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1. $x_1 :=$ 1000 barrels of oil.

$x_2 :=$ 1000 barrels of aviation fuel.

$x_3 :=$ 1000 barrels of heating oil

$x_4 :=$ 1000 barrels of processed aviation fuel.

$x_5 :=$ 1000 barrels of processed heating oil.

$$\max \quad \underbrace{-40x_1}_{\text{cost}} + \underbrace{60(x_2 - x_4) + 40(x_3 - x_5) + 130x_4 + 90x_5}_{\text{profit}}$$

s.t.

$x_1 \le 20$ — we can only buy 20,000 barrels oil a day.

$0.5x_1 - x_2 \ge 0$ — 1000 barrels of oil yields 500 aviation fuel

$0.5x_1 - x_3 \ge 0$ — " 500 heating oil.

$60x_4 + 45x_5 \le 480$ — only have 8 hr a day.

$x_2 - x_4 \ge 0$ — make sure cracked aviation fuel originate from aviation fuel

$x_3 - x_5 \ge 0$ — "

$x_1, x_2, x_3, x_4, x_5 \ge 0$

⇒ by gurobi

the optimal obj: 260 daily profit

where $x_1 = 20$ (1000 barrels of oil)

$x_2 = 10$ (1000 barrels of aviation fuel).

$x_3 = 10$ (1000 barrels of heating oil).

$x_4 = 8$ (1000 barrels of processed aviation fuel)

$x_5 = 0$ (1000 barrels of processed heating oil).

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2. X_1 := times of process 1

X_2 := Times of process 2

X_3 := hiring hours.

supply : $2X_1 + 5X_2$

demand : $200X_3 + 1000$.

labor used : $X_1 + 2X_2$

chem used : $2X_1 + 3X_2$

$$\max : 5(3X_1 + 5X_2) - 3(X_1 + 2X_2) - 2(2X_1 + 3X_2) - \overset{\text{hiring cost}}{100X_3}$$

$$\text{s.t.} \quad X_1 + 2X_2 \le 20000$$

$$2X_1 + 3X_2 - 200X_3 = 1000$$

$$X_1, X_2, X_3 \ge 0$$

⇒ by gurobi

X_1 = 10000 times of process 1

X_2 : 5000 times of process 2

X_3 : 270 hiring hours.

obj value = 118000. #.

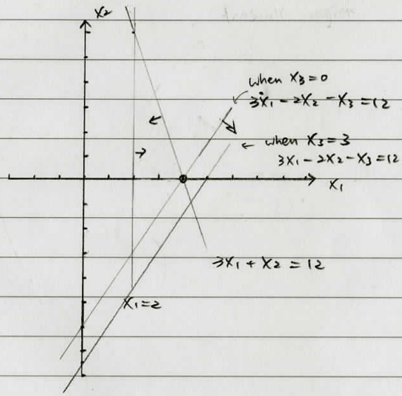
3. $\min z = 3x_1 - 2x_2$

s.t. $3x_1 + x_2 \le 12$

$3x_1 - 2x_2 - x_3 = 12 \Rightarrow 3x_1 - 2x_2 \ge 12$ apply $x_3 \ge 0$.

$x_1 \ge 2$

$x_1, x_2, x_3 \ge 0$



By above graph, the only optimal solution is

when $x_1 = 4$, $x_2 = x_3 = 0$

where z is the minimum value 12.

there isn't feasible region

4. $X_{c,1}$:= crude oil used in method 1

$X_{c,2}$:= " 2

$X_{c,3}$:= " 3

$X_{g,6}$:= barrels of 6 grade gas.

