Contents

2 Game-Theoretic Background **3**

2.1 Basic Game Theory and Rationality 3
2.2 Normal Form Games and Strategy 4
2.3 Sequential Form Games 9
2.4 Expected Utility and Information States 13
2.5 Efficiency, Risk, and Public Goods 17
2.6 Signaling, Conventions, and Communication 21
2.7 Conclusion 24

Chapter 2

Game-Theoretic Background

The hallmark of life is this: a struggle among an immense variety of organisms weighing next to nothing for a vanishingly small amount of energy.
– E.O. Wilson [Wilson, 1992]

Brown and Levinson mentioned being hesitant of ”reviving the economic homunculus” in their seminal work on politeness theory. In this chapter we will begin to do just that. Game theory gives a mathematical model of strategic interaction between agents. We begin by presenting canonical examples, techniques, and notions from the field. Major themes will include representations of games, distinguishing coordination from cooperation, understanding rationality and expected utility, and constructing exogenous mechanisms for arriving at optimal outcomes. The discussion in **??** will delineate exactly what those mechanisms should be.

2.1 Basic Game Theory and Rationality

Informally, a game can be thought of as an interaction decision problem, where each person involved, henceforth called *agents*, has a set of beliefs, actions, and preferences available to them. One of the main foundations of game theory is the idea of the **rational** agent, i.e. an agent who acts according to his preferences. These preferences are over outcomes of the game, and we will represent them with *utility functions.*

> *There was a most ingenious architect, who had contrived a new method for building houses, by beginning at the roof, and working downward to the foundation.* –Jonathan Swift

We will build our notion of a game in the same way, attempting to highlight those most salient features necessary to our analysis. Before we begin with analyzing a game, we should lay out the groundwork for rational agency and preferences. According to Myerson [1997], a rational agent prefers better outcomes over worse ones. A rational agent should attempt to maximize his utility. Formally,

Definition 1. *Given a set A of actions in a game, a **utility function** is a map from A to the real numbers $\Re$.*

As a basic example, consider finding a \$20 bill on the street. There are two options, picking the bill up (P) or not ($\neg P$). We can now derive a utility function that models the agent's payoffs [1] in the situation.

$$U(P) = 20$$
$$U(\neg P) = 0$$

Notice that this utility function does not predict what the agent will do. To understand that, we need a definition for what an agent prefers.

Definition 2. *Given two actions in a game A and B and the respective utility function $U(x)$, we say that an agent **prefers** A to B iff $U(A) > U(B)$.*

Now that we have a definition of preferences, we proceed to rationality. Are they different? We could certainly imagine people acting in accordance with their own preferences, and yet again we see examples in modern life where people act against their own preferences in irrational fashion.

Definition 3. *We say an agent is **rational** iff $U(A) > U(B)$ implies that the agent would choose action A over B.*

Notice that this definitions mean that, given a belief about the actions of the other players, a rational player would choose responses that maximize his own utility. These actions are called *best responses.* Now that we have these basic notions for understanding decisions, we can move on to how agents might interact with each other.

2.2 Normal Form Games and Strategy

Games are multi-agent decision problems. Moving from individual preferences to group decisions requires ranking outcomes based on an agent's individual preferences and his beliefs about the preferences of others. One way to represent the outcomes based on the choices available to each agent is in a table, known as a *normal form* game. In these games, we think of agents acting simultaneously and without knowledge of the other agent's move, much as people in different rooms might.

Example: The Prisoner's Dilemma

One of the most highly researched games is the *Prisoner's Dilemma*, invented by the RAND corporation to model the nuclear arms race during the Cold War

[1] A crucial point in this dissertation will be unraveling payoffs and preference. We devote an entire chapter, Chapter **??** to this topic later. In this chapter the notions are used interchangeably.

Era[Axelrod, 1984]. Its general theme is the existence of an outcome that is better off for both players, and yet arriving at that outcome presents a dilemma, based on the perverse incentives surrounding the other options. On a theoretical level, these incentives challenged the notion of rationality seen in the previous section, as an application of the rules for making decisions under bare-boned rationality leads the players to a sub-optimal outcome.

The normal form representation for the Prisoner's Dilemma is given in the table below. The numbers reflect the respective utilities of the players in choosing their strategies. As mentioned before, a rational player has a higher incentive to choose strategies leading to higher utilities. The utility of the Row Player (A) is given first, the Column Player (B) second.

	C	D
C	-2;-2	-5;0
D	0;-5	-3;-3

Table 2.1: Prisoner's Dilemma: Two criminals are arrested, with the options to COOPERATE(C) with the police or DEFECT(D) on each other and deny their complicity in the crime. The numbers represent the possible outcomes in terms of years in prison that each prisoner might lose. Outcomes are listed $Row, Column$.

Given our previous definitions of rationality, utility, and preferences, we can begin to analyze what rational players in this situation might do. A should make his decision between cooperating and defecting by considering each scenario that B might choose. On one hand, if B cooperates, A should defect, as it is better to spend no years in prison as opposed to two. On the other, if B defects, A should still choose to defect, as it is better to spend three years in prison as opposed to five.

B's possible decisions are *symmetric* to A's. I.e. if we were to change their roles, their utilities based on each outcome would be the same. This means that a rational player B would also choose to defect regardless of A's actions. The resulting outcome would thus be (D, D), which by looking at the table we also see to be the worst outcome for the group.

One-Shot Games and Equilibrium

One objection to this outcome might be that clearly the players can see that a better outcome exists for both of them. Keep in mind, however, that this situation will only happen once. Once each prisoner plays his strategy, the police now have the sufficient evidence to bring a case. This is what we call a *one-shot* scenario, a concept that will come into play later. Games can be repeated as *iterated games*, and this mechanism will feature prominently in our analysis later. The behavior we see in a one-shot game may differ drastically from what we see in a repeated one.

This result may seem surprising, and yet it forms the basis for much of the study of *cooperation* in game theory. It also highlights a key concept in the field, known as a *Nash Equilibrium.* Informally, a Nash equilibrium is an outcome of a game from which no rational player would elect to deviate on his own, given the belief that the other players are also rational. To understand this concept, we should now specify what we mean by an action profile.

