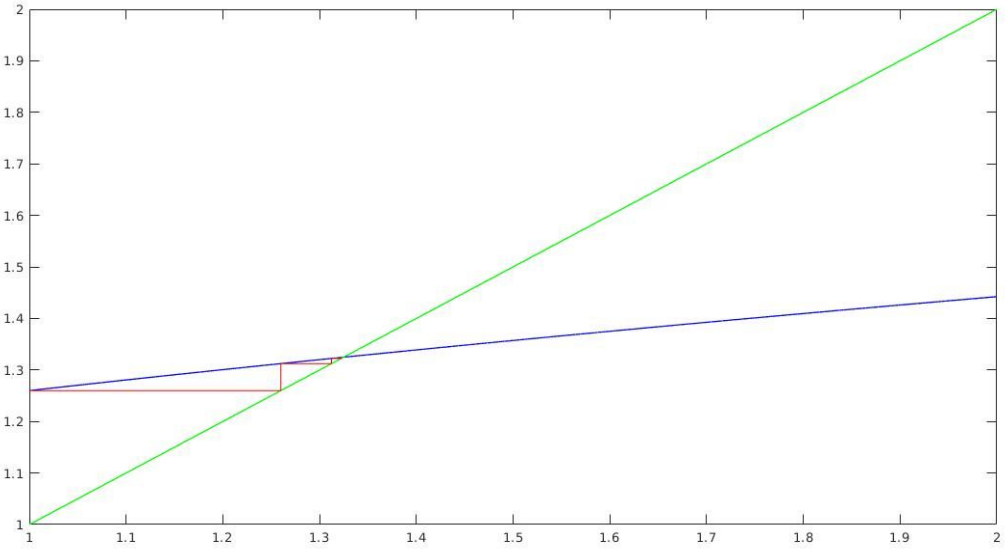
Lab 2 Output

Q1. $f(x) = x^3 - x - 1$, $x_0 = 1$

n	x(n)	f(x(n))-x(n)
1	1.000000	0.259921
2	1.259921	0.052373
3	1.312294	0.010060
4	1.322354	0.001915



Q2

The number of iterations required, using the Banach's fixed point theorem, can be calculated as, $|\alpha - x_n| < k/(1-k)|x_n - x_{n-1}| < k^n/(1-k)|x_1 - x_0|$

Where $k = \max|f'(x)| < 1$ in the interval where we are trying to find the root

(a) $x = -\sqrt{e^{x}/3}$

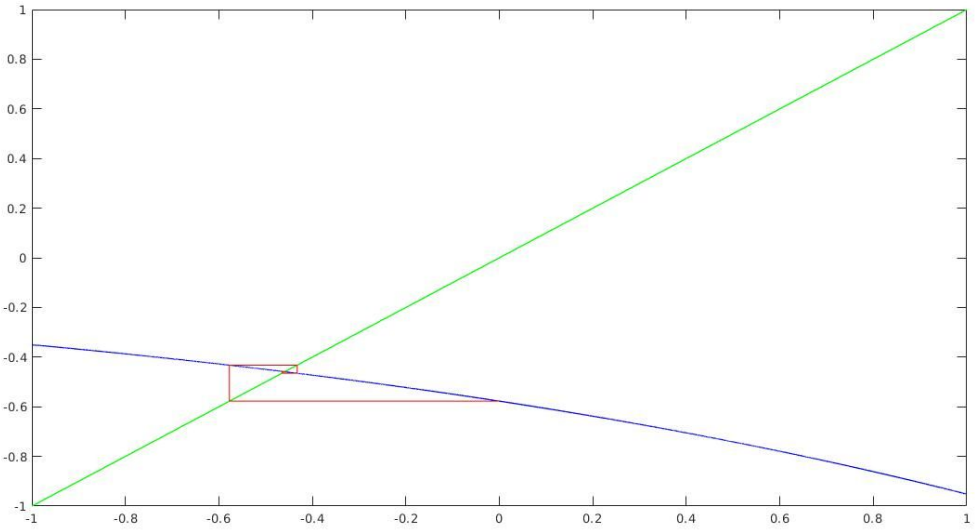
In the interval [-1,0] , $|f'(x)| < 1$

Estimate of Number of Iterations = 5.3922

First root at:-0.465

n	x(n)	f(x(n))-x(n)
1	0.000000	-0.577350
2	-0.577350	0.144767
3	-0.432583	-0.032473

