Sequence Form Solver Documentation and Test Cases

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1 Code Explained

The file `sequence_form.py` contains the following main methods:

- `decorate_sequences(bt)`
  This function consumes a tree, and performs a breadth-first search so as to decorate each node with the unique sequences of player 1 and player 2's movement decisions from the root to itself. A decision is encoded as the index of the child, enumerating from left to right. (For example, if a given node has two children, and the sequence elects to move to the leftmost child, a '0' is stored). The method then returns a list of nodes.

- `extensive_to_strategic_form(game)`
  As it sounds, this method intakes a decorated game, and populates the A and B matrices, corresponding to players' payoff matrices.

- `extensive_to_sequence_form(game)`
  This method, similarly, intakes a decorated game, and returns the A, B, E, F, e, f, matrices.

- `solve_strategic_form(game)`
  This method computes player 1's strategy for inputted strategic form matrices A, B.

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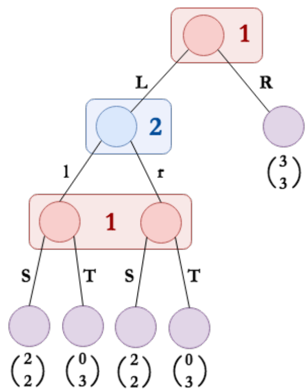
The file `sequence_form_test.py` contains various named tests, annotated in the ensuing pages.

2 Tests

2.1 Von Stengel Test

In this test, we solve the same tree as in the paper: [1]



The associated payoff matrices for players A and B are as follows:

$$
A = \begin{array}{c} \\ \end{array}
\begin{array}{c|ccc|c}
 & \varnothing & l & r & \\
\hline
 & & & & \varnothing \\
 & & & & L \\
 & 3 & & & R \\
 & & 2 & 5 & LS \\
 & & 0 & 6 & LT
\end{array}
\qquad
B = \begin{array}{c|ccc|c}
 & \varnothing & l & r & \\
\hline
 & & & & \varnothing \\
 & & & & L \\
 & 3 & & & R \\
 & & 2 & 6 & LS \\
 & & 3 & 1 & LT
\end{array}
$$

The associated constraint matrices are as follows:

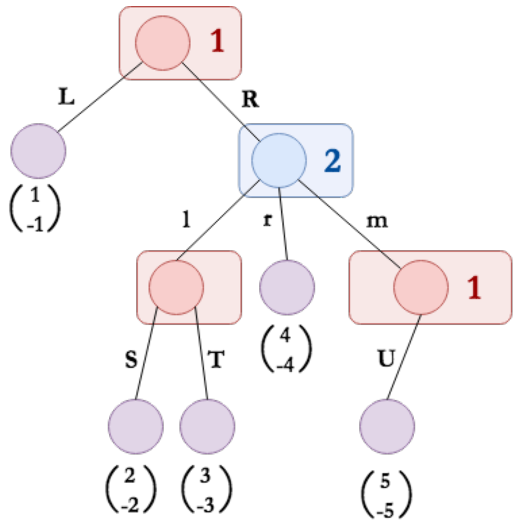
$$
E = \begin{array}{c}
\begin{array}{ccccc} \varnothing & L & R & LS & LT \end{array} \\
\begin{bmatrix} 1 & & & & \\ -1 & 1 & 1 & & \\ & -1 & & 1 & 1 \end{bmatrix}
\end{array}, \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
F = \begin{array}{c}
\begin{array}{ccc} \varnothing & l & r \end{array} \\
\begin{bmatrix} 1 & & \\ -1 & 1 & 1 \end{bmatrix}
\end{array}, \qquad f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
$$

[1] Colored blocks indicate information sets, where 1 implies player A's turn, and 2 implies player B's.

2.2 *Test 2: Three Branches, Unbalanced*

In this test, we solve the following zero-sum game:



The associated payoff matrices for players A and B are as follows:

$$A = \begin{array}{cc} \begin{matrix} \varnothing & l & r & m \end{matrix} & \\ \begin{bmatrix} & & & \\ 1 & & & \\ & & 4 & \\ & 2 & & \\ & 3 & & \\ & & & 5 \end{bmatrix} & \begin{matrix} \varnothing \\ L \\ R \\ RS \\ RT \\ RU \end{matrix} \end{array} \qquad B = \begin{array}{cc} \begin{matrix} \varnothing & l & r & m \end{matrix} & \\ \begin{bmatrix} & & & \\ -1 & & & \\ & & -4 & \\ & -2 & & \\ & -3 & & \\ & & & -5 \end{bmatrix} & \begin{matrix} \varnothing \\ L \\ R \\ RS \\ RT \\ RU \end{matrix} \end{array}$$

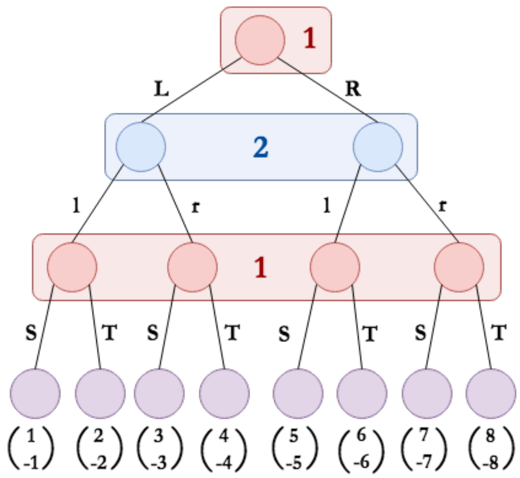
The associated constraint matrices are as follows:

$$E = \begin{array}{c} \begin{matrix} \varnothing & L & R & RS & RT & RU \end{matrix} \\ \begin{bmatrix} 1 & & & & & \\ -1 & 1 & 1 & & & \\ & & -1 & 1 & 1 & \\ & & -1 & & & 1 \end{bmatrix} \end{array}, \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{array}{c} \begin{matrix} \varnothing & l & r & m \end{matrix} \\ \begin{bmatrix} 1 & & & \\ -1 & 1 & 1 & 1 \end{bmatrix} \end{array}, \qquad f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2.3 *Test 3: Complete Binary Tree, Depth 3*

