Chapter 03 Risk Evaluation (Part II)

Financial Risk Management

February 23, 2016

Outline

- Stochastic Dominance: Theory and Applications
- Index of Riskiness: Theory and Applications
- Almost Stochastic Dominance: Theory and Applications

Stochastic Dominance (SD)

- First-degree Stochastic Dominance (FSD)
- Second-degree Stochastic Dominance (SSD)
- Applications

First-degree Stochastic Dominance (FSD) (1/3)

- Assume that $\tilde{x} \in [a, b]$.
- Denote $F(x) = \text{Prob}_F(\tilde{x} \leq x)$ and $G = \text{Prob}_G(\tilde{x} \leq x)$.
- Definition: F FSD G if

$$F(x) \leq G(x) \ \forall x.$$

- Note that $F(x) \leq G(x) \ \forall x$ implies $E_F(x) \geq E_G(x)$.

First-degree Stochastic Dominance (FSD) (2/3)

- Suppose that $x = -1,000$. $F(x) \leq G(x)$ at $x = -1000$ means that

$$\Pr ob_F(\tilde{x} \leq -1000) \leq \Pr ob_G(\tilde{x} \leq -1000).$$

- On the other hand, suppose that $x = 1,000$. $F(x) \leq G(x)$ at $x = 1000$ means that

$$\Pr ob_F(\tilde{x} \leq 1000) \leq \Pr ob_G(\tilde{x} \leq 1000).$$

- In other words,

$$\Pr ob_F(\tilde{x} > 1000) > \Pr ob_G(\tilde{x} > 1000).$$

First-degree Stochastic Dominance (FSD) (3/3)

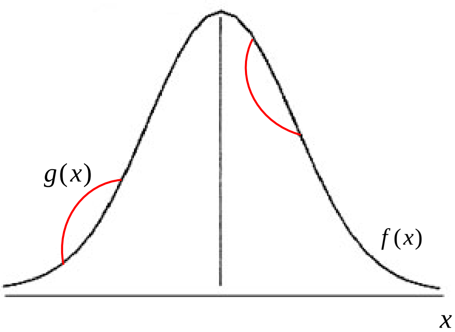
- Theorem: $Eu_F(x) \geq Eu_G(x)$ for all individuals with $u' > 0$ iff

$$F(x) \leq G(x) \ \forall x.$$

- Proof:

$$\begin{aligned} Eu_F(x) - Eu_G(x) &= \int u(x) d\left[F(x) - G(x)\right] \\ &= -\int u'(x)\left[F(x) - G(x)\right] dx. \end{aligned}$$

Figures of FSD (1/2)



Figures of FSD (2/2)

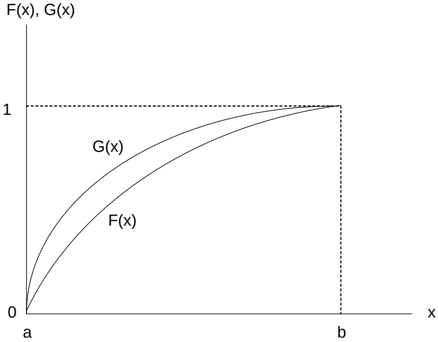


Figure of Example 1

- $F \equiv (0.5, 0.5; -1, 11); G \equiv (0.75, 0.25; -1, 11)$

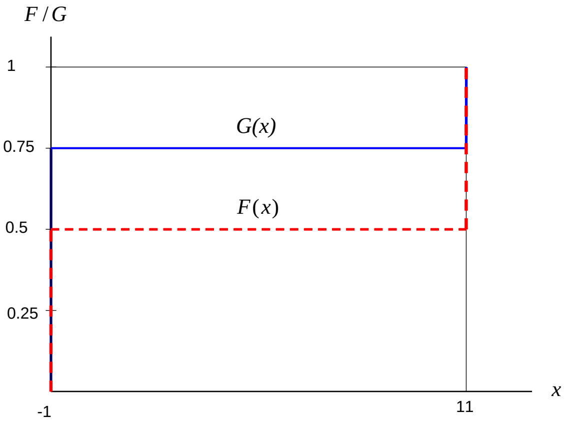
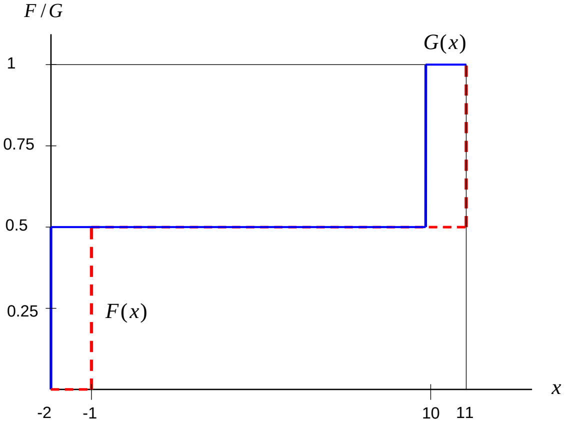


