{ for } D^C_{ref_{ijt}} = 1, \tag{19}$$

where $D^C_{ref_{ijt}}$ is an indicator variable for refinancing by household i on mortgage type j in period t. In periods subsequent to early mortgage termination or default, the nominal cash flows received by the lender are zero. This assumes that, in the case of FRM interest rate refinancing, borrowers take out a loan with a different mortgage provider.

We calculate the present value of the cash flows that the mortgage lender receives by discounting them using a risk-adjusted discount rate. We describe the pricing kernel in the Internet Appendix. Let $PV_1[CF_{ijt}](S_1, S_2, \ldots, S_t)$ denote the present value (at the initial date) of the period t cash flow for loan type j taken out by household i. This present value depends not only on the value of the date t state variables, but also on the value of the state variables in previous periods since they affect the rate that is appropriate for discounting the profits.[12] We scale the present value of the sum of the cash flows by the loan amount to calculate risk-adjusted profitability,

$$PR_{ij}(S) = \frac{\sum_{t=1}^{T} PV_1[CF_{ijt}](S_1, S_2, \ldots, S_t)}{(1 - d_i)H_i}, \tag{20}$$

where $S = [S_1, S_2, \ldots, S_T]$. This gives us a measure of the return on each loan type, $j = ARM, FRM$, for a given borrower type i and a given realization of the state variables. If at the initial date we take expectations across all possible realizations of the state variables,

$$PR_{ij}(S_1) = E_1[PR_{ij}(S)], \tag{21}$$

[12] Only a subset of the state variables will affect the discount rate, namely, the aggregate variables in the model (real interest rates, inflation rate, and house prices).

we obtain a measure of expected profitability of mortgage loan type j to borrower type i conditional on the values of the state variables at the time the mortgage is taken out.

These profitability calculations do not subtract administrative expenses and should be interpreted accordingly. Computationally it would be straightforward to subtract expenses when calculating the profits of mortgage providers, but one would need to specify the type of expenses (per period or up front, fixed or as a proportion of the loan value).

B. Model Summary and Solution

B.1. Model Summary

The state variables of the household's problem are age (t), cash-on-hand (X_{it}), whether the household has previously terminated the mortgage through prepayment or default ($D^S_{term_{ijt}}$, equal to one given a previous prepayment or default and zero otherwise), real house prices (P^H_t), the nominal price level (P_t), inflation (π_t), the real interest rate (r_{1t}), and the level of permanent income (v_{it}). For the FRM there is an additional state variable, namely, whether the household has previously refinanced the loan ($D^S_{refi_{ijt}}$, equal to one given a previous refinancing and zero otherwise).

The choice variables are consumption (C_{it}), whether to default on the mortgage loan if no default has occurred before ($D^C_{defi_{ijt}}$, equal to one if household i chooses to default on loan j in period t and zero otherwise), and in the case of positive home equity whether to prepay ($D^C_{prepay_{ijt}}$, equal to one if the household chooses to prepay the mortgage in period t and zero otherwise). For the FRM, there is an additional choice variable, namely, whether to refinance the loan ($D^C_{refi_{ijt}}$, equal to one if household i chooses to refinance the mortgage in period t and zero otherwise).

In all periods before the last, if the household has not defaulted on or terminated its mortgage, its cash-on-hand evolves as follows for the case of an ARM:

$$\begin{aligned} X^j_{i,t+1} = {} & (X_{it} - C_{it})\frac{(1 + Y_{1t}(1-\tau))}{(1+\pi_t)} - M^i_{it} - (m_p + \tau_p)P^H_t H_i + L_{i,t+1}(1-\tau) \\ & + \frac{Y^{ij}_{1t} D_t \tau}{P_t} + \tau_p P^H_t H_i \tau, \end{aligned} \tag{22}$$

for $j = ARM$. Savings earn interest that is taxed at rate τ. Next period's cash-on-hand is equal to savings plus after-tax interest, minus real mortgage payments (made at the end of the period), minus property taxes and maintenance expenses, plus next period's labor income and the tax deduction on both nominal mortgage interest and property taxes.

The equation describing the evolution of cash-on-hand for the FRM in periods in which there is no refinance is similar, except that the mortgage interest tax deduction is calculated using the interest rate on that mortgage. In periods in which the FRM is refinanced, we need to subtract the refinancing cost and an amount equal to the difference between the loan amount on the new loan and the amount outstanding on the refinanced loan.[13]

If the household has defaulted on or prepaid its mortgage and moved to rental housing, the evolution of cash-on-hand is given by

$$X_{i,t+1}^{Rent} = (X_{it} - C_{it})\frac{(1 + Y_{1t}(1 - \tau))}{(1 + \pi_t)} - U_{it} + L_{i,t+1}(1 - \tau), \tag{23}$$

where U_{it} denotes the date t real rental payment.

Terminal, that is, date $T+1$, wealth is given by

$$\begin{aligned} W_{i,T+1}^{j} &= \frac{P_{T+1}X_{i,T+1} + P_{T+1}P_{T+1}^{H}H_i}{P_{T+1}^{Composite}}, \\ \text{for } j &= ARM, FRM \text{ and } D_{term_{ij,T+1}}^{S} = 0 \end{aligned} \tag{24}$$

$$W_{i,T+1}^{Rent} = \frac{P_{T+1}X_{i,T+1}}{P_{T+1}^{Composite}}, \text{ for } D_{term_{ij,T+1}}^{S} = 1. \tag{25}$$

If the household has not previously defaulted or terminated the mortgage contract, terminal wealth is equal to financial wealth plus housing wealth. In the rental state, households only have financial wealth at the terminal date.

Households derive utility from real terminal wealth, so that in all of the above cases nominal terminal wealth is divided by a composite price index, denoted by $P_{T+1}^{Composite}$. This index is given by

$$P_{T+1}^{Composite} = \left[(P_{T+1})^{1-\frac{1}{\gamma_i}} + \theta_i^{\frac{1}{\gamma_i}}(P_{T+1}P_{T+1}^{H})^{1-\frac{1}{\gamma_i}}\right]^{\frac{\gamma_i}{\gamma_i - 1}}, \tag{26}$$

where (recall that) γ_i is the coefficient of relative risk aversion and θ_i measures the preference for housing relative to other goods in the preference specification (2). The above composite price index is consistent with our assumptions regarding preferences (Piazzesi, Schneider, and Tuzel (2007)). The fact that nominal terminal wealth is scaled by a price index that depends on the price of housing implies that, even in the penultimate period, homeownership serves as an hedge against house price fluctuations (Sinai and Souleles (2005)). The larger is θ_i, the stronger is such a hedging motive for homeownership.

B.2. Solution Technique

Our model cannot be solved analytically. The numerical techniques that we use for solving it are standard. Since the mortgage premium depends on mortgage type, borrower characteristics, and the initial values of the aggregate state

[13] The speed at which FRM principal is repaid depends on the initial interest rate. We take this difference into account when the loan is refinanced.

variables, we solve the model separately for each of these cases. Recall that we normalize the initial price level and house prices to one, so that, as far as the aggregate variables are concerned, we need to calculate the mortgage premium for different initial levels of the inflation rate and the real interest rate.

The expected risk-adjusted profitability of each mortgage contract depends on the mortgage premium, which affects the default and prepayment decisions of borrowers, which in turn affect the expected risk-adjusted profitability of the loan. Therefore, for each case, we need to solve several iterations of our model to find a fixed point. We do so using a grid for mortgage premia with steps of five basis points. We start by making a guess for the mortgage premium, and then solve the borrower's problem given that premium. Once we have the borrower's optimal decisions, we use the transition probabilities and pricing kernel to calculate expected risk-adjusted profitability. We then iterate: if the expected risk-adjusted profitability is too high (low), we decrease (increase) the mortgage premium and solve the household's problem again.

For each possible mortgage premium, we solve the borrower's problem by discretizing the state space and using backwards induction starting from period $T + 1$. The shocks are approximated using Gaussian quadrature, assuming two possible outcomes for each of them. This simplifies the numerical solution of the problem since for each period t we only need to keep track of the number of past high (low) inflation shocks, high (low) permanent income shocks, and high (low) house price shocks to determine the date t price level, permanent income, and house prices. For each combination of the state variables, we optimize with respect to the choice variables. We use cubic spline or, in the areas in which there is less curvature in the value function, linear interpolation to evaluate the value function for outcomes that do not lie on the grid for the state variables. In addition, we use a log scale for cash-on-hand. This ensures that there are more grid points at lower levels of cash-on-hand.

To handle the refinancing option for FRMs, we solve the model sequentially, starting with the lowest level of initial interest rates, and save the value function. We then use this value function in each period t subsequent to mortgage origination as an input for the borrower's refinancing decision in the solution for the case of higher initial interest rates.

C. Parameterization

C.1. Time and Preference Parameters

To parameterize the model we assume that each period corresponds to one year. We set the initial age to 30 and the terminal age to 50; thus, mortgage maturity is 20 years. In the baseline parameterization we set the discount factor β equal to 0.98 and the coefficient of relative risk aversion γ equal to two. The parameter θ, which measures the preference for housing relative to other consumption, is set to 0.3. To account for household heterogeneity with respect to the preference for housing and other parameters, we solve the model for alternative parameter values. The parameter that measures the relative

Table I
Baseline Parameters

This table reports the parameter values used in the baseline case.

Description	Parameter	Value
Panel A: Time and Preference Parameters		
Discount factor	β	0.98
Risk aversion	γ	2
Preference for housing	θ	0.3
Initial age		20
Terminal age		50
Bequest motive	b	400
Panel B: Inflation and Real Interest Rate		
Mean log inflation	μ_π	0.029
St. dev. of the inflation rate	σ_ϵ	0.009
Log inflation AR(1) coefficient	ϕ_π	0.891
Mean log real rate	μ_r	0.012
St. dev. of the real rate	σ_r	0.018
Log real rate AR(1) coefficient	ϕ_r	0.825
Correl. inflation and real rate	$\rho_{\pi,r}$	0.597
Panel C: Labor Income and House Prices		
Mean log real income growth	$\overline{\Delta l_t}$	0.008
St. dev. permanent income shocks	σ_η	0.063
St. dev. temporary income shocks	σ_ω	0.225
Mean log real house price growth	g	0.003
St. dev. house price return	σ_δ	0.162
Correl. perm. inc. and house price shocks	$\rho_{\eta,\delta}$	0.191
Correl. real int. rate and house price shocks	$\rho_{\varepsilon,\delta}$	0.300
Correl. temp. inc. and inflation shocks	$\rho_{\omega,\epsilon}$	0.000
Panel D: Tax Rates and Other Parameters		
Income tax rate	τ	0.25
Property tax rate	τ_p	0.015
Property maintenance	m_p	0.025
Lower bound on cash-on-hand	$\underline{X}$	\$1,000
Transaction costs of house sale	c_s	0.060
Exogenous moving probability	φ	0.040
Exogenous mov. prob. if negative equity	φ'	0.008
Panel E: Loan Parameters		
Initial loan to income	LTI	4.5
Initial loan to value	LTV	0.90
FRM refinancing cost	c_r	0.01

importance of terminal wealth, b, is set to 400. This value is large enough to ensure that households have an incentive to save, and that our model generates reasonable values for wealth accumulation. Panel A of Table I summarizes baseline time and preference parameters.

C.2. Interest and Inflation Rates

We use data from the Livingston survey of inflation expectations to parameterize the stochastic process for expected inflation (median one-year forecast, sample period 1987 to 2012). We obtain information on one-year nominal bond yields from the Federal Reserve and calculate the expected one-year real interest rate by deflating the nominal yield by expected inflation. The estimated parameters for the AR(1) processes for the logarithm of expected inflation and the logarithm of the expected real rate are reported in Panel B of Table I. The implied half-life of expected inflation shocks is six years, while the half-life for real interest rate shocks is 3.6 years.

C.3. Labor Income

We use data from the Panel Study of Income Dynamics (PSID) for the years 1970 to 2005 to calibrate the labor income process. Our income measure is broadly defined to include total reported labor income plus unemployment compensation, workers compensation, Social Security transfers, and other transfers for both the head of the household and his spouse. We use such a broad measure to implicitly allow for the several ways in which households insure themselves against risks of labor income that is more narrowly defined. Labor income is deflated using the Consumer Price Index.

It is widely documented that an individual's income profile varies with educational attainment (see, for example, Gourinchas and Parker (2002)). To control for this relation, we follow existing literature and partition the sample into three education groups based on the educational attainment of the head of the household. For each education group we regress the log of real labor income on age dummies, controlling for demographic characteristics such as marital status and household size, and allowing for household fixed effects. We use this smoothed income profile to calculate, for each education group, the average household income for age 30 and the average annual growth rate in household income for ages 30 to 50. The estimated real labor income growth rate for households with a high-school degree is 0.8%, and we use this value in the benchmark case. The assumption of a constant income growth rate is a simplification of the true income profile that makes it easier to carry out comparative statics and investigate the role of future income prospects on the default decision.

We use the residuals of the above panel regressions to estimate labor income risk. To mitigate the effects of measurement error on estimated income risk, we winsorize the income residuals at the 5^{th} and 95^{th} percentiles. We follow the procedure of Carroll and Samwick (1997) to decompose the variance of the winsorized residuals into transitory and permanent components. The estimated parameter values reported in Panel C of Table I should be interpreted as possible parameter values. To account for heterogeneity in labor income characteristics we solve our model for alternative parameter values for expected labor income growth and income risk.

C.4. House Prices

To calibrate the parameters of the house price process, also reported in Panel C of Table I, we use PSID data and Case-Shiller house price indices. The advantage of PSID data is that they contain both house price and labor income information. However, annual data, which we need to calculate annual house price returns, are only available until 1997. Furthermore, PSID house prices are self-reported and vulnerable to measurement error.

Using PSID household-level data, we obtain real house prices by dividing self-reported house prices by the consumer price index, and we calculate changes in house prices as the first difference of log real house prices for individuals who are present in consecutive annual interviews and who report not having moved since the previous year. To address potential measurement error, and parallel to our treatment of labor income, we winsorize the logarithm of real house price changes at the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentiles (-36.6% and 40.3%, respectively). We use the winsorized data to calculate the expected value and the standard deviation of real house price changes, which are equal to 1.6% and 16.2%, respectively (as before we also consider alternative parameterizations). Next we estimate the correlation between labor income shocks and house price shocks. To do so we first calculate

$$\Delta(l_{it} - \hat{f}_{it}) = [l_{it} - \hat{f}(t, Z_{it})] - [l_{i,t-1} - \hat{f}(t-1, Z_{i,t-1})] = \eta_{it} + \omega_{it} - \omega_{i,t-1}, \tag{27}$$

where $\hat{f}$ denotes the predicted regression values. The correlation between (27) and the first differences in log house prices, δ_t, is positive and statistically significant, and equal to 0.037. Under the model assumption that temporary labor income shocks, ω_{it}, are serially uncorrelated and uncorrelated with house price shocks, this value implies a correlation between permanent labor income shocks, η_{it}, and house price shocks, δ_t, equal to 0.191. This value reflects the fact that a significant component of the innovations to permanent labor income shocks is idiosyncratic (and therefore uncorrelated with house prices).

We also use the S&P/Case-Shiller 10-City Composite Home Price Index to parameterize the model. The sample period is 1987 to 2012. We are particularly interested in the relation between house prices and real interest rates. As before, we deflate the house price index by the Consumer Price Index and calculate the logarithm of annual real house price growth. The mean log real house price growth, 0.005, is higher than that estimated in the PSID data due to the differences in the period covered. The standard deviation of log real house price growth, 0.09, is somewhat lower than in the PSID data. We estimate a positive correlation between innovations to the logarithm of real interest rates and log real house price returns equal to 0.38, with a p-value of 0.07. We parameterize the model using a somewhat lower value of 0.30. We set the remaining model correlations to zero.[14]

[14] We have estimated the correlation between log real house price returns and expected inflation, but the estimated value was not significantly different from zero.

