Bitcoin is dead: Long live Bitcoin?

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Abstract

1 Introduction

2 Data

I gathered transaction-level data on Bitcoin, Ether, Ripple, and Litecoin from the European coin exchange Bitstamp. The raw data includes information about each transaction in the history of the exchange, including the unix-timestamp[1], trade price, and trade quantity. Several points to consider:

— The different coins were offered at different points in time. For example, Bitcoin was the first coin offered on the exchange, while Ether was the most recent addition. Thus, for consistency of the analysis I used the earliest date such that trade information exists for each coin.

— Each transaction is marked to the second. In order to use a smooth time series without missing or repeated observations, I found the smallest period of time such that at least one transaction took place for each series. In this case, Litecoin was the limiting factor in calculating the time periods - there was a maximum gap of 5525 seconds between subsequent transactions. Thus, I made a dataset using this as the period length as well as a second dataset that used one-hour as the period length[2]. Ultimately, for the analysis I used the hourly dataset.

The final series consisted of hourly periods ranging from Thursday, August 17, 2017 2:00:03 pm to Tuesday, April 24, 2018 4:00:03 am GMT. Figure 1 show the raw series, and table 1 shows the descriptive statistics.

[1]The unix-timestamp is given by the number of seconds that have passed since January 1, 1970 at 12:00:00 am GMT

[2]Using one hour as the period length created a single missing observation in the Litecoin series.

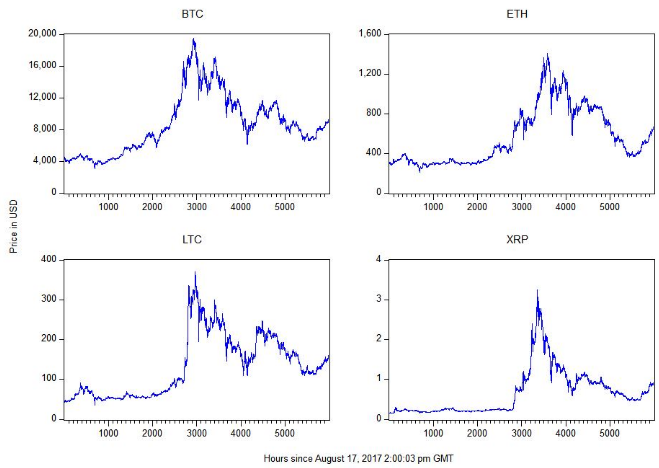

Figure 1: Time series of each coin in dataset.

	Bitcoin	Ethereum	Ripple	Litecoin
Maximum	\$19537.83	\$1405.34	\$3.25	\$370.28
Minimum	\$3009.53	\$204.85	\$0.15	\$36.17
Mean	\$8648.10	\$566.08	\$0.65	\$133.05
St. Dev	\$3749.17	\$284.70	\$0.57	\$76.66
Observations	5991	5990	5991	5991

Table 1: Descriptive statistics. Prices in USD.

Interestingly, the price for each coin are on different orders of magnitude and include an order of magnitude variation from the minimum to maximum. As the descriptive statistics show in table 1, Bitcoin has a maximum price of \$19,537.83 per BTC and minimum price of \$3009.53 per BTC. On the other end of the spectrum, Ripple has a maximum price of \$3.25 per XRP, and a minimum price of \$0.15 per XRP. Ethereum and Litecoin have similar relative variation. In order to better understand how the series moved in relative terms, I created a normalized series, $\{s^n\}$, by dividing the price at a given point in time by the series mean, μ_s. Mathematically this is to say:

$$s_t^n = \frac{s_t}{\mu_s} \tag{1}$$

Since this helps to normalize the series deviations, we can view this graphically below:

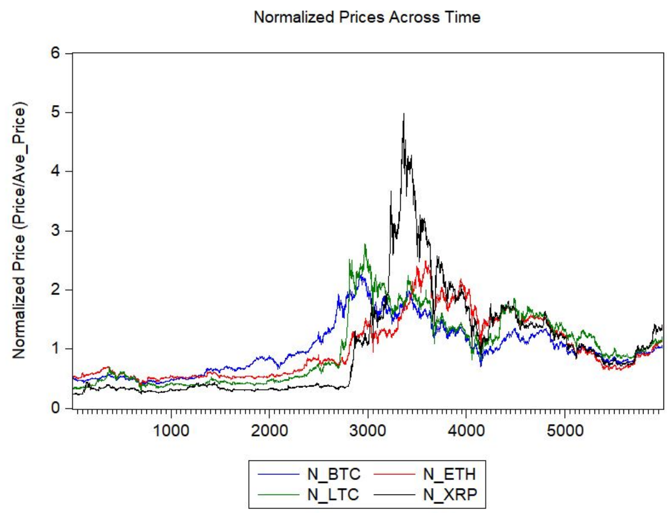

Figure 2: Overlay of each normalized series in dataset.

While the graph is a bit cluttered, what stands out is that Ripple experienced the largest percent increase at its peak, while Litcoin, Ether, and Bitcoin are all very similar. To get a clearer idea of how the series respond to each other, I restrict the graph to only the two largest coins by market cap - Bitcoin and Ether:

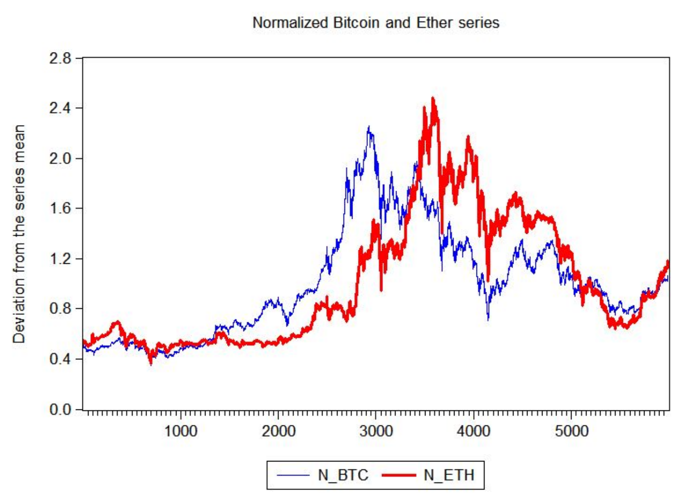

Figure 3: Overlay of Bitcoin and Ether's normalized series.

Here we see that, in normalized terms, Bitcoin's largest surge and drop seems to occur about a month (i.e. approximately 750 hours) before

Ether. Interesting, however, the last month or so in the dataset show much tighter price movements, suggesting a potential change in investor perspective.

3 Analysis and Results

Since the series levels are clearly not stationary, I calculated the series of first differences ($\Delta s_t = s_t - s_{t-1}$) and performed the Augmented Dicky-Fuller test for unit roots. In each differenced series, I am able to reject the null hypothesis that a unit root exists and accept that the level series are all integrated of degree one (i.e. I(1)). Visually we can see these differenced series below:

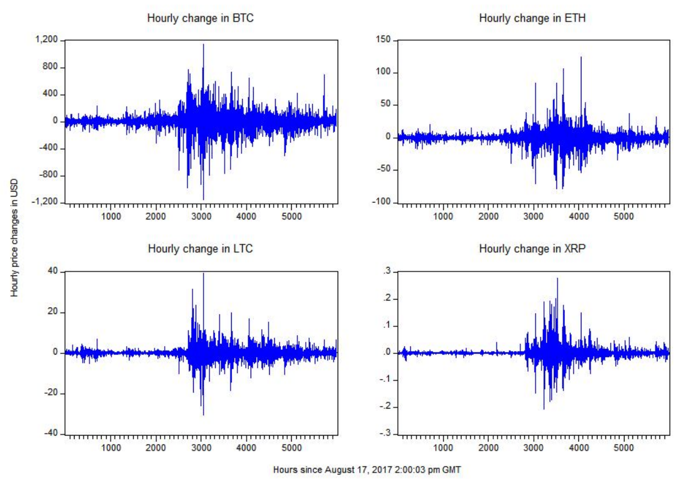

Figure 4: Time series of each differenced coin series in dataset.