4 -0.465056 0.007490



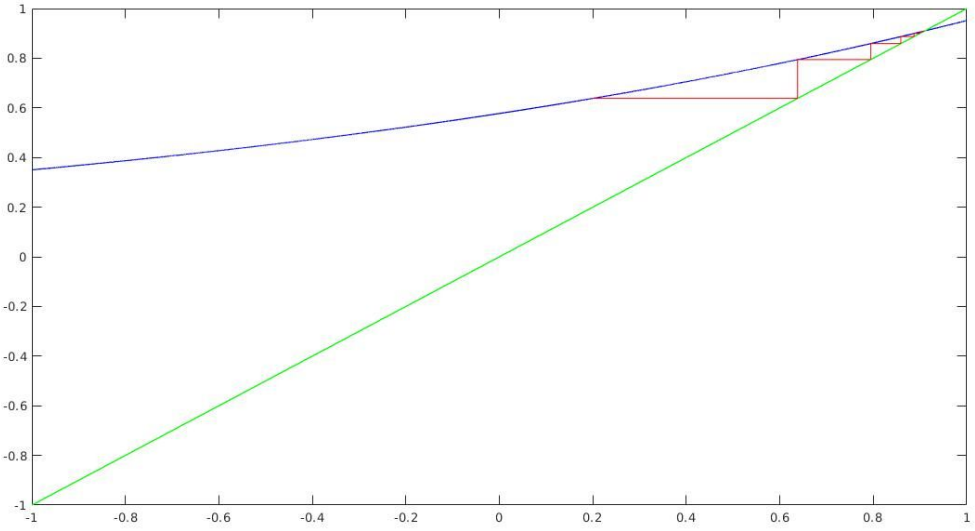
x = $\sqrt{e^{x}/3}$

In the interval [0,1] |f'(x)| < 1

The approximation for number of iterations = 139

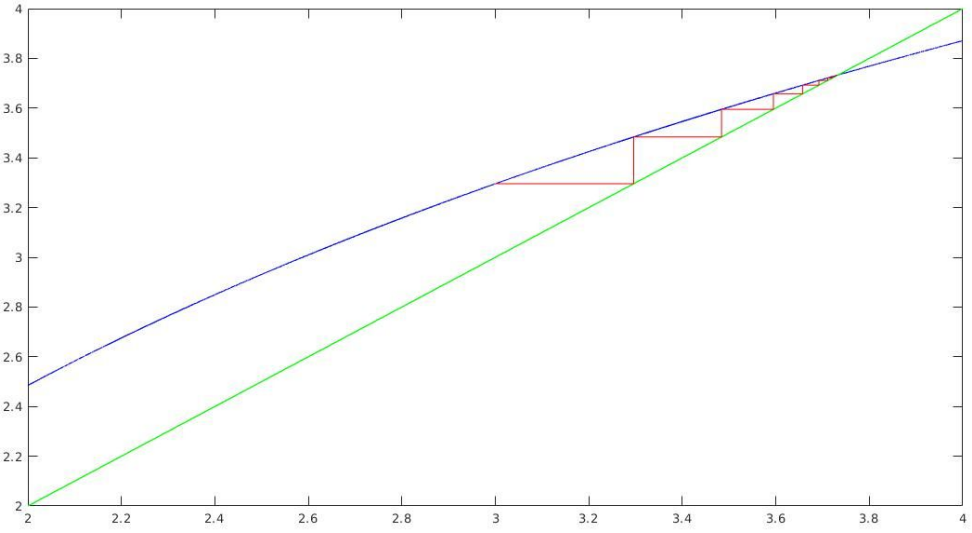
Second rooot at:0.899

n	x(n)	f(x(n))-x(n)
1	0.200000	0.438071
2	0.638071	0.156248
3	0.794318	0.064544
4	0.858862	0.028169
5	0.887031	0.012582
6	0.899613	0.005677



$X = \ln(3x^2)$
In the interval [3,4] $|f'(x)| < 1$
With max value k at x = 3, k = 0.6667
Approximate number of Iterations = 11
Third root: 3.721