$X_{g,8}$:= " 8 "

$X_{g,10}$:= " 10 "

$X_{h,6}$:= bartels of 6 grade heating oil

$X_{h,8}$:= " 8 "

$X_{h,10}$:= " 10 "

$X_{6\to8}$:= barrels of grade 6 products cracked to grade 8

$X_{8\to10}$:= " 8 " " 10.

sell: $12(X_{g,6} + X_{g,8} + X_{g,10}) + 5(X_{h,6} + X_{h,8} + X_{h,10})$.

cost: $3.4X_{c,1} + 3X_{c,2} + 2.6X_{c,3} + X_{6\to8} + 1.5X_{8\to10}$.

max sell - cost

s.t.

$$X_{g,6} + X_{g,8} + X_{g,10} \le 2100.$$

$$X_{h,6} + X_{h,8} + X_{h,10} \le 600.$$

$$6X_{g,6} + 8X_{g,8} + 10X_{g,10} \ge 9(X_{g,6} + X_{g,8} + X_{g,10}).$$

$$6X_{h,6} + 8X_{h,8} + 10X_{h,10} \ge 7(X_{h,6} + X_{h,8} + X_{h,10}).$$

$$0.3X_{c,1} + 0.4X_{c,2} + 0.3X_{c,3} = X_{g,6} + X_{h,6} + X_{6\to8}$$

$$0.5X_{c,1} + 0.2X_{c,2} + 0.3X_{c3} = X_{g,8} + X_{h,8} + X_{8\to10} - X_{6\to8}$$

$$0.8X_{c1} + 0.4X_{c2} + 0.2X_{c3} = X_{g,10} + X_{h,10} - X_{8\to10}.$$

⇒ by gurobi

obj value = 21475

where $X_{c1} = 1625$ $X_{c2} = 0$ $X_{c3} = 0$

$X_{g6} = 300$ $X_{g8} = 400$ $X_{g10} = 1200$

$X_{h6} = 187.5$ $X_{h8} = 412.5$ $X_{h10} = 0$.

$X_{6\to8} = 0$ $X_{8\to10} = 0$. #

5. x_i := represent the remaining unit of stock i

$100 - x_i$:= represents the selling unit of stock i

	purchase p	current c	future f
x_1	20	30	36
x_2	25	34	39
x_3	30	43	42
x_4.	35	47	45
x_5	40	49	51
x_6	45	53	55
x_7	50	60	63
x_8	55	62	64
x_9	60	64	66
x_{10}	65	66	70.

max $f^T x$

s.t. $\Big(\underbrace{0.99\,c}_{\text{all current price minus 0.01 transaction fee}} - \underbrace{0.3(c-p)}_{\text{tax}}\Big)\Big(\underbrace{100\,\mathbb{1} - x}_{\text{selling unit of each stock}}\Big) = 30\,000$

$x_1, x_2, \dots, x_{10} \le 100$ remaining unit should not exceed 100

⇒ by gurobi

optimal objective :

$x_1 = 100$ ⇒ remaining 100 unit of stock 1
$x_2 = 100$ 〃 100 〃 2
$x_3 = 0$ 〃 0 〃 3
$x_4 = 0$ 〃 0 〃 4.
$x_5 = 100$ 〃 100 〃 5
$x_6 = 36.249$ 〃 36.249 〃 6.
$x_7 = 100$ 〃 100 〃 7
$x_8 = 0$ 〃 0 〃 8
$x_9 = 0$ 〃 0 〃 9
$x_{10} = 0$ 〃 0 〃 10. ☆.

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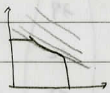
6. (1). False, because LP optimize by linear equation
so the optimal Solution is either a point or a line (multiple solutions)

(2). True, for example, it can have two corner point and two optimal corner solutions.



(3). False. feasible LP may be unbounded, so there might not be an optimal solution that is a corner point.

(4). True

variables : $x_1, \ldots, x_n$
each x_i can be replaced by $y_i \pm y_j$.

and m constraints make m new variables.
$\Rightarrow 2 \times n + m$ variables at most.

constraints : if $2n$ and m new variables.
then there might be $2n+m$ bound constraints.

original constraints.
$\Rightarrow 2n+m+m$ constraints at most