3.1 Individual Series Analysis

While the differenced series do seem to have a time constant mean, the variance is definitely not constant across time. With that in mind, I tested various specifications using ARCH and GARCH errors to see how the data moves. For each series, the ACF and PACF suggest significant autocorrelation for two-periods (i.e. two hours). However, after finding the best-fit ARCH and GARCH, the series of squared residuals still indicates significant autocorrelation and partial autocorrelation. Thus, I finally settled on an AR model with TGARCH errors, suggesting that traders are affected by whether the prices are rising or falling. The results for Bitcoin, Ether, and Ripple are presented below.

3.1.1 Bitcoin

The best fitting model for the Bitcoin data is an AR(3) with TGARCH error terms. After fitting I find the following model[3]:

$$\underset{}{\Delta BTC_t} = \underset{(15.34)}{0.20\Delta BTC_{t-1}} - \underset{(-8.25)}{0.12\Delta BTC_{t-2}} + \underset{(2.36)}{0.03\Delta BTC_{t-3}} + \epsilon_t \tag{2}$$

With an error process given by:

$$E_{t-1}(\epsilon_t^2) = \underset{(14.23)}{23.25} + \underset{(23.13)}{0.068\epsilon_{t-1}^2} + \underset{(3.82)}{0.02D_{t-1}\epsilon_{t-1}^2} + \underset{(530.00)}{0.926h_{t-1}} \tag{3}$$

Next we look at the ACF and PACF of the residual and squared residual series from this estimation. We find that they mirror a white noise process.

Correlogram of Standardized Residuals

Date: 04/30/18 Time: 18:34
Sample: 1 5991
Included observations: 5987
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.011	0.011	0.7479	
		2	0.010	0.010	1.4037	
		3	-0.005	-0.006	1.5704	
		4	0.003	0.003	1.6170	0.204
		5	-0.021	-0.021	4.2073	0.122
		6	0.010	0.010	4.7922	0.188
		7	0.001	0.002	4.8030	0.308
		8	0.007	0.007	5.1241	0.401
		9	-0.006	-0.006	5.3773	0.496
		10	0.013	0.012	6.3401	0.501
		11	-0.004	-0.004	6.4383	0.598
		12	0.018	0.018	8.4914	0.485
		13	0.005	0.005	8.6480	0.566
		14	0.031	0.030	14.363	0.214
		15	0.006	0.006	14.590	0.265
		16	0.042	0.041	25.069	0.023
		17	0.024	0.024	28.463	0.012
		18	0.033	0.031	34.820	0.003
		19	0.008	0.008	35.174	0.004
		20	0.016	0.015	36.795	0.004
		21	0.010	0.012	37.379	0.005
		22	-0.003	-0.004	37.438	0.007
		23	0.011	0.012	38.128	0.009
		24	-0.005	-0.007	38.300	0.012

*Probabilities may not be valid for this equation specification.

Figure 5: ACF and PACF of the residuals from the above AR(2) model with TGARCH errors on the differenced Bitcoin series.

[3]Z-statistics in parentheses.

Correlogram of Standardized Residuals Squared

Date: 04/30/18 Time: 18:35
Sample: 1 5991
Included observations: 5987

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.020	0.020	2.3194	0.128
		2	-0.007	-0.007	2.5956	0.273
		3	-0.003	-0.003	2.6666	0.446
		4	-0.004	-0.004	2.7745	0.596
		5	0.022	0.023	5.7744	0.329
		6	-0.010	-0.011	6.3935	0.381
		7	-0.010	-0.010	7.0496	0.424
		8	-0.008	-0.008	7.4547	0.488
		9	-0.005	-0.005	7.5967	0.575
		10	0.000	-0.000	7.5969	0.668
		11	-0.007	-0.007	7.8774	0.724
		12	0.007	0.007	8.1515	0.773
		13	-0.008	-0.008	8.5430	0.807
		14	-0.003	-0.002	8.5877	0.857
		15	-0.004	-0.004	8.6808	0.894
		16	-0.002	-0.001	8.6974	0.925
		17	-0.007	-0.008	8.9957	0.940
		18	-0.004	-0.003	9.0766	0.958
		19	-0.004	-0.004	9.1892	0.970
		20	-0.005	-0.005	9.3353	0.979
		21	-0.006	-0.006	9.5572	0.984
		22	0.013	0.013	10.517	0.981
		23	-0.008	-0.008	10.879	0.984
		24	-0.006	-0.006	11.112	0.988

*Probabilities may not be valid for this equation specification.