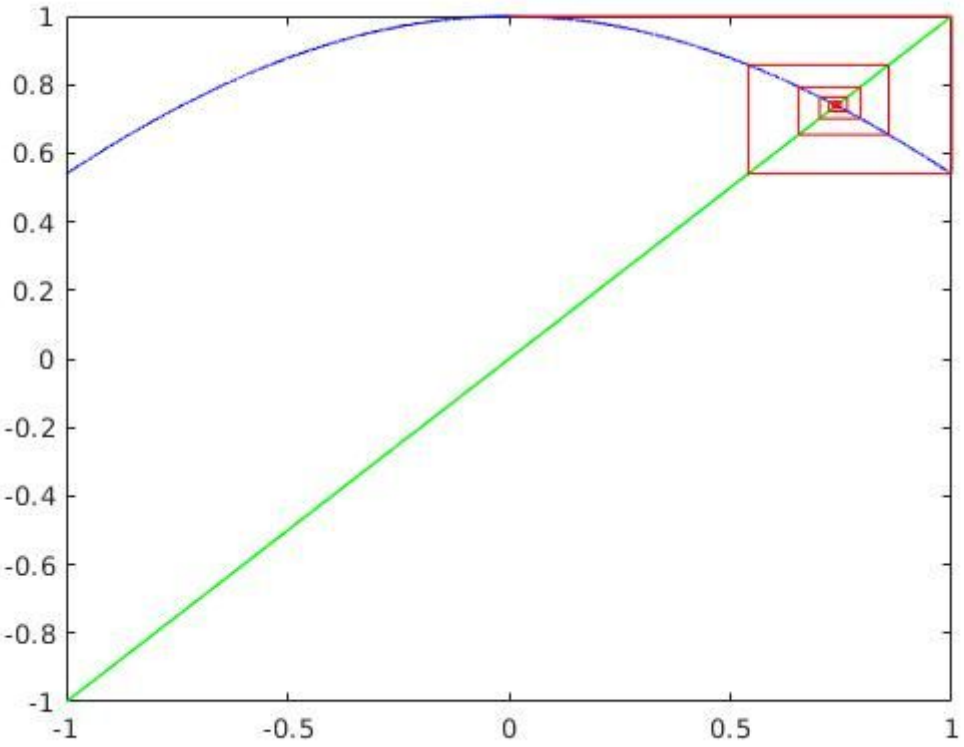
n	x(n)	f(x(n))-x(n)
1	3.000000	0.295837
2	3.295837	0.188096
3	3.483933	0.111003
4	3.594936	0.062729
5	3.657664	0.034597
6	3.692262	0.018829
7	3.711091	0.010173
8	3.721264	0.005475



Q2(b) f(x) = x - cos(x)
X = cos(x)
$|f'(x)| < 1$ for all intervals
Estimate of Number of Iterations in the interval [0,0.78] is 21

n	x(n)	f(x(n))-x(n)
1	0.000000	1.000000
2	1.000000	-0.459698
3	0.540302	0.317251
4	0.857553	-0.203263
5	0.654290	0.139191
6	0.793480	-0.092112
7	0.701369	0.062591

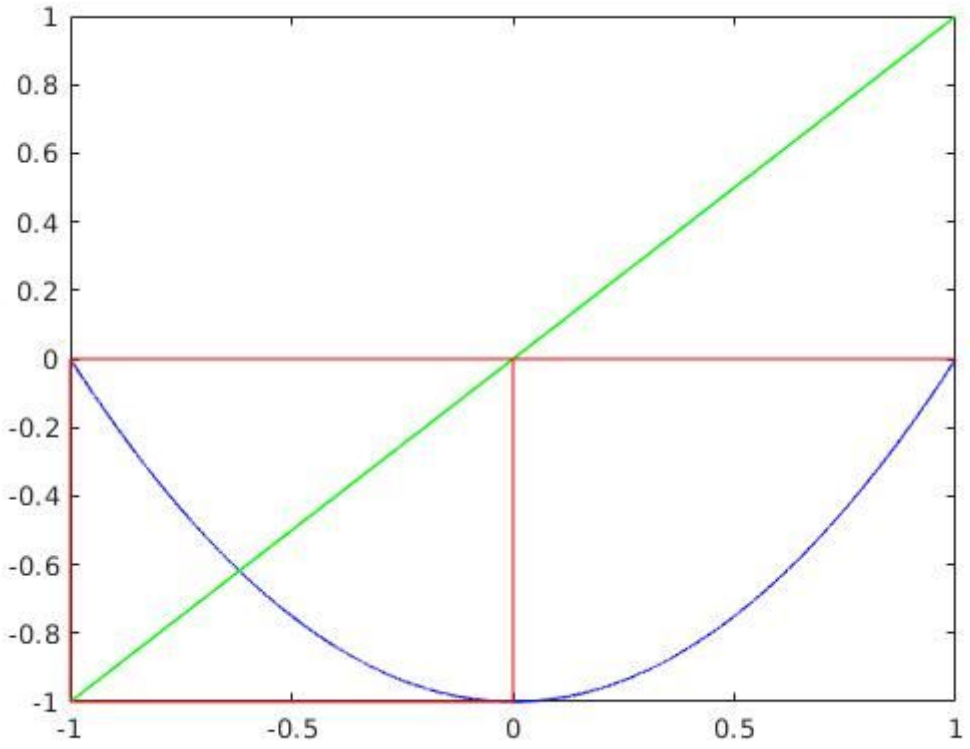
8	0.763960	-0.041857
9	0.722102	0.028315
10	0.750418	-0.019014
11	0.731404	0.012833
12	0.744237	-0.008633