The S&P/Case-Shiller composite house price index is less volatile than self-reported house prices from the PSID, because idiosyncratic house price variation diversifies away in the composite index.[15] Our model abstracts from idiosyncratic house price variation. Nonetheless, we calibrate it using an estimate of total house price volatility since all movements in house prices, not just aggregate movements, affect homeowners' incentives to default on their mortgages.

C.5. Tax Rates and Other Parameters

We follow Himmelberg, Mayer, and Sinai (2005) in setting the values for the tax rates. More specifically, we set the income tax rate, τ, equal to 0.25, the property tax rate, τ_p, equal to 0.015, and property maintenance expenses, m_p, equal to 0.025. In addition, we assume that a house sale is subject to a realtor commission, t_c, equal to 6% of the value of the house, which is a fairly standard value. We set the lower bound on (real) cash-on-hand to \$1,000. We set the exogenous probability of a house move for borrowers with positive home equity to 0.04. Chan (2001) estimates that an increase in LTV to over 95% would result in a moving probability that is 20% of the original. Therefore, in the case of negative home equity we set the exogenous moving probability to 0.008. We report these parameters in Panel D of Table I.

C.6. Loan Parameters

We consider alternative values for the down payment/initial LTV and LTI, but to facilitate discussion we refer to the case in which the LTV ratio is 0.9 and the LTI is 4.5 as the baseline. We set the costs of refinancing the FRM contract t_r to 1% of the loan amount. The credit risk premium on each of the mortgage loans, ψ^{ij}, where i denotes the borrower and $j = FRM, ARM$, is determined endogenously. We report the baseline loan parameters in Panel E of Table I.

D. Simulated Data

We solve the model separately for each mortgage type, set of borrower characteristics, and combination of initial values for the aggregate variables. Since we normalize the initial price level and real house prices to one, the aggregate variables we need to consider are expected inflation and the real interest rate. In the numerical solution we assume two possible states for each of these, which implies four different values for the initial one-year nominal rates. For each case, once we find a fixed point for the problem, we use the optimal policy functions to generate simulated data.

Agents in our model are subject to both aggregate and idiosyncratic shocks. Aggregate shocks are to real house prices, the inflation rate, and real interest

[15] This diversification effect is also visible in data on median U.S. house prices from the Monthly Interest Rate Survey. Over the period 1991 to 2007, the average growth rate in real (nominal) house prices was 1.2% (3.9%), with a standard deviation of only 4.8%.

rates. Idiosyncratic shocks are innovations to the permanent component of the labor income process (which also have an aggregate component since labor income is positively correlated with house price shocks) and temporary labor income shocks.

We first generate one realization for the aggregate shocks. Next, for this realization of aggregate shocks, we generate realizations for the shocks to the labor income process for 50 individuals. We then use the model policy functions, the one path for the aggregate variables, and the individual income shocks to simulate optimal consumption, prepayment, refinancing, and default behavior for these 50 individuals. We repeat this process for 800 paths for the aggregate variables, and for 50 individuals for each of these paths, which yields for each initial value of the aggregate variables, mortgage type, and borrower type a total of 40,000 different paths. We use the same realizations for the shocks to simulate consumption and default behavior for each of the different mortgage types that we study.

To understand the basic properties of the simulated data, in the Internet Appendix we plot the age profiles of cross-sectional average real gross income, consumption, and cash-on-hand. Real consumption is on average considerably lower than real gross income. The reason is that part of gross income must be paid in taxes, and the individual must also make mortgage payments and other housing-related expenditures such as property taxes and maintenance expenses. Part of income is also saved.[16]

In the next section, we use the simulated data to predict unconditional default, prepayment, and refinancing probabilities. These probabilities, calculated across the different paths for the aggregate and idiosyncratic variables in the model, are expected probabilities calculated at the initial date. Ex post, only one of the many possible paths for the aggregate variables will be realized. Section III studies probabilities conditional on a specific path. This analysis allows us to determine the relative contributions of aggregate and idiosyncratic shocks to default. Of particular interest is a path of low interest rates and declining house prices, which replicates the economic conditions that followed the recent U.S. crisis.

II. Unconditional Default Rates

A. Mortgage Default Triggers for ARMs and FRMs

We are interested in determining what triggers default in our model. We focus our attention on home equity and the MTI ratio. The empirical literature on mortgage default emphasizes the importance of home equity for the default

[16] Although not completely visible in the figure provided in the Internet Appendix, there is a slight decline in the average real consumption profile with age. This happens for two main reasons. First, the consumption profile is an average across many aggregate states, including those with declining house prices (and income). Second, we estimate an average growth rate of house prices higher than labor income (in levels, not in logs), and house price increases also drive up housing-related expenses.

decision (see, for example, Deng, Quigley, and Van Order (2000), or more recently Foote, Gerardi, and Willen (2008), and Bajari, Chu, and Park (2008)).

To measure home equity, we calculate for each household i and each date t the current debt outstanding as a fraction of current house value

$$LTV_{ijt} = \frac{D_{ijt}}{P_t P_t^H H_{it}}, \tag{28}$$

where D_{ijt} denotes the loan principal amount outstanding on mortgage j at date t, P_t the price level, and P_t^H the real price of housing. An LTV_{ijt} greater than one corresponds to negative home equity. Equation (28) shows that negative home equity tends to occur for a particular combination of the state variables: house prices are declining, the price level is low, and there are large mortgage balances outstanding (i.e., early in the life of the loan).[17]

In Figure 2 we plot default probabilities for ARMs conditional on the level of negative equity. These probabilities are shown as solid lines in four alternative cases. In panels A and B we plot the results for the baseline level of income risk (a standard deviation of temporary income shocks of 0.225), and in Panels C and D we plot results for a higher level of income risk (a standard deviation of 0.35). Further, Panels A and C show the results for a low initial interest rate (defined as the lowest interest rate in our discretization of the model), while Panels B and D show the results for a high initial interest rate (defined as the second-highest discrete interest rate, since the highest rate is extreme and rarely observed). We use these two levels of interest rates throughout our presentation of results to illustrate the properties of the model.

The default probabilities in Figure 2 are calculated using one observation per mortgage, so that, for those households that choose never to default, even in the face of negative equity, we calculate these probabilities using the lowest level of equity that the household observes over the life of the mortgage. This approach is similar to that used by Bhutta, Dokko, and Shan (2010), who study default rates for nonprime borrowers from Arizona, California, Florida, and Nevada.

Figure 2 shows that few households default at low levels of negative home equity. For most of the cases considered, the probability of default is less than 10% for LTVs up to 1.3. Thus, households only exercise their option to default when it is considerably in the money. This prediction of our model is consistent with Bhutta, Dokko, and Shan (2010), who find that the median homeowner does not default until equity falls to −62% of their home's value, and with Foote, Gerardi, and Willen (2008), who study 100,000 homeowners in Massachusetts who had negative equity during the early 1990s and find that fewer than 10% of these owners lost their home to foreclosure.

[17] In our model the probability of negative equity first rises, as negative shocks have time to erode initially positive home equity, then declines later in the life of the mortgage, as the loan is repaid, as inflation erodes the value of the outstanding nominal debt, and as real house prices (on average) increase. This explains why most defaults occur in the first half of the life of the loan. Schwartz and Torous (2003) find in regressions aimed at explaining default rates that the age of the mortgage plays an important role.

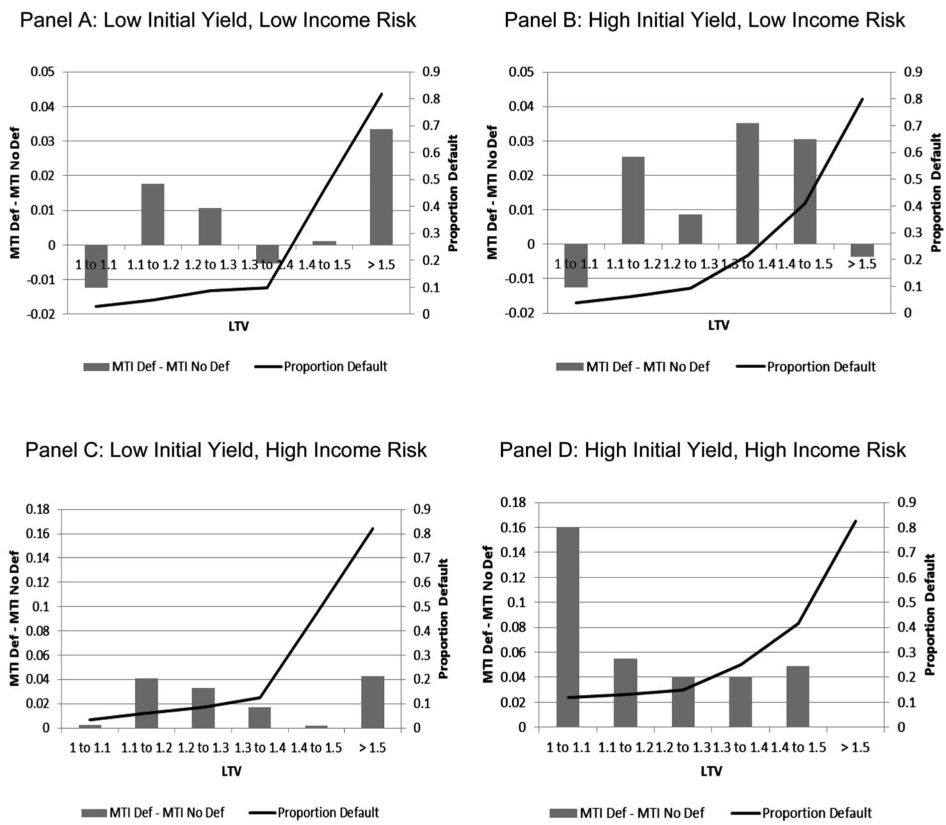


Figure 2. Difference in MTI between households that default and households that do not default and the proportion of defaults as a function of home equity for the ARM contract. The data are obtained by simulating the model for the ARM contract. Low (high) income risk refers to households with a standard deviation of temporary income shocks equal to 0.225 (0.35). LTV is the ratio of current house value to the outstanding loan principal. The vertical bars plot, for each level of negative home equity, MTI for those households that choose to default minus MTI for those households that choose not to default.

The prediction that borrowers do not default as soon as home equity becomes negative is a prediction of all default models based on real option theory. A special feature of our model is that the MTI ratio also plays an important role:

$$MTI_{ijt} = \frac{M_{ijt}}{L_{it}}. \tag{29}$$

At the most basic level this is illustrated by the fact that ARM default rates are higher for borrowers with high labor income risk who take out ARMs at high initial rates (Panel D of Figure 2).

The bars in Figure 2 show, for each level of negative equity, the difference in current MTI between those households who choose to default and those who choose not to default. Focusing first on the case of low income risk, at very low levels of negative home equity the few borrowers who default do so because they are forced to move. This explains the fairly small (and even slightly negative) differences in MTI between the two groups of borrowers. When home equity becomes more negative and initial interest rates are high (Panel B), the MTI ratio becomes more important for the default decision. Its importance is most visible in Panel D, where the combination of high initial rates and high income risk leads households to endogenously default at relatively low levels of negative home equity, and where there are large differences in current MTI between defaulting and nondefaulting borrowers. Large MTI, in the presence of borrowing constraints and low savings, forces a choice between severe consumption cutbacks and mortgage default. Elul et al. (2010) provide empirical evidence of the importance of liquidity considerations for mortgage default decisions.

The default probabilities in Figure 2 show that, at high levels of negative home equity, the vast majority of borrowers decide to default. At these levels, wealth motives tend to be an important determinant of default decisions. This is consistent with the empirical findings of Haughwout, Okah, and Tracy (2014). They study mortgage redefault using data on subprime mortgage modifications for borrowers who were seriously delinquent and whose monthly mortgage payment was reduced as part of the modification. They find that the redefault rate declines relatively more when the payment reduction is achieved through principal forgiveness as compared to lower interest rates. The empirical analysis of Doviak and MacDonald (2012) also emphasizes the role of modifications that reduce loan balances in preventing default.[18]

To better understand the importance of wealth and cash-flow motives for mortgage decisions, Table II reports the means of several variables for ARM borrowers who choose to default, for borrowers with negative home equity but who choose not to default, for borrowers who choose to cash out, and for borrowers who take no action (regardless of whether they have negative home equity). In contrast with Figure 2, in this table each household-date pair is an observation, so any given mortgage is observed multiple times and possibly in multiple states.

As before, we report results for low and high initial interest rates, and for low and high income risk. Across these four cases, we see that households with negative home equity that default tend to have more negative home equity than those with negative home equity but who choose not to default. In addition, households that choose to default are those with lower income and larger MTI. The larger MTIs are also the result of higher nominal interest rates. The difference in MTIs is larger when initial interest rates and income risk are high: in this case the average MTI is equal to 0.40 for households that default compared to an average MTI ratio of 0.34 for households with negative equity that choose not to default. Table II also reports the difference between mortgage and rent payments scaled by household income. For households significantly underwater that choose to default, doing so allows for a reduction in

[18] Das (2012) and Foote, et al. (2009) provide model-based analysis of mortgage modification.

Table II

Means for Different Variables for the ARM Contract by Household Action, for Different Levels of Income Risk, and Conditional on Initial Interest Rates

This table reports the mean for several variables for the ARM contract by household action (default, no default given negative home equity, cash-out, no action). The table reports means across aggregate states and individual shocks, conditional on the initial level of interest rates. Low (High) initial rate corresponds to the state with the lowest (second-highest) level of interest rates in our model. The top (bottom) panels report results for the case in which the standard deviation of income shocks is equal to 0.225 (0.35). For each case the first column reports means for observations in which individuals choose to default, the second column reports means for observations in which individuals have negative home equity but choose not to default, the third column reports means for observations in which individuals choose to cash out, and the last column reports means for observations in which individuals choose neither to default nor to cash out (in case they have not done so before). Current LTV is the loan-to-value at the time of the action (or no action). In the means reported, each observation corresponds to an individual and time period. The probabilities of default and cash-out prepayment are the proportion of households who choose to default or cash out over the life of the mortgage.

	Panel A: Low Initial Rate, Low Income Risk				Panel B: High Initial Rate, Low Income Risk			
	Def	No def|Eq< 0	Cash-out	No act	Def	No def|Eq<0	Cash-out	No act
Current LTV	1.43	1.11	0.42	0.55	1.41	1.15	0.43	0.54
Price level	1.17	1.08	1.29	1.26	1.23	1.17	1.35	1.32
Real house price	0.45	0.69	1.34	1.08	0.45	0.59	1.32	1.07
Real income	46.6	47.7	48.8	52.5	46.3	48.0	48.5	52.6
Real cons t − 1	13.9	15.1	13.4	14.9	13.4	14.2	12.9	14.5
Mort/Inc	0.33	0.29	0.31	0.28	0.33	0.33	0.33	0.30
(Mort-Rent)/Inc	0.26	0.22	0.05	0.12	0.26	0.24	0.05	0.11
Nom int rate	0.037	0.022	0.041	0.037	0.039	0.038	0.047	0.044
Age	36.4	33.2	39.3	38.3	36.4	34.6	39.0	38.3
Probability	0.044		0.583		0.037		0.595	
	Panel C: Low Initial Rate, High Income Risk				**Panel D: High Initial Rate, High Income Risk**			
	Def	No def|Eq<0	Cash-out	No act	Def	No def|Eq<0	Cash-out	No act
Current LTV	1.41	1.11	0.44	0.56	1.33	1.15	0.46	0.55
Price level	1.16	1.08	1.28	1.25	1.20	1.17	1.32	1.31
Real house price	0.46	0.69	1.32	1.07	0.51	0.59	1.29	1.07
Real income	47.3	49.5	48.2	54.4	43.3	50.3	46.8	54.7
Real cons t − 1	14.1	15.4	13.5	15.5	12.6	14.4	12.8	15.2
Mort/Inc	0.35	0.30	0.35	0.29	0.40	0.34	0.38	0.31
(Mort-Rent)/Inc	0.28	0.23	0.07	0.12	0.30	0.25	0.08	0.12
Nom int rate	0.037	0.022	0.041	0.036	0.040	0.038	0.047	0.044
Age	36.4	33.2	39.0	38.2	35.5	34.5	38.2	38.2
Probability	0.046		0.602		0.048		0.625	

current expenditure of between 26% and 30% of income (depending on the case considered).

