CVX demo

Jun 4 2018, CCIMI short-course on optimisation, Cambridge. Prof. Becker. This demonstrates basic usage of cvx (http://cvxr.org/cvx/) and how it obviates the need to re-write problems in standard form.

Make some sample problems

```
n = 10;
m = 5;
rng(0); % set the seed
A = randn(m,n);
b = randn(m,1);
```

Solve

$$\min_x \|x\|_1 \quad \text{s.t.} \quad Ax = b$$

This is a linear program, but it is not in standard form, so standard solvers cannot solve it without rewriting it. Let us rewrite it (by hand):

$$\min_{s,x} \sum_i s_i \quad \text{s.t.} \quad Ax = b, x_i \leq s_i, \ -x_i \leq s_i$$

and to get this into standard form, we have still more work. Want it like:

$$\min_z c^T z \quad \text{s.t. } Bz \leq f, Dz = d$$

where

$$z = [x, s]$$

Doing this in CVX is:

```
c = [zeros(n,1); ones(n,1)];
B = [ eye(n), -eye(n);
     -eye(n), -eye(n) ];
f = zeros(2*n,1);
D = [ A, zeros(m,n) ];
d = b;

cvx_begin
  variable z(2*n)
  cvx_quiet false  % optional
  cvx_precision best  % optional
  minimize dot(c,z)
  subject to
      B*z <= f
      D*z == b
cvx_end
```

```
Calling SDPT3 4.0: 30 variables, 15 equality constraints
------------------------------------------------------------

 num. of constraints = 15
 dim. of linear var  = 20
 dim. of free   var  = 10
 *** convert ublk to linear blk
**********************************************************************************************
   SDPT3: homogeneous self-dual path-following algorithms
**********************************************************************************************
 version  predcorr  gam  expon
    NT      1      0.000   1
it pstep dstep pinfeas dinfeas  gap     mean(obj)    cputime    kap   tau    theta
-----------------------------------------------------------------------------------------
 0|0.000|0.000|1.6e+00|4.7e+00|7.6e+01| 4.818221e+00| 0:0:00|7.6e+01|1.0e+00|1.0e+00| chol 1   1
 1|0.960|0.960|2.1e-01|6.0e-01|1.1e+01| 3.136489e+00| 0:0:00|2.3e+00|1.1e+00|1.3e-01| chol 1   1
 2|0.732|0.732|6.0e-02|1.9e-01|3.1e+00| 2.267812e+00| 0:0:00|7.1e-01|1.1e+00|4.2e-02| chol 1   1
 3|1.000|1.000|2.0e-02|7.1e-02|1.5e+00| 2.085581e+00| 0:0:00|7.3e-02|1.2e+00|1.4e-02| chol 1   1
 4|0.866|0.866|3.7e-03|2.4e-02|2.2e-01| 1.895690e+00| 0:0:00|3.5e-02|1.4e+00|3.2e-03| chol 1   1
 5|0.970|0.970|6.4e-04|1.5e-02|3.2e-02| 1.866420e+00| 0:0:00|3.9e-03|1.6e+00|6.3e-04| chol 1   1
 6|0.930|0.930|2.6e-04|1.3e-02|1.4e-02| 1.864885e+00| 0:0:00|1.5e-03|1.6e+00|2.6e-04| chol 1   1
 7|0.907|0.907|3.0e-05|1.1e-02|1.3e-03| 1.862748e+00| 0:0:00|6.4e-04|1.6e+00|3.1e-05| chol 1   1
 8|0.989|0.989|2.1e-06|9.6e-03|7.6e-05| 1.862126e+00| 0:0:00|7.2e-05|1.6e+00|2.2e-06| chol 1   1
 9|0.990|0.990|1.5e-07|3.9e-03|5.4e-06| 1.859704e+00| 0:0:00|5.5e-06|1.6e+00|1.5e-07| chol 1   1