Formally, we have

Definition 4. *An outcome of a finite game is known as a* ***Nash Equilibrium*** *iff each player plays a strategy that is a best response to the belief that the other players are playing strategies that would yield that outcome.*

Normal form games offer a succinct way to visualize the payoffs of the players and the potential outcomes of the game. Perhaps it is this reason that much of game theory has focused on the notion of equilibrium. The major result proved by Nash [Myerson, 1997] showed that as long as a game could be represented as a finite game in normal form, a (mixed strategy) Nash Equilibrium always exists. Understanding this notion of a mixed strategy is highly critical because we want to understand what makes one action preferable to another. By finding the mechanisms that lead to a mixed strategy equilibrium, we can derive the parameters that might influence an agent to take one action over another.

Strategic Equivalence

An important point should be made about strategy. Although the utilities listed in the Prisoner's Dilemma above in Table 2.1 match what we could perceive to be a potential prison sentence, this iconicity is not necessary. From Myerson [1997] we know that two games can be *strategically equivalent* if a linear transformation can map between their respective utility functions. What this means is that only the ordering between the utilities within the outcomes need be preserved. From a modeling perspective, this means that we need not pay so much attention to the precise derivation of the utilities but rather to the comparison between them over the different actions of the game. More precisely,

Definition 5. *Given games G_1 and G_2 with respective utility functions U and V, we say that games G_1 and G_2 are* ***strategically equivalent*** *iff $U = aV + b$ for two constants a and b.*

This definition will come in handy later in several cases. The first important thing to note is that many games differ in what their nominal payoffs are, and yet they are strategically equivalent. This will help us to understand later the idea of relationships as a form of a game. This will also help us to understand how mechanisms like sympathy can transform a game on the mathematical modeling level or a relationship on a human level in later chapters. [2]

[2] Two games are strategically equivalent if they induce the same preference ordering over the space of "mixed" strategies. This is why we require a linear transformation and not simply a monotone function, an insight pointed out by my advisor and found in Myerson [1997].

Example: Prisoner's Dilemma with Positive Payoffs

Using this notion, we can transform the payoffs in the Prisoner's Dilemma to positive values seen in the example inspired by the film *Rocky III* in Table 2.2. [3]

	C	D
C	3;3	0;5
D	5;0	2;2

Table 2.2: Prisoner's Dilemma (Positive Utilities): Two boxers, Rocky (the Row Player) and Clubber (the Column Player) are evenly matched in a fight for a $6 million purse. This means their expected payoff for fighting clean (C) would be $3 million. As a twist, they each have the chance to take a drug (D) that will cost them $1 million in legal and medical fees, but will guarantee their victory if the other one does not take it. Should they both take it, they will remain evenly matched but be forced to pay the fees nonetheless.

Consider Rocky's situation. If Clubber fights clean (C)Cooperates, Rocky can obtain a payoff of 5 if he chooses the drugs (D) versus 3 if he also fights clean (C). If Clubber uses the drugs (D), Rocky would rationally choose the higher payoff of 2 versus 0. Just as we saw before, no matter what action Clubber chooses, it behooves Rocky to choose D. The situation for Clubber is symmetric. Thus rationality would lead them to both use drugs and fight for a lower expected payoff than before. Just as before, this is a one-shot game, as we cannot expect them to fight an indefinite number of times.

Observe that this pattern of defection only holds however for a one–shot scenario and that if we repeated the game, new strategies might emerge. Defection now becomes irrational when other players know of our past via direct observation or even word of mouth. So repetition and reputation, therefore, are innately tied together and form one of the bases for building cooperative social structures. These notions will be explained formally in **??**

Justifying Strategic Equivalence

As we mentioned that these two games are strategically equivalent, we should take a moment time to see why they are and how one determines this. We begin with the games in Figure 2.1:

	C	D
C	-4;-4	-10;0
D	0;-10	-6;-6

	C	D
C	3;3	0;5
D	5;0	2;2

Figure 2.1: Two strategicallly equivalent forms of the Prisoner's Dilemma.

[3] I owe inspiration for this example to my colleague Roland Mühlenbernd. That this is quite literally an arms race escaped us at first.

We want to derive a linear transformation (an equation resembling $y = mx+b$) that will take as inputs entries in the first game and transform them into entries in the second. We will denote entries in the first game from Figure 2.1 as U_1 and entries in the second game from Figure 2.1 as U_2. If we were to consider the utility of the row player, we can now think of the utility U_1 as an input and U_2 as the output of an equation

$$U_2 = aU_1 + b.$$

with two points sufficient for determining the equation of the linear transformation, namely $(-4, 3)$ and $(0, 5)$. We now have an equation for the transformation of

$$U_2 = \frac{U_1}{2} + 5.$$

On this note, we make the following provisos. First, it is not always clear from what constraints the utilities in a game derive. A pattern therefore seen often is that the utilities are placed arbitrarily in terms of cardinal value but not in terms of ordinal value. That is to say that what matters most in a game is often just that one quantity is greater than another, not necessarily the degree of the difference. To connect this to the Prisoner's Dilemma, it is only important that defection dominates the other action of cooperation, not so much that it dominates cooperation by two "points". The second point is that we now have a way to compare scenarios that may look entirely different from the ground up. As is the case with the Prisoner's Dilemma, its strategies cover scenarios ranging from the arms race in the Cold War to buying health insurance. The third point is that mechanisms added to a game may change a) its expected payoffs or b) the players' preferences over those payoffs. This will be especially true in the case of sympathy seen in **??**. To get a better understanding of how a cooperative dilemma can arise, we turn to a more general form of the Prisoner's Dilemma.

Example: Prisoner's Dilemma in General Form

In the following case, we pursue a more general formalization of the Prisoner's Dilemma that gives us a way of understanding cooperation as a costly endeavor.

	C	D
C	b-c;b-c	-c;b
D	b;-c	0;0

Figure 2.2: General Form Prisoner's Dilemma : Choosing COOPERATE(C) entails paying a cost c. Being cooperated with entails a benefit b. DEFECT(D) is the dominant strategy for both players, as $0 > -c$ and $b > b-c$.

