FHHPS Code structure

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Abstract

Code structure

1 Second moments

As opposed to the paper, in the code we work with the centered second moments. To make notation simpler, let $I = ([cond])$, and let $\mathrm{Var}_I[X_i] = \mathrm{Var}\left[X_i \,\middle|\, I\right]$ and .

$$\begin{aligned}\mathrm{Var}_I[Y_1] = {} & \mathrm{Var}_I[A_1] + \mathrm{Var}_I[B_1]X_1^2 + \mathrm{Var}_I[C_1]Z_1^2 \\ & + 2\,\mathrm{Cov}_I[A_1, B_1]X_1^2 + 2\,\mathrm{Cov}_I[A_1, C_1]Z_1 \\ & + 2\,\mathrm{Cov}_I[B_1, C_1]X_1Z_1\end{aligned} \tag{1}$$

$$\begin{aligned}\mathrm{Var}_I[Y_2] = {} & \mathrm{Var}_I[A_1] + \mathrm{Var}_I[B_1]X_2^2 + \mathrm{Var}_I[C_1]Z_2^2 \\ & + 2\,\mathrm{Cov}_I[A_1, B_1]X_2 + 2\,\mathrm{Cov}_I[A_1, C_1]Z_2 \\ & + 2\,\mathrm{Cov}_I[B_1, C_1]X_2Z_2 \\ & \mathrm{Var}_I[U_2] + \mathrm{Var}_I[V_2]X_2^2 + \mathrm{Var}_I[W_2]Z_2^2 \\ & + 2\,\mathrm{Cov}_I[U_2, V_2]X_2 + 2\,\mathrm{Cov}_I[U_2, W_2]Z_2 \\ & + 2\,\mathrm{Cov}_I[V_2, W_2]X_2Z_2\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{Var}_I[Y_3] = {} & \mathrm{Var}_I[A_1] + \mathrm{Var}_I[B_1]X_3^2 + \mathrm{Var}_I[C_1]Z_3^2 \\ & + 2\,\mathrm{Cov}_I[A_1, B_1]X_3 + 2\,\mathrm{Cov}_I[A_1, C_1]Z_3 \\ & + 2\,\mathrm{Cov}_I[B_1, C_1]X_3Z_3 \\ & + \mathrm{Var}_I[U_2] + \mathrm{Var}_I[V_2]X_3^2 + \mathrm{Var}_I[W_2]Z_3^2 \\ & + 2\,\mathrm{Cov}_I[U_2, V_2]X_3 + 2\,\mathrm{Cov}_I[U_2, W_2]Z_3 \\ & + 2\,\mathrm{Cov}_I[V_2, W_2]X_3Z_3 \\ & + \mathrm{Var}_I[U_3] + \mathrm{Var}_I[V_3]X_3^2 + \mathrm{Var}_I[W_3]Z_3^2 \\ & + 2\,\mathrm{Cov}_I[U_3, V_3]X_3 + 2\,\mathrm{Cov}_I[U_3, W_3]Z_3 \\ & + 2\,\mathrm{Cov}_I[V_3, W_3]X_3Z_3\end{aligned} \tag{3}$$

$$\begin{aligned}\mathrm{Cov}_I[Y_1, Y_2] = {} & \mathrm{Var}_I[A_1] + \mathrm{Var}_I[B_1]X_1X_2 + \mathrm{Var}_I[C_1]Z_1Z_2 \\ & + \mathrm{Cov}_I[A_1, B_1](X_1 + X_2) + \mathrm{Cov}_I[A_1, C_1](Z_1 + Z_2) \\ & + \mathrm{Cov}_I[B_1, C_1](X_1Z_2 + X_2Z_1)\end{aligned} \tag{4}$$

$$
\begin{aligned}
\operatorname{Cov}_I[Y_1,Y_3] = {} & \operatorname{Var}_I[A_1] + \operatorname{Var}_I[B_1]\, X_1X_3 + \operatorname{Var}_I[C_1]\, Z_1Z_3 \\
& + \operatorname{Cov}_I[A_1,B_1](X_1+X_3) + \operatorname{Cov}_I[A_1,C_1](Z_1+Z_3) \\
& + \operatorname{Cov}_I[B_1,C_1](X_1Z_3+X_3Z_1)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\operatorname{Cov}_I[Y_2,Y_3] = {} & \operatorname{Var}_I[A_1] + \operatorname{Var}_I[B_1]\, X_2X_3 + \operatorname{Var}_I[C_1]\, Z_2Z_3 \\
& + \operatorname{Cov}_I[A_1,B_1](X_2+X_3) + \operatorname{Cov}_I[A_1,C_1](Z_2+Z_3) \\
& + \operatorname{Cov}_I[B_1,C_1](X_2Z_3+X_3Z_2) \\
& \operatorname{Var}_I[U_2] + \operatorname{Var}_I[V_2]\, X_2X_3 + \operatorname{Var}_I[W_2]\, Z_2Z_3 \\
& + \operatorname{Cov}_I[U_2,V_2](X_2+X_3) + \operatorname{Cov}_I[U_2,W_2](Z_2+Z_3) \\
& + \operatorname{Cov}_I[V_2,W_2](X_2Z_3+X_3Z_2)
\end{aligned} \tag{6}
$$