These results illustrate the fact that, in our model, default is driven by both wealth and cash-flow considerations. House price declines lead to negative home equity. Households that face larger house price declines, particularly when outstanding debt is large, are more likely to default. Since house price shocks are correlated with permanent income shocks, larger house price declines tend to be associated with larger decreases in household income. This forces households to cut back on nondurable consumption. For ARMs such cutbacks are more severe when interest rates are high, since they lead to an increase in mortgage payments. This can be seen in Table II, as the average level of consumption is lowest among high-income-risk borrowers just prior to default (Panel D). The last row of each panel in Table II reports probabilities of default. They are higher when income risk is higher, but the increase is more pronounced for ARMs taken at times when initial interest rates are high. Interestingly, higher income risk means that borrowers default on average at lower LTVs.

Table II also characterizes those households that decide to access their home equity (i.e., cash out). Compared to no action, these households have on average more home equity, mainly as a result of larger increases in house prices. Furthermore, they face higher interest rates and higher MTI, and have lower levels of income and consumption prior to the decision to cash out. This combination motivates their decision to tap into their home equity. When income risk is higher, households on average tap into their home equity at slightly higher LTVs.

Turning to FRMs, in Table III we see that, when initial interest rates are low, default rates for FRMs are lower than for ARMs. However, the reverse is true when initial interest rates are high. The reason is simple. When initial interest rates are high, mortgage providers must charge borrowers for the option to refinance the loan. This increases the premium and the average payments of FRMs, which makes them particularly expensive in an environment of declining house prices and low interest rates. Negative home equity prevents borrowers from refinancing the loan, while low interest rates lead to a lower user cost of housing and lower rental payments compared to mortgage payments. On the other hand, for ARMs default tends to occur when nominal interest rates are high, since high interest rates lead to large mortgage payments.

Table III also reports summary statistics for those borrowers who decide to cash out or to refinance their FRMs. The determinants of the decision to cash out are similar to those for ARMs: large house price increases, lower income, and higher MTI motivate borrowers to tap into their home equity. When initial interest rates are high, the probability of early mortgage termination as a result of a cash out is considerably smaller. The reason is that the mortgage is more likely to be terminated as a result of an interest rate refinancing. Not surprisingly, borrowers tend to refinance when interest rates are low. Borrowers who face higher income risk are more likely to default or cash out, and less likely to terminate their loan with a refinancing.

Table III

Means for Different Variables for the FRM Contract by Household Action, for Different Levels of Income Risk, and Conditional on Initial Interest Rates

This table reports the mean for several variables for the FRM contract by household action (default, no default given negative home equity, cash-out, interest rate refinance, no action). The table reports means across aggregate states and individual shocks, conditional on the initial level of interest rates. Low (High) initial rate corresponds to the state with the lowest (second-highest) level of interest rates in our model. The top (bottom) panels report results for the case in which the standard deviation of income shocks is equal to 0.225 (0.35). For each case the first column reports means for observations in which individuals choose to default, the second column reports means for observations in which individuals have negative home equity but choose not to default, the third column reports means for observations in which individuals choose to cash out, the fourth column reports means for observations in which individuals choose to refinance to take advantage of lower interest rates, and the last column reports means for observations in which individuals choose not to default, cash out, or refinance (in case they have not done so before). Current LTV is the loan-to-value at the time of the action (or no action). In the means reported, each observation corresponds to an individual and time period. The probabilities of default, cash-out prepayment, and refinancing are the proportion of households who choose to default, cash out, or refinance over the life of the mortgage.

	Panel A: Low Initial Rate, Low Income Risk				Panel B: High Initial Rate, Low Income Risk				
	Def	No def|Eq<0	Cash-out	No act	Def	No def|Eq<0	Cash-out	Ref	No act
Current LTV	1.42	1.11	0.41	0.54	1.41	1.17	0.47	0.61	0.57
Price level	1.16	1.08	1.29	1.26	1.22	1.17	1.31	1.24	1.31
Real house price	0.47	0.68	1.34	1.08	0.46	0.59	1.38	0.98	1.15
Real income	46.8	47.7	49.3	52.4	46.9	47.4	46.9	54.0	51.5
Real cons t − 1	13.9	15.0	13.7	14.8	13.0	13.6	12.2	14.3	13.9
Mort/Inc	0.32	0.33	0.28	0.27	0.38	0.39	0.37	0.26	0.34
(Mort-Rent)/Inc	0.26	0.26	0.03	0.10	0.32	0.29	0.06	0.21	0.10
Nom int rate	0.027	0.023	0.040	0.037	0.031	0.042	0.050	0.008	0.053
Age	36.1	33.3	39.5	38.5	36.0	34.3	37.5	36.1	37.3
Probability	0.034		0.572		0.051		0.369	0.471	

	Panel C: Low Initial Rate, High Income Risk				Panel D: High Initial Rate, High Income Risk				
	Def	No def|Eq<0	Cash-out	No act	Def	No def|Eq<0	Cash-out	Ref	No act
Current LTV	1.40	1.11	0.43	0.55	1.32	1.17	0.52	0.60	0.61
Price level	1.15	1.08	1.28	1.26	1.19	1.17	1.26	1.24	1.29
Real house price	0.48	0.68	1.32	1.07	0.52	0.60	1.31	1.01	1.12
Real income	47.2	49.5	48.7	54.5	44.0	49.7	44.5	56.9	53.5
Real cons t − 1	13.8	15.1	13.7	15.5	11.8	13.8	11.9	14.7	14.7
Mort/Inc	0.34	0.34	0.31	0.28	0.46	0.41	0.44	0.26	0.36
(Mort-Rent)/Inc	0.28	0.27	0.05	0.11	0.37	0.31	0.12	0.21	0.12
Nom int rate	0.027	0.023	0.039	0.037	0.034	0.042	0.050	0.008	0.054
Age	36.0	33.3	39.0	38.4	35.2	34.3	36.4	36.0	36.8
Probability	0.035		0.593		0.068		0.397	0.453	

Table IV

Probabilities of Default, Cash-Out Prepayment, and Interest-Rate Refinancing, Lender Profitability, and Mortgage Premia, Conditional on Initial Interest Rates

This table reports results for mortgage contracts with LTV=0.9 and LTI=4.5 and for a standard deviation of temporary income shocks equal to 0.225, for low and high initial one-year bond yields. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. For each of these levels, and for both ARM and FRM contracts, the table reports the mortgage premium required by the lender, the initial mortgage payments relative to income, the probability of default, and the probability of cash-out prepayment. For the FRM contract, the table also reports the probability of interest rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The table also reports lenders' average profitability as a function of households' decisions. Profitability is calculated as the present discounted value of the cash flows that lenders receive divided by the initial loan amount. The last row reports the welfare gains of ARMs relative to FRMs, under consumption-equivalent variations. The table reports the percentage difference in the constant consumption stream that makes the idividual as well off in the ARM contract as in the FRM contract.

Initial One-Year Bond Yield	Low	High
Panel A: ARM		
Prem over one-year bond yield	1.50%	1.60%
Initial mort payment/Inc	0.234	0.410
Prob(Default)	0.044	0.037
Prob(Cash-out)	0.583	0.595
Profitability(Default)	−0.175	−0.139
Profitability(Cash-out)	0.079	0.077
Profitability(Other)	0.162	0.160
Panel B: FRM		
Prem over 20-year bond yield	0.75%	2.85%
Prem over 20-year annuity yield	1.69%	2.63%
Initial mort payment/Inc	0.344	0.433
Prob(Default)	0.034	0.051
Prob(Cash-out)	0.572	0.369
Prob(Refinancing)	0.000	0.471
Profitability(Default)	−0.122	−0.094
Profitability(Cash-out)	0.086	0.077
Profitability(Refinancing)	0.000	0.122
Profitability(Other)	0.137	0.166
Panel C: ARM/FRM		
Welfare gain of ARM	−0.12%	1.12%

B. Mortgage Premia and Profitability

Table IV reports mortgage premia for the same two initial levels of one-year bond yields that we used in Figure 2. The column "low initial yield" reports the results for the lowest level of interest rates in our model, corresponding to a positively sloped term structure. The column "high initial yield" reports results for the second-highest level of initial interest rates, corresponding to an almost flat term structure. Results for other levels of interest rates are reported in the Internet Appendix.

The mortgage premia reported in Table IV are determined endogenously so that mortgage providers are able to achieve risk-adjusted discounted profitability of 10%.[19] This is gross profitability (before expenses incurred by banks) expected at the initial date, that is, averaging across the different possible paths for the aggregate variables. Ex post, only one of these aggregate paths will be realized. The table also reports conditional probabilities of default, cash-out prepayment, and FRM refinancing, and the profitability associated with each of these cases (which is lowest in the event of default, intermediate for cash-out prepayment and FRM refinancing, and highest if none of these events occur).

Focusing first on the results for ARMs, Panel A shows that the required mortgage premium is almost constant but slightly increasing in the level of initial interest rates. This pattern results from three offsetting effects. First, the ARM default probability declines with the level of initial interest rates. Although high initial interest rates imply a high initial MTI ratio as shown in the table, reducing mortgage affordability, high initial rates and inflation also imply that outstanding nominal mortgage balances are eroded faster by inflation, so households are likely to have lower LTVs later in the life of the mortgage. For the baseline parameters the latter effect dominates (but in Section II.D below we will show that the mortgage affordability effect dominates for households that face higher income risk, so for these borrowers the default probability increases with the level of initial interest rates). Second, the probability of an ARM cash-out increases with the level of initial interest rates. Third, the profits generated by the mortgage premium are discounted more heavily when interest rates are initially high. The first effect makes the ARM premium decrease with the level of initial interest rates, but the second and third effects make it increase, and these dominate in the benchmark case.

Panel B reports the results for FRMs. We report endogenously determined mortgage premia calculated over two different benchmark yields. The first is the premium over the yield on a 20-year zero-coupon bond. The second is the premium over the yield on a 20-year annuity priced using the initial term structure of interest rates. The latter is a more reasonable benchmark since mortgages make constant payments like annuities, and therefore have lower duration than zero-coupon bonds of the same maturity. For this reason we focus the discussion on annuity-relative premia to capture the pure compensation that mortgage providers require for default, prepayment, and refinancing risk.

The required mortgage premium for FRMs increases with the level of initial yields much more steeply than it does for ARMs. The main reason is the presence of the interest rate refinancing option. A higher initial yield increases both

[19] We chose this level to try to quantitatively match the average premia observed in the data. We report results for other levels of risk-adjusted profitability in Section II.E and compare the model with the data in Section III.C.

the value of this option and the probability that it will be exercised. Lenders must be compensated for refinancing risk through a higher mortgage premium. The higher premium increases the likelihood of default when borrowers face negative home equity that prevents them from exercising the option. This explains why default probabilities now increase with the level of initial rates, from 0.034 to 0.051. Further, profitability in the case of default is higher than for ARM contracts. This is mainly due to the fact that FRM borrowers tend to default when interest rates are low, in which case the present value of the recovered house is higher. Also, higher initial interest rates mean that FRMs are more likely to be terminated as a result of interest rate refinancing and less likely to be terminated as a result of borrowers wishing to tap into their home equity.

Panel C of Table IV reports borrower welfare benefits for ARMs relative to FRMs. These welfare benefits are calculated as consumption-equivalent variations, or the percentage of the (constant) consumption-equivalent stream that individuals would be willing to give up to have an ARM contract instead of an FRM contract. These calculations also tell us what the mortgage choices of individuals at the initial date would be. When initial rates are low, borrowers are less likely to prefer an ARM, but the difference relative to an FRM is not large at -0.12%. When initial rates and MTI are low, borrowers are better positioned to meet the relatively higher initial mortgage payments of FRMs. At the same time, the likelihood that interest rates will increase is large, which reduces the appeal of ARMs, and increases the incentives to lock in the low initial rate. As initial interest rates and the MTI ratio increase, borrowers become less willing to pay the additional premium that FRMs require.

B.1. FRM Refinancing Inertia

In our model households exercise the interest rate refinancing option of FRMs optimally. At higher levels of initial interest rates the value of the option is larger, which together with the expectation of optimal household exercise implies that mortgage providers require a much larger premium at origination. However, Miles (2004) and Campbell (2006) present evidence that many households do not refinance when it would be optimal to do so, which implies that there is some degree of household refinancing inertia. We evaluate, in the context of our model, the effects of such inertia on mortgage premia. We model inertia in a simple way, assuming that in each period there is a probability, $p_{inertia}$, that households do not refinance even though it would be optimal to do so. If as a result of inertia households do not refinance immediately, they may do so in the following period provided that it still is optimal to refinance and they do not suffer from further inertia. Households are aware of their degree of inertia and make consumption and mortgage decisions taking it into account. Mortgage providers are also aware of the degree of household inertia and price mortgages accordingly.

The results for different levels of inertia are shown in Table V. In the first column we report the results for the lowest level of initial interest rates for

Table V

Inertia in Interest Rate FRM Refinancing

This table reports results for different levels of inertia in interest rate FRM refinancing. The first row reports the initial level of one-year rates. Low (High) initial yield corresponds to the state with the lowest (second-highest) level of interest rates in our model. This table reports results for the baseline case in which there is no inertia, and for the cases in which in each period 50% of the individuals who would benefit from refinancing do so, in each period 30% of the individuals who would benefit from refinancing do so, and no individual refinances. For each of these cases the table reports the morgage premium, the ratio of initial MTI, the probability of default, the probability of cash-out prepayment, and the probability of interest rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs, under consumption-equivalent variations.

Initial One-Year Bond Yield	Low Initial Yield	High Initial Yield			
Level of Inertia	Baseline	Baseline	Inert = 0.5	Inert = 0.7	Inert = 1.0
Prem over 20-y ann yield	1.69%	2.63%	2.18%	1.88%	1.38%
Initial mort payment/Inc	0.344	0.433	0.428	0.418	0.401
Prob(Default)	0.034	0.051	0.046	0.043	0.039
Prob(Cash-out)	0.572	0.369	0.424	0.482	0.584
Prob(Refinance)	0.000	0.471	0.367	0.250	0.000
Welfare gain of ARM	−0.12%	1.12%	0.72%	0.35%	−0.30%

which the refinancing option is not relevant. In the columns to the right we report the results for a high initial yield, for the baseline parameters in which there is no inertia, and for $p_{inertia}$ equal to 0.5, 0.7, and one. The latter extreme case of inertia corresponds to a situation in which the option to refinance the FRM is not available.

The first row of Table V shows that the mortgage premia required by lenders decrease considerably with mortgage inertia. As expected, households with greater inertia are less likely to terminate their mortgage contract as a result of interest rate refinancing, but are more likely to terminate it as a result of a cash-out. In addition, due to the decrease in initial mortgage premia, default probabilities decrease as inertia increases. The last row of Table V reports the welfare gains of ARMs relative to FRMs. As inertia increases and the required FRM premium decreases, the welfare gains of ARMs decrease and become negative. This illustrates the interesting point that households may be better off with FRMs that are harder to refinance, because such mortgages are cheaper in equilibrium and the refinancing option may not justify its interest cost.