Figure of Example 2

- $F \equiv (0.5, 0.5; -1, 11)$; $G \equiv (0.5, 0.5; -2, 10)$



Second-degree Stochastic Dominance (SSD) (1/4)

- Define $F^{(2)}(x) = \int_a^x F(t)dt$ and $G^{(2)}(x) = \int_a^x G(t)dt$.
- Note that

$$\begin{aligned} F^{(2)}(x) &= \int_a^x F(t)dt = xF(x) - \int_a^x tdF(t) \\ &= \int_a^x (x-t)dF(t) \\ &= F(x)\int_a^x (x-t)\frac{f(t)}{F(x)}dt. \end{aligned}$$

Second-degree Stochastic Dominance (SSD) (2/4)

- Definition: F SSD G if

$$F^{(2)}(x) \leq G^{(2)}(x) \ \forall x.$$

- Note that $F(x) \leq G(x) \ \forall x$ implies $F^{(2)}(x) \leq G^{(2)}(x) \ \forall x$, which means that FSD implies SSD.
- Further, $F^{(2)}(x) \leq G^{(2)}(x) \ \forall x$ requires $F^{(2)}(b) \leq G^{(2)}(b)$ which is equivalent to

$$b - E_F(x) \leq b - E_G(x).$$

  Thus, SSD requires

$$E_F(x) \geq E_G(x).$$

Second-degree Stochastic Dominance (SSD) (3/4)

- Suppose that $x = -1,000$. $F^{(2)}(x) \leq G^{(2)}(x)$ at $x = -1000$ means that

$$\begin{aligned} \Pr ob_F(\tilde{x} &\leq -1000) \times E_F(x|\, \tilde{x} \leq -1000) \\ &\leq \Pr ob_G(\tilde{x} \leq -1000) \times E_G(x|\, \tilde{x} \leq -1000). \end{aligned}$$

- On the other hand, suppose that $x = 1,000$. $F^{(2)}(x) \leq G^{(2)}(x)$ at $x = 1000$ means that

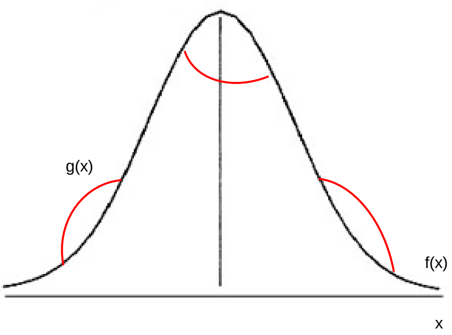
$$\begin{aligned} \Pr ob_F(\tilde{x} &\leq 1000) \times E_F(x|\, \tilde{x} \leq 1000) \\ &\leq \Pr ob_G(\tilde{x} \leq 1000) \times E_G(x|\, \tilde{x} \leq 1000). \end{aligned}$$

Second-degree Stochastic Dominance (SSD) (4/4)

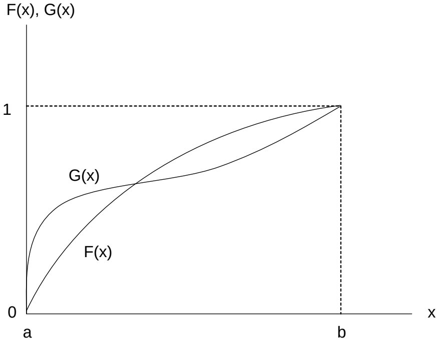
- Theorem: $Eu_F(x) \geq Eu_G(x)$ for all individuals with $u' > 0$ and $u'' < 0$ iff
$$F^{(2)}(x) \leq G^{(2)}(x) \ \forall x.$$
- Sketch of proof

$$\begin{aligned} & Eu_F(x) - Eu_G(x) \\ = \quad & -\int_a^b u'(x) \left[F(x) - G(x)\right] dx \\ = \quad & -u'(b) \left[F^{(2)}(b) - G^{(2)}(b)\right] \\ & + \int_a^b u''(x) \left[F^{(2)}(x) - G^{(2)}(x)\right] dx. \end{aligned}$$

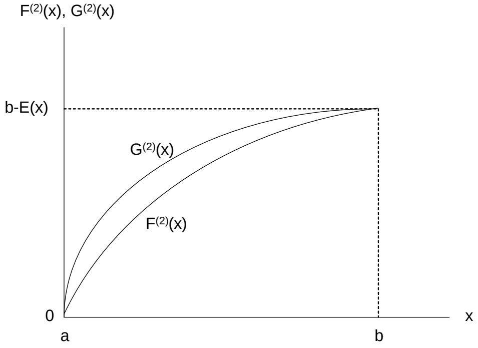
Figures of SSD (1/3)