Cooperation occurs on every level of the biological, political, and economic strata [Fehr and Gachter, 2000], and mechanisms like reputation, reciprocity, repetition, and selection increase the likelihood of its emergence Nowak [2006], Mailath and Samuelson [2006]. Like cooperation in the Prisoner's Dilemma, polite language seemingly confounds theories of rational communication. Yet it bears the same flavor of reciprocal exchange and tit-for-tat standards of repeated behavior. For this reason, some of the games similar to the Prisoner's Dilemma may hold promise for investigating speech acts like requests, and some of the mechanisms found to induce cooperation may also play a role in encouraging linguistic politeness. One thing we want to consider more is the process by which agents reason dynamically within a game. Within a single dialog, speech acts occur in sequence, hence we now consider sequential form games.

2.3 Sequential Form Games

We will focus on the difference between cooperation and coordination in sequential play. Instead of a model where players act simultaneously, players in a *sequential* or *extensive form game* make their choices based on their belief of what choices were made before. Sequential play allows for an optimal outcome under rational behavior in cases of coordination, but not necessarily for cooperation.

Per Fudenberg and Tirole [1991] , an *extensive form game* is

- A set of players $N = \{1, 2, \ldots, n\}$
- An order O of players' moves, given by a game tree T
- Payoff profiles as a function of the game's *history* (sequence of moves)
- A set of choices per player when they move, called *actions*
- What each player knows when they make choices, termed *information sets*
- A probability distribution over any moves by an outside force, *Nature*

Sequential Form with Perfect Information

These examples are each cases of *perfect information.* The semantics of *perfect* here is more like its classical meaning of *complete.* That is, every agent perfectly informed of the actions previously taken. A game of chess has this quality. Another way to say this is to invoke the idea of common knowledge, a concept that will play a role later. Once play begins, the players are not only aware of each move of the game, they are also aware that each other agent is aware of each move of the game, ad infinitum. This was not the case in the normal form games, as each agent was thought to move simultaneously. To connect these two, we will later discuss expected utility and information states.

Example: Sequential Prisoner's Dilemma

The Prisoner's Dilemma (PD) offers the canonical example of choosing between cooperation and defection. Let us observe how this game differs slightly from its counterpart in normal form. Just as before, the game reflects a scenario wherein two prisoners must choose between cooperatively staying silent (C), telling the police nothing about their crime, or defecting on each other (D) and confessing the details to the police. Jointly, both prisoners do better if they remain silent, but individually, they each would improve their fortunes by ratting out their accomplice.

We represent the structure of the Prisoner's Dilemma in *extensive form* in Figure Figure 2.3. Each node in the game is labeled with the letter of the player whose turn it is to take an action, $O = \langle X, Y \rangle$. The payoffs, as determined by the utility functions, are listed as (U_X, U_Y) at the bottom of the tree.

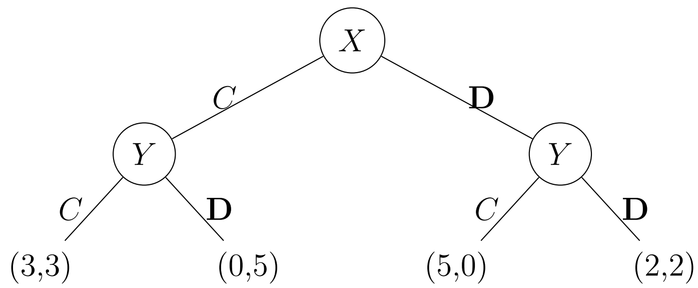

Figure 2.3: A Sequential Prisoner's Dilemma: D is in **Bold**, as it represents the preferred strategy at each choice point.

This game differs in the information states of the prisoners, a subtle distinction for now. X could choose to cooperate or defect like before, but in this case Y would know what move X made. We can imagine a possible scenario in which X is asked about his involvement in the crime before Y and in the same room. [4] We now have to ask in this case whether that dynamic would change the eventual outcome among rational players for one-shot games like this one.

To find out, we will use the notion of *backwards induction* to examine expected behavior in the Prisoner's Dilemma. The reasoning proceeds as follows. We begin by considering the lowest nodes in the game and putting in bold the best action available. Notice that in these cases Y will make the move. For any node where Y can make a choice, she should choose D as it is always the better option. This action strictly dominates cooperation in every case. Knowing that this is the case, X should always choose D, as it always the best option based on what Y will do. That is, cooperation will only ever be met with defection, so no player would cooperate.

As mentioned before, we will refer to those instances where players have diverging interests but could come together to yield the optimal outcome for all as instances of cooperation. In instances of cooperation, as in the Prisoner's Dilemma, reaching the best outcome for the players as a group requires a potential

[4] To preclude complications of deception on the part of the police, we would assume that Y has perfect access to observing the actions of X.

sacrifice in terms of individual payoff or security. That is, each player must forgo the temptation of defecting in order to maintain cooperation. Mechanisms for enforcing or promoting cooperation will be discussed later.

Example: Sequential Coordination Game

In contrast to cooperation where the efficient state might not be the equilibrium outcome, players' incentives do not conflict in cases of coordination. Consider the case of a *Pure Coordination Game* (PCG) in Figure 2.4. Here the players do not have a specific preference for one action over the other; rather, they only prefer to take the same action. An example might be a scenario where two friends want to meet up for lunch at noon at one of two equally preferable restaurants. If one player openly claims he will visit a restaurant A, then the other should indeed go to that restaurant. In the game, if X plays A (B), then Y should play A (B). Sequential games allow for both players to arrive at the optimal outcome in pure coordination games. Contrast this to the previous examples of normal form game, where the players stood a strong chance of not coordinating.

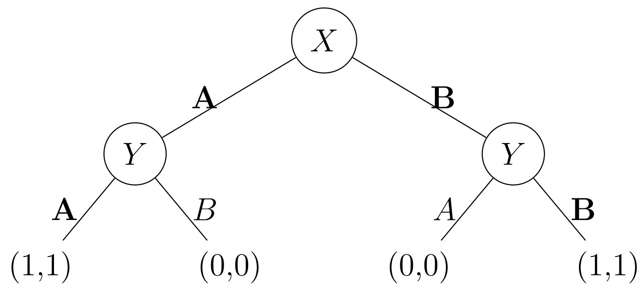

Figure 2.4: A Sequential Pure Coordination Game: Two equilibrium paths in **Bold** result in (A, A) or (B, B).