C. The Effects of Initial LTV and LTI on Default

We now ask how LTV and LTI ratios at mortgage origination relate to mortgage premia and default rates. We are particularly interested in LTI given the significant increase in average LTI during the 2000s illustrated in Figure 1. One important advantage of using a model to study the effect of LTI is that we can compare outcomes across LTI for a common set of shocks to the households in the model.

With our analysis of mortgage default triggers in mind, we write the probability of default as the probability that the household faces negative home equity times the probability of default conditional on negative home equity,

$$\Pr(Default) = \Pr(Equity < 0) \times \Pr(Default|Equity < 0). \tag{30}$$

When calculating these probabilities, we classify as having negative home equity those households whose house value net of the transaction costs of a house sale is lower than outstanding debt. Since there are a few instances of default when house value is slightly higher than remaining debt, the classification of negative home equity using house value net of transaction costs ensures that the above equation holds exactly. Also, the probability of negative home equity is calculated as the probability that the borrower faces at least one period of negative equity during the life of the mortgage.

The results are reported in Table VI. Panel A shows the results for ARMs, and Panel B for FRMs, with a low initial interest rate scenario at the left and a high initial interest rate scenario at the right. For each scenario, we consider three cases. In the first column we report the results for the baseline case, with an LTV of 0.90 and an LTI of 4.5. The probability of negative home equity tends to decline with the level of initial rates. There are two opposing effects. The higher the initial rates, the higher the proportion of mortgage payments that cover interest payments and the lower the initial reduction in principal outstanding. On the other hand, higher initial expected inflation and nominal interest rates mean that nominal house prices are more likely to increase, which reduces the likelihood of negative home equity. Interestingly, we see that the probability of default conditional on negative equity is higher for ARMs than for FRMs for low initial rates, but the reverse is true for high initial rates. For low initial rates, the ARM borrowers who eventually default tend to be those who subsequently face house price declines and higher interest rates. FRM borrowers who locked in a low rate at the initial date are not as affected by the subsequent increase in rates and therefore are less likely to default. In Panel C we report the welfare gains of ARMs relative to FRMs.

In the second column we report results for a lower LTI equal to 3.5. Focusing first on the ARM, we see that default probabilities are now lower. The main reason is the lower probability of default in the case of negative equity, with a smaller effect on the probability of negative equity. The lower the initial LTI, the lower are MTI, which makes liquidity constraints less severe and also makes it less likely that households default when facing negative equity. Furthermore, due to the lower MTI, households in a lower LTI loan have less of an incentive to tap into their home equity. The reduction in default and cash-out probabilities contribute to a reduction in mortgage premia that is larger for a high initial yield.

For FRM contracts, and similar to ARM contracts, borrowers in lower LTI loans are less likely to default or to tap into their home equity. However, there

Table VI

Initial LTI and LTV

This table reports results for different initial levels of LTI and LTV and different initial values of the one-year bond yield. The first row reports the initial level of one-year rates. Low (High) initial yield corresponds to the state with the lowest (second-highest) level of interest rates in our model. The table shows results for different values of the initial LTI and LTV. The baseline case is LTI=4.5, LTV=0.90. Lower LTI corresponds to LTI=3.5, LTV=0.90, and lower LTV to LTI=4.5, LTV=0.80. This table reports results for households facing a standard deviation of temporary income shocks equal to the baseline value of 0.225. The table reports the mortgage premium required by lenders, the ratio of initial MTI, and the probability of default decomposed into the probability of negative equity and the probability of default conditional on negative home equity. It also reports the probabilities of cash-out prepayment, and for the FRM contract, interest rate refinancing. The table reports probabilities calculated across aggregate states and individual shocks. Negative home equity corresponds to situations in which $(1-c) \times$ Nominal house value < Outstanding debt. The last row of the table reports welfare gains of ARMs relative to FRMs, under consumption-equivalent variations.

Initial One-Year Bond Yield	Low Initial Yield			High Initial Yield		
Initial LTI, LTV	Baseline	Lower LTI	Lower LTV	Baseline	Lower LTI	Lower LTV
Panel A: ARM						
Prem over one-y yield	1.50%	1.45%	1.45%	1.60%	1.50%	1.55%
Initial mort payment/Inc	0.234	0.181	0.232	0.410	0.316	0.408
Prob(Default)	0.044	0.041	0.023	0.037	0.034	0.018
Prob(Equity< 0)	0.554	0.548	0.278	0.538	0.538	0.271
Prob(Def|Equity<0)	0.080	0.075	0.082	0.069	0.063	0.067
Prob(Cash-out)	0.583	0.508	0.646	0.595	0.513	0.656
Panel B: FRM						
Prem over 20-y ann yield	1.69%	1.69%	1.64%	2.63%	2.73%	2.28%
Initial mort payment/Inc	0.344	0.267	0.342	0.433	0.339	0.421
Prob(Default)	0.034	0.033	0.013	0.051	0.049	0.023
Prob(Equity<0)	0.548	0.548	0.275	0.548	0.548	0.268
Prob(Def|Equity<0)	0.061	0.061	0.047	0.094	0.090	0.087
Prob(Cash-out)	0.572	0.508	0.628	0.369	0.281	0.473
Prob(Refinance)	0.000	0.000	0.000	0.471	0.543	0.389
Panel C: ARM/FRM						
Welfare gain of ARM	−0.12%	−0.10%	−0.04%	1.12%	1.01%	0.87%

is an additional effect: when initial interest rates are high, borrowers are now much more likely to refinance the loan if interest rates subsequently decline, an event for which lenders must be compensated ex ante by a higher mortgage premium. In spite of the increase in FRM premia for high initial yields, the welfare gains of ARMs for lower LTI borrowers are lower than in the baseline case. Lower LTIs and lower initial mortgage payments mean that households are less borrowing constrained and benefit less from the relatively lower initial MTI of ARMs.

In the third column of each Table VI scenario we study the effects of a reduction in LTV from 0.9 to 0.8. Not surprisingly, a lower LTV reduces the probability of negative home equity. Quantitatively, this leads to a very large reduction in the probability of default. Krainer, Leroy, and Mungpyung (2009) develop an equilibrium valuation model that emphasizes the role of the initial LTV for mortgage default. The lower default rate means that the ARM premia required by lenders are generally lower. We say "generally" because a lower LTV implies higher home equity for households and increases the probability of early mortgage termination through a cash-out. Since cash-out prepayment is unprofitable for mortgage lenders, a higher probability of a cash-out leads to an increase in premium that offsets the decrease due to the lower probability of default. However, for the cases considered in Table VI, the reduction in default probability is the dominant effect.

For FRMs the reduction in LTV also leads to a reduction in default probabilities and an increase in the probability of a cash-out. In addition, there is a significant decrease in the probability that the loan will be refinanced. The latter effect explains why, when initial rates are high, the reduction in the mortgage premia required by lenders when LTV decreases from 0.9 to 0.8 is larger for FRMs than for ARMs. This differential reduction also explains the decrease in the welfare benefits of ARMs relative to FRMs.

The results in Table VI show that there is a differential sensitivity of default rates of FRMs and ARMs to LTI and LTV ratios. On the one hand, default rates for FRMs decrease less with a decrease in LTI than do default rates for ARMs, particularly at low levels of initial interest rates. On the other hand, default probabilities for FRMs are more sensitive to LTV than are default probabilities for ARMs. This differential sensitivity can be understood in light of our previous analysis of default triggers. For ARMs a higher proportion of individuals default for cash-flow reasons. A higher LTI implies larger MTI, which makes borrowing constraints more likely to bind. On the other hand, for FRMs, a higher proportion of individuals default for wealth reasons. This makes default rates for these mortgages more sensitive to the LTV ratio. This distinction between the cash-flow risk of ARMs and the wealth risk of FRMs is emphasized by Campbell and Cocco (2003).

D. Borrower Heterogeneity

In the previous sections, we study mortgage default for different initial LTV and LTI ratios, and for different mortgage types, but for fixed household preference parameters. In reality borrowers are heterogenous, which affects portfolio choice (Curcuru et al. (2010)) and is also likely to affect mortgage choice. With this in mind, in this section we investigate further the effects of household characteristics on mortgage premia, default rates, and borrower welfare. Recall that we assume that banks can observe household characteristics, and price loans accordingly.[20]

D.1. Labor Income Risk

Table VII shows the results for the case in which borrowers face a higher standard deviation of temporary labor income shocks, equal to 0.35 (in the column labeled "Higher"). In Panel A we see that, when labor income risk is higher, ARM borrowers are more likely to default if house prices subsequently decline, or to cash out if house prices subsequently increase. The increases in these probabilities are larger when initial rates are high, and so are the additional ARM premia that banks require to lend to riskier borrowers. Qualitatively, the effects are similar for FRMs (Panel B). However, quantitatively there are interesting differences. For high initial rates the increases in default probabilities and mortgage premia are higher for FRMs than for ARMs. The additional mortgage premia that lenders require from riskier borrowers increase the average level of mortgage payments. When initial rates are high, this affects FRM borrowers more than ARM borrowers, since the former need to meet higher initial mortgage payments due to the refinancing option, which increases the premium on FRMs. This also explains why, when initial interest rates are high, the benefits of ARMs relative to FRMs are larger for riskier borrowers (Panel C).

In the columns labeled "Correlated" we study the effects of allowing labor income realizations to depend on the level of interest rates. As in the case of higher income risk, we set the overall standard deviation of temporary labor income shocks equal to 0.35. In the higher income risk scenario the average level of the log temporary income shock is zero for all levels of the short rate. In the correlated scenario, the average level of the log temporary labor income shock is related to the level of interest rates. It is equal to -0.49 when one-year yields are at their lowest level, it increases to -0.07 for the second-lowest level of one-year yields, followed by 0.07 and 0.49. The motivation for this type of income risk is simple: interest rates tend to be lower in recessions, which reduce the labor income that some households receive. Naturally, recessions affect some workers more than others, so that the scenario with higher income risk related to the short rate should not be seen as representative of the situation of all borrowers.

Some interesting patterns emerge. First, the probabilities of default and cash-out refinancing are higher for lower initial rates, and so is the

[20] Furthermore, we assume that the pricing kernel is the same as the one previously derived. The assumption is that the representative agent has our baseline preferences and other parameters. It would be interesting to investigate mortgage pricing for a population of heterogeneous households whose characteristics can only be imperfectly observed by banks.

Table VII

Different Levels of Labor Income Risk

This table reports results for different types of income risk and different initial values of the one-year bond yield. The first row reports the initial level of one-year rates. Low (High) initial yield corresponds to the state with the lowest (second-highest) level of interest rates in our model. This table reports results for households facing a standard deviation of temporary income shocks equal to 0.225, for those facing a higher income risk (a standard deviation of temporary labor income shocks equal to 0.35), and for those facing higher income risk that is correlated with the level of real interest rates (correlated). For each of these cases, and for the ARM and FRM contracts, the table reports the mortgage premium, the probability of default, the probability of cash-out prepayment, and for the FRM contract, the probability of interest rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs under consumption-equivalent variations.

Initial One-Year Bond Yield	Low Initial Yield			High Initial Yield		
Income Risk	Baseline	Higher	Correlated	Baseline	Higher	Correlated
			Panel A: ARM			
Prem over one-y yield	1.50%	1.55%	1.85%	1.60%	1.75%	1.75%
Prob(Default)	0.044	0.046	0.064	0.037	0.048	0.051
Prob(Cash-out)	0.583	0.602	0.659	0.595	0.625	0.621
			Panel B: FRM			
Prem over 20-y ann yield	1.69%	1.74%	3.29%	2.63%	2.98%	3.68%
Prob(Default)	0.034	0.035	0.133	0.051	0.068	0.102
Prob(Cash-out)	0.572	0.593	0.695	0.369	0.397	0.361
Prob(Refinance)	0.000	0.000	0.000	0.471	0.453	0.479
			Panel C: ARM/FRM			
Welfare gain of ARM	−0.12%	−0.13%	3.53%	1.12%	1.29%	3.30%

mortgage premium that lenders require on ARMs. Second, the default probabilities for FRMs are significantly higher than for ARMs. This is due to the hedging properties of ARMs: when interest rates and income are low, so are mortgage payments. The same is not true for FRMs. Furthermore, due to the relation with interest rates, low income realizations tend to occur at times when the rental cost of housing is low, which increases the incentives for FRM borrowers to default. This leads to an increase in the required FRM premia, which is higher for low initial rates. Borrowers who face labor income risk related to interest rates benefit the most from ARMs, particularly so for low initial rates. These results may help explain why in the recent financial crisis, and in spite of the low interest rate environment, ARM borrowers defaulted more than FRM borrowers. We investigate this possibility in Section III.B.

D.2. Labor Income Growth

Households differ in their expected growth rate of labor income. We investigate the impact of this parameter on default, cash-out, and refinance probabilities. In Table VIII we report results for average income growth equal to 1.2% (higher than the baseline value of 0.8%). Compared to the base case, we see that the probabilities of default and cash-out prepayment are now only slightly lower, if affected at all, for both the ARM and the FRM contracts. When expected income growth is higher, there are two effects. On the one hand, households have a lower incentive to save early on, which increases the likelihood of default and cash-out refinancing. On the other hand, the higher income growth leads to a lower future MTI, which improves mortgage affordability. The results in Table VIII show that the latter effect is stronger, and that a crucial parameter when thinking of mortgage affordability is expected income growth. Because of the opposing effects of a higher income growth rate, the quantitative effects on mortgage premia and welfare are very small.

D.3. Discount Factor and Moving Probability

Another potential source of borrower heterogeneity is the discount factor. We report the effects of a lower value, equal to 0.92, in the third column of Table VIII. Due to the lower incentives to save, default and cash-out probabilities are now higher, and so is the ARM premia required by banks to lend to more myopic borrowers. For FRMs we also need to take into account borrower incentives to refinance the loan, which are reduced compared to the base case. The reduction in refinancing probability on lender profits more than offsets the effects of the increase in default and cash-out probabilities, so that for high initial yields the required FRM mortgage premia are lower. The reduction in FRM premia and the increase in ARM premia at intermediate levels of initial rates makes the welfare gains of ARMs smaller for more myopic borrowers.[21]

[21] The effects of a reduction in the parameter b that measures the relative importance of terminal wealth are similar to the effects of a reduction in the discount factor. The average financial savings

Table VIII

Other Household Parameters

This table reports results for different household parameters and different initial values of the one-year bond yield. Low (High) initial yield corresponds to the state with the lowest (second-highest) level of interest rates in our model. This table reports results for households facing the baseline parameters, for those facing a higher growth rate of labor income (equal to 0.012), for those with a lower discount factor (equal to 0.92), and for those facing a higher probability of an exogenous house move (equal to 0.06) and a disutility from default. For each of these cases, and for the ARM and FRM contracts, the table reports the mortgage premium, the probability of default, the probability of cash-out prepayment, and for the FRM contract the probability of interest rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The table reports welfare gains of ARMs relative to FRMs under consumption-equivalent variations.