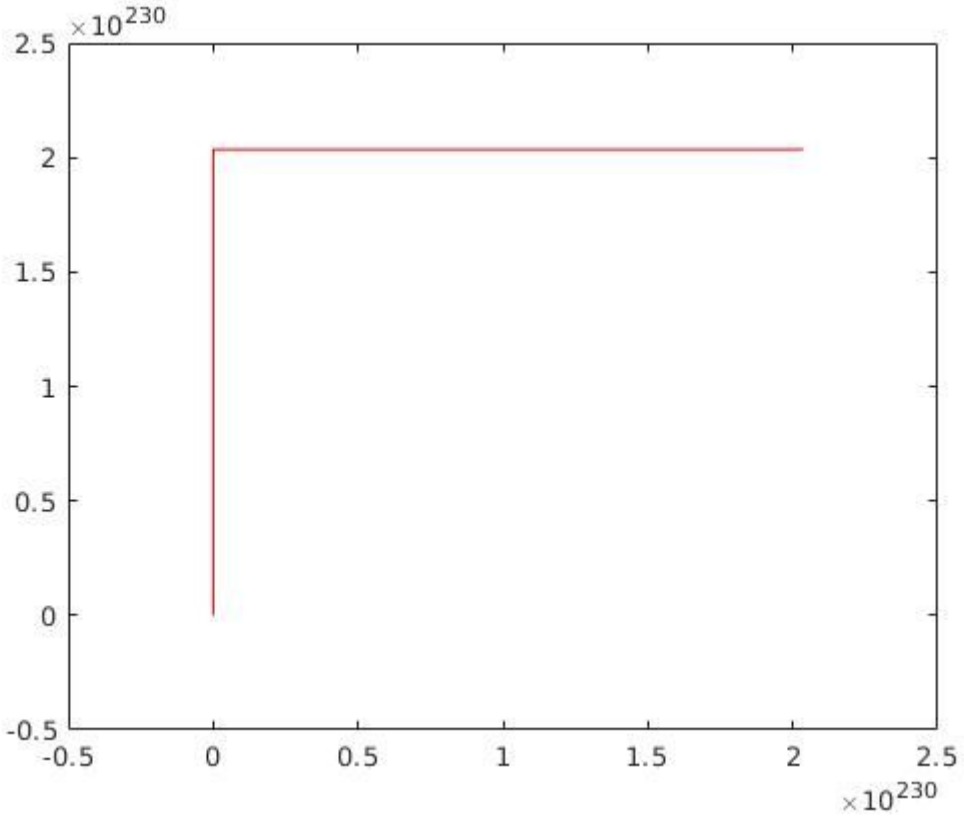


Q3.