Asymmetric Games

The games seen previously were known as *symmetric games*, where each player's utility function was identical over permutations of identical actions. This is certainly not always the case however, and the crux of our discussion will contain several asymmetric games, the most prominent of which is the trust game. The motivation for using asymmetric games is that in a verbal exchange, participants engage in turn-taking behavior where each plays the role of speaker or listener. These roles are necessarily asymmetric. Moreover, as dialog occurs sequentially, the nature of speech acts should lead us sequential play with asymmetric roles. This is expanded on in the centipede game, which provides a preview of how dialog structures can go. It also gives us a feel for the dynamics of repeated games, which include a sequential component.

Example: Trust Games

We provide here a formal model that will later allow us to analyze requests through the mechanisms of asymmetric bargaining and exchange games. This

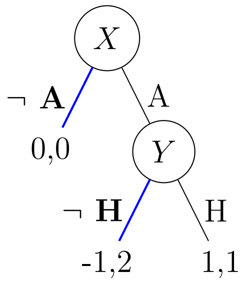

Figure 2.5: Trust Game Backward Induction: The **Bold** actions give us the equilbrium. Y would prefer 2 to 1, so Y will choose $\neg H$. As X knows that he would receive -1 in this case, he will choose $\neg A$, as $0 > -1$.

will feature prominently in **??**. An exchange game can be thought of as a formal model of two or more agents sending gifts to one another; the Prisoner's Dilemma is an example of a symmetric exchange game. To model this, we incorporate the literature on the burgeoning field of *trust games* [Bicchieri, 1990, Zak et al., 2005, Zak, 2008].

Trust games, seen in Figure 2.5 depict a scenario where Player X has an initial option to defer to Player Y for a potentially larger payoff for both. However, similar to the Prisoner's Dilemma, Player Y could defect on Player X and keep more money for himself. For a one-shot game, this act of deference will not occur for a rational Player X. Hence, a signal granting yes-no power to a hearer would not be rational *prima facie*.

We can formalize this by finding the Nash Equilibrium of a trust game via backwards induction, seen in Figure 2.5. This involves examining the final nodes of the game tree and tracing the optimal paths back to the initial node. What is rational for the players to do? First, think of Y's options. Given the choice, it makes sense for him to choose Defect, as this leads to a higher payoff. But if Y were to not help, then X would never ask for Y's help in the first place.

Example: Centipede and Backward Induction

A game with structure similar to a trust game is that of *Centipede*, seen in the following figure Figure 2.6. Centipede gives us both a sense of the complications involved within a single interaction and a preview of repeated interactions and extended trust games. [5]

Just as before, we can solve this with backward induction. At each node, the players have a chance to continue(C) or stop(S). As player 2 chooses last, he would prefer to take action S to action C. Player 1 knows this however, and he would rather take 5 points for playing S in the previous round. Continuing this line of reasoning, we land at an outcome $(1, 0)$ where player 1 would choose S, one clearly worse than the $(6, 5)$ outcome seen at the last branch.

[5] Thanks to Haiyun Chen for the majority of the code for the diagram.

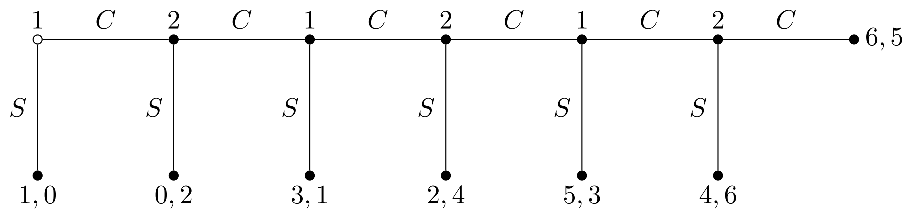

Figure 2.6: Centipede: A game of partially aligned interests?

2.4 Expected Utility and Information States

We are not done with the theme of scarcity, nor will we soon finish. As we think back to the exposition in normal form games, we see that one of the problems with strategic interaction is that we may lack information about what our partner in a game might do. Here we have not a scarcity of resources but a scarcity of information. To remedy that, we might form a *belief* about our partner.

Definition 6. *In a normal form game, a* ***belief*** *of player i is a probability distribution over the strategy space of each player j for $j \neq i$.*

If we extrapolate from our previous definition of rationality, we realize that given a belief about another player's strategic profile, a rational player will attempt to maximize his own expected utility as a weighted average of the possible utilities a player could receive for playing various actions.

Calculating Expected Utility

Let us begin with calculating the expected utility of an action a for a player j given the possibilities of actions for a player i. We denote the sets of possible actions for each player as A_i and A_j, seen in general form below:

$$EU_j(a) = \sum_{b \in A_i} Pr(b)U_j(a, b) \tag{2.1}$$

The expected utility of this player must be compared against the possible moves of his opponent and the respective probabilities of each. We mentioned the idea of a mixed strategy earlier, but why exactly would an agent choose such a strategy? If we go back to the idea of rationality, we can see that if an agent would obtain the same payoff for two different strategies, he should be indifferent to them.

In the 2×2 case, if an agent i is playing strategies S and T with probabilities p and $1-p$, then we can define the expected utility of his opponent j for choosing an action A as

$$EU_j(A) = pU_j(A, S) + (1 - p)U_j(A, T) \tag{2.2}$$

Likewise, if j chooses an action B, we would have similarly

$$EU_j(B) = pU_j(B, S) + (1 - p)U_j(B, T) \tag{2.3}$$

More generally, a *mixed strategy* σ_i is a probability distribution function on the action space A_i, $\sigma_i : A - I \to [0, 1]$. For each player i, we have

$$\sum_{a_i \in A_i} \sigma_i(a_i) = 1$$

If then it is the case that $EU_j(A) = EU_j(B)$, we can say that j is indifferent between the two options. This forms the basis for understanding a *mixed strategy Nash Equilibrium.* Consider the set of mixed strategy profiles Σ_i for each player i and the payoff profiles available.