Initial One-Year Bond Yield	Low Initial Yield				
Parameter	Baseline	Inc Growth	Disc Factor	Mov Prob	Stigma
Panel A: ARM					
Prob(Default)	0.044	0.044	0.048	0.055	0.034
Prob(Cash-out)	0.583	0.569	0.637	0.690	0.586
Prem over one-y yield	1.50%	1.50%	1.60%	1.70%	1.45%
Panel B: FRM					
Prob(Default)	0.034	0.034	0.034	0.042	0.025
Prob(Cash-out)	0.572	0.558	0.603	0.685	0.573
Prob(Refinance)	0.000	0.000	0.000	0.000	0.000
Prem over 20-y ann yield	1.69%	1.69%	1.72%	1.81%	1.64%
Panel C: ARM/FRM					
Welfare gain of ARM	−0.12%	−0.13%	−0.22%	−0.07%	−0.14%
Initial One-Year Bond Yield	High Initial Yield				
Parameter	Baseline	Inc growth	Disc factor	Mov prob	Stigma
Panel D: ARM					
Prob(Default)	0.037	0.036	0.041	0.049	0.029
Prob(Cash-out)	0.595	0.581	0.654	0.702	0.596
Prem over one-y yield	1.60%	1.58%	1.75%	1.80%	1.55%
Panel E: FRM					
Prob(Default)	0.051	0.051	0.052	0.061	0.040
Prob(Cash-out)	0.369	0.356	0.472	0.430	0.371
Prob(Refinance)	0.471	0.477	0.362	0.441	0.475
Prem over 20-y ann yield	2.63%	2.63%	2.48%	2.93%	2.53%
Panel F: ARM/FRM					
Welfare gain of ARM	1.12%	1.08%	0.63%	1.27%	1.07%

The effects of a higher probability of an exogenous move on ARMs are similar to those of a lower discount factor. The probabilities of default and cash-out prepayment increase, and so does the ARM mortgage premium required by lenders. For FRMs the increase in the probability of a cash-out is offset by a reduction in the probability of interest rate refinancing, but this reduction is not very large and thus the FRM premium still increases relative to the baseline case.

D.4. Stigma from Mortgage Default

In a recent empirical paper, Guiso, Sapienza, and Zingales (2013) find that moral and social considerations play an important role in the default decision. We can adapt our model to investigate how such considerations affect default rates for different mortgage types. We assume that in the case of default the household incurs a utility loss, *Stigma*. The household will choose to default, setting $Def^{C}_{ijt} = 1$, whenever the continuation utility with default less the stigma cost is higher than the utility without default:

$$V_{it}(State_t \mid Def^{c}_{ijt} = 1) - Stigma > V_{it}(State_t \mid Def^{C}_{ijt} = 0). \tag{31}$$

The main difficulty with this extension of our model is determining an appropriate value for *Stigma*. In the last column of Table VIII we report the results for $Stigma = 0.05$. To give the reader an idea of what this means, we translate this value into an equivalent per-period consumption loss. For the ARM mortgage, $Stigma = 0.05$ is equal to a decrease in the constant equivalent-consumption stream of 2% per period. The results in Table VIII show that this level of *Stigma* has a significant effect on both default probabilities and mortgage premia that is larger for FRMs than for ARMs.

E. Alternative Mortgages and Lender Profitability

During the recent financial crisis, mortgage delinquency and default rates were particularly high for alternative mortgage products. These come in many different forms, but generally share the feature that they postpone repayments to later in the life of the loan. We use our model to study these mortgages and to compare them to more traditional principal-repayment mortgages.[22]

We model a common type of alternative mortgage, an ARM with a teaser rate. More specifically, we set the mortgage premium equal to zero in the first year, but allow it to increase in subsequent years. The value that it increases to is determined endogenously such that mortgage providers receive the same

at the terminal age for the more myopic ARM borrowers is $99,348. As expected, this value is lower than the financial savings accumulated in the base case, which are equal to $122,405. These values should be compared to the financial wealth held by households in checking and saving accounts, mutual funds, and retirement accounts.

[22] Amromin et al. (2011), Cocco (2013), and Narita (2011) characterize households that borrow using these alternative mortgage products.

Table IX

ARM Contract with a Teaser Rate and Lower Target Profitability

This table reports results for different levels of lender profitability (equal to 0.08) and for an ARM with a teaser rate, for different initial values of the one-year bond yield. Low (High) initial yield corresponds to the state with the lowest (second-highest) level of interest rates in our model. The ARM contract with a teaser rate has an interest rate equal to the one-year bond yield for the first year of the contract, that is reset to a higher value in subsequent years. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs, and of ARMs relative to the ARM teaser, under consumption-equivalent variations.

Initial One-Year Bond Yield	Low Initial Yield		High Initial Yield	
Profitability	Baseline	Lower	Baseline	Lower
		Panel A: ARM		
Prem over one-y yield	1.50%	1.25%	1.60%	1.30%
Prob(Default)	0.044	0.041	0.037	0.034
Prob(Cash-out)	0.583	0.582	0.595	0.592
		Panel B: FRM		
Prem over 20-y ann yield	1.69%	1.44%	2.63%	2.18%
Prob(Default)	0.034	0.031	0.051	0.045
Prob(Cash-out)	0.572	0.570	0.369	0.368
Prob(Refinancing)	0.000	0.000	0.471	0.471
		Panel C: ARM Teaser		
Prem over one-y yield	0%/1.75%		0%/1.85%	
Prob(Default)	0.046		0.040	
Prob(Cash-out)	0.584		0.596	
		Panel D: ARM/FRM/ARM Teaser		
Welfare gain of ARM/FRM	−0.12%	−0.10%	1.12%	1.04%
Welf gain ARM/ARM teaser	0.09%		−0.04%	

risk-adjusted level of expected profitability as in the other mortgages. Panel C of Table IX reports the results; for comparison Panel A (B) reports the results for the ARM (FRM). As expected, after the first year the ARM teaser premium must increase to a level higher than the ARM premium, with a difference that is larger for higher initial rates. The increase in mortgage premia after the first year leads to slightly higher expected default probabilities for the ARM with a teaser rate compared to the plain-vanilla ARM.

The last row of Panel D reports the welfare gains of ARMs relative to ARMs with a teaser rate. Interestingly, for the combination of parameters considered, for high initial interest rates ARM teasers tend to be preferred to plain-vanilla ARMs. At the lowest level of initial rates, mortgage payments relative to income are low, and hence the benefits of the teaser rate are relatively low. There is the risk that interest rates will subsequently increase when the mortgage premium will also be higher, but overall the welfare differences between the plain-vanilla ARM and the ARM with a teaser rate are not very large.

In the baseline case we assume a risk-adjusted level of profitability of 10%. We choose this value so that the mortgage premia that our model predicts on average (roughly) match the mortgage premia observed in the data. But, as we will see in Section III.C below, there is considerable time-series variation in mortgage premia. This variation may in part be supply driven, with mortgage suppliers acting more competitively at some times than at others. We try to capture this by solving our model for a lower level of risk-adjusted profitability of 8%. The results are shown in the columns labeled "Lower" in Table IX.

As expected, mortgage premia are lower when the target level of profitability is lower. Default rates are also lower. Therefore, if the supply of credit is more competitive when interest rates are high than when they are low (for example, because low interest rates tend to coincide with periods of economic weakness), the model predicts that the ARM premium should be declining in the level of interest rates. If low initial rates correspond to target profitability of 10%, the ARM premium with low rates should equal 1.50%, while if high initial rates correspond to target profitability of 8%, the ARM premium with high rates should be 1.30%. We return to this issue in Section III.C, where we compare the model predictions to the data.

III. Conditional Default Rates

In the previous section, we characterize for each level of initial interest rates the mortgage premia, default, cash-out, and refinancing rates predicted by our model, calculated as average rates across the 800 different paths for the aggregate variables that we generate (and across the realizations for the individual labor income shocks). Of course, ex post only one of the paths for the aggregate variables will be realized, and the realized default rates may be higher or lower than those reported.

We now focus on the conditional default probabilities predicted by our model, that is on how default probabilities differ across the different paths for the aggregate variables. From a policy maker's point of view, the concern relates to those states with a large incidence of mortgage default. This analysis also allows us to study the relative contribution of aggregate and idiosyncratic shocks to the default decision. In Section III.A, we study differences in default rates across aggregate states, focusing on those states in which there is a high incidence of mortgage default. In Section III.B, we study default rates for a path of the aggregate variables that resembles the recent U.S. experience, with low interest rates and declining house prices—this path is useful for understanding the financial crisis. In Section III.C, we compare the model predictions to the data.

A. Differences in Default across Aggregate States

Recall that in our model the aggregate shocks are shocks to real house prices, the inflation rate, and the real interest rate. Past realizations of house price and inflation shocks determine the current level of real house prices and the current price level, respectively. When we refer to an aggregate state, we mean one possible combination of these aggregate shocks out of the 800 that we generate.

To characterize the differences in default rates across aggregate states, in Figure 3 we plot the proportion of aggregate states with given default frequencies. Panel A conditions on a low initial interest rate, and Panel B on a high initial interest rate. Results are shown both for the baseline level and for a higher level of labor income risk. The black bars in the figure show the proportion of aggregate states with any defaults at all (in practice, these are states in which house prices fall below the initial level for some period of time). The pale gray bars show the expected default rate, conditional on any defaults occurring in a state, and the dark gray bars (plotted against the different vertical scale shown on the right axis of the figure) show the proportion of aggregate states with an extreme default wave, defined as defaults by at least 80% of outstanding mortgages.

The comparison of Panels A and B, which differ in initial rates, shows that both expected default rates and the probability of a default wave are higher for ARMs when initial rates are low, but higher for FRMs when initial rates are high. The first result is due to the fact that interest rate increases combined with house price declines can trigger ARM default waves. The second result is due to the fact that high initial rates imply large FRM premia. If house prices and interest rates subsequently decline, many FRM borrowers who cannot refinance their loans decide to default. Higher labor income risk increases the number of states in which borrowers find it optimal to default, but the effects are much more pronounced when initial rates are high than when they are low.

To further characterize the different aggregate states, in Figure 4 we plot the average evolution of nominal house prices and interest rates for states with extreme default waves. We plot such averages for both ARMs and FRMs and for two different levels of initial rates (in Panels A and B, respectively) up to age 45, since no default occurs after this age. Not surprisingly, for both ARMs and FRMs, default waves tend to occur in aggregate states with large house prices declines, of roughly 50% on average. Default waves occur 5 to 10 years after mortgage initiation, because it takes time for house prices to decline this far. In addition, for low initial yields, default waves for ARMs tend to occur at higher interest rates than for FRMs. On the other hand, for high initial yields, a larger decline in interest rates triggers FRM default. However, the differences between ARM and FRM interest rates are not very large, reflecting the fact that, in states of large house price declines, house prices and the level of negative equity become the most important determinants of the default decision.

B. Recent U.S. Experience

A path for the aggregate variables that is particularly interesting to analyze is one that matches the recent U.S. experience, which is characterized by declining house values after 2006 and low interest rates after 2007 and

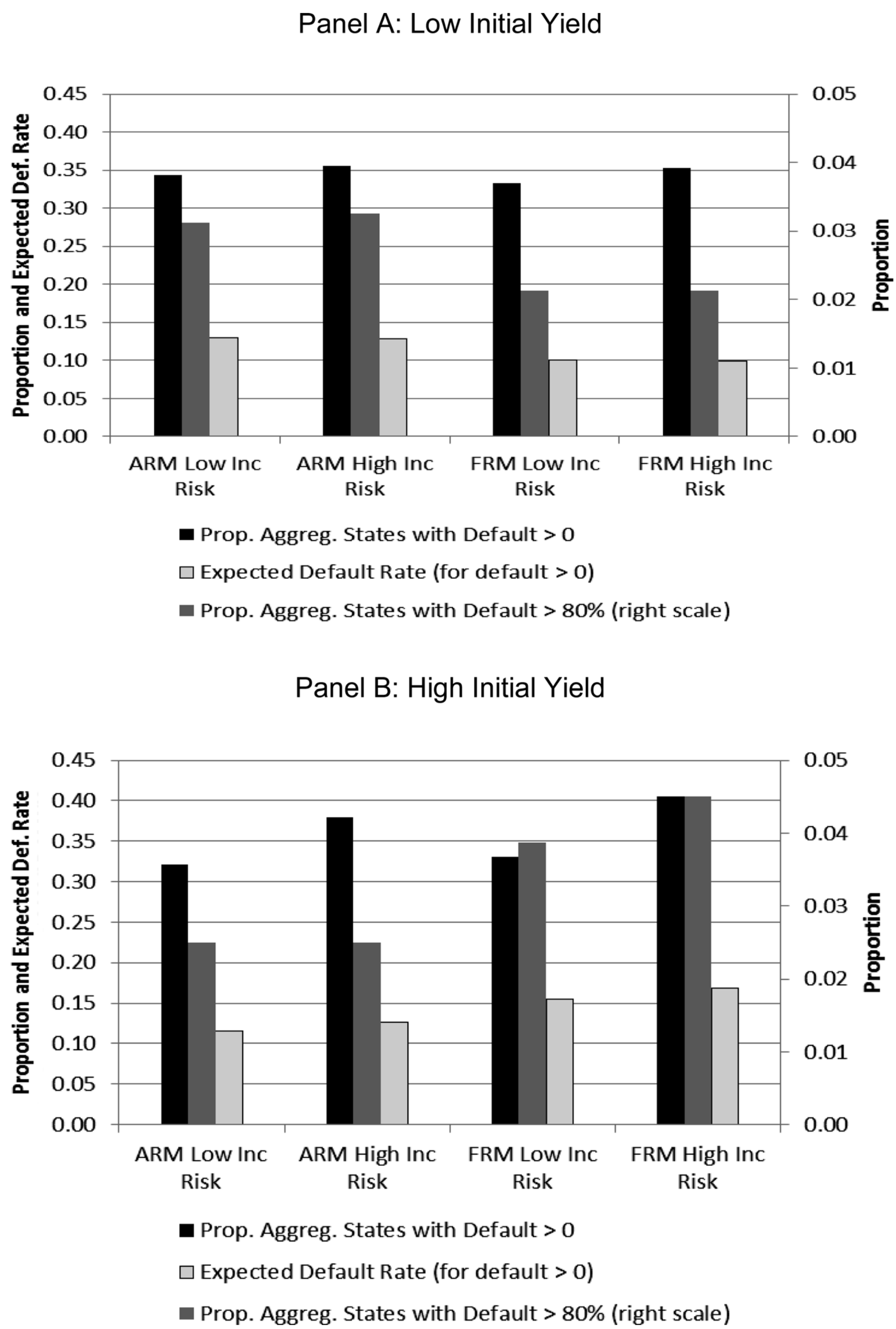
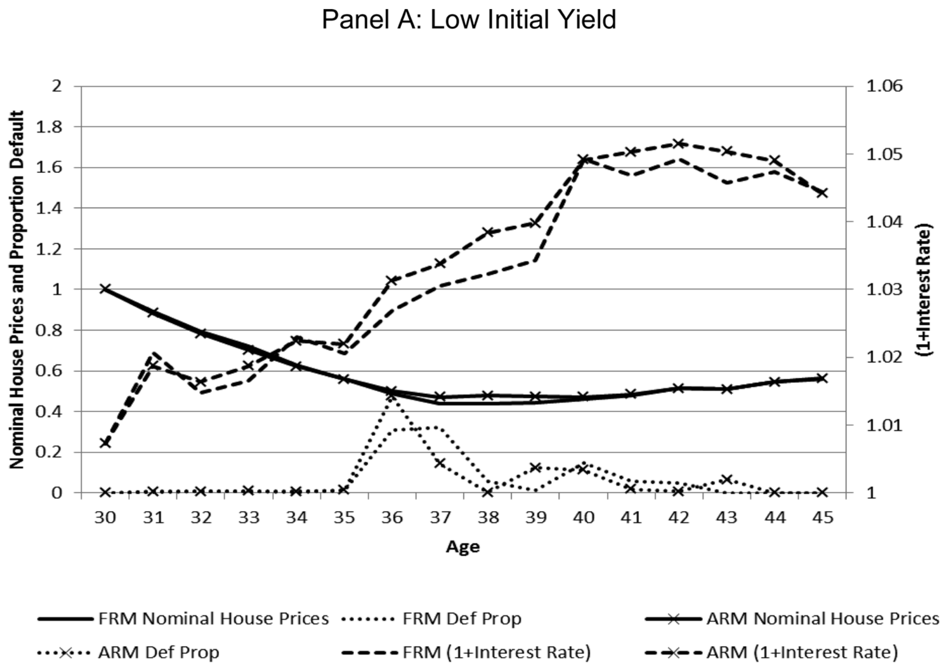


Figure 3. Proportion of aggregate states with given default frequencies. This figure reports the proportion of aggregate states with a positive mortgage default and with a default rate higher than 80%, by initial yield and mortgage type for different levels of income risk. Low (high) income risk refers to households with a standard deviation of temporary income shocks equal to 0.225 (0.35). In addition, the figure reports the average default rate across states with positive default. The data are obtained by simulating the model with the parameters shown in Table I.