(a) No convergence

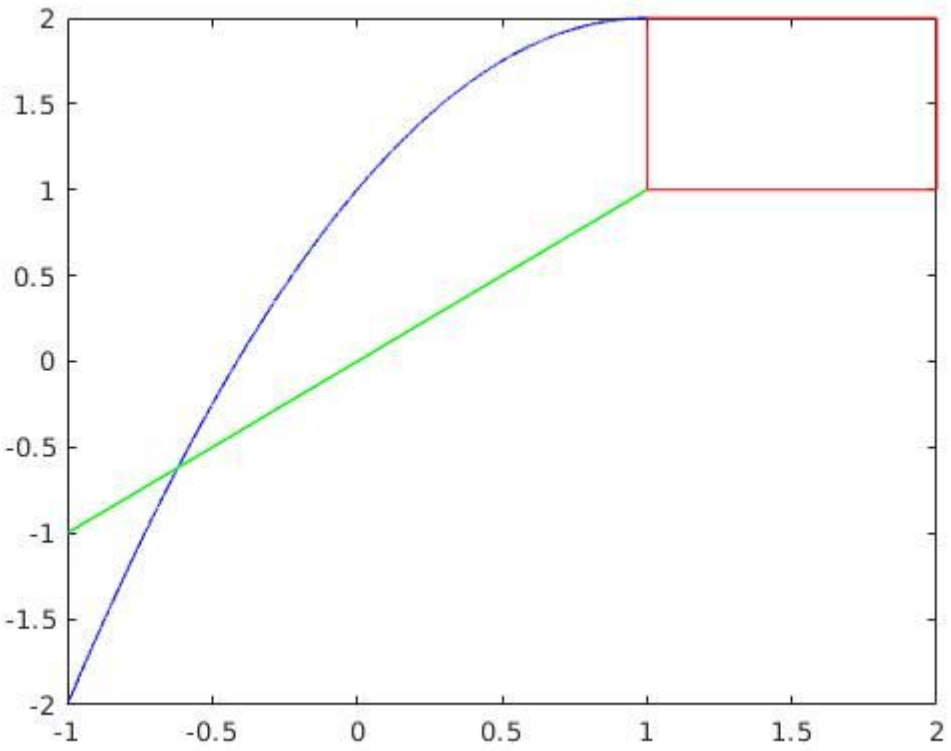
For x_0 = 1,2

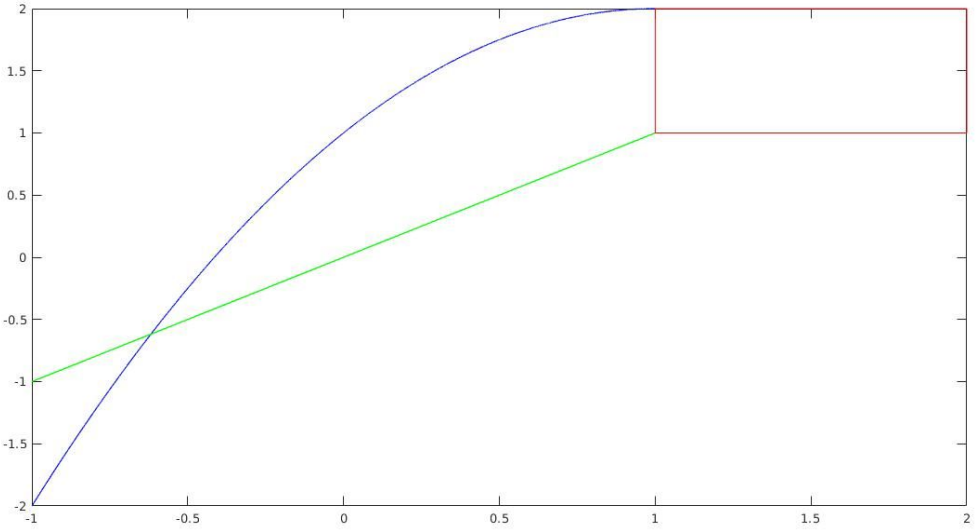




(b)