Definition 7. *We call a strategy profile (σ_i, σ_j) a **mixed strategy Nash Equilibrium** iff for all mixed strategy profiles $\sigma_i' \in \Sigma_i$ and players i, we have that $EU_i(\sigma_i, \sigma_j) \geq EU_i(\sigma_i', \sigma_j)$.*

Note that this payoff profile must hold for all players in the game, not merely one. As mentioned before, an agent might prefer or be indifferent to another outcome, but the quality of the Nash Equilibrium is that the agent would not change strategy from the equilibrium, given the belief that his opponent would keep his strategy constant. In the case of 2×2 games, where $Pr(S) = p$ and $Pr(T) = 1-p$ for player i and $Pr(A) = q$ and $Pr(B) = 1-q$ for player j, we can write the mixed strategy Nash Equilibrium as (p, q). For instance, this might be $(p = \frac{1}{2}, q = \frac{2}{3})$. We can now think of a pure strategy Nash Equilibrium as a special case of a mixed strategy Nash Equilibrium, where an equilibrium $(S.B)$ could be written as $(p = 1, q = 0)$.

This idea captures why external mechanisms are needed to promote the efficient, cooperative outcome in the Prisoner's Dilemma and why two populations arrive at mutually exclusive but equally efficient conventions through coordination. This will be important later as we examine why a speaker might choose one utterance over another or why a hearer might interpret utterances differently. As our games in the later sections will feature multiple strategies and probabilistic beliefs, we should be able to extend this definition of expected utility over a larger space.

Given a game G with sets of possible actions for each player denoted A_i and A_j, we can now calculate a player's expected utility of playing a mixed strategy when the other player is also playing a mixed strategy as:

$$EU_i = \sum_{a \in A_i} Pr(a) \sum_{b \in A_j} Pr(b) U_i(a, b) \tag{2.4}$$

In a mixed strategy Nash Equilibrium, this utility will be equivalent to each player's utility from playing their respective pure strategies, given the mixing from the other. How might such an outcome arise? One case is where players do not know the choice made by their opponents, as is the case in games of *imperfect information.*

Example: Battle of the Sexes

Another coordination game is known as the *Battle of the Sexes* (BoS). As seen in Table 2.3, it depicts a stereotypical attempt at coordination between players each having their own preferred outcome, which we will denote *Sports* or *Musical*. They also prefer taking the same action as their partner, as both would get a zero payoff otherwise. If both were certain that the other would play S, neither would unilaterally deviate; likewise for M. This gives us Nash equilibria of (S, S) and (M, M).

	S	M
S	2;1	0;0
M	0;0	1;2

	p	$1-p$
$EU(S)$	2p	0(1-p)
$EU(M)$	0p	1(1-p)

Table 2.3: Battle of the Sexes (left) and Expected Utilities against a player with a mixed strategy.

As the players are acting simultaneously without communicating, we could also have uncertainty over the outcomes. We can imagine that the column player might play according to $Pr(S) = p$ and $Pr(M) = 1-p$. This would give the row player different expected utilities for each action, as seen on the right in Table 2.3. The row player would be indifferent between these two strategies when

$$EU(S) = EU(M) \Rightarrow 2p + 0(1-p) = 0p + 1(1-p) \Rightarrow p = \frac{1}{3}$$

This would give the row player an expected utility of $\frac{2}{3}$ for any combination of either choice. Thus a belief that the column player will play according to these probabilities would not give the row player any reason to deviate from his current strategy. The same is true in reverse for the row player playing according to $Pr(S) = q$ and $Pr(M) = 1-q$, and thus we have a mixed strategy equilibrium of $(p = \frac{1}{3}, q = \frac{2}{3})$. We can use this to visualize the probability that the game will be in each state and the respective utilities.

	$Pr(S) = \frac{1}{3}$	$Pr(M) = \frac{2}{3}$
$Pr(S) = \frac{2}{3}$	$\frac{2}{9}$	$\frac{4}{9}$
$Pr(M) = \frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$

	$EU(S)$	$EU(M)$
$EU(S)$	$\frac{4}{9}; \frac{2}{9}$	0;0
$EU(M)$	0;0	$\frac{2}{9}; \frac{4}{9}$

Table 2.4: Battle of the Sexes: Mixed Strategy Equilibrium Probabilities(left) and Expected Utilities (right).

As we can see here, the mixed strategy equilibrium does not reward the players to the same degree as either of the pure strategy equilibria, which we can now think of as $(p = 1, q = 1)$ or $(p = 0, q = 0)$.

Sequential Games with Imperfect Information

If we take a look back at the sequential form games and their corresponding representations in the normal form games, we might recognize that they are not exactly equivalent, at least not yet. One salient detail missing from the representation of the extensive form or sequential games is a *lack of information.* In other words, it is often the case that our partner may have taken an action and that we would not know which action they took. Intuitively, the previously seen sequential games played out like a game of chess, where each player could see the moves made by the other. A normal form game, with players acting simultaneously, plays out more like a silent auction, where I might only have a belief about one of many actions my opponent may have taken.

Example: The Sequential Prisoner's Dilemma

We will use that idea to model the Prisoner's Dilemma again, except for this time we will make the second player unsure of the action taken by the first. To model this, we place a dotted line between the two nodes representing the action taken by the first player, as seen in Figure 2.7.

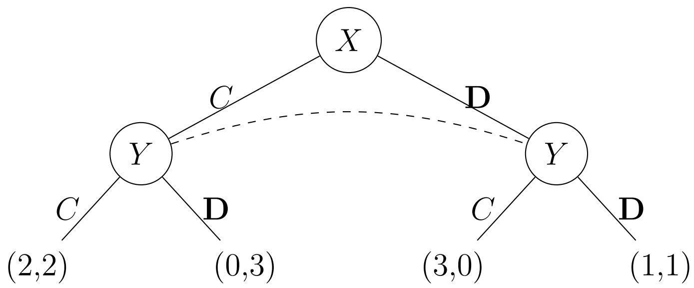

Figure 2.7: A Sequential PD: The dotted line means Y is unsure of what action X took. Dominant strategy of D for both players means equilibrium payoffs of $(1, 1)$ and no mixed strategy equilibria.