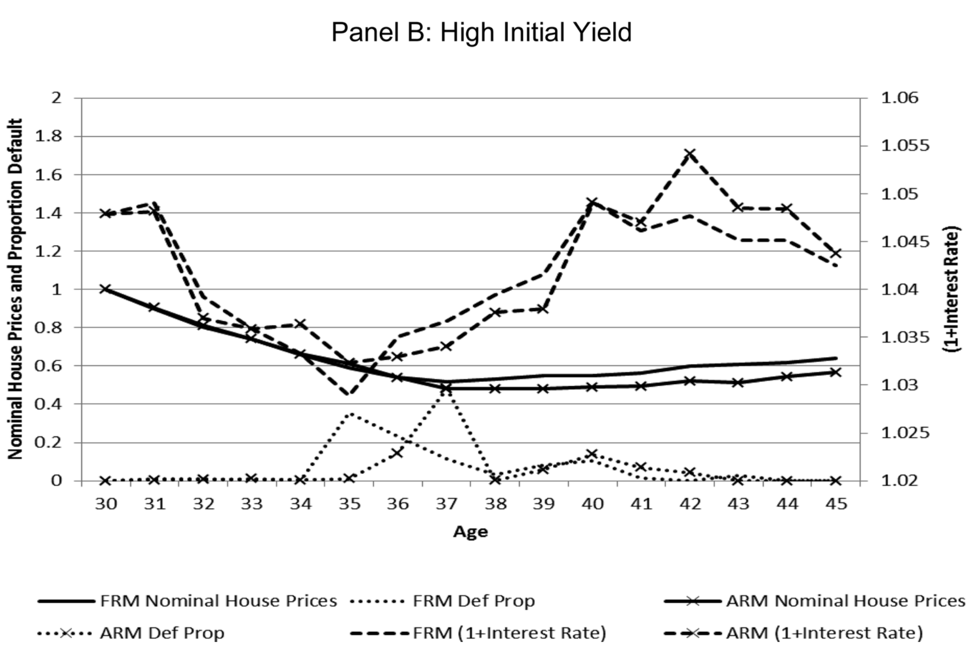


Figure 4. Aggregate characteristics of default waves. This figure plots nominal house prices, nominal interest rates, and the proportion of defaults for aggregate states with a default rate over 80% by mortgage type. Low (high) initial yield refers to the initial interest rates. The data are obtained by simulating the model with the parameters shown in Table I. The figure plots the data until age 45 since no default occurs after this age.

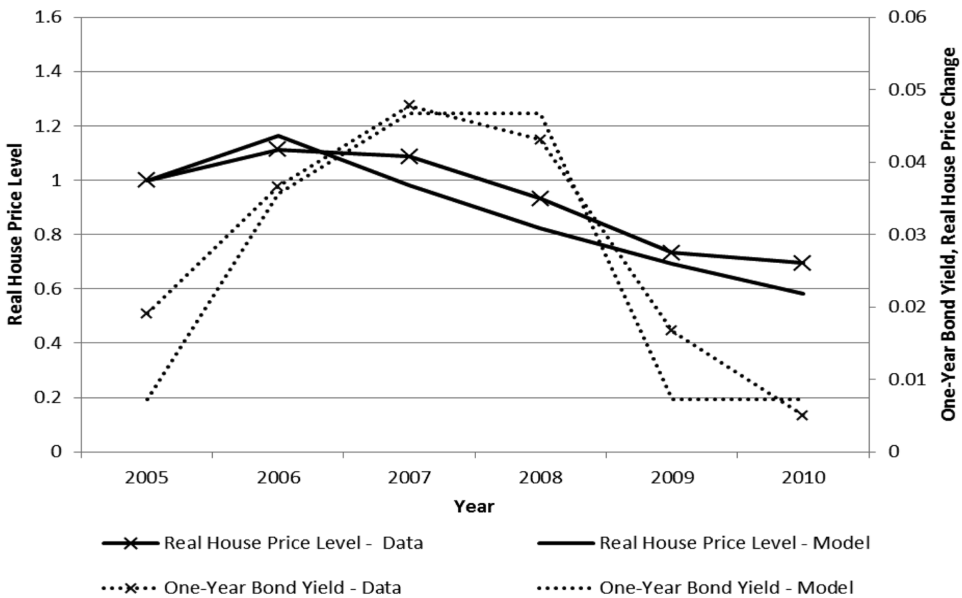


Figure 5. Real house prices and nominal interest rates in the model and in the data. This figure plots real house prices and nominal interest rates over time in the data and in the model. House price data come from the Case-Shiller US 10-City Composite Home Price Index and interest rate data are from the Federal Reserve. House prices in 2005 are normalized to one.

particularly after 2008. With this in mind we set the aggregate variables in our model to values that more closely match those observed in the data. In Figure 5 we plot real house prices and nominal interest rates both in the model and in the data. In this figure we normalize the initial real price of housing to one. The house price data come from the S&P/Case 10-City Composite Home Price Index. Therefore, any variation in the evolution of house prices across cities is not captured by our experiment. Furthermore, because each period in our model corresponds to one year, we cannot capture intra-year variation. Nonetheless, we evaluate the extent to which our model is able to capture the main patterns observed in the data.

We consider hypothetical mortgages originated in 2005 and 2006 and use our model to calculate their performance during the global financial crisis. More specifically, in Table X we report cash-out, default, and refinance probabilities for ARMs and FRMs originated in each of above years. The reported probabilities are cumulative probabilities through the end of each year from 2006 to 2009. The first observation that emerges from Table X is the difference in outcomes for mortgages originated in different years. Given the house price increases that took place in 2005, a considerable proportion of the mortgages that began in 2005 are terminated due to borrowers wishing to tap into their home equity. The cash-outs occur in both 2006 and 2007.

Table X

Cumulative Probabilities of Default, Cash-Out Prepayment, and Refinancing, by Mortgage Type and Income Risk Characteristics, for a Path of the Aggregate Variables that Matches the U.S. Experience

This table reports the probabilities of default, cash-out, and refinancing for a given path of the aggregate variables that matches the U.S. historical experience. The table reports results for LTI=4.5 and LTV = 0.90, for different household income risk characteristics, and for mortgages that began in 2005 and in 2006.

	Year mort 2005			Year mort 2006
Probability through end-2006				
ARM		Cash-out	Default	Default
Base case		0.078	0.000	0.008
Higher inc risk		0.106	0.000	0.008
Correl inc risk		0.078	0.000	0.008
FRM	Refinance	Cash-out	Default	Default
Base case	0.000	0.078	0.000	0.008
Higher inc risk	0.000	0.164	0.000	0.008
Correl inc risk	0.000	0.078	0.000	0.008
Probability through end-2007				
ARM		Cash-out	Default	Default
Base case		0.115	0.000	0.016
Higher inc risk		0.159	0.000	0.060
Correl inc risk		0.216	0.000	0.016
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.000	0.016
Higher inc risk	0.677	0.198	0.000	0.060
Correl inc risk	0.000	0.166	0.000	0.278
Probability through end-2008				
ARM		Cash-out	Default	Default
Base case		0.115	0.008	0.023
Higher inc risk		0.159	0.007	0.067
Correl inc risk		0.216	0.030	0.023
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.001	0.023
Higher inc risk	0.677	0.198	0.009	0.067
Correl inc risk	0.000	0.166	0.246	0.937
Probability through end-2009				
ARM		Cash-out	Default	Default
Base case		0.115	0.015	0.032
Higher inc risk		0.159	0.014	0.075
Correl inc risk		0.216	0.044	0.032
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.002	0.032
Higher inc risk	0.677	0.198	0.010	0.087
Correl inc risk	0.000	0.166	0.289	1.000

Khandani, Lo, and Merton (2013) emphasize the role of cash-out refinancing in the recent U.S. financial crisis. They show how the interplay between house price increases, low interest rates, and the availability of refinancing opportunities led to a large increase in household cash-out refinancing in the years prior to the crisis. This generated a "ratchet effect," that is, an increase in mortgage principal when home values appreciated without the possibility of a decrease in mortgage debt when house prices subsequently declined. This mechanism synchronized borrowers' default decisions, creating systemic risk.[23]

Comparing across borrower types in Table X, we see that the cash-out probabilities predicted by our model tend to be larger for borrowers who face large income risk, particularly when the level of income is correlated with interest rates. For these borrowers, the decline in interest rates and incomes in 2007 leads them to tap into their home equity. One way to think of borrowers tapping into home equity is that they take out a new mortgage in a later year with a higher LTV than their existing mortgage. Our model predicts that riskier borrowers were more likely to do so, which suggests that, in the years leading up to the financial crisis, the pool of new borrowers in the market may have had an increasing proportion of riskier borrowers. Further, recall that our model predicts that borrowers who face large income risk are more likely to prefer ARMs. Thus, the pool of ARM borrowers may have become increasingly risky.

Table X also shows that in 2007, as interest rates start to decline, many FRMs initially taken out in 2005 are refinanced. After 2007, we start to see some defaults. Default rates are on average higher for ARMs than FRMs, with the exception of the case in which high income risk is correlated with interest rates. However, recall that the model predicts that borrowers with this type of income risk would benefit the most from ARMs relative to FRMs (Table VII). Thus, the results for the FRM with correlated income risk should be seen as a hypothetical case shown for comparison, but that we would not expect to observe in reality.

The final column of Table X reports results for the mortgages originated in 2006, just before house prices started to decline. Given the immediate decline in house prices, we do not observe any cash-outs for these mortgages. Furthermore, negative home equity prevents FRM borrowers from refinancing their mortgages. We observe some defaults as early as 2006, but they only become prevalent in later years, as house prices decline further and borrowers find themselves with more negative home equity. Among the different borrower types considered, default rates tend to be larger for borrowers with higher

[23] Miltersen and Torous (2012) also investigate the effects of cash-out refinancing and the synchronization of borrowers' decisions, focusing on how the risks of first-lien mortgages and collateralized debt obligations change when homeowners take second mortgages. Mian and Sufi (2011) use individual-level data on homeowner debt and defaults to show that borrowing against rising home values can explain a significant fraction of the increase in household leverage prior to the crisis and a significant part of the subsequent default. Importantly, they use land supply elasticity measures based on land topology as an instrument for house price growth. They show that home equity–based borrowing is stronger for younger households and for those with low credit scores.

income risk and for income correlated with interest rates. Again, the model predicts that these are the borrowers who, when taking out a mortgage in 2006, would benefit the most from an ARM relative to an FRM.

In the next section, we compare these model predictions to the data. Before we do so, we note that, in the experiments reported in Table X, the initial LTI and LTV are equal to the baseline values of 4.5 and 0.90, respectively. The corresponding real house value is $231.8 thousand. In 2005, given that we normalize real house prices to one, this is also the house size. However, in 2006 real house prices are higher than in 2005, equal to 1.16. Thus, for same house value, it has to be the case that borrowers in 2006 are buying houses of smaller size, equal to a house of value $231.8/1.16 = \$198.9$ thousand in 2005. In this case, since we assume separability between housing and nondurable consumption, and we take house size to be fixed throughout, our calculations go through. An alternative approach would be to assume that individuals buy the same-sized house in 2006 as in 2005 and that they use the same dollar amount for the down payment, but they need to take out a loan with a higher initial LTV and a higher multiple of labor income.

C. Empirical Evidence

We can evaluate the extent to which our model is able to capture the patterns observed in the data along at least four dimensions: mortgage premia, mortgage choice, refinancing patterns, and default rates. We discuss each in turn. For the first two we use data from the MIRS of the FHFA. We use information on the effective rates for ARMs and FRMs, their LTV, the loan amount, and also the proportion of new mortgages that are of the FRM type (FRM share). To calculate mortgage premia, we use the yields on zero-coupon bonds from Gurkaynak, Sack, and Wright (2006). We calculate the premium for ARMs as the difference relative to the yield on one-year zero-coupon bonds, and the premium for FRMs as the difference relative to the yield on a 20-year annuity (consistent with our model analysis in the previous section). We also use CPI data from the Bureau of Labor Statistics to calculate real loan amounts.

C.1. Mortgage Premia and Choice

Figure 6, Panel A plots the history of mortgage premia on ARMs as well as one-year bond yields. Panel B provides a corresponding plot for FRMs, adding the slope of the term structure of Treasury yields. Both panels cover the sample period for which data are available, 1986:01 to 2008:10. It is immediately apparent from the figure that the ARM premium is much more volatile than the FRM premium, and there is a strong tendency for the ARM premium to decline with the level of interest rates. In contrast, the FRM premium seems to increase modestly with the level of interest rates. To facilitate comparison with the model, in Panel A of Table XI we report the average mortgage premia that we observe in the data. We do so in two different ways. In Panel A we match the observations in the data to those in the model based on the level of

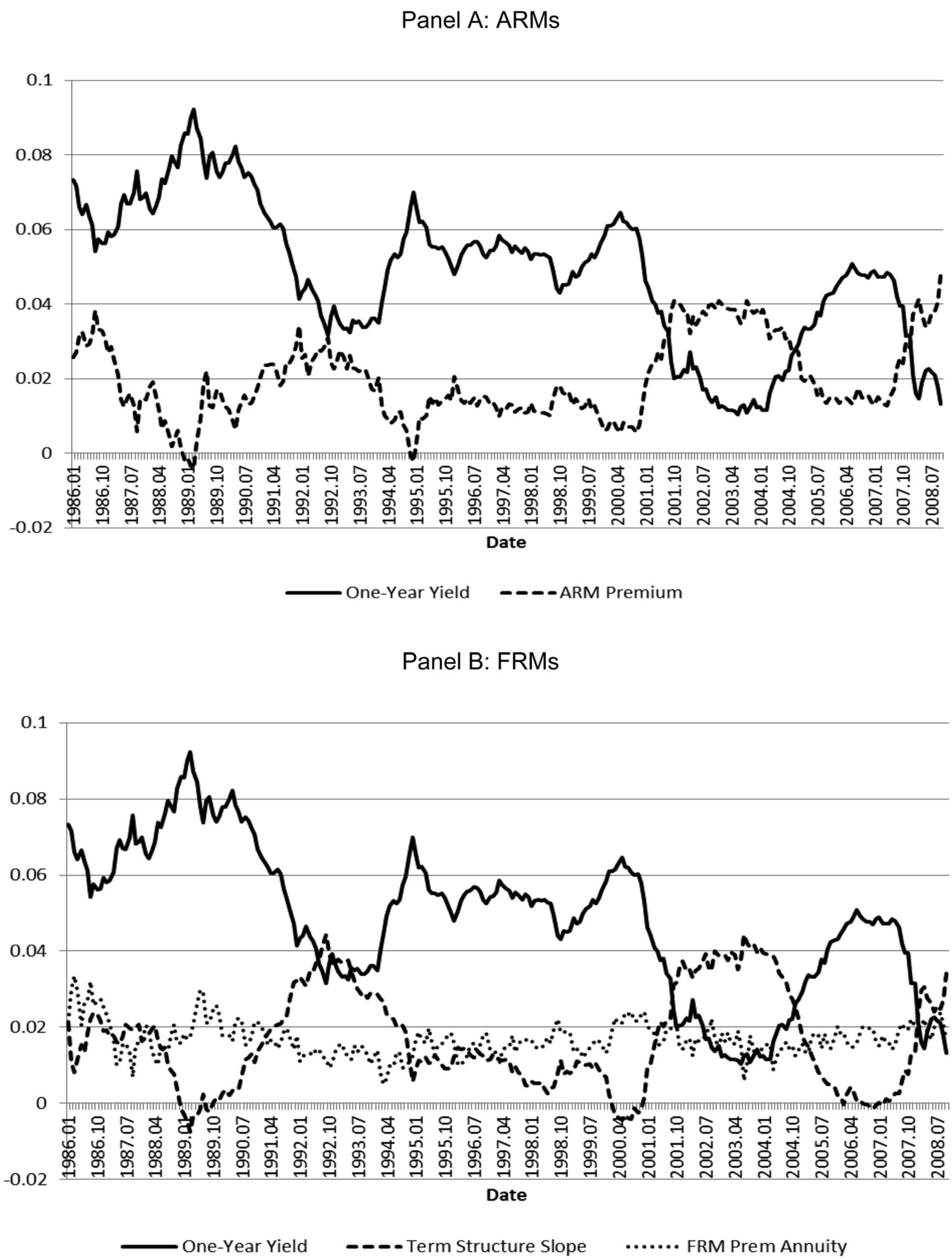


Figure 6. Evolution of mortgage premia over time in the data. This figure plots the evolution over time of one-year zero-coupon bond yields, the slope of the term structure, and mortgage premia. Mortgage premia for ARMs are calculated as the difference between the effective ARM rate and the yield on one-year zero-coupon bonds. Mortgage premia for FRMs are calculated as the difference between the effective FRM rate and the yield on a 20-year annuity. The mortgage data are from the MIRS of the FHFA. The data on yields are from the Federal Reserve Board.