Figure 6: ACF and PACF of the squared residuals from AR(2) model with TGARCH errors on the differenced Bitcoin series.

3.1.2 Ether

The best fitting model for the Ether data is an AR(2) with TGARCH error terms. After fitting I find the following model[4]:

$$\begin{aligned} \Delta ETH_t = 0.15 & \Delta ETH_{t-1} - 0.10 \Delta ETH_{t-2} + \epsilon_t \\ (12.02) & \qquad\qquad (-6.71) \end{aligned} \tag{4}$$

With an error process given by:

$$\begin{aligned} E_{t-1}(\epsilon_t^2) = 0.12 + 0.11\epsilon_{t-1}^2 + 0.02 D_{t-1}\epsilon_{t-1}^2 + 0.89 h_{t-1} \\ (12.72)\ (21.36) \qquad (2.84) \qquad\qquad (270.10) \end{aligned} \tag{5}$$

We again look at the ACF and PACF of the residual and squared residual series from this estimation and confirm that they mirror a white noise process.

[4]Z-statistics in parentheses.

Date: 05/01/18 Time: 10:03
Sample: 1 5991
Included observations: 5988
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.038	0.038	8.7696	
		2	0.021	0.020	11.537	
		3	-0.000	-0.002	11.537	0.001
		4	0.022	0.021	14.369	0.001
		5	-0.016	-0.018	15.898	0.001
		6	0.012	0.013	16.811	0.002
		7	0.014	0.014	18.055	0.003
		8	-0.004	-0.006	18.137	0.006
		9	-0.002	-0.002	18.166	0.011
		10	0.023	0.022	21.253	0.007
		11	0.011	0.010	22.024	0.009
		12	0.007	0.006	22.303	0.014
		13	0.009	0.008	22.775	0.019
		14	0.032	0.030	28.964	0.004
		15	0.014	0.012	30.221	0.004
		16	0.034	0.032	37.226	0.001
		17	0.027	0.024	41.743	0.000
		18	0.015	0.011	43.178	0.000
		19	0.015	0.014	44.616	0.000
		20	0.048	0.045	58.514	0.000
		21	0.015	0.010	59.846	0.000
		22	-0.006	-0.010	60.098	0.000
		23	-0.001	-0.002	60.104	0.000
		24	-0.029	-0.032	64.999	0.000

*Probabilities may not be valid for this equation specification.

Figure 7: ACF and PACF of the residuals from the above AR(2) model with TGARCH errors on the differenced Ether series.

Date: 05/01/18 Time: 10:03
Sample: 1 5991
Included observations: 5988

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.040	0.040	9.5561	0.002
		2	-0.009	-0.010	10.009	0.007
		3	-0.014	-0.013	11.128	0.011
		4	-0.021	-0.020	13.828	0.008
		5	0.005	0.006	13.960	0.016
		6	-0.012	-0.013	14.756	0.022
		7	-0.014	-0.013	15.882	0.026
		8	-0.003	-0.002	15.938	0.043
		9	-0.010	-0.010	16.536	0.057
		10	-0.002	-0.002	16.569	0.084
		11	0.001	0.000	16.571	0.121
		12	0.003	0.003	16.632	0.164
		13	-0.019	-0.020	18.836	0.128
		14	0.002	0.004	18.868	0.170
		15	0.005	0.004	19.025	0.213
		16	0.004	0.003	19.102	0.263
		17	-0.017	-0.018	20.788	0.236
		18	-0.014	-0.013	22.039	0.230
		19	0.012	0.013	22.947	0.240
		20	-0.009	-0.011	23.453	0.267
		21	0.004	0.004	23.558	0.315
		22	-0.003	-0.004	23.630	0.367
		23	-0.000	0.000	23.631	0.424
		24	0.001	0.000	23.641	0.482

*Probabilities may not be valid for this equation specification.

