

Title

Author

Today

$$\frac{dz}{dt} = z \left( \alpha + i\omega + \beta_1 |z|^2 + \frac{\epsilon \beta_2 |z|^4}{1 - \epsilon |z|^2} \right) + F e^{i\theta}$$

where  $z = r e^{i\phi}$ ,  $\psi = \phi - \theta$ ,  $\theta = \omega_0 t + \theta_0$ , and  $\Omega = \omega - \omega_0$

To find the steady-state amplitude we solve  $\dot{r} = 0$

$$\begin{aligned}\dot{r} &= \alpha r + \beta_1 r^3 + \frac{\epsilon \beta_2 r^5}{1 - \epsilon r^2} \\ 0 &= \alpha r + \beta_1 r^3 + \frac{\epsilon \beta_2 r^5}{1 - \epsilon r^2} \\ -\frac{\epsilon \beta_2 r^5}{1 - \epsilon r^2} &= \alpha r + \beta_1 r^3 \\ -\epsilon \beta_2 r^5 &= (\alpha r + \beta_1 r^3)(1 - \epsilon r^2) \\ &= \alpha r + \beta_1 r^3 - \epsilon \alpha r^3 - \epsilon \beta_1 r^5 \\ 0 &= \alpha r + \beta_1 r^3 - \epsilon \alpha r^3 - \epsilon \beta_1 r^5 + \epsilon \beta_2 r^5 \\ &= \alpha r + (\beta_1 - \epsilon \alpha) r^3 - \epsilon (\beta_2 - \beta_1) r^5\end{aligned}$$