Table XI

Mortgage Premia in the Data and in the Model

Panels A and B report the average ARM and FRM premia in the data for different levels of the one-year bond yield and the slope of the term structure of interest rates. The data are from the Monthly Interest Rate Survey from 1986.01 to 2008.10. Panels C (and D) report some of the model-predicted mortgage premia for ARMs and FRMs, respectively.

One-Year Bond Yield	<0.021	0.021 to 0.041	0.041 to 0.061	>0.061
Panel A: Premium in the Data, Based on One-Year Bond Yield				
ARM premium	3.80%	2.59%	1.55%	1.27%
FRM premium	1.58%	1.53%	1.55%	1.89%
Term structure slope	>0.016	0.0 to 0.016	-0.016 to 0	<-0.016
Panel B: Premium in the Data, Based on Slope of Term Structure				
ARM premium	2.78%	1.45%	0.82%	
FRM premium	1.56%	1.71%	2.09%	
Panel C: ARM Premium in the Model				
Baseline	1.50%	1.60%	1.60%	1.70%
Lower LTV	1.45%	1.55%	1.55%	1.75%
Higher inc risk	1.55%	1.65%	1.75%	1.95%
Correlated inc risk	1.85%	1.70%	1.75%	1.65%
Lower profitability	1.25%	1.30%	1.30%	1.40%
Panel D: FRM Premium in the Model				
Baseline	1.69%	2.26%	2.63%	4.69%
Lower LTV	1.64%	1.72%	2.28%	3.39%
Inertia = 0.5	1.69%	1.66%	2.18%	2.99%
Lower profitability	1.44%	1.71%	2.18%	3.59%

one-year bond yields, which is reported in the first line of the panel, and then calculate the average premia. In Panel B we repeat the same exercise based on the slope of the term structure of interest rates. The differences between the two are mainly due to the fact that in our model, for the highest level of interest rates, the slope of the term structure is steeply negative, which has not happened in the data during the sample period.

For comparison, in Panels C and D we report the mortgage premia predicted by our model. In Panel C we report results for ARMs and in Panel D we report results for FRMs. Recall that in our model short rates do not vary independently from the slope of the term structure. Focusing first on FRMs, we see that in general our model predicts mortgage premia that increase more steeply with the level of interest rates than the increase that we observe in the data. This is due to the refinancing option being increasingly valuable as interest rates increase. The model with refinancing inertia predicts an increase in FRM mortgage premia that is more in line with that observed in the data. In addition, our model seems to do a reasonably good job at matching FRM

Table XII
Predicting Mortgage Premia

The dependent variable is the mortgage premia for ARMs (specifications (1) and (2)) and for FRMs (specifications (3) and (4)). Standard errors are reported in brackets below the estimated coefficients. The data are monthly from 1986.01 to 2008.10. The yields data are from the Federal Reserve Board. The mortgage related data are from the Monthly Interest Rate Survey. LTV for ARM–FRM is the difference in LTV between ARM and FRM mortgages initiated during the month (in percentage points). All regressions include a constant (not reported).

	(1) ARM	(2) ARM	(3) FRM	(4) FRM
Yield on one-year zero-coupon bonds	−0.409	−0.415	0.045	0.189
	[0.019]	[0.016]	[0.012]	[0.022]
Yield on 20-year annuity				−0.227
				[0.029]
LTV for ARM − FRM	0.119	0.130	−0.074	−0.066
	[0.038]	[0.015]	[0.019]	[0.017]
LTV for ARM		−0.187		
		[0.020]		
LTV for FRM			−0.086	−0.108
			[0.015]	[0.142]
Number of observations	274	274	274	274
Adjusted R-squared	0.671	0.75	0.136	0.292

mortgage premia for a LTI equal to 4.5 and a lower LTV equal to 0.80 when we match it to the data using the slope of the term structure. Alternatively, if the pool of FRM borrowers becomes safer when interest rates increase or if target profitability is lower when interest rates are high, then this will contribute to relatively lower FRM premia when interest rates are high.

The differences between the model and the data are much more pronounced for ARM mortgage premia. In the data there is significantly more variation than in the model, and, contrary to the model baseline case, ARM premia in the data decrease significantly with the level of short rates and the slope of the term structure. Of all the model cases that we consider, the only one that delivers such a decreasing pattern is the case in which borrowers face higher income risk that is positively correlated with the short-term interest rate. This case is particularly interesting because, for such borrowers, ARMs provide large hedging benefits, which makes it much more likely that they choose to borrow using an ARM. Even in this case, however, the decrease in mortgage premia that the model predicts is smaller than that observed in the data.

The much higher mortgage premia for ARMs when interest rates are low may potentially be explained by the selection of riskier borrowers into ARMs when interest rates are low. If such selection occurs and mortgage lenders are aware of this phenomenon, they may increase ARM premia to compensate during periods of low interest rates. The FHFA data do not have information on borrower characteristics, but they do provide LTV ratios. Table XII reports regression results of mortgage premia on bond yields and LTV. As Figure 6 shows, the mortgage premium for ARMs is higher when short-term yields

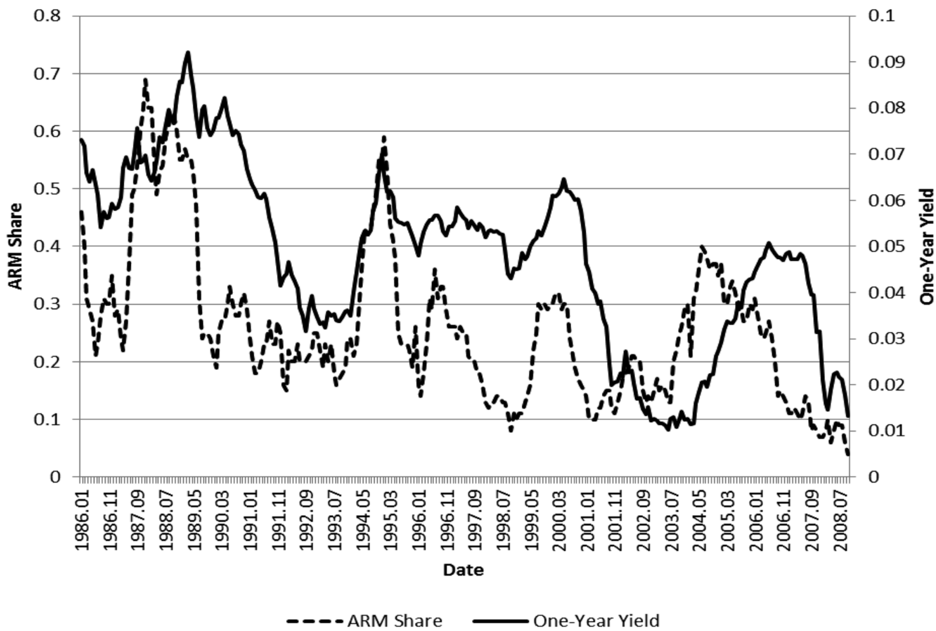


Figure 7. Evolution of the ARM share of new mortgages over time in the data. This figure plots the evolution over time of the proportion of new mortgages that are of the ARM type. The figure also plots one-year zero-coupon bond yields, and the difference between the effective rates on ARMs and FRMs. The mortgage data are from the MIRS of the FHFA. The data on yields are from the Federal Reserve Board.

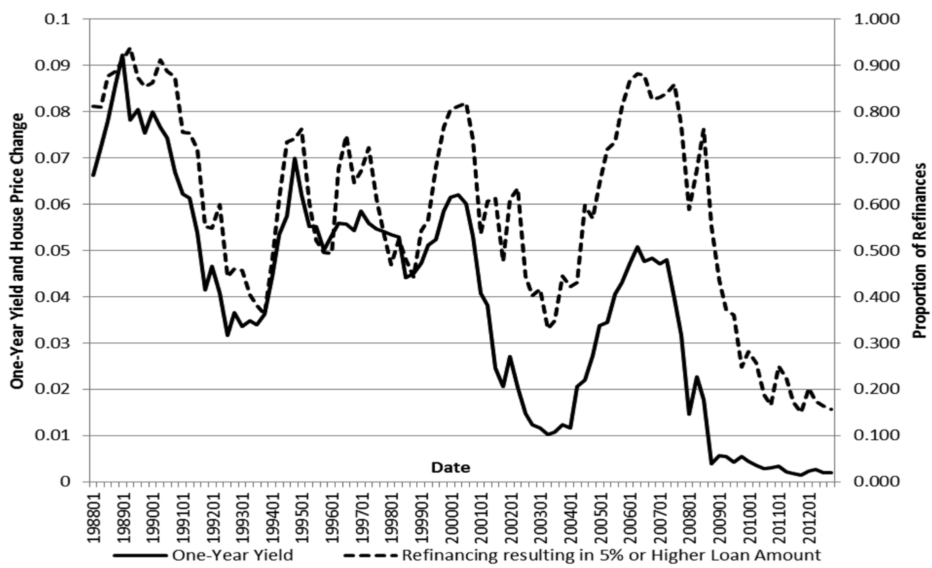
are low. It is also higher when the LTV on ARMs is large relative to that on FRMs, consistent with the notion that riskier borrower composition increases the mortgage premium.

However, in Table XII the mortgage premium for ARMs is negatively related to the LTV for ARMs. Similarly, the mortgage premium for FRMs is negatively related to the LTV for FRMs. One plausible explanation for this result is that easing credit conditions both reduce credit spreads and increase LTV. A possible (if fairly crude) way to think about easing credit conditions in the context of our model is to assume that at such times there is increased competition among lenders, which leads to a lower level of target profitability. If that happens when interest rates are high, then this may help explain the reduction in premia that we observe in the data for high interest rates.

Although our model is partially successful at generating the qualitative patterns of mortgage premia that we observe in the data, the quantitative variation in mortgage premia for ARMs is much larger in the data than in any of our model-based experiments. To achieve larger variation in mortgage premia, we would need to consider more extreme cases than those considered.

Figure 7 plots the evolution over time of the ARM share, illustrating the fact that the popularity of ARMs is positively correlated with the one-year yield.

Panel A: Proportion of Refinances Resulting in a 5% or Higher Loan Amount



Panel B: Cash-Out Amounts

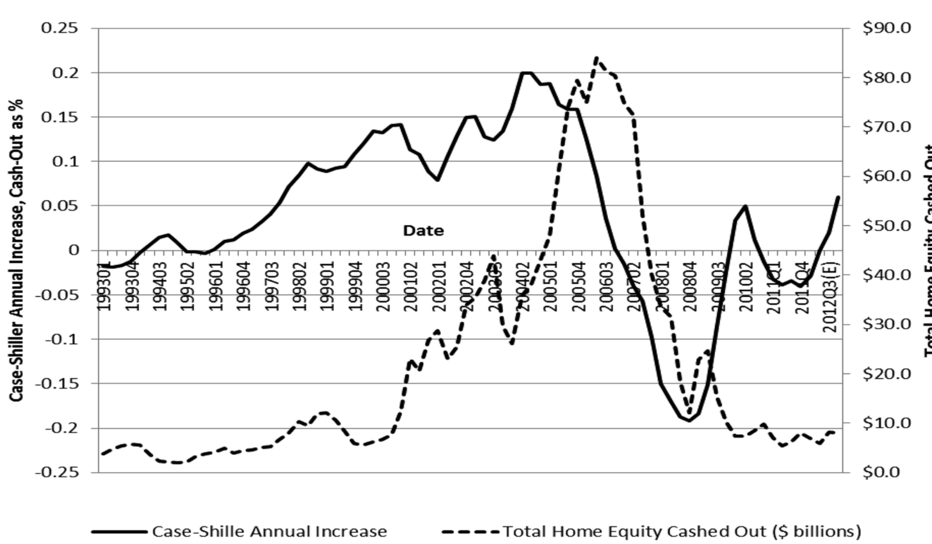


Figure 8. Refinancing activity. The refinancing data are from the Freddie Mac Cash-Out Refinance Report.

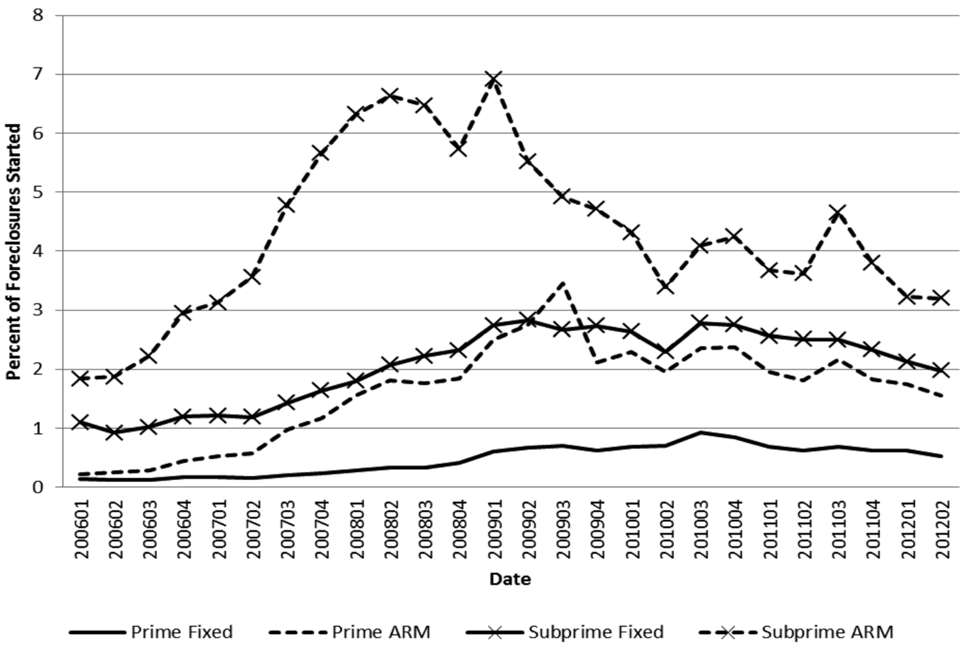


Figure 9. Foreclosures started by loan type (percent). The data are from the National Delinquency Survey of the Mortgage Bankers Association.