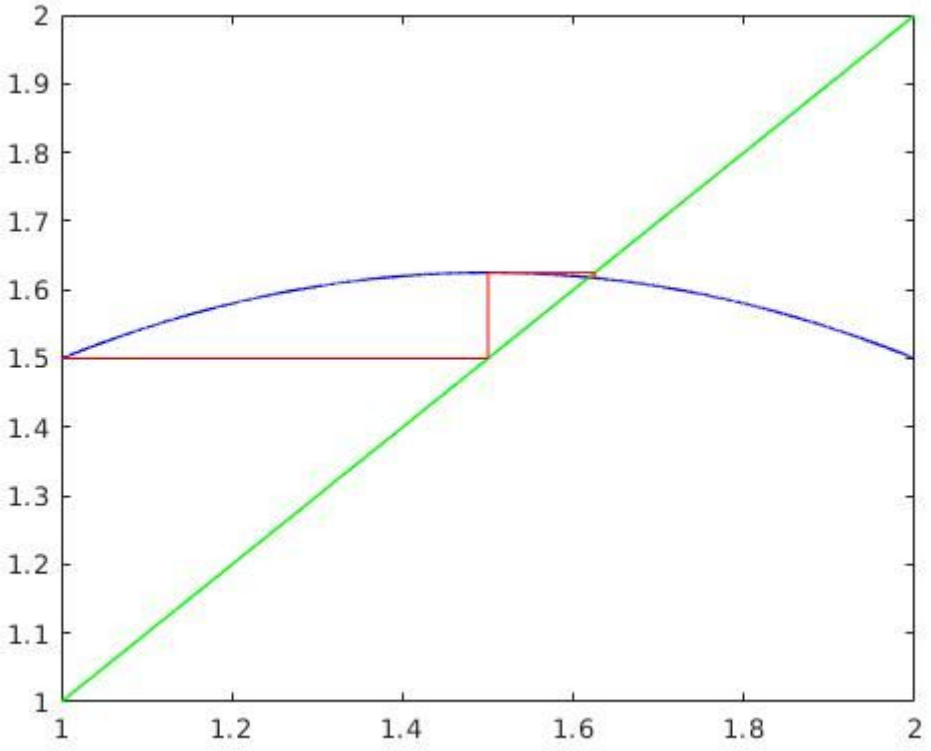
No convergence for x_0 = 1,2

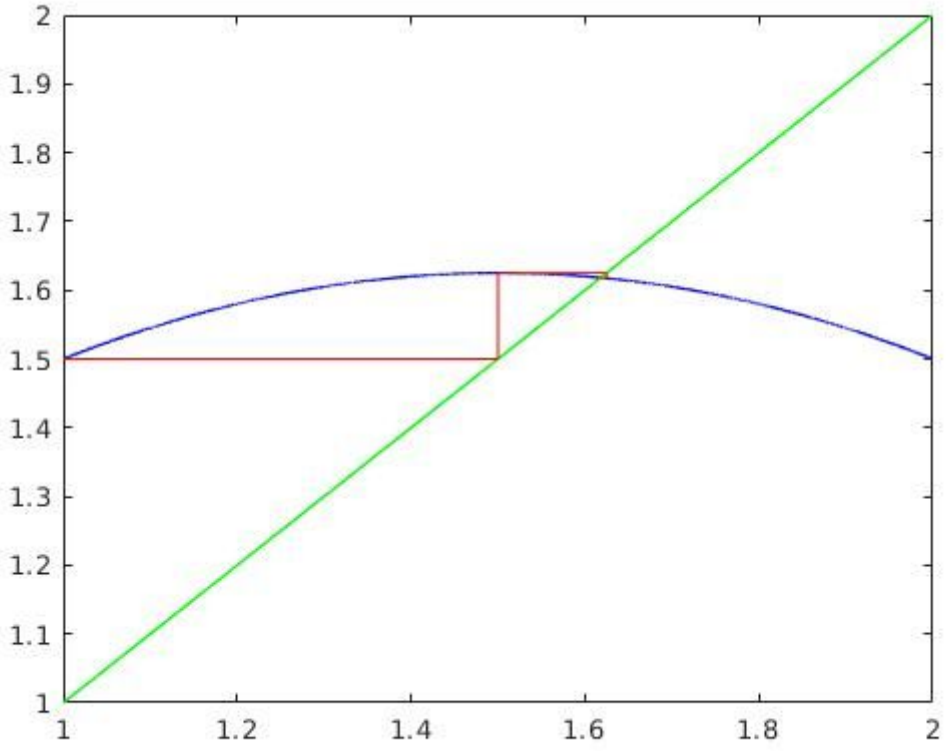




(c)

