IEORE4004: Optimization Models and Methods - Prof. Christian Kroer 09/30/2019

Assignment 2

Deadline: 10/10/2019, h: 11:40am

- This assignment sheet has 6 exercises. You must submit your solution via Canvas before the deadline. The time constraint is strict. Late submissions will not be accepted.
- You are allowed to discuss the assignment with others but the write-up must be individual work. Please mention in your write-up all the people you have discussed the solution with.

Problem 1: (This is – essentially – exercise 3, pag. 149 from the textbook). Starting from the basic feasible solution with basic variables $(4, 5, 6)$, find the optimal solution to the following LP by hand. Verify that your calculations are correct using your favourite software.

$$
\begin{array}{lllllll}
\max & 2x_1 & -x_2 & +x_3 & & & \\
\text{s.t.} & & & & & & \\
 & 3x_1 & +x_2 & +x_3 & +x_4 & = & 60 \\
 & x_1 & -x_2 & +2x_3 & +x_5 & = & 10 \\
 & x_1 & +x_2 & -x_3 & +x_6 & = & 20 \\
 & x & & & & \geq & 0
\end{array}
$$

Problem 2: As seen in the lecture, put the following LP in standard form (P) and then construct the auxiliary problem needed to find a starting basic feasible solution (P). Then solve it by hand. Verify that your calculations are correct using your favourite software.

$$
\begin{array}{lllll}
\max & 2x_1 & +3x_2 & & \\
\text{s.t.} & & & & \\
 & .5x_1 & +.25x_2 & \leq & 4 \\
 & x_1 & +3x_2 & \geq & 20 \\
 & x_1 & +x_2 & = & 4 \\
 & x & & \geq & 0
\end{array}
$$

Problem 3: Consider the partial tableau given below wrt the basic solution $\bar{x}$ with exactly 3 non-zero entries.

$$
\begin{array}{cccccccc}
z & -x_1 & * & * & * & * & -x_6 & = & 7 \\
\hline
 & 0 & 0 & 0 & 0 & * & * & = & * \\
 & * & * & x_3 & * & * & +2x_6 & = & 5 \\
 & x_1 & * & * & * & * & 0 & = & *
\end{array}
$$

For each of the following properties, complete the tableau as to satisfy it, or argue why this cannot be done:

1. The basic variables are $1, 3, 5$.
2. The basic variables are $2, 3, 4$.
3. $\bar{x}$ is feasible but not optimal.
4. $\bar{x}$ is optimal.
5. $\bar{x}$ is the unique optimal solution to the problem.
6. $\bar{x}$ is infeasible.

Problem 4: Suppose you want to solve the following LP:

$$\max c^T x : \; Ax \leq b,$$

but unfortunately it is infeasible (think e.g. about an inventory problem where the demand cannot be satisfied by the warehouse). Let $A \in \mathbb{R}^{m \times n}$. Now suppose you can "augment" your LP by buying some more slack in your problem (think e.g. of buying some of the product from other warehouses). In particular, for $i = 1, \ldots, m$, if you want to increase the right-hand side of the i-th constraint by some value λ_i, you will pay $d_i \lambda_i$ for some fixed number d_i. Suppose moreover that the right-hand side of the i-th constraint can be augmented by at most k_i, for $i = 1, \ldots, m$. How can you find the optimal augmentation, i.e. the one that maximizes the profit of the optimal solution of the augmented LP minus the cost for the augmentation?

Problem 5: Consider the following feasible region:

$$P = \{x : Ax = b, \; x \geq 0\}.$$

Now suppose that you have two objective functions: a "primary" $c^T x$ and a "secondary" $d^T x$ (think of the primary objective function as the main goal of the company, say profit, and of the secondary one as some organization-related issue, say a measure of how fairly the tasks are divided among the employees). Describe an algorithm that finds, among all vectors that maximize the primary objective function, the one that achieves the maximum value of the secondary objective function.

Problem 6: We saw in class that each LP can be transformed into an equivalent LP in any of the following two forms below:

$$(1) \; \max c^T x : \; Ax = b, \; x \geq 0 \qquad\qquad (2) \; \max c^T x : \; Ax \leq b.$$

Can we always transform any LP in an LP of the form

$$\max c^T x : \; Ax = b \; ?$$

Prove your answer correct.