In general, this change is not merely a cosmetic effect. When realized into the strategic space of the game, it renders the actions in the extensive form equivalent to the normal form. Let us examine why. If the second player does not know what the first player did, then the first player's actions cannot influence the second player. This means that they could in effect act at the same time, just like the model found in the normal form game. In some games like the the Prisoner's Dilemma, it matters little what the first player does, as both players have a strictly better choice in choosing D.

Example: Comparing Coordination Games

If we were to take a game like the Coordination Game in Figure 2.8, we would see that choosing sequentially with perfect information as done in the left figure renders a much better outcome for rational players, as they will always coordinate on a $(1, 1)$ outcome, simply based on what choice the first player makes. If the players do not know what the other one is doing, then their expected utility could be significantly reduced, as they may play a mixed strategy.

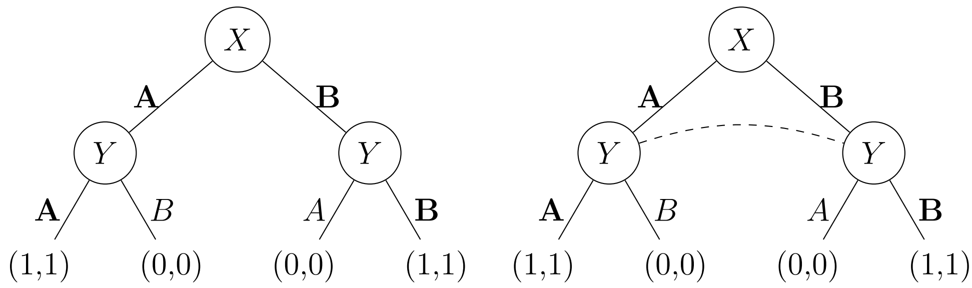

Figure 2.8: Pure Coordination Games: Perfect (left) and Imperfect Information (right). The dotted line represents an unsure belief on which action X takes. With perfect information, Y should always coordinate with the observed move X makes. With imperfect information, there are pure strategy equilibria of (A, A) or (B, B) or a mixed strategy of playing 50% for each.

As the Coordination Game is symmetric, the mixed strategy Nash Equilibrium yields the same payoffs for both of $\frac{1}{2}(\frac{1}{2}(1) + \frac{1}{2}(0)) + \frac{1}{2}(\frac{1}{2}(0) + \frac{1}{2}(1)) = \frac{1}{2}$. This would make either player indifferent to their own choices, given that the other player was mixing strategies according to this split, as choosing either option would yield 1 half of the time. Note this is less efficient than the case with perfect information and is equivalent to the mixed strategy outcome in the normal form game, seen in Table 2.5. .

	A	B
A	1,1	0,0
B	0,0	1,1

Table 2.5: The Pure Coordination Game in normal form.

2.5 Efficiency, Risk, and Public Goods

Part of the allure of game theory is how it manages to account for the emergence of group outcomes through the actions of self-interested individuals. This is especially true in the case of games with partially or fully conflicting interests like the Prisoner's Dilemma or Hawks & Doves, where the selfish nature of the agents and perverse incentives of the game make it more difficult to achieve efficient outcomes. We have seen that these games can end up in equilibrium, but the final outcome may not be so simple. To quote Myerson [1997] on this issue:

> Two general observations on Nash equilibria: A game may have equilibria that are *inefficient*, and a game may have *multiple* equilibria.

While the equilibrium in the Prisoner's Dilemma is inefficient from the point of view of the group's total utility, the Stag Hunt is an example of a game with multiple equilibria, one of which is more efficient than the other. [6] and the multiple equilibria in the Stag Hunt confound us when the most efficient equilibrium is also the one that requires a level of trust in the other player. We will use the Stag Hunt to get an intuition for these notions of efficiency and risk before pursuing them more formally.

Example: The Stag Hunt

The Stag Hunt presents us with a hybrid case of cooperation and coordination. Its original purpose was as a model of the social contract given by Rousseau, and it was later elaborated on in Skyrms [2004]. It gives us an example for understanding coordination, risk, expected utility, and signaling as well. This is not its only use however. Later chapters like **??** will also show how understanding the Stag Hunt also helps us understand the ways in which we negotiate the language of commitment and preferences regarding social contracts.

> *If it was a matter of hunting a deer, everyone well realized that he must remain faithful to his post; but if a hare happened to pass within reach of one of them, we cannot doubt that he would have gone off in pursuit of it without scruple*
> —Rousseau

If we look at the game in normal form (Table 2.6) , we see that two pure strategy equilibria arise. Among these equilibria, we can classify them as such: the *payoff-dominant* (S, S) outcome or the *risk-dominant* (H, H) outcome. To see why the first label is as such needs little effort, as its payoffs are best for the group. The second of these labels comes from the fact that if we had no idea about the play of our partner, deviating from *Stag* (S) unilaterally results only in a decrease of 1 from the equilibrium payoff, whereas deviating from $Hare$ (H) unilaterally would result in a decrease of 2 from the equilibrium payoff. As players move simultaneously, this means they do not know what move the other would make. Thus, the normal form is strategically equivalent to the sequential form with imperfect information, as seen in Figure 2.9.

	S	H
S	5,5	0,4
H	4,0	2,2

Table 2.6: The Stag Hunt in normal form with actions *Hare*(H) and *Stag*(H). Pure equilibria of (S, S) and (H, H). We can imagine its utilities as motivated by the amount of meat in a hunt.

[6] More efficient outcomes have higher total utilities for the group.

Despite the potential attractiveness of the payoff-dominant outcome, without an additional mechanism to ensure we arrive at it, the Stag Hunt maintains an element of risk. One such element is signaling. The deceptive part about the game is that the signaling must be in some way credible. This can be done through the content of the signal or by revelation of one agent's preferences Skyrms [2004], Fehr and Schmidt [1998]

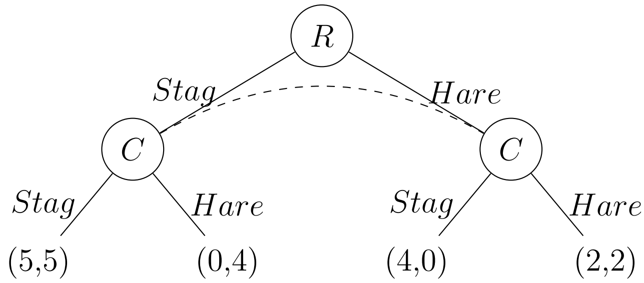

Figure 2.9: Stag Hunt (SH) in extensive form. Pure equilibria of both playing *Stag*- (S, S) or both playing *Hare*- (H, H). The second player is unsure of the first's move.