Converging to the root for both x_0 = 1,2



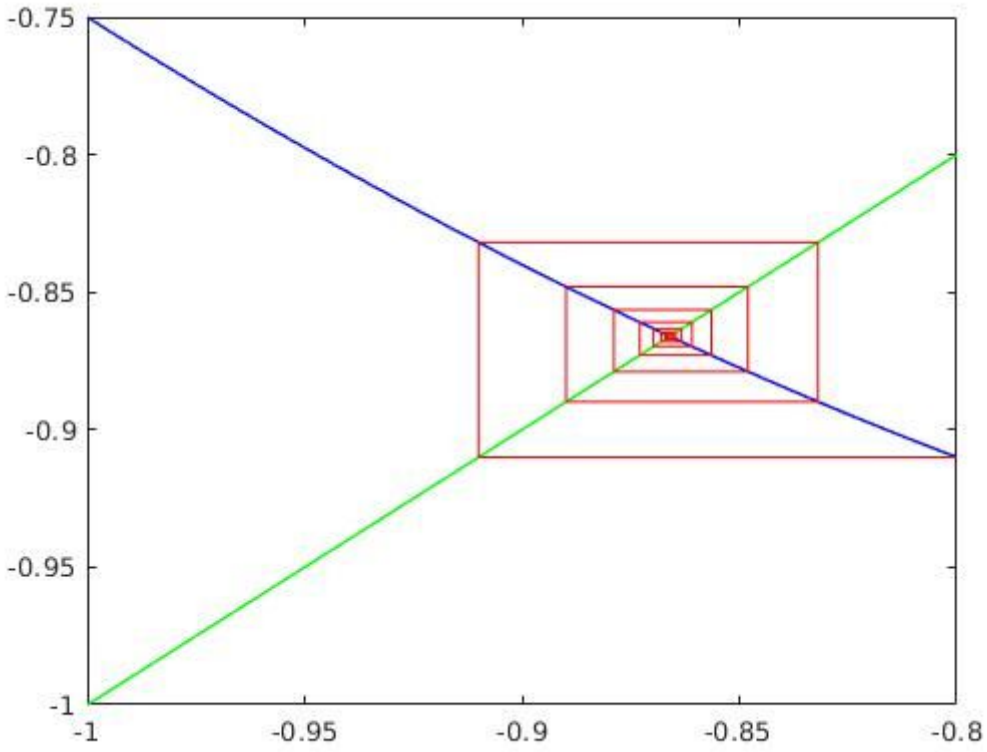


Q4

Solving $x^2+x-0.75 = x$

Initial Point -0.8

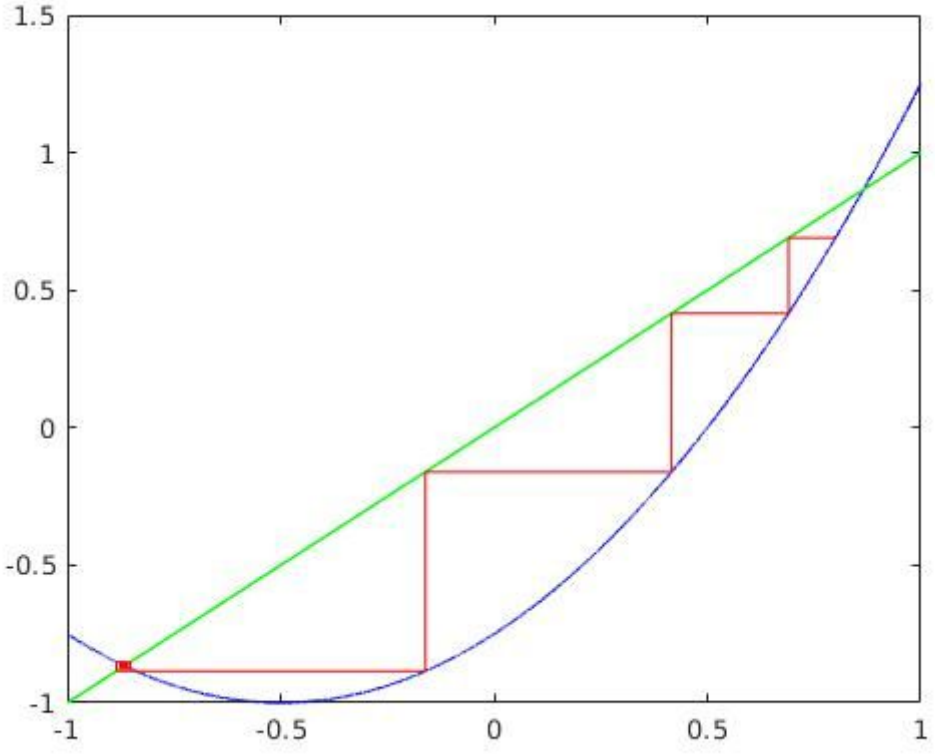
n	x(n)	f(x(n))-x(n)
1	-0.800000	-0.110000
2	-0.910000	0.078100
3	-0.831900	-0.057942
4	-0.889842	0.041819
5	-0.848023	-0.030857
6	-0.878880	0.022430
7	-0.856450	-0.016494
8	-0.872943	0.012030
9	-0.860913	-0.008829
10	-0.869742	0.006451
11	-0.863291	-0.004728
12	-0.868020	0.003458
13	-0.864562	-0.002533
14	-0.867095	0.001853
15	-0.865241	-0.001357
16	-0.866599	0.000993