Figure 8: ACF and PACF of the squared residuals from AR(2) model with TGARCH errors on the differenced Ether series.

3.1.3 Ripple

The best fitting model for the Ether data is an AR(2) with TGARCH error terms. After fitting I find the following model[5]:

$$\underset{}{\Delta XRP_t} = \underset{(9.67)}{0.13\Delta XRP_{t-1}} - \underset{(-6.68)}{0.10\Delta XRP_{t-2}} + \epsilon_t \tag{6}$$

With an error process given by:

$$E_{t-1}(\epsilon_t^2) = \underset{(35.14)}{0.000000312} + \underset{(26.60)}{0.14\epsilon_{t-1}^2} + \underset{(1.97)}{0.014D_{t-1}\epsilon_{t-1}^2} + \underset{(261.57)}{0.87h_{t-1}} \tag{7}$$

We again look at the ACF and PACF of the residual and squared residual series from this estimation and confirm that they mirror a white noise process.

[5]Z-statistics in parentheses.

Date: 05/01/18 Time: 10:10
Sample: 1 5991
Included observations: 5988
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.026	0.026	4.1606	
		2	0.026	0.025	8.2374	
		3	0.009	0.008	8.7675	0.003
		4	0.015	0.014	10.143	0.006
		5	-0.001	-0.002	10.149	0.017
		6	0.017	0.016	11.906	0.018
		7	0.013	0.012	12.887	0.024
		8	-0.011	-0.013	13.671	0.034
		9	0.020	0.020	16.104	0.024
		10	0.019	0.018	18.292	0.019
		11	-0.006	-0.008	18.496	0.030
		12	-0.002	-0.003	18.519	0.047
		13	0.017	0.017	20.349	0.041
		14	0.046	0.046	33.197	0.001
		15	0.022	0.019	36.002	0.001
		16	0.036	0.031	43.774	0.000
		17	0.028	0.024	48.405	0.000
		18	0.025	0.021	52.226	0.000
		19	0.015	0.010	53.558	0.000
		20	0.035	0.030	60.754	0.000
		21	0.009	0.005	61.238	0.000
		22	0.023	0.020	64.503	0.000
		23	0.027	0.022	68.859	0.000
		24	-0.027	-0.032	73.179	0.000

*Probabilities may not be valid for this equation specification.

Figure 9: ACF and PACF of the residuals from the above AR(2) model with TGARCH errors on the differenced Ripple series.

Date: 05/01/18 Time: 10:10
Sample: 1 5991
Included observations: 5988

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.001	0.001	0.0042	0.948
		2	-0.002	-0.002	0.0281	0.986
		3	-0.001	-0.001	0.0376	0.998
		4	-0.005	-0.005	0.1760	0.996
		5	-0.004	-0.004	0.2817	0.998
		6	-0.005	-0.005	0.4317	0.999
		7	-0.002	-0.002	0.4503	1.000
		8	-0.003	-0.003	0.5028	1.000
		9	-0.001	-0.001	0.5115	1.000
		10	-0.002	-0.002	0.5301	1.000
		11	-0.002	-0.002	0.5627	1.000
		12	0.001	0.001	0.5687	1.000
		13	0.001	0.001	0.5708	1.000
		14	-0.002	-0.002	0.5872	1.000
		15	-0.001	-0.002	0.6007	1.000
		16	-0.001	-0.001	0.6073	1.000
		17	-0.002	-0.002	0.6356	1.000
		18	-0.002	-0.002	0.6697	1.000
		19	-0.003	-0.003	0.7376	1.000
		20	-0.001	-0.001	0.7433	1.000
		21	0.001	0.000	0.7452	1.000
		22	0.004	0.004	0.8374	1.000
		23	-0.001	-0.001	0.8489	1.000
		24	-0.002	-0.002	0.8826	1.000

*Probabilities may not be valid for this equation specification.