The Stag Hunt maintains an interesting mixed strategy equilibrium. For the payoffs in Figure 2.9, we have that an opponent's mix of playing *Stag* for $\frac{2}{3}$ of the time would make each player indifferent to their own actions, as they would each get a payoff of $\frac{10}{3}$ from any mix of S and H. Note again that this mix outperforms the risk-dominant payoffs of $(2, 2)$. This is still not the most efficient state however, and thus we would like other mechanisms for promoting the profile of $(5, 5)$.

When agents come together to achieve something better for the group, despite reasons against the choice individually, this is often known as *public good.*[Cárdenas and Ostrom, 2001]. The classic public goods game is the Prisoner's Dilemma: its payoff structure gives us a situation that would be better for all if the agents could choose against the dominant strategy. The trust game is another example of such a game, whose variants we will examine in detail. In later chapters, we will consider the idea that information and face are the public goods that drive the strategic incentives behind pragmatic strategies. The Stag Hunt is also a public goods game, but here the operative distraction is not temptation so much as uncertainty. Someone with an aversion to a zero payoff might chose *Hare* despite its lower payoff. In general, the concept of the public good being better for the group gives us the notion of efficiency.

Efficiency

There are multiple ways of measuring how well a group performs in a game. We should know what they are so as to be able to construct decision procedures based on them. One form of measurement is (weak) *Pareto-efficiency* or *Pareto-optimality.* We consider the two-player case.

Definition 8. *We say an outcome of actions* (a_i, a_j) *is* ***Pareto-efficient*** *if there* *does not exist an alternative outcome (b_i, b_j), where $U_i(b_i, b_j) \geq U_i(a_i, a_j)$ for all players i and strictly better for at least one player $x \in \{i, j\}$.*

Geometrically, we can think of the frontier of efficiency as a line in the two-player case, given by the maximum of $U_X + U_Y$ for players X and Y. Measuring efficiency by this sum of utilities is known as *Kaldor-Hicks Optimality* [Ellickson, 2001]. Outcomes that are optimal under this metric are immediately Pareto-optimal. If we look at Figure 2.10, we can see the payoffs in the Stag Hunt and the frontier of maximum efficiency.

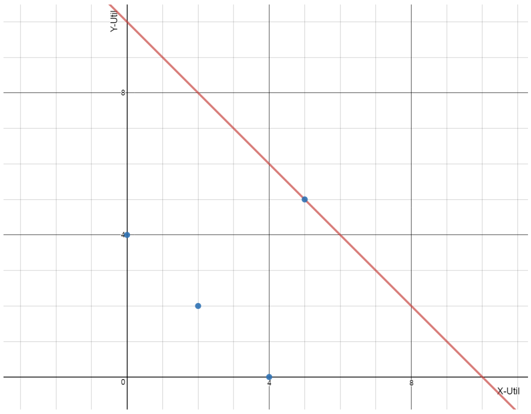

Figure 2.10: Graph of outcomes (5;5),(0;4), (4;0), and (2;2) in Stag Hunt with the line representing the frontier of Pareto-optimal outcomes. Any movement towards it represents an increase in efficiency.

We can see that states like (S, S) in the Stag Hunt or (C, C) in the Prisoner's Dilemma give us an example of attaining the public good in that they are both Pareto-optimal. The problem with (C, C) is that it is not an equilibrium because there is an individual incentive against choosing it.

Risk

The (S, S) outcome in the Stag Hunt is both Pareto-optimal and an equilibrium. It is not, however, the only equilibrium that can arise. The (H, H) Nash Equilibrium, while inefficient, can be termed *risk-dominant*, because the group's losses magnify when deviating from this state.

	A	B
A	P,P	Q,R
B	R,Q	S,S

Table 2.7: Generic $2x2$ Symmetric Game in normal form.

Taking the notions in Harsanyi and Selten [1988] and Samuelson [1998] , we can begin with a symmetric game as seen in Table 2.7, with equilibria (A, A) and (B, B). We now consider the joint loss from deviating from either state for the

	S	H
S	5;5	0;4
H	4;0	2;2

	C	H
S	9;9	0;8
H	8;0	7;7

Table 2.8: Stag Hunt as seen in Skyrms [2004] vs. Aumann [1990]. The risk-dominant outcome in Aumann's game has a larger basin of attraction and joint-loss value.

two players. For example, for deviating from (A, A), we have a a loss of $(P - R)$ for both players, and thus a joint-loss of $(P - R)^2$. [7]

Definition 9. *Given a symmetric* 2×2 *game as in Table 2.7, we say an equilibrium* (A, A) ***risk-dominates*** *an equilibrium* (B, B) *iff the joint-loss for deviating from* (A, A) *is greater than that from* (B, B)*. In this case,* $(P - R)^2 > (S - Q)^2$*.*

Consider the payoffs in Table 2.8. In the first case, when considering how the group's losses would compound when moving away from (H, H) or (S, S) unilaterally, we see that $(2 - 0)^2 > (5 - 4)^2$. The second case is more dramatic. When moving from (H, H) or (S, S), we see that $(7 - 0)^2 > (9 - 8)^2$. Thus even a low degree of uncertainty in the second case over whether a partner would coordinate might promote both agents to remain in (H, H), so as to minimize potential losses.

In the evolutionary sense [Samuelson, 1998], the outcome (H, H) has a much larger basin of attraction. I.e. if we are rewarded by our choice in one round of playing the game proportional to our payoffs, a larger set of initial variations in strategy will drive convergence of a population towards (H, H), especially in the game from Aumann [1990]. This would also mean that we would have a low incentive towards deviating from this state, as the losses for the group would be higher. This will play in later in our discussion of norms and how a population might want to push away from a risk-dominant default and towards a Pareto-optimal state. [8] Although Evolutionary Game Theory is a very active and applicable branch of game theory, we limit our discussion of it to this remark.