Initial Point 0.8

n	x(n)	f(x(n))-x(n)
1	0.800000	-0.110000
2	0.690000	-0.273900
3	0.416100	-0.576861
4	-0.160761	-0.724156
5	-0.884917	0.033078
6	-0.851839	-0.024370

7	-0.876209	0.017743
8	-0.858467	-0.013035
9	-0.871502	0.009515
10	-0.861986	-0.006979
11	-0.868966	0.005102
12	-0.863864	-0.003739
13	-0.867603	0.002735
14	-0.864868	-0.002003
15	-0.866871	0.001466
16	-0.865406	-0.001073
17	-0.866479	0.000786



Q5

Q5(a)

x^3 - x - 2

n	x(n)	f(x(n))
3	1.400000	-0.656000
4	1.524956	0.021316
5	1.521356	-0.000140
6	1.521380	-0.000000

Q5(b)

1 + 2x - tanx

n	x(n)	f(x(n))
3	1.300000	-0.002102
4	1.300200	-0.004498
5	1.299824	0.000002

Q6

Q6(a)

f(x) = x^2 + exp(x) -5

Root near -2:

n	x(n)	f(x(n))
3	-1.400000	-2.793403
4	-2.205182	-0.026942
5	-2.211407	-0.000132
6	-2.211438	-0.000000

Root near 1:

n	x(n)	f(x(n))
3	0.700000	-2.496247
4	1.257924	0.100483
5	1.240706	-0.002593
6	1.241142	-0.000002

Q6(b)

f(x) = x^2 - sin(x)

Root near 0.9:

n	x(n)	f(x(n))
3	0.700000	-0.154218
4	0.878935	0.002466
5	0.876715	-0.000013
6	0.876726	-0.000000

Q7

Roots of z^4 - 2z^3 - 2iz^2 + 4iz = 0

n	x(n)	f(x(n))
3	0.200000+0.000000j	-0.014400+0.720000j
4	0.000000+0.000000j	-0.000000+0.000000j

n	x(n)	f(x(n))
3	-1.300000+0.000000j	7.250100+-8.580000j
4	-0.879303+-0.691211j	2.644703+-0.843783j
5	-0.888203+-0.918863j	1.180663+-0.825112j
6	-0.980444+-1.016473j	-0.108481+-0.302532j
7	-1.000633+-0.999874j	-0.001014+0.008101j
8	-1.000000+-1.000001j	-0.000015+-0.000005j

9	-1.000000+-1.000000j	0.000000+0.000000j

n	x(n)	f(x(n))
3	1.600000+0.000000j	-1.638400+1.280000j
4	2.067330+0.052894j	0.783681+0.239286j
5	1.995697+-0.009917j	-0.074871+-0.060935j
6	2.000152+-0.000176j	0.000508+-0.002015j
7	2.000000+0.000000j	0.000001+0.000001j

n	x(n)	f(x(n))
3	3.600000+0.000000j	74.649600+-11.520000j
4	2.598146+0.658040j	4.096749+14.878367j
5	2.235749+0.770362j	-3.042998+8.291213j
6	1.848268+0.836988j	-4.217322+2.782855j
7	1.451581+0.896193j	-2.572381+0.092998j
8	1.188906+0.932833j	-1.073581+-0.208621j
9	1.040898+0.974459j	-0.263102+-0.045088j
10	1.002072+0.997324j	-0.018931+0.002472j
11	1.000001+0.999974j	-0.000106+0.000100j
12	1.000000+1.000000j	0.000000+0.000000j

(b)

Roots of z = e^z

n	x(n)	f(x(n))
3	0.800000+0.000000j	-0.077356+0.000000j
4	0.860088+0.000000j	-0.018148+0.000000j
5	0.873281+0.000000j	-0.003820+0.000000j
6	0.876015+0.000000j	-0.000791+0.000000j
7	0.876580+0.000000j	-0.000163+0.000000j
8	0.876696+0.000000j	-0.000034+0.000000j
9	0.876720+0.000000j	-0.000007+0.000000j