Figures of SSD (2/3)

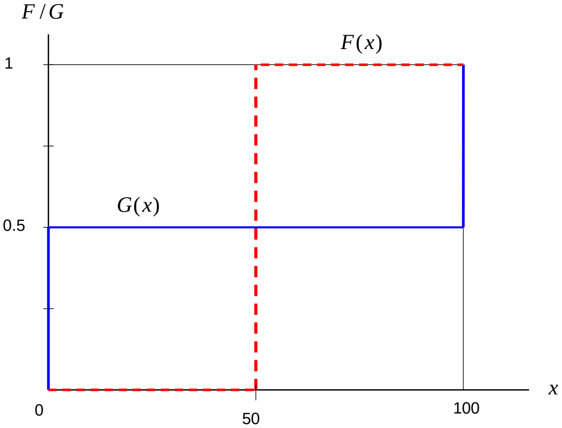


Figures of SSD (3/3)

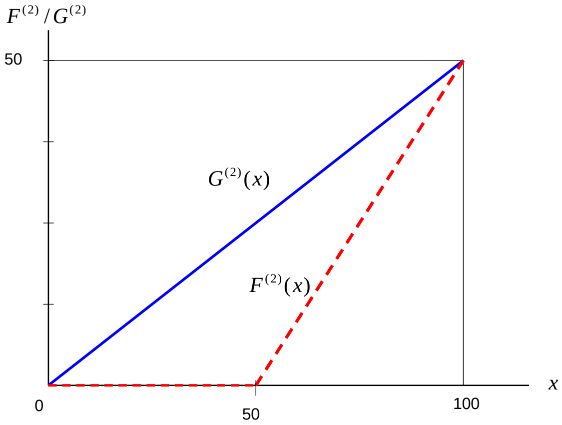


An Example of SSD (1/2)

- $F \equiv (1;50);\ G \equiv (0.5, 0.5; 0, 100)$



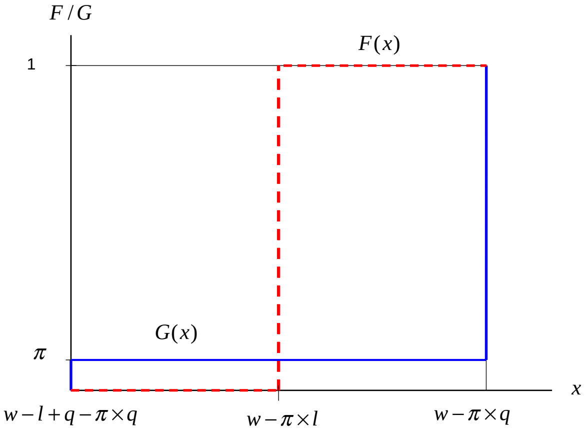
An Example of SSD (2/2)

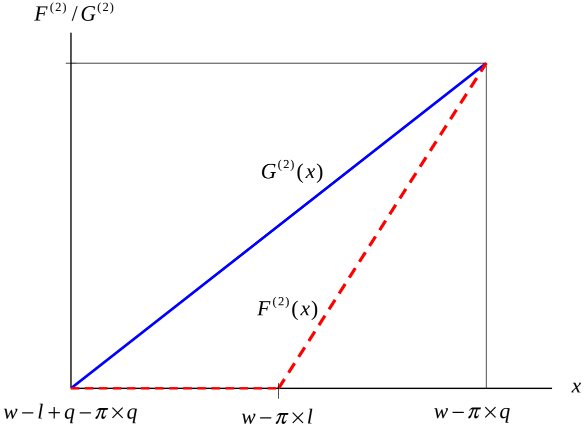


An Example in Insurance: Schlesinger (1997, JRI) (1/3)

- Let an individual with initial wealth w face a random loss l with probability π.
- She can purchase an insurance with coverage q by paying premium πq.
- Purchasing full coverage is a SSD strategy than purchasing any coverage less than l.

An Example in Insurance: Schlesinger (1997, JRI) (2/3)

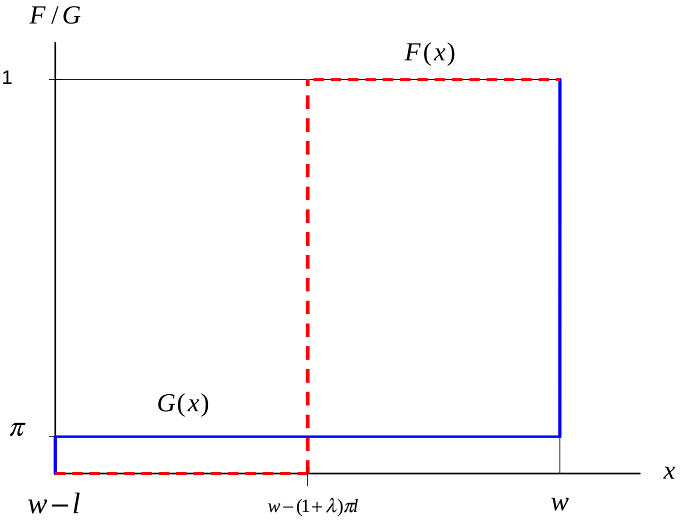



$F^{(2)}/G^{(2)}$
$G^{(2)}(x)$
$F^{(2)}(x)$
x
$w-l+q-\pi\times q$
$w-\pi\times l$
$w-\pi\times q$