The correlation between the two series is equal to 0.56. As in the data, the model predicts that borrowers are more likely to prefer an ARM when interest rates are high and an FRM when interest rates are low. But Figure 7 also shows that in the data there is variation in the ARM share that is independent of the short rate. This may reflect changes in the composition of borrowers.

C.2. Mortgage Refinancing and Default

To measure refinancing activity, we use quarterly data from the Freddie Mac Cash-Out Refinance report. In Panel A of Figure 8 we plot the proportion of refinancings that result in a 5% or greater increase in loan amount, along with one-year bond yields. The figure shows that the proportion of refinancings that involve a significant cash-out is strongly positively correlated with the level of interest rates. The correlation between the two series is as high as 0.80. Thus, not surprisingly, interest rate refinancing tends to occur when interest rates are low. In Panel B we plot the total quantity of home equity cashed out and the annual change in the Case-Shiller 10-City Composite U.S. House Price Index. In this figure we see that the large increases in house prices in the early to mid 2000s are followed by a significant volume of cash-out refinancing, particularly from 2005 through early 2007, when house prices were already starting to drop. These patterns are broadly consistent with what our model predicts in the experiments that we carry out in Table X.

Figure 9 plots quarterly foreclosures initiated by loan type, using data from the National Delinquency Survey. An intriguing feature of the data clearly visible in the figure is that, in spite of the low interest rate environment that followed the onset of the crisis, default rates for ARMs are considerably higher than for FRMs, among both prime and subprime borrowers. Figure 9 also shows that some foreclosures are initiated in 2006 and early 2007, but the number of initiations greatly increases in 2008 and 2009. Our model is consistent with both of these features of the data. Table X shows that our model is able to generate higher default rates for ARMs than for FRMs if those borrowers who took out ARMs have higher labor income risk. Importantly, we also find that these same borrowers tend to benefit more from ARMs ex ante, that is, at the time the mortgage was chosen. Table X also shows that some defaults occur in 2006, but much higher default rates are observed in later years.

IV. Conclusion

In this paper we propose a rational life-cycle model of household behavior, incorporating risks to labor income, house prices, inflation, and interest rates, to understand the types of mortgages that borrowers take out and their subsequent decisions to refinance, cash out, or default on those mortgages. In our model, competitive lenders set mortgage rates to achieve a target level of risk-adjusted profitability. The model takes into account the two-way feedback between mortgage rates and borrower decisions.

Our model highlights the fact that the default decision depends not only on the extent to which a borrower has negative home equity, but also on the extent to which borrowers are constrained by low current resources. These two factors are sometimes described by mortgage practitioners as "dual triggers" of default. In our model, constraints shift the threshold at which a borrower optimally decides to exercise the irreversible option to default.

We use our model to explore several policy issues concerning mortgages. The relative merits of ARMs and FRMs have been much debated, with some commentators arguing that ARMs are inherently more prone to default, a position that seems to be supported by high ARM default rates during the recent U.S. housing downturn. In our model, ARMs and FRMs have similar overall default rates and similar sensitivities to the level of house prices, but the other drivers of default are different. ARM defaults tend to occur when interest rates and inflation increase, driving up required payments on ARMs, while FRM defaults tend to occur when interest rates and inflation decrease. For this reason ARM default risk is highest for mortgages originated at low rates, while FRM default risk is highest for mortgages originated at high rates.

This raises the question of why ARM defaults were so high during the U.S. housing downturn, even while interest rates were declining. We argue that one plausible explanation is the selection of borrowers with riskier income, and income positively correlated with interest rates, into ARMs. In our model such borrowers favor ARMs, particularly when interest rates are initially low. Another contributory factor may have been the modification of plain-vanilla ARMs to incorporate teaser rates and other devices to defer mortgage repayment. Not surprisingly, we show that such deferral of principal repayment tends to increase default rates.

Our model also has implications for the pattern of mortgage premia as interest rates vary. The model implies that FRM premia tend to increase with the initial level of interest rates, because high initial interest rates increase the value of the borrower's options to refinance, or to default if refinancing is prevented by declines in house prices. This increase in FRM premia makes FRMs relatively less attractive to borrowers when interest rates are high, consistent with the U.S. experience during the early 1980s. Using a constant composition of borrowers, our model implies that ARM premia are slightly increasing with the initial level of interest rates. However, this pattern can be reversed, and thereby improve the fit to historical U.S. data, by the selection of riskier borrowers into ARMs when initial rates are low. Our baseline model with fully rational households generates excessively strong responses of mortgage premia to interest rates, but these responses are moderated if households refinance more slowly than is optimal.

Although our model has a rich stochastic structure that includes many realistic aspects of mortgage markets, the need to economize on state variables prevents us from capturing certain phenomena that we leave for future research. In the paper we assume an exogenously fixed house size, and we present only limited results for a model of endogenous housing choice in the Internet Appendix. Further, in the paper we assume that mortgage lenders have no recourse in the event of default, an assumption that is accurate for some U.S. states but not for others, and we present only limited results for recourse mortgages in the Internet Appendix. We also allow a limited form of cash-out refinancing, modeled as selling a house to tap positive home equity and moving to rental accommodation, but we do not study second mortgages or home equity lines of credit. Moreover, our model does not include unsecured borrowing or bankruptcy, and it does not allow households to invest in risky assets such as long-term bonds or equities.

Beyond addressing these limitations of the current model, there are several other interesting directions for future research. First, we can use microeconomic data on mortgage choice, mortgage premia, delinquency, and default to structurally estimate our model parameters and test the predictions of the model across households and mortgage types. Second, we can assess the risk, systemic and otherwise, of portfolios of mortgages. Of particular interest is the differential response of FRM and ARM defaults to interest rate movements. This is relevant for monetary authorities in areas such as the eurozone in which these types of mortgages co-exist. Third, we can study the effects of structural changes in mortgage markets such as declining underwriting standards (which could be captured by lower profitability targets for mortgage lenders) or changes in implicit subsidies from government mortgage credit guarantees. We can also study mortgage features that are standard in other countries but not in the

United States, such as the ability of FRM borrowers in Denmark to refinance (without increasing mortgage principal) even when they have negative home equity. Finally, we can use our model to analyze mortgage modification policies intended to reduce the incidence of default in the aftermath of severe declines in house prices.

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REFERENCES

Amromin, Gene, Jennifer Huang, Clemens Sialm, and Edward Zhong, 2011, Complex mortgages, NBER Working Paper 17315.

Bajari, Patrick, Chenghuan Sean Chu, and Minjung Park, 2008, An empirical model of subprime mortgage default from 2000 to 2007, NBER Working Paper 14625.

Bhutta, Neil, Jane Dokko, and Hui Shan, 2010, The depth of negative equity and mortgage default decisions, Finance and Economics Discussion Series 2010-35, Federal Reserve Board.

Brueckner, Jan K., 1994, Borrower mobility, adverse selection, and mortgage points, *Journal of Financial Intermediation* 3, 416–441.

Campbell, John Y., 2006, Household finance, *Journal of Finance* 61, 1553–1604.

Campbell, John Y., and João F. Cocco, 2003, Household risk management and optimal mortgage choice, *Quarterly Journal of Economics* 118, 1449–1494.

Campbell, John Y., Stefano Giglio, and Parag Pathak, 2011, Forced sales and house prices, *American Economic Review* 101, 2108–2131.

Campbell, Sean D., Morris A. Davis, Joshua Gallin, and Robert M. Martin, 2009, What moves housing markets: A variance decomposition of the rent-price ratio, *Journal of Urban Economics* 66, 90–102.

Campbell, Tim S., and J. Kimball Dietrich, 1983, The determinants of default on insured conventional residential mortgage loans, *The Journal of Finance* 37, 5.

Carroll, Christopher D., and Andrew Samwick, 1997, The nature of precautionary wealth, *Journal of Monetary Economics* 40, 41–71.

Chan, Sewin, 2001, Spatial lock-in: Do falling house prices constrain residential mobility?, *Journal of Urban Economics* 49, 567–586.

Chatterjee, Satyajit, and Burcu Eyigungor, 2009, Foreclosures and house price dynamics: A quantitative analysis of the mortgage crisis and the foreclosure prevention policy, Working paper, Federal Reserve Bank of Philadelphia.

Chen, Hui, Michael Michaux, and Nikolai Roussanov, 2013, Houses as ATMs? Mortgage refinancing and macroeconomic uncertainty, Working paper, MIT Sloan, USC, and Wharton.

Cocco, João F., 2013, Evidence on the benefits of alternative mortgage products, *Journal of Finance* 68, 1663–1690.

Corbae, Dean, and Erwan Quintin, 2015, Leverage and the foreclosure crisis, *Journal of Political Economy* 123, 1–65.

Corradin, Stefano, 2014, Household leverage, *Journal of Money, Credit, and Banking* 46, 567–613.

Curcuru, Stephanie, John Heaton, Deborah Lucas, and Damien Moore, 2010, Heterogeneity and portfolio choice: Theory and evidence, in Yacine Ait-Sahalia and Lars Peter Hansen eds. *The Handbook of Financial Econometrics* (North Holland, Amsterdam).

Das, Sanjiv R., 2012, The principal principle, *Journal of Financial and Quantitative Analysis* 47, 1215–1246.

Demyanyk, Yuliya, Ralph S.J. Koijen, and Otto Van Hemert, 2010, Determinants and consequences of mortgage default, Working paper, Federal Reserve Bank of Cleveland and University of Chicago.

Deng, Yongheng, John M. Quigley, and Robert Van Order, 2000, Mortgage terminations, heterogeneity and the exercise of mortgage options, *Econometrica* 68, 275–307.

Diaz, Antonia, and Maria-Jose Luengo-Prado, 2008, On the user cost and homeownership, *Review of Economic Dynamics* 11, 584–613.

Doviak, Eric, and Sean MacDonald, 2012, Who defaults? Who goes into foreclosure? Working paper, City University of New York.

Elul, Ronen, Nicholas Souleles, Souphala Chomsisengphet, Dennis Glennon, and Robert Hunt, 2010, What "triggers" mortgage default? *American Economic Review Papers and Proceedings* 100, 490–494.

Financial Services Authority, 2009, Mortgage market review, Discussion Paper 09/3.

Foote, Christopher, Kristopher Gerardi, Lorenz Goette, and Paul Willen, 2009, Reducing foreclosures: No easy answers, in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, ed.: *NBER Macroeconomics Annual 2009* (University of Chicago Press, Chicago).

Foote, Christopher, Kristopher Gerardi, and Paul Willen, 2008, Negative equity and foreclosure: Theory and evidence, *Journal of Urban Economics* 6, 234–245.

Garriga, Carlos, and Don E. Schlagenhauf, 2009, Home equity, foreclosures, and bail-outs, Working paper, Federal Reserve Bank of St. Louis and Florida State University.

Ghent, Andra, 2013, Home ownership, household leverage and hyperbolic discounting, *Real Estate Economics*, forthcoming.

Ghent, Andra, and Marianna Kudlyak, 2011, Recourse and residential mortgage default: Evidence from U.S. states, *Review of Financial Studies* 24, 3139–3186.

Glaeser, Edward L., and Jesse M. Shapiro, 2003, The benefits of the home mortgage interest deduction, *Tax Policy and the Economy* 17, 37–82.

Gourinchas, Pierre-Olivier, and Jonathan Parker, 2002, Consumption over the life cycle, *Econometrica* 70, 47–89.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales, 2013, The determinants of attitudes towards strategic default on mortgages, *Journal of Finance* 68, 1473–1515.

Guiso, Luigi, and Paolo Sodini, 2013, Household finance: An emerging field, in George Constantinides, Milton Harris, and Rene Stulz, eds.: *Handbook of the Economics of Finance* (North-Holland, Amsterdam).

Gurkaynak, Refet, Brian Sack, and Jonathan H. Wright, 2006, The U.S. Treasury yield curve: 1961 to the present, Finance and Economics Discussion Series, Federal Reserve Board.

Haughwout, Andrew, Ebiere Okah, and Joseph Tracy, 2014, Second chances: Subprime mortgage modification and re-default, *Journal of Money, Credit, and Banking*, forthcoming.

Himmelberg, Charles, Christopher Mayer, and Todd Sinai, 2005, Assessing high house prices: Bubbles, fundamentals and misperceptions, *Journal of Economic Perspectives* 19, 67–92.

Khandani, Amir E., Andrew W. Lo, and Robert C. Merton, 2013, Systemic risk and the refinancing ratchet effect, *Journal of Financial Economics* 108, 29–45

Koijen, Ralph S.J., Otto Van Hemert, and Stijn Van Nieuwerburgh, 2009, Mortgage timing, *Journal of Financial Economics* 93, 292–324.

Krainer, John, Stephen Leroy, and Munpyung O, 2009, Mortgage default and mortgage valuation, Working paper, Federal Reserve Bank of San Francisco and University of California Santa Barbara.

Li, Wenli, Michelle J. White, and Ning Zhu, 2011, Did bankruptcy reform cause mortgage default to rise? *American Economic Journal: Economic Policy* 3(4), 123–147.

Mayer, Christopher, Karen Pence, and Shane Sherlund, 2009, The rise in mortgage defaults, *Journal of Economic Perspectives* 23, 27–50.

Mian, Atif, and Amir Sufi, 2009, The consequences of mortgage credit expansion: Evidence from the U.S. mortgage default crisis, *Quarterly Journal of Economics* 124, 1449–1496.

Mian, Atif, and Amir Sufi, 2011, House prices, home equity-based borrowing, and the U.S. household leverage crisis, *American Economic Review* 101, 2131–2156.

Miles, David, 2004, *The UK Mortgage Market: Taking a Longer-Term View* (HM Treasury, London).

Miltersen, Kristian R., and Walter N. Torous, 2012, Second mortgages: Valuation and implications for the performance of structured financial products, Working paper, Copenhagen Business School and MIT.

Mitman, Kurt, 2012, The Macroeconomic effects of bankruptcy and foreclosure policies, Working paper, University of Pennsylvania.

Narita, Machiko, 2011, Understanding the housing bust: Disentangling credit crunch and severe recession, Working paper, University of Minnesota and Federal Reserve Bank of Minneapolis.
Piazzesi, Monika, and Martin Schneider, 2012, Inflation and the price of real assets, Working paper, Stanford University.
Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2007, Housing, consumption, and asset pricing, *Journal of Financial Economics* 83, 531–569.
Piskorski, Tomasz, and Alexei Tchistyi, 2010, Optimal mortgage design, *Review of Financial Studies* 23, 3098–3140.
Piskorski, Tomasz, and Alexei Tchistyi, 2011, Stochastic house appreciation and optimal subprime lending, *Review of Financial Studies* 24, 1407–1446.
Poterba, James M., 1984, Tax subsidies to owner-occupied housing: An asset-market approach, *Quarterly Journal of Economics* 99, 729–752.
Poterba, James M., and Todd Sinai, 2011, Revenue costs and incentive effects of mortgage interest deduction for owner-occupied housing, *National Tax Journal* 64, 531–564.
Schwartz, Eduardo S., and Walter N. Torous, 2003, Mortgage prepayment and default decisions: A Poisson regression approach, *Real Estate Economics* 21, 431–449.
Sinai, Todd, and Nicholas Souleles, 2005, Owner-occupied housing as a hedge against rent risk, *Quarterly Journal of Economics* 120, 763–789.
Stanton, Richard, and Nancy Wallace, 1998, Mortgage choice: What's the point? *Real Estate Economics* 26, 173–205.
Stanton, Richard, and Nancy Wallace, 1999, Anatomy of an ARM: The interest-rate risk of adjustable-rate mortgages, *Journal of Real Estate and Finance and Economics* 19, 49–67.
Vandell, Kerry D., 1978, Default risk under alternative mortgage instruments, *Journal of Finance* 33, 1279–1296.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix