

Mingjia Yu (My2638)

Programming Hw

1. (a) set $v = \max(|y_1 - w^T x_1 - b|, |y_2 - w^T x_2 - b|, \dots)$
 $\min v$

$$|y_1 - w^T x_1 - b| - v \leq 0$$

$$|y_2 - w^T x_2 - b| - v \leq 0$$

\vdots

$$|y_n - w^T x_n - b| - v \leq 0$$

\Downarrow

$$y_1 - w^T x_1 - b - v \leq 0 \Rightarrow w^T x_1 + b + v \geq y_1$$

$$-(y_1 - w^T x_1 - b) - v \leq 0 \Rightarrow w^T x_1 + b - v \leq y_1$$

$$y_2 - w^T x_2 - b - v \leq 0 \Rightarrow w^T x_2 + b + v \geq y_2$$

$$-(y_2 - w^T x_2 - b) - v \leq 0 \Rightarrow w^T x_2 + b - v \leq y_2$$

objective value = 50, ...

optimal value: $w_1 = -9, w_2 = 0, w_3 = -12, w_4 = 28, w_5 = -25, w_6 = 6, w_7 = 21, w_8 = 64$

$w_9 = 36, w_{10} = 100$

running time = 0.17 sec

$v = 50, b = 27$

(b) $\min(t_1 + t_2 + t_3 + \dots + t_n)$

$$|y_1 - w_1^T x_1 - b| \leq t_1$$

$$y_1 - w_1^T x_1 - b \leq t_1 \Rightarrow t_1 + w_1^T x_1 + b \geq y_1$$

$$-(y_1 - w_1^T x_1 - b) \leq t_1$$

$$y_1 - w_1^T x_1 - b \geq t_1 \Rightarrow w_1^T x_1 + b - t_1 \leq y_1$$

$$w_1^T x_1 + b + t_1 \geq y_1$$

$$w_1^T x_1 + b - t_1 \leq y_1$$

$$w^T x_2 + b + t_2 \geq y_2$$

$$w^T x_2 + b - t_2 \leq y_2$$

running time: 4.94 sec

optimal: objective = 1000

$w_1 = -9, w_2 = 0, w_3 = -12, w_4 = 28, w_5 = -25, w_6 = 6, w_7 = 21, w_8 = 64, w_9 = 36, w_{10} = 100, b = 27$

$$2 \text{ a) Dual} = \max y_1 a_1 + y_2 a_2 + \dots + y_N a_N - y_1 a_{N+1} - y_2 a_{N+2} - \dots - y_N a_{2N}$$

$$\text{s.t. } a_1 + a_2 + \dots + a_{2N} = 1$$

$$x_{1,1} a_1 + x_{2,1} a_2 + \dots + x_{N,1} a_N - x_{1,1} a_{N+1} - x_{2,1} a_{N+2} - \dots