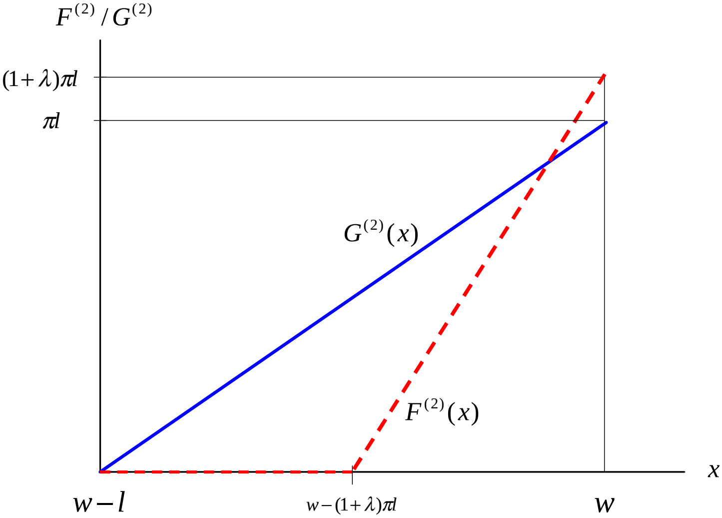
Another Example in Insurance: Unfair Premium (1/3)

- Let an individual with initial wealth w face a random loss l with probability π.
- She can purchase an insurance with coverage q by paying premium $(1 + \lambda)\, \pi q$.
- Will a risk-averse agent prefer purchasing full coverage to no insurance?

An Example in Insurance: Unfair Premium (2/3)



An Example in Insurance: Unfair Premium (3/3)



Applications: Yen Carry Trades v.s. Stock (Fong, 2010, JBF)

- Yen carry trades have made headline news for over a decade. We examine the profitability of such trades for the period 2001–2009. Yen carry trades generated high mean returns and Sharpe ratios prior to the recent financial crisis. They continued to outperform major stock markets for the full sample period. Given the non-normality of carry trade returns, we apply non-parametric tests based on stochastic dominance (SD) to evaluate whether the high returns of yen carry trades are compatible with risk as reflected in returns on US and global stock market indices. We apply a general test for SD developed recently by Linton, Maasoumi and Whang (2005) to six currencies as well as portfolios of these currencies. For a large class of risk-averse investors, profits from yen carry trades cannot be attributed to risks.

Applications: Portfolio Insurance (Annaert, Van Osselaer and Verstraete, 2009, JBF)

- This paper evaluates the performance of the stop-loss, synthetic put and constant proportion portfolio insurance techniques based on a block-bootstrap simulation. We consider not only traditional performance measures, but also some recently developed measures that capture the non-normality of the return distribution (value-at-risk, expected shortfall, and the Omega measure). We compare them to the more comprehensive stochastic dominance criteria. The impact of changing the rebalancing frequency and level of capital protection is examined. We find that, even though a buy-and-hold strategy generates higher average excess returns, it does not stochastically dominate the portfolio insurance strategies, nor vice versa. Our results indicate that a 100% floor value should be preferred to lower floor values and that daily-rebalanced synthetic put and CPPI strategies dominate their counterparts with less frequent rebalancing.

Applications: Value versus Growth (Abhyankar, Ho and Zhao, 2008, QF)

- In this paper, we study the relative performance of value versus growth strategies from the perspective of stochastic dominance. Using half a century US data on value and growth stocks, we find no evidence against the widely documented fact that value stocks stochastically dominate growth stocks in all three orders of stochastic dominance relations over the full sample period as well as during economic boom (good) periods. However, we observe no significant stochastic dominance relation between value and growth stocks during recession (bad) periods, which is inconsistent with the risk-based predictions but is better explained by behavioural models.

Applications: Initial Public Offerings (Abhyankar, Chen, Ho, 2006, QREF)

- We examine the long-run performance of initial public offerings (IPOs) using the idea of stochastic dominance. We find that there is no first-order stochastic dominance relation between the IPO portfolio and the benchmark of a broad index or a portfolio including either small size or low book-to-market stocks. However, those benchmarks second-order stochastically dominate the IPO portfolio. When using a portfolio including both small size and low book-to-market stocks as benchmark, there is a clear dominance of the IPO portfolio over the benchmark for both orders. Our findings generally imply that the question of assessing portfolio performance between IPO firms and benchmark portfolios depends critically on the specific construction or the cumulative distribution function of the benchmark portfolios.