2.6 Signaling, Conventions, and Communication

Let us discuss briefly signaling games and pre-play communication. If we look back to the sections on normal and extensive form games, we see that each of the games presented before gave us not only a model of strategic interaction, but also a dilemma in some way or another. These dilemmas might arise a lack of efficient options in equilibrium like the Prisoner's Dilemma or from a lack of information in the Coordination Game. One difficulty in a coordination game is

[7] This joint-loss function is also called the *Nash Product* in discussions of bargaining.

[8] This is our claim as to why implicatures and face-saving are not the default strategy but rather are valuable norms that can stabilize in a cohesive population.

that the players have no idea what the other will do. The coordinative dilemma's property's have been much discussed as of late in the context of *signaling games*, sequential games of imperfect information.

In **??** we will see that repeating games with reputation monitoring is one way to improve the outcomes for both players. In the real world, such repeated interaction and reputation monitoring takes time and effort. Thankfully, we have another mechanism that, given the right incentive structure, can short-circuit this process: signaling.

Signaling games arose in the work of David Lewis when he attacked the problem of *convention* in the eponymous book [**?**]. His question centered on how conventions can arise without the benefit of prior agreement. To answer that question, he minted a sequential form game called a *signaling game* that models a sender with private information sending messages to a receiver who then interprets that message. Signaling games therefore provide a link between semantic meaning and pragmatic usage, and much of our discussion builds on works like Benz et al. [2005], Jäger [2011], Franke [2009], van Rooij [2003], and Clark [2013].

Signaling Game Example

Imagine a pair of friends Robin and Sam. Robin wants to know if Sam (S) will go to a party, but they are too far across the room to speak. Same wants to convey a proposition from "Yes" or "No" to Robin (R) without using words. Sam has two available gestures to send: he can shake his side to side or nod his head up and down. In many Western cultures, the conventional meaning for these gestures is obvious, and yet in Bulgaria we see exactly the opposite convention [Jakobson, 1972]. A signaling game like the one in Figure 2.11 helps us understand the dynamics of this process. [9]

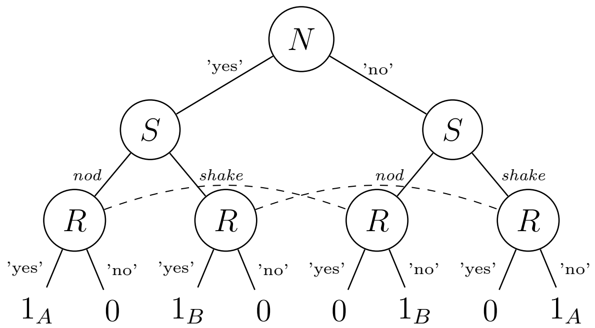

Figure 2.11: Extensive form for the canonical 2×2 signaling game's example *nod or shake*. Path with A represent one convention, those with B another.

Note that utility structure in the signaling game is much like that of a sequential Coordination Game except for a few key differences. It differs in that we first have a random move by *Nature*, which can be seen only by the Sender. Next, the Receiver's information states are certain about the Sender's move but

[9] Thanks to my colleague Roland Mühlenbernd for help with the figure.

not Nature's move. The utility function rewards pairing the action with the appropriate type. While the Sender or Receiver could choose the same message or action repeatedly in a *pooling equilbrium*, the more efficient equilibrium is to arrive at a convention associating a unique message with a type-action pairing. In a setting with types occurring equiprobably and message of no cost, which message is associated does not impact the efficiency as much as the *separating equilibrium* of a one-to-one correspondence between the types and actions. In the example from Figure 2.11, we could have one of two conventions between an intention and interpretation:

A: American

- *Yes* → **Nod** → *Yes*
- *No* → **Shake** → *No*

B: Bulgarian

- *Yes* → **Shake** → *Yes*
- *No* → **Nod** → *No*

The model of the signaling game can become considerably more complex. One wrinkle that we can attach to the signaling game is that of *costly signals*, messages that cost something to send. Costly signaling has a literature of its own, especially in terms of evolutionary dynamics, much of which we will not have space to cover. We will late, however, address several key points and results that will be of use when we discuss whether politeness truly is a costly signal in the sense of works like Zahavi [1975], Spence [1973], and Zahavi [1993]. This will help us to address the first attempt at modeling politeness game-theoretically from van Rooij [2003].

Cheap Talk vs. Costly Signaling

Communication can present problems, even when the messages are without cost. This is especially true in the case of pre-play comunication[Farrell, 1988]. To distinguish this from signaling, we should keep in mind that in a signaling game the strategies are over functions from types to messages and messages to actions. In other words, the players are determining the meaning of the messages through the game, where the meaning is a stable equilbrium of correspondence between the type of S and the action of R.

In a cheap talk game, messages arrive with pre-determined meanings. The players then have the option to engage in a costless round of communication before playing a game at hand.[Farrell and Rabin, 1996]. This can help agents resolve coordination problems simply like the Coordination Game, but in a cooperatiopn problem like the Prisoner's Dilemma, messages without cost still leave room for deception.

While other works in the intersection of game theory and pragmatics have focused primarily on signaling to derive conventionalized implicatures under coordination, we will focus more on the dynamics of norms that bring about situations of coordination. This will involve understanding speech acts, relationship dynamics, and their connections with social dilemmas. This is why, in some cases, we will assume the conventionalization of some utterances and attempt to understand their place under the larger umbrella of societal norms.

2.7 Conclusion

This chapter has been a warm-up of sorts to prepare us for a deeper look at trust games and their properties when repeated. In general, we would like to point out the following concepts and ideas:

- Games can be depicted in normal form or extensive form. Games can also be symmetric or asymmetric.
- The trust game is an asymmetric game in extensive form. We will symmetrize it in normal form to analyze population-level behavior.
- Performing backwards induction on games like the trust game and centipede gives us a way to find equilibria in an extensive form game.
- In a symmetric game like the Prisoner's Dilemma, we see that efficient outcomes may not always be equilbria, in contrast to the Stag Hunt.
- In the coming chapters, we will show how to manipulate games like the Prisoner's Dilemma into a strategically equivalent version of the Stag Hunt.
- Under incentive structures in coordination games like the Stag Hunt, meaningful communication is possible, and thus conventionalization can occur.

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