Figure 10: ACF and PACF of the squared residuals from AR(2) model with TGARCH errors on the differenced Ripple series.

3.2 Cointegrated Series Analysis

After finding a model for Bitcoin, Ether, and Ripple that only rely on their own history, next, I test for Granger causality in both the level-series and log-series[6] to determine if information of the other coins can improve estimates for a given coin. Since both are I(1), this should be appropriate and reveal similar results. Interestingly, however, the direction of Granger causality for the Ripple-Bitcoin pair is flipped when compared to a log-differenced price series (i.e. for a series $\{s\}$, the log-differenced series would be found by $\Delta ls_t = ln(s_t) - ln(s_{t-1})$). Additionally, the granger causality appears to be bi-directional in the majority of coin pairs. We see the results below:

[6]The lag length for the level series was determined by optimizing the AIC in a standard VAR, and the lag length for the log series was determined by optimizing the Schwarz criteria in a standard VAR.

Pairwise Granger Causality Tests
Date: 04/30/18 Time: 19:08
Sample: 1 5991
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
D_ETH does not Granger Cause D_BTC	5988	10.3061	3.E-05
D_BTC does not Granger Cause D_ETH		6.76282	0.0012
D_LTC does not Granger Cause D_BTC	5984	1.94498	0.1431
D_BTC does not Granger Cause D_LTC		10.2860	3.E-05
D_XRP does not Granger Cause D_BTC	5988	0.90736	0.4036
D_BTC does not Granger Cause D_XRP		0.03955	0.9612
D_LTC does not Granger Cause D_ETH	5984	6.99999	0.0009
D_ETH does not Granger Cause D_LTC		11.4547	1.E-05
D_XRP does not Granger Cause D_ETH	5988	3.03104	0.0483
D_ETH does not Granger Cause D_XRP		4.84651	0.0079
D_XRP does not Granger Cause D_LTC	5984	1.40093	0.2464
D_LTC does not Granger Cause D_XRP		0.71664	0.4884

Figure 11: Granger causality tests using the stationary differenced series.

Pairwise Granger Causality Tests
Date: 04/30/18 Time: 19:04
Sample: 1 5991
Lags: 26

Null Hypothesis:	Obs	F-Statistic	Prob.
DL_ETH does not Granger Cause DL_BTC	5964	2.54392	3.E-05
DL_BTC does not Granger Cause DL_ETH		2.16442	0.0005
DL_LTC does not Granger Cause DL_BTC	5936	1.84321	0.0057
DL_BTC does not Granger Cause DL_LTC		2.92259	1.E-06
DL_XRP does not Granger Cause DL_BTC	5964	2.46926	5.E-05
DL_BTC does not Granger Cause DL_XRP		1.43792	0.0696
DL_LTC does not Granger Cause DL_ETH	5936	2.79835	3.E-06
DL_ETH does not Granger Cause DL_LTC		2.03907	0.0014
DL_XRP does not Granger Cause DL_ETH	5964	2.56637	2.E-05
DL_ETH does not Granger Cause DL_XRP		3.18161	9.E-08
DL_XRP does not Granger Cause DL_LTC	5936	2.01291	0.0017
DL_LTC does not Granger Cause DL_XRP		1.86838	0.0048

Figure 12: Granger causality tests using $DL\_ETH_t = ln(ETH_t) - ln(ETH_{t-1})$.