Applications: Technical Trading (Taylor, 2014, JBF)

- The purpose of this paper is to examine the performance of an important set of momentum-based technical trading rules (TTRs) applied to all members of the Dow Jones Industrial Average (DJIA) stock index over the period 1928–2012. Using a set of econometric models that permit time-variation in riskadjusted returns to TTR portfolios, the results reveal that profits evolve slowly over time, are confined to particular episodes primarily from the mid-1960s to mid-1980s, and rely on the ability of investors to short-sell stocks. These findings are demonstrated to be consistent with theoretical models that predict a relationship between TTR performance and market conditions.

Index of Riskiness

- Aumann and Serrano (2008, JPE)
- Foster and Hart (2009, JPE)
- Bali et al. (2011, JPE)
- Applications

Motivation for New Index of Riskiness

- Most of the commonly used indices of riskiness in the literature do not satisfy monotonicity with respect to SSD.
  - Monotonicity with respect to SSD: If F SSD G, then the risk index of F is smaller than that of G.
- For example, $F \equiv (0.1, 0.9; -10, 100)$ and $G \equiv (0.9, 0.1; -10, 100)$. Obviously, F SSD G. However, variance of F = 1089 = variance of G.
- Thus, an index satisfying monotonicity with respect to SSD is needed in the literature.

Aumann and Serrano (2008, JPE)

- Let $\tilde{x}$ have some positive and negative values and the expected value of the gamble is positive.
- Definition:

$$E\left[e^{-\frac{\tilde{x}}{R^{AS}}}\right] = 1$$

- For example: If $\tilde{x}$ follows a normal distribution $N(\mu, \sigma^2)$, then

$$R^{AS} = \frac{\sigma^2}{2\mu}.$$

Foster and Hart (2009, JPE)

- Definition:

$$E\left[\ln(\frac{R^{FH}+x}{R^{FH}})\right]=0.$$

- In other words,

$$E\left[\ln(R^{FH}+x)\right]=\ln(R^{FH}).$$

Bali, Cakici, and Chabi-Yo (2011, MS)

- Definition:

$$\frac{1}{\gamma}\left[E\left(\frac{R^{BCCY}+x}{R^{BCCY}}\right)^{\gamma}-1\right]=0.$$

In other words,

$$\frac{1}{\gamma}E\left[(R^{BCCY}+x)^{\gamma}\right]=\frac{1}{\gamma}(R^{BCCY})^{\gamma}.$$

Numerical Example

- Let $x \equiv (0.5, 0.5; -10, 100)$
- AS:

$$0.5 \times e^{-\frac{-10}{R^{AS}}} + 0.5 \times e^{-\frac{100}{R^{AS}}} = 1$$

- FH:

$$0.5 \times \ln(\frac{R^{FH} - 10}{R^{FH}}) + 0.5 \times \ln(\frac{R^{FH} + 100}{R^{FH}}) = 0.$$

- BCCY:

$$0.5 \times \left(\frac{R^{BCCY} - 10}{R^{BCCY}}\right)^{\gamma} + 0.5 \times \left(\frac{R^{BCCY} + 100}{R^{BCCY}}\right)^{\gamma} - 1 = 0.$$

Applications: Hedge Ratio (Chen, Ho, Tzeng, 2014, JBF)

- In this paper, we propose a new spot-futures hedging method that determines the optimal hedge ratio by minimizing the riskiness of hedged portfolio returns, where the riskiness is measured by the index of Aumann and Serrano (2008). Unlike the risk measurements widely used in the literature, the riskiness index employed in our method satisfies monotonicity with respect to stochastic dominance. We also provide an empirical example to demonstrate how to estimate and test this optimal hedge ratio in equity data by the method-of-moments.

Applications: Performance Measurement (Homm and Pigorsch, 2012, JBF)

- We propose a performance measure that generalizes the Sharpe ratio. The new performance measure is monotone with respect to stochastic dominance and consistently accounts for mean, variance and higher moments of the return distribution. It is equivalent to the Sharpe ratio if returns are normally distributed. Moreover, the two performance measures are asymptotically equivalent as the underlying distributions converge to the normal distribution. We suggest a parametric and a non-parametric estimator for the new performance measure and provide an empirical illustration using mutual funds and hedge funds data.

Applications: Option Implied Measure of Riskiness (Bali, Cakici, and Chabi-Yo, 2015, JBF)

- We introduce a new approach to measuring riskiness in the equity market. We propose option implied and physical measures of riskiness and investigate their performance in predicting future market returns. The predictive regressions indicate a positive and significant relation between time-varying riskiness and expected market returns. The significantly positive link between aggregate riskiness and market risk premium remains intact after controlling for the S&P 500 index option implied volatility (VIX), aggregate idiosyncratic volatility, and a large set of macroeconomic variables. We also provide alternative explanations for the positive relation by showing that aggregate riskiness is higher during economic downturns characterized by high aggregate risk aversion and high expected returns.

