Title

Author

Today

$$\frac{dz}{dt} = z\left(\alpha + \mathrm{i}\omega + \beta_1|z|^2 + \frac{\epsilon\beta_2|z|^4}{1-\epsilon|z|^2}\right) + Fe^{\mathrm{i}\theta}$$

where $z = re^{\mathrm{i}\phi}, \psi = \phi - \theta, \theta = \omega_0 t + \theta_0,$ *and* $\Omega = \omega - \omega_0$

To find the steady-state amplitude we solve $\dot{r} = 0$

$$\begin{aligned}
\dot{r} &= \alpha r + \beta_1 r^3 + \frac{\epsilon\beta_2 r^5}{1-\epsilon r^2} \\
0 &= \alpha r + \beta_1 r^3 + \frac{\epsilon\beta_2 r^5}{1-\epsilon r^2} \\
-\frac{\epsilon\beta_2 r^5}{1-\epsilon r^2} &= \alpha r + \beta_1 r^3 \\
-\epsilon\beta_2 r^5 &= (\alpha r + \beta_1 r^3)(1-\epsilon r^2) \\
&= \alpha r + \beta_1 r^3 - \epsilon\alpha r^3 - \epsilon\beta_1 r^5 \\
0 &= \alpha r + \beta_1 r^3 - \epsilon\alpha r^3 - \epsilon\beta_1 r^5 + \epsilon\beta_2 r^5 \\
&= \alpha r + (\beta_1 - \epsilon\alpha) r^3 - \epsilon(\beta_2 - \beta_1) r^5
\end{aligned}$$