

Handout

APPM 4720/5720 Spring 2017

Advanced Convex Optimization

Date: Wed, Jan 25 2017

Instructions Solve the following exercises using CVX (MATLAB) or similar. Use the following matrices:

$$A = \begin{bmatrix} 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 & 8 & 2 \\ 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 \\ 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 \\ 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 \\ 5 & 10 & 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 \end{bmatrix}$$

$$y = [1, 2, 3, 4, 5]^T$$

Exercises

1. Solve $\min_x \|x\|_2$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.294216.
2. Solve $\min_x \|x\|_2^2$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.0865633.
3. Solve $\min_x \|x\|_1$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.787669.
4. Resolve the equation in Exer. 1 and request the dual variable λ , and use this value so that your solution of $\min_x \|x\|_2 + \lambda \|Ax - y\|_2$ coincides with that of Exer. 1.
5. Matrices in objective: Solve $\min_{x \in \mathbb{R}^5} g(A - x\mathbf{1}^T)$ where $g(A) = \sum_{j=1}^n \sqrt{\sum_{i=1}^m A_{ij}^2}$ is the sum of column ℓ_2 norms. The minimum should be 63.9551.
6. Matrices in objective: Solve $\min_{x \in \mathbb{R}^5} h(A - x\mathbf{1}^T)$ where $h(A) = \|A\|$ is the spectral norm of A (largest singular value). The minimum should be 14.3922.
7. Matrix variables: Solve $\min_{X \in \mathbb{R}^{5 \times 10}} \|X - A\|_F$ subject to $\mathbf{1}_5^T X \mathbf{1}_{10} = 1$. The minimum should be 40.1637.
8. Matrix variables: Let B be the first 5 columns of A . Solve $\min_{X \in \mathbb{R}^{5 \times 5}} \|X - B\|_F$ subject to $X \succeq 0$ (i.e., X is positive semi-definite). The minimum should be 14.436.

Remarks With CVX and Matlab, note that if X is a matrix, then `norm(X,1)` and `norm(X(:),1)` are different (same for the 2 norm), as the former gives you the operator norm. The CVX command `norms(X,1)` is also different — try all three versions with the A matrix given above.