Applications: CAPM (Chen, Huang, Shih and Tzeng, 2015, Working)

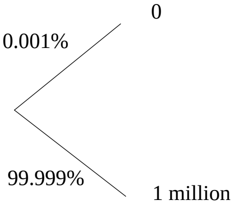
- This paper estiblishes a generalized index which includes the risk measures proposed by Aumann and Serrano (2008), Foster and Hart (2009) and Bali et al. (2011) as special cases. This generalized index satisfies several desirable properties such as convexity, zero-payoff condition, and monotonicity with respect to second-degree stochastic dominance. Based on the index, we derive the capital pricing equilibrium in a reward-risk framwork. Furthermore, the statictic tests are proposed. Using the tests, the empirical studies show that our model can better capture the cross section of stock return than the traditional mean-variance capital asset pricing model. The findings also suggest that comparing to our model, using the traditional pricing model could over estimate the mangitude of the excess portfolio returns by mis-evaluating the systematic risk.

Almost Stochastic Dominance (ASD)

- Almost First-degree Stochastic Dominance (AFSD): Leshno and Levy (2002, MS)
- Almost Second-degree Stochastic Dominance (ASSD): Tzeng, Huang and Shih (2013, MS)
- Applications

Motivation for ASD (1/3)

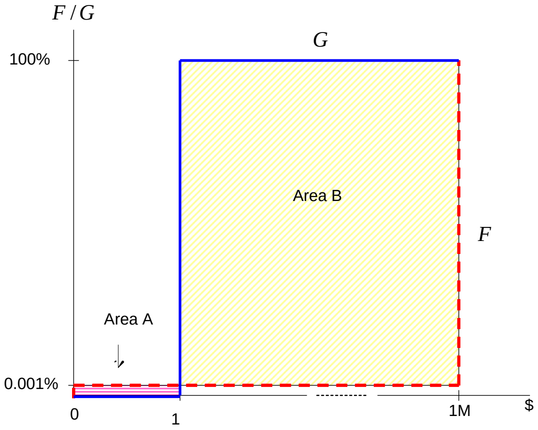
Lottery X
CDF: F(x)



Lottery Y
CDF: G(x)

1 for sure

Motivation for ASD (2/3)



Motivation for ASD (3/3)

- Who would prefer Y to X?

$$u(x) = \begin{cases} x & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

  - For lottery X: $u = 0.001\% \times 0 + 99.999\% \times 1 = 99.999\%$
  - For lottery Y: $u = 1 \times 1 = 1$

Almost First-degree Stochastic Dominance (AFSD) (1/3)

- Definition
  - For $\varepsilon_1 \in (0, 0.5)$, F ε_1-AFSD G if

$$\int_{F \geq G} [F(x) - G(x)]\, dx \leq \varepsilon_1 \times \int_a^b |F(x) - G(x)|\, dx$$

  - In the previous example, X ε_1-AFSD Y if

$$\frac{\text{Area } A}{\text{Area } A + \text{Area } B} \leq \varepsilon_1.$$

- Note that if F ε_1-AFSD G, then $E_F(x) \geq E_G(x)$.

Almost First-degree Stochastic Dominance (AFSD) (2/3)

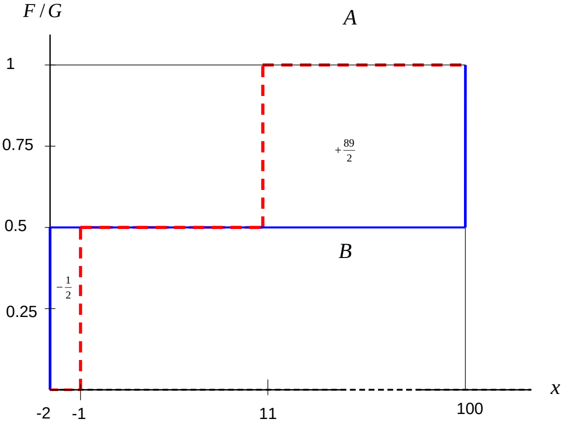
- Theorem
  - $Eu_F(x) \geq Eu_G(x)$ for all individuals with $u' > 0$ and $\frac{\sup\{u'\}}{\inf\{u'\}} \leq \left(\frac{1}{\varepsilon_1} - 1\right)$ iff F ε_1-AFSD G.
- If $\varepsilon_1 \rightarrow 0$, then $\frac{\sup\{u'\}}{\inf\{u'\}} \leq \left(\frac{1}{\varepsilon_1} - 1\right)$ always holds and AFSD reduces to FSD.
- If $\varepsilon_1 \rightarrow 0.5$, then $\frac{\sup\{u'\}}{\inf\{u'\}} \leq 1$, which implies that u' is a constant (risk neutral).