In this test, we solve the following zero-sum game on a complete binary tree:



The associated payoff matrices for players A and B are as follows:

$$A = \begin{array}{c} \begin{array}{ccc} \varnothing & l & r \end{array} \\ \begin{bmatrix} & & \\ & & \\ & & \\ & 1 & 3 \\ & 2 & 4 \\ & 5 & 7 \\ & 6 & 8 \end{bmatrix} \end{array} \begin{array}{c} \\ \varnothing \\ L \\ R \\ LS \\ LT \\ RS \\ RT \end{array} \qquad B = \begin{array}{c} \begin{array}{ccc} \varnothing & l & r \end{array} \\ \begin{bmatrix} & & \\ & & \\ & & \\ & -1 & -3 \\ & -2 & -4 \\ & -5 & -7 \\ & -6 & -8 \end{bmatrix} \end{array} \begin{array}{c} \\ \varnothing \\ L \\ R \\ LS \\ LT \\ RS \\ RT \end{array}$$

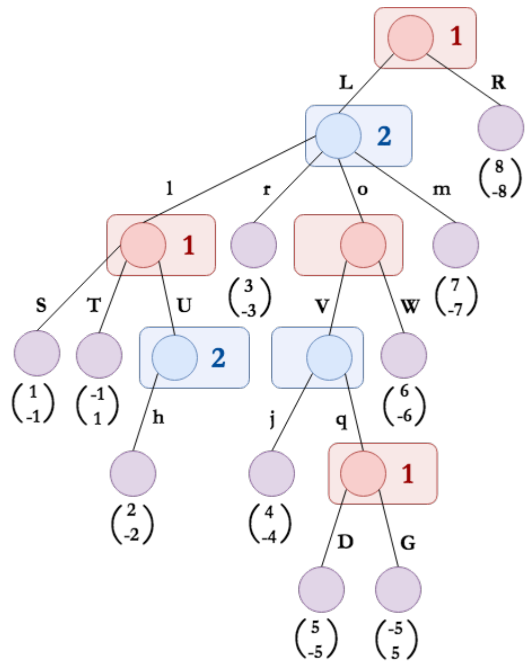
The associated constraint matrices are as follows:

$$E = \begin{array}{c} \begin{array}{ccccccc} \varnothing & L & R & LS & LT & RS & RT \end{array} \\ \begin{bmatrix} 1 & & & & & & \\ -1 & 1 & 1 & & & & \\ & -1 & & 1 & 1 & & \\ & & -1 & & & 1 & 1 \end{bmatrix} \end{array}, \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{array}{c} \begin{array}{ccc} \varnothing & l & r \end{array} \\ \begin{bmatrix} 1 & & \\ -1 & 1 & 1 \end{bmatrix} \end{array}, \qquad f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2.4 *Test 4: Highly Uneven Tree, Depth 5*

In this test, we solve the following zero-sum game on a highly uneven tree:



The associated payoff matrices for players A and B are as follows:

$$
A = \begin{array}{cc}
\begin{array}{cccccccc} \varnothing & l & r & o & m & lh & oj & oq \end{array} & \\
\begin{bmatrix}
 & & & & & & & \\
 & & 3 & & 7 & & & \\
8 & & & & & & & \\
 & 1 & & & & & & \\
 & -1 & & & & & & \\
 & & & & & 2 & & \\
 & & & & & & 4 & \\
 & & & 6 & & & & \\
 & & & & & & & 5 \\
 & & & & & & & -5
\end{bmatrix} &
\begin{array}{c} \varnothing \\ L \\ R \\ LS \\ LT \\ LU \\ LV \\ LW \\ LVD \\ LVG \end{array}
\end{array}
$$

$$B = \begin{array}{cc} \begin{matrix} \varnothing & l & r & o & m & lh & oj & oq \end{matrix} & \\ \begin{bmatrix} & & & & & & & \\ & & -3 & & -7 & & & \\ -8 & & & & & & & \\ & -1 & & & & & & \\ & 1 & & & & & & \\ & & & & & -2 & & \\ & & & & & & -4 & \\ & & & -6 & & & & \\ & & & & & & & -5 \\ & & & & & & & 5 \end{bmatrix} & \begin{matrix} \varnothing \\ L \\ R \\ LS \\ LT \\ LU \\ LV \\ LW \\ LVD \\ LVG \end{matrix} \end{array}$$

The associated constraint matrices are as follows:

$$E = \begin{array}{c} \begin{matrix} \varnothing & L & R & LS & LT & LU & LV & LW & LVD & LVG \end{matrix} \\ \begin{bmatrix} 1 & & & & & & & & & \\ -1 & 1 & 1 & & & & & & & \\ & -1 & & 1 & 1 & 1 & & & & \\ & -1 & & & & & 1 & 1 & & \\ & & & & & & -1 & & 1 & 1 \end{bmatrix} \end{array}, \quad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{array}{c} \begin{matrix} \varnothing & l & r & o & m & lh & oj & oq \end{matrix} \\ \begin{bmatrix} 1 & & & & & & & \\ -1 & 1 & 1 & 1 & 1 & & & \\ & -1 & & & & 1 & & \\ & & & -1 & & & 1 & 1 \end{bmatrix} \end{array}, \quad f = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$