Putting (1)-(6) together:

$$
\begin{bmatrix}
\operatorname{Var}_I[Y_1] \\ \operatorname{Var}_I[Y_2] \\ \operatorname{Var}_I[Y_3] \\ \operatorname{Cov}_I[Y_1,Y_2] \\ \operatorname{Cov}_I[Y_1,Y_3] \\ \operatorname{Cov}_I[Y_2,Y_3]
\end{bmatrix}
=
\begin{bmatrix}
1 & X_1^2 & Z_1^2 & 2X_1 & 2Z_1 & 2X_1Z_1 \\
1 & X_2^2 & Z_2^2 & 2X_2 & 2Z_2 & 2X_2Z_2 \\
1 & X_3^2 & Z_3^2 & 2X_3 & 2Z_3 & 2X_3Z_3 \\
1 & X_1X_2 & Z_1Z_2 & X_1+X_2 & Z_1+Z_2 & X_1Z_2+X_2Z_1 \\
1 & X_1X_3 & Z_1Z_3 & X_1+X_3 & Z_1+Z_3 & X_1Z_3+X_3Z_1 \\
1 & X_2X_3 & Z_2Z_3 & X_2+X_3 & Z_2+Z_3 & X_2Z_3+X_3Z_2
\end{bmatrix}
\begin{bmatrix}
\operatorname{Var}_I[A_1] \\ \operatorname{Var}_I[B_1] \\ \operatorname{Var}_I[C_1] \\ \operatorname{Cov}_I[A_1,B_1] \\ \operatorname{Cov}_I[A_1,C_1] \\ \operatorname{Cov}_I[B_1,C_1]
\end{bmatrix}
+
\begin{bmatrix}
E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6
\end{bmatrix} \tag{7}
$$

where the E_k are the second moment *excess terms* that involve shock moments and must be subtracted from the left-hand side before the inversion step. By inspection, $E_1 = E_5 = E_6 = 0$ and

$$
\begin{aligned}
E_2 = {} & \operatorname{Var}_I[U_2] + \operatorname{Var}_I[V_2]\, X_2^2 + \operatorname{Var}_I[W_2]\, Z_2^2 && (8) \\
& + 2\operatorname{Cov}_I[U_2,V_2]\, X_2 + 2\operatorname{Cov}_I[U_2,W_2]\, Z_2 && (9) \\
& + 2\operatorname{Cov}_I[V_2,W_2]\, X_2Z_2 && (10)
\end{aligned}
$$

$$
\begin{aligned}
E_3 = {} & \operatorname{Var}_I[U_2] + \operatorname{Var}_I[V_2]\, X_3^2 + \operatorname{Var}_I[W_2]\, Z_3^2 && (11) \\
& + 2\operatorname{Cov}_I[U_2,V_2]\, X_3 + 2\operatorname{Cov}_I[U_2,W_2]\, Z_3 && (12) \\
& + 2\operatorname{Cov}_I[V_2,W_2]\, X_3Z_3 && (13) \\
& + \operatorname{Var}_I[U_3] + \operatorname{Var}_I[V_3]\, X_3^2 + \operatorname{Var}_I[W_3]\, Z_3^2 && (14) \\
& + 2\operatorname{Cov}_I[U_3,V_3]\, X_3 + 2\operatorname{Cov}_I[U_3,W_3]\, Z_3 && (15) \\
& + 2\operatorname{Cov}_I[V_3,W_3]\, X_3Z_3 && (16)
\end{aligned}
$$

$$
\begin{aligned}
E_6 = {} & \operatorname{Var}_I[U_2] + \operatorname{Var}_I[V_2]\, X_2X_3 + \operatorname{Var}_I[W_2]\, Z_2Z_3 && (17) \\
& + \operatorname{Cov}_I[U_2,V_2](X_2+X_3) + \operatorname{Cov}_I[U_2,W_2](Z_2+Z_3) && (18) \\
& + \operatorname{Cov}_I[V_2,W_2](X_2Z_3+X_3Z_2) && (19)
\end{aligned}
$$