Almost First-degree Stochastic Dominance (AFSD) (3/3)

- Let $x \in [a, b]$. If $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, then $u'(x) = x^{-\gamma}$, $u''(x) = -\gamma x^{-\gamma-1}$ and $-x\frac{u''(x)}{u'(x)} = \gamma$ (constant relative risk aversion).
  - When $\gamma \geq 0$, $\frac{\sup\{u'\}}{\inf\{u'\}} = \frac{a^{-\gamma}}{b^{-\gamma}} \leq \left(\frac{1}{\varepsilon_1} - 1\right) \Rightarrow \gamma \leq -\frac{\ln\left(\frac{1}{\varepsilon_1}-1\right)}{\ln\left(\frac{a}{b}\right)}$.
  - When $\gamma < 0$, $\frac{\sup\{u'\}}{\inf\{u'\}} = \frac{b^{-\gamma}}{a^{-\gamma}} \leq \left(\frac{1}{\varepsilon_1} - 1\right) \Rightarrow \gamma \geq -\frac{\ln\left(\frac{1}{\varepsilon_1}-1\right)}{\ln\left(\frac{a}{b}\right)}$.

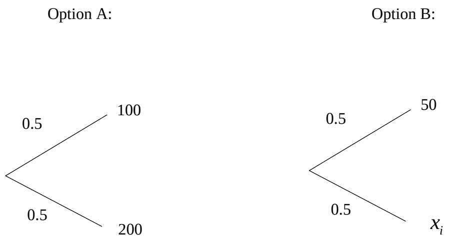
An Example of AFSD

- $A \equiv (0.5, 0.5; -1, 11)$; $B \equiv (0.5, 0.5; -2, 100)$
- Thus, if $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{89}{2}} \leq \varepsilon_1$, or $\varepsilon_1 \geq \frac{1}{90}$, then B ε_1-AFSD A.



The estimation: Levy, Leshno and Leibovitch (2010, AOR)

- They suggest $\varepsilon_1 = 5.9\%$.



Almost Second-degree Stochastic Dominance (ASSD) (1/3)

- Definition
    - For $\varepsilon_2 \in (0, 0.5)$, F ε_2-ASSD G if

$$\int_{F^{(2)} \geq G^{(2)}} \left[F^{(2)}(x) - G^{(2)}(x) \right] dx \leq \varepsilon_2 \times \int_a^b \left| F^{(2)}(x) - G^{(2)}(x) \right| dx$$

and

$$E_F(x) - E_G(x) \geq 0.$$

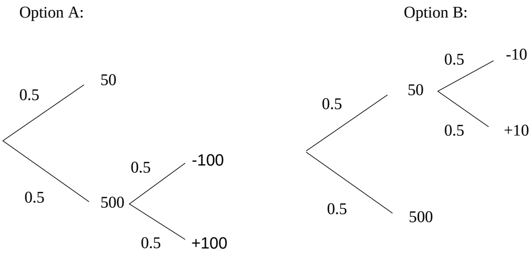
Almost Second-degree Stochastic Dominance (ASSD) (2/3)

- Theorem
  - $Eu_F(x) \geq Eu_G(x)$ for all individuals with $u' > 0$, $u'' < 0$ and $\frac{\sup\{-u''\}}{\inf\{-u''\}} \leq \left(\frac{1}{\varepsilon_2} - 1\right)$ iff F ε_2-ASSD G.
- If $\varepsilon_2 \to 0$, then $\frac{\sup\{-u''\}}{\inf\{-u''\}} \leq \left(\frac{1}{\varepsilon_2} - 1\right)$ always holds and ASSD reduces to SSD.
- If $\varepsilon_1 \to 0.5$, then $\frac{\sup\{-u''\}}{\inf\{-u''\}} \leq 1$, which implies that $-u''$ is a constant (quadratic utility).

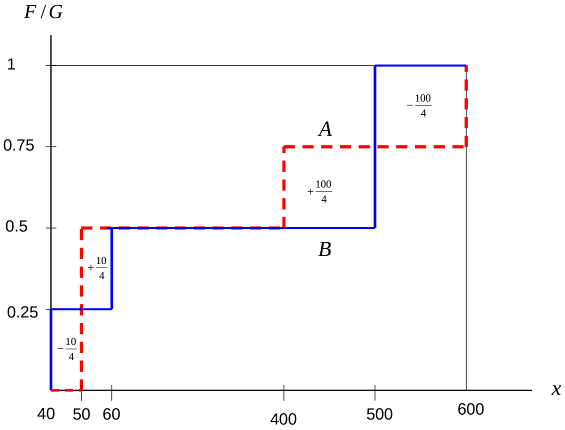
Almost Second-degree Stochastic Dominance (ASSD) (3/3)