Next we look for co-integrating equations using the Johansen system test with two lags as described above. The results from both the trace and maximum eigenvalue statistics indicate three co-integrating equations at the 5% level. To further motivate and understand the possibility of co-integration, we can look at the relative prices of each series. Specifically, I calculate the price of a given coin as denominated in another coin. For example, the Bitcoin and Ether pair would tell us how many Bitcoin it costs to buy a single Ether across our sample. The idea is to check if there exists a relatively consistent relationship between the coins. A graph of each pair is presented below:

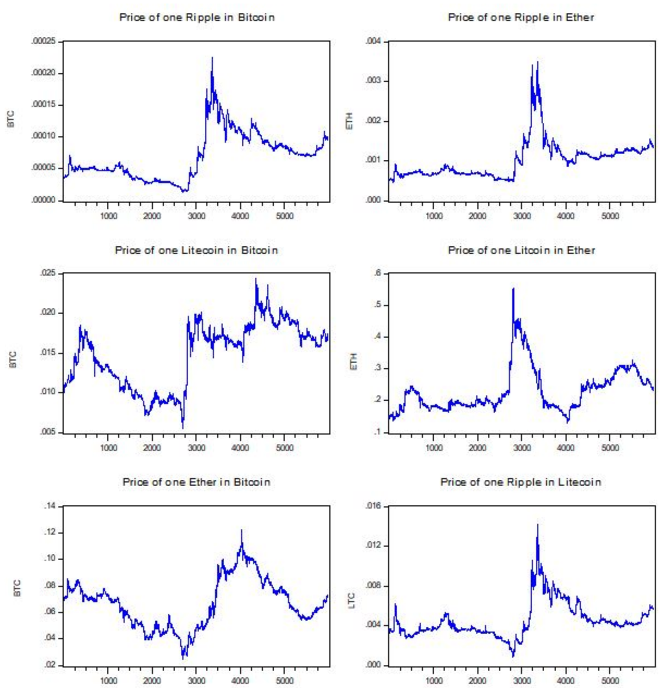

Figure 13: The relative price for each coin pair.

These graphs, while undoubtedly noisy, tell the story of different relative price jumps, but also of a potential correction to a long-run relationship. Specifically, we see that each coin was “cheap” to buy with Bitcoin up until about the 2750th hour in the dataset (this corresponds to approximately December 10, 2017). At this point, Litecoin, Ripple, and Ether all began to increase in price (and in that respective order) relative to Bitcoin. The order of the price increases are exemplified in the graphs on the right-hand-side, which show that Ripple and Litecoin both became relatively “expensive” to buy with Ether, before returning to the previous level. Thus, while volatile indeed, there does appear to be the possibility of persistent long-run relationships.

With that said, I run the vector error correction model with three co-integrating equations, and no trend or intercept, as suggested by the Johansen system test. The results are presented in Figure 14 below:

Vector Error Correction Estimates
Date: 05/01/18 Time: 20:57
Sample (adjusted): 5 5991
Included observations: 5982 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	CointEq2	CointEq3	
ETH(-1)	1.000000	0.000000	0.000000	
XRP(-1)	0.000000	1.000000	0.000000	
LTC(-1)	0.000000	0.000000	1.000000	
BTC(-1)	-0.068185	-8.21E-05	-0.015995	
	(0.00479)	(8.7E-06)	(0.00083)	
	[-14.2310]	[-9.42743]	[-19.1596]	
Error Correction:	**D(ETH)**	**D(XRP)**	**D(LTC)**	**D(BTC)**
CointEq1	-0.003404	-6.91E-06	-0.000360	-0.008474
	(0.00079)	(1.8E-06)	(0.00023)	(0.01028)
	[-4.32193]	[-3.91229]	[-1.54972]	[-0.82448]
CointEq2	1.400112	-0.000194	-0.007361	1.825967
	(0.39368)	(0.00088)	(0.11607)	(5.13791)
	[3.55650]	[-0.21929]	[-0.06342]	[0.35539]
CointEq3	-0.002783	2.22E-05	-0.002850	-0.122030
	(0.00396)	(8.9E-06)	(0.00117)	(0.05170)
	[-0.70263]	[2.49730]	[-2.44053]	[-2.36041]

Figure 14: Vector Error Correction model with three co-integrating equations and no time-trend or intercept.

While this table will take me some time to explain in detail, the parts that stand out is the non-zero speed of adjustment for both Litecoin and Bitcoin in cointegrating equation 3. Besides that, nothing stands out as statistically and economically significant. Based on the graphs above, the next steps are to explore longer lag lengths, or by using a restricted dataset.

4 Conclusion