PROBLEM STATEMENT

We[1] consider the problem of modeling a market consists of only stocks. Specifically, let $\{s_{d,t}\}_{t=0}^{T}$ being the time series of realized daily prices of the d-th stock, for $d = 1, \ldots, N$. We would like to understand the distributions of the next-day stock prices $S_{d,T+1}$ for stock allocations.

DETAILS OF RECIPE STEPS

1. **Identify the risk drivers.** $X_{d,t} = \log(S_{d,t})$.
2. **Extract invariants from risk drivers.** We assume each risk driver $X_{d,t}$ follows a GARCH(1,1) process

$$dX_{d,t} = X_{d,t} - X_{d,t-1} = \mu_d + \sigma_{d,t}\varepsilon_{d,t} \tag{1}$$

$$\sigma_{d,t}^2 = c_d + b_d\sigma_{d,t-1}^2 + a_d(dX_{d,t-1} - \mu_d)^2 \tag{2}$$

   The $\varepsilon_{d,t}$ are the i.i.d. invariants (i.e., shocks). We use the `garch` model in MATLAB to fit our data $\{x_{d,t} = \log(s_{d,t})\}_{t=0}^{T}$ for each $d = 1, \ldots, N$ to estimate the parameters μ_d (the offset), c_d (the constant variance), b_d (the GARCH coefficient), and a_d (the ARCH coefficient). Using the estimated parameters of GARCH(1,1), we can extract the invariants $\{\epsilon_{d,t}\}_{t=1}^{T}$ for $d = 1, \ldots, N$.
3. **Estimation.** We follow a marginal-copula approach for estimating the joint distribution of $\{\varepsilon_d\}_{d=1}^{N}$. First we model the marginal distribution of each ε_d by its empirical distribution given the realizations $\{\epsilon_{d,t}\}_{t=1}^{T}$. Then we use `copulafit` in MATLAB to fit a t-copula to the joint grades $\{u_{d,t}\}_{d=1}^{N}$ from empirical CDF for $t = 1, \ldots, T$. The fitted t-copula is parametrized by correlation matrix $\boldsymbol{\rho}$ and degree of freedom ν.
4. **Project the next-day's risk drivers.** We would like to generate a total of J scenarios for the next step shocks. From the t-copula parameters $(\boldsymbol{\rho}, \nu)$ and using the `copularnd` in MATLAB, we first generate the scenarios of joint grades $\{u_d^{(j)}\}_{d=1}^{N}$ for $j = 1, \ldots, J$. We then invert the empirical CDF defined by $\{\epsilon_{d,t}\}_{t=1}^{T}$ to find the corresponding $\{\epsilon_d^{(j)}\}_{j=1}^{J}$ for $d = 1, \ldots, N$. Put $\{\epsilon_d^{(j)}\}_{j=1}^{J}$ back into the GARCH(1,1) equations (1) and (2), we generate the scenarios of next-day risk drivers $\{x_d^{(j)}\}_{j=1}^{J}$ for $d = 1, \ldots, N$.
5. **Pricing.** From the scenarios of next-day risk drivers $\{x_d^{(j)}\}_{j=1}^{J}$, we can compute the next-day's stock prices $\{s_d^{(j)} = \exp(x_d^{(j)})\}_{j=1}^{J}$, P&L $\{\pi_d^{(j)} = s_d^{(j)} - s_{d,T}\}_{j=1}^{J}$, and returns $\{r_d^{(j)} = \pi_d^{(j)}/s_{d,T}\}_{j=1}^{J}$, for $d = 1, \ldots, N$.
6. **View processing.** The prior distribution on the generated J scenarios are defined as $p_0(j) = 1/J$ for $j = 1, \ldots, J$. We express the following two views on the invariant shocks under the posterior distribution $p_\_$: $\mathbb{E}^{p_\_}[\varepsilon_1 - \varepsilon_2] = -0.03$, and $\mathbb{E}^{p_\_}[\varepsilon_3 - \varepsilon_2] = 0.15$. We can find $p_\_$ by minimizing the relative entropy $\mathcal{E}(f, p_0)$ under the two view constraints. This is achieved by calling `EntropyProg.m` by Attilio Meucci. We plot the posterior $p_\_$ from using the entire dataset in the left panel of Figure 1.
7. **Portfolio optimization and evaluation.** From the scenarios of joint returns $\{r_d^{(j)}\}_{d=1}^{N}$, for $j = 1, \ldots, J$, we generate the mean-variance frontier of allocations under $p_\_$. We constrain the allocations to be no shorting. We plot the mean-variance frontier from using the entire dataset on the right panel of Figure 1, with the left most being the min-variance allocation, and the right most being the max-return allocation. Finally, to choose one allocation on the mean-variance frontier, we use the CVaR (with the definition using the negative sign) of P&L under $p_\_$. The minimizer of CVaR for a total wealth of 1 million dollars on the efficient frontier has a return of 0.17% and volatility of 1.4%, with CVaR=$27637.57.

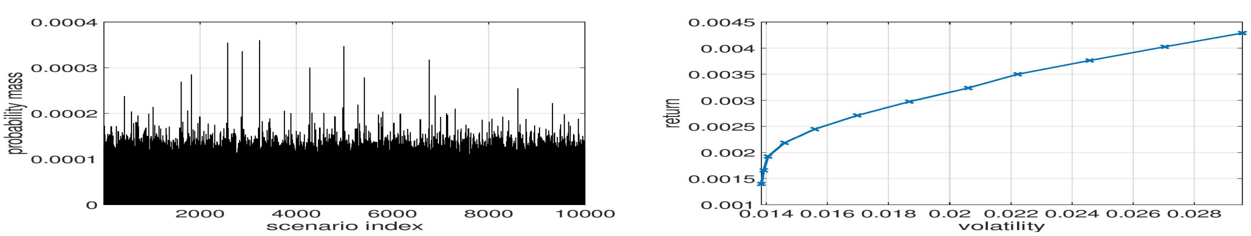

Figure 1: The left panel shows the posterior distribution $p_\_$ satisfying the view constraints, which is apparently different from the uniform prior p_0. The right panel plots the mean-variance efficient frontier under $p_\_$.

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