- Let $x \in [a, b]$. If $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, then $u'(x) = x^{-\gamma}$, $u''(x) = -\gamma x^{-\gamma-1}$, $u''' = \gamma(\gamma+1)x^{-\gamma-2}$ and $-x\frac{u''(x)}{u'(x)} = \gamma$ (constant relative risk aversion).
  - Degree of relative prudence: $-x\frac{u'''(x)}{u''(x)} = (\gamma + 1)$
  - $\frac{\sup\{-u''\}}{\inf\{-u''\}} = \frac{-\gamma a^{-\gamma-1}}{-\gamma b^{-\gamma-1}} \leq \left(\frac{1}{\varepsilon_1} - 1\right) \Rightarrow \gamma + 1 \leq -\frac{\ln\left(\frac{1}{\varepsilon_1} - 1\right)}{\ln\left(\frac{a}{b}\right)}$.

An Example for ASSD (1/3)

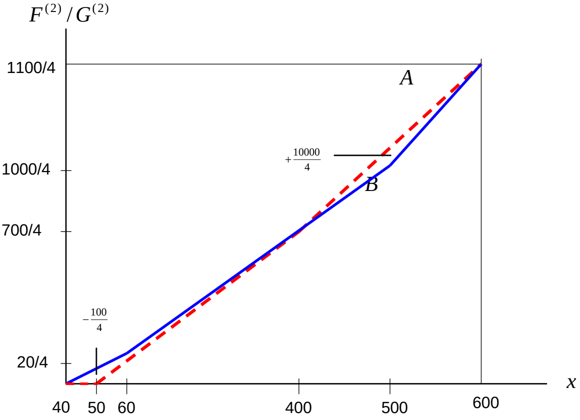


An Example for ASSD (2/3)



An Example for ASSD (3/3)

- Thus, if $\frac{\frac{100}{4}}{\frac{100}{4}+\frac{10000}{4}} \leq \varepsilon_2$, or $\varepsilon_2 \geq \frac{1}{101}$, then B ε_2-ASSD A.



The estimation: Huang, Huang and Tzeng (2015, Working)

- They suggest
  - $\varepsilon_1 < 5.9\%$.
  - $\varepsilon_2 = 2.2\%$.

Applications: Hedge funds v.s. stock: (Bali, Brown and Ozgur Demirtas, 2013, MS)

- Hedge funds' extensive use of derivatives, short selling, and leverage and their dynamic trading strategies create significant nonnormalities in their return distributions. Hence, the traditional performance measures fail to provide an accurate characterization of the relative strength of hedge fund portfolios. This paper uses the utility-based nonparametric and parametric performance measures to determine which hedge fund strategies outperform the U.S. equity and/or bond markets. The results from the realized and simulated return distributions indicate that the long/short equity hedge and emerging markets hedge fund strategies outperform the U.S. equity market, and the long/short equity hedge, multistrategy, managed futures, and global macro hedge fund strategies dominate the U.S. Treasury market.

Applications: Long-term investment in stock v.s. bond (Bali, Ozgur Demirtas, Levy and Wolf, 2009, JME)

- It has become increasingly popular to advise investors to relocate their funds from a primarily stock portfolio to a primarily bond portfolio as they get older. However, the well-known decision rules such as mean–variance or stochastic dominance rules are unable to explain this common practice. Almost stochastic dominance (ASD) and almost mean–variance (AMV) approaches are used to examine the dominance of stock and bond portfolios. ASD and AMV rules unambiguously support the popular practice of advising higher stock to bond ratio for long investment horizons. Hence, we provide an explanation to the practitioners' recommendation within the expected utility paradigm.

Applications: Portfolio Insurance (Fu, Hsu, Huang, and Tzeng, 2015, Working)

- The purpose of this paper is to derive an efficient frontier according to generalized almost stochastic dominance (GASD) rules proposed by Tsetlin et al. (2015), which can effectively delete the choices which are not preferred by most economically important decision makers. We first respectively propose tests for portfolio admissibility and portfolio optimality under generalized almost first-degree stochastic dominance. We then propose tests for efficient diversification under generalized almost second-degree stochastic dominance. In each test, we demonstrate how to use computational and tractable linear programming to implement the tests and provide their applications in the stock markets.

Applications: Estimation (Huang, Huang, and Tzeng, 2015, Working)

- Almost stochastic dominance (ASD) as proposed by Leshno and Levy (2002) has been widely applied in decision theory and in practice. It can help to identify the preferred distribution for the majority of decision makers when a small violation in stochastic dominance is involved. The purpose of this paper is to experimentally define the preference parameters in the set of most decision makers. By adopting almost Nth-degree risk defined by Tsetlin et al. (2014), the parameters for economically relevant risk-averse and prudent decision makers are estimated.