10|1.000|1.000|3.3e-08|1.5e-03|1.7e-06| 1.858706e+00| 0:0:00|3.5e-07|1.6e+00|3.3e-08| chol 1   1
11|1.000|1.000|2.4e-09|6.2e-05|1.0e-07| 1.858083e+00| 0:0:00|7.6e-08|1.6e+00|2.5e-09| chol 1   1
12|1.000|1.000|1.4e-10|1.2e-06|4.5e-09| 1.858058e+00| 0:0:00|5.9e-09|1.6e+00|1.4e-10| chol 1   1
13|1.000|1.000|4.9e-10|2.5e-07|3.7e-10| 1.858057e+00| 0:0:00|3.2e-10|1.6e+00|0.0e+00| chol 1   1
14|1.000|1.000|1.9e-09|2.8e-09|1.5e-11| 1.858057e+00| 0:0:00|2.3e-11|1.6e+00|1.6e-10| chol 1   1
15|1.000|1.000|4.0e-09|9.6e-10|1.1e-12| 1.858057e+00| 0:0:00|1.2e-12|1.6e+00|3.3e-10| chol 1   1
16|1.000|1.000|2.6e-10|6.0e-11|6.7e-14| 1.858057e+00| 0:0:00|7.2e-14|1.6e+00|2.1e-11| chol 1   1
17|1.000|1.000|1.4e-10|3.7e-12|4.1e-15| 1.858057e+00| 0:0:00|4.4e-15|1.6e+00|1.3e-12| chol 1   1
18|1.000|1.000|1.4e-10|2.2e-13|2.5e-16| 1.858057e+00| 0:0:01|2.7e-16|1.6e+00|7.8e-14| chol 1   1
19|0.541|0.541|8.0e-11|2.0e-13|2.3e-16| 1.858057e+00| 0:0:01|1.3e-16|1.6e+00|7.0e-14| chol 1   1
20|0.363|0.363|5.2e-11|2.0e-13|2.4e-16| 1.858057e+00| 0:0:01|9.0e-17|1.6e+00|7.0e-14| chol 1   1
21|0.276|0.276|3.6e-11|2.0e-13|2.6e-16| 1.858057e+00| 0:0:01|6.8e-17|1.6e+00|7.1e-14| chol 1   1
22|0.225|0.225|2.5e-11|2.1e-13|2.8e-16| 1.858057e+00| 0:0:01|5.5e-17|1.6e+00|7.3e-14| chol 1   1
23|0.192|0.192|1.7e-11|2.1e-13|3.0e-16| 1.858057e+00| 0:0:01|4.7e-17|1.6e+00|7.5e-14| chol 1   1
24|0.169|0.169|1.1e-11|2.2e-13|3.2e-16| 1.858057e+00| 0:0:01|4.1e-17|1.6e+00|7.8e-14| chol 1   1
25|0.152|0.152|6.6e-12|2.3e-13|3.5e-16| 1.858057e+00| 0:0:01|3.7e-17|1.6e+00|8.2e-14| chol 1   1
26|0.139|0.139|3.0e-12|2.4e-13|3.8e-16| 1.858057e+00| 0:0:01|3.4e-17|1.6e+00|8.6e-14| chol 1   1
27|0.129|0.129|2.0e-12|2.6e-13|4.1e-16| 1.858057e+00| 0:0:01|3.2e-17|1.6e+00|9.0e-14| chol 1   1
28|0.122|0.122|4.1e-12|2.7e-13|4.5e-16| 1.858057e+00| 0:0:01|3.0e-17|1.6e+00|9.5e-14|
  Stop: relative gap < infeasibility
  lack of progress in infeas
-----------------------------------------------------------------
 number of iterations   = 28
 primal objective value =  1.85805710e+00
 dual   objective value =  1.85805710e+00
 gap := trace(XZ)       = 4.10e-16
 relative gap           = 1.43e-16
 actual relative gap    = 8.76e-13
 rel. primal infeas     = 1.95e-12
 rel. dual   infeas     = 2.54e-13
 norm(X), norm(y), norm(Z) = 2.5e+00, 2.6e+00, 2.8e+00
 norm(A), norm(b), norm(C) = 2.0e+01, 1.9e+00, 5.5e+00
 Total CPU time (secs)  = 0.65
 CPU time per iteration = 0.02
 termination code       = -9
 DIMACS: 2.0e-12  0.0e+00  2.5e-13  0.0e+00  8.8e-13  8.7e-17
-----------------------------------------------------------------

------------------------------------------------------------
```

```
Status: Solved
Optimal value (cvx_optval): +1.85806
```

and look at solution

```
x = z(1:n);
x_slowAndPainful = x
```

```
x_slowAndPainful =
    0.0000
    0.3023
   -0.0000
   -0.9867
   -0.1822
   -0.0204
   -0.0000
   -0.3664
    0.0000
    0.0000
```

Try using some of the power of CVX

```
cvx_begin
  variables x(n) s(n)
  minimize sum(s)
  subject to
      abs(x) <= s
      A*x == b
cvx_end
```

```
Calling SDPT3 4.0: 20 variables, 5 equality constraints
------------------------------------------------------------

 num. of constraints =  5
 dim. of linear var  = 20
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|6.5e-01|1.0e+01|2.0e+03| 1.414214e+02  0.000000e+00| 0:0:00| chol  1  1
 1|1.000|1.000|9.3e-08|1.0e-01|1.4e+02| 1.197941e+02  9.949180e-02| 0:0:00| chol  1  1
 2|0.974|1.000|2.9e-07|1.0e-02|4.6e+00| 4.720695e+00  1.881665e-01| 0:0:00| chol  1  1
 3|1.000|0.791|9.1e-08|2.9e-03|1.6e+00| 3.136293e+00  1.586169e+00| 0:0:00| chol  1  1
 4|0.900|1.000|2.3e-08|1.0e-04|4.1e-01| 2.176454e+00  1.770504e+00| 0:0:00| chol  1  1
 5|0.978|0.900|6.3e-10|1.9e-05|2.2e-02| 1.874406e+00  1.852129e+00| 0:0:00| chol  1  1
 6|0.905|0.983|2.5e-09|1.3e-06|1.7e-03| 1.859294e+00  1.857570e+00| 0:0:00| chol  1  1
 7|1.000|0.956|7.6e-08|1.5e-07|1.5e-04| 1.858165e+00  1.858015e+00| 0:0:00| chol  1  1
 8|0.988|0.988|3.4e-09|2.1e-09|1.8e-06| 1.858058e+00  1.858057e+00| 0:0:00| chol  1  1
 9|0.994|0.995|5.5e-11|4.4e-10|3.1e-08| 1.858057e+00  1.858057e+00| 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   =  9
```

```
 primal objective value =  1.85805712e+00
 dual   objective value =  1.85805709e+00
 gap := trace(XZ)       = 3.11e-08
 relative gap           = 6.60e-09
 actual relative gap    = 6.35e-09
 rel. primal infeas (scaled problem)   = 5.48e-11
 rel. dual     "        "       "      = 4.36e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+00, 1.3e+00, 4.0e+00
 norm(A), norm(b), norm(C) = 1.0e+01, 2.9e+00, 4.2e+00
 Total CPU time (secs)  = 0.33
 CPU time per iteration = 0.04
 termination code       =  0
 DIMACS: 7.1e-11  0.0e+00  1.1e-09  0.0e+00  6.3e-09  6.6e-09
------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +1.85806
```

```
x_lessPainful = x;
norm( x_lessPainful - x_slowAndPainful )
```

```
ans = 3.1990e-08
```

Now try with the full power of CVX

```
cvx_begin
  variable x(n)
  minimize norm(x,1)
  subject to
      A*x==b
cvx_end
```

```
Calling SDPT3 4.0: 20 variables, 5 equality constraints
------------------------------------------------------------

 num. of constraints =  5
 dim. of linear var  = 20
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|6.5e-01|1.0e+01|2.0e+03| 1.414214e+02  0.000000e+00| 0:0:00| chol  1  1
 1|1.000|1.000|9.3e-08|1.0e-01|1.4e+02| 1.197941e+02  9.949180e-02| 0:0:00| chol  1  1
 2|0.974|1.000|2.9e-07|1.0e-02|4.6e+00| 4.720695e+00  1.881665e-01| 0:0:00| chol  1  1
 3|1.000|0.791|9.1e-08|2.9e-03|1.6e+00| 3.136293e+00  1.586169e+00| 0:0:00| chol  1  1
 4|0.900|1.000|2.3e-08|1.0e-04|4.1e-01| 2.176454e+00  1.770504e+00| 0:0:00| chol  1  1
 5|0.978|0.900|6.3e-10|1.9e-05|2.2e-02| 1.874406e+00  1.852129e+00| 0:0:00| chol  1  1
 6|0.905|0.983|2.5e-09|1.3e-06|1.7e-03| 1.859294e+00  1.857570e+00| 0:0:00| chol  1  1
 7|1.000|0.956|7.6e-08|1.5e-07|1.5e-04| 1.858165e+00  1.858015e+00| 0:0:00| chol  1  1
 8|0.988|0.988|3.4e-09|2.1e-09|1.8e-06| 1.858058e+00  1.858057e+00| 0:0:00| chol  1  1
 9|0.994|0.995|5.5e-11|4.4e-10|3.1e-08| 1.858057e+00  1.858057e+00| 0:0:00|
```

```
  stop: max(relative gap, infeasibilities) < 1.49e-08
------------------------------------------------------------------
 number of iterations   =  9
 primal objective value =  1.85805712e+00
 dual   objective value =  1.85805709e+00
 gap := trace(XZ)       = 3.11e-08
 relative gap           = 6.60e-09
 actual relative gap    = 6.35e-09
 rel. primal infeas (scaled problem)   = 5.48e-11
 rel. dual     "        "       "      = 4.36e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+00, 1.3e+00, 4.0e+00
 norm(A), norm(b), norm(C) = 1.0e+01, 2.9e+00, 4.2e+00
 Total CPU time (secs)  = 0.17
 CPU time per iteration = 0.02
 termination code       =  0
 DIMACS: 7.1e-11  0.0e+00  1.1e-09  0.0e+00  6.3e-09  6.6e-09
------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +1.85806
```

```
norm( x - x_slowAndPainful )
```

```
ans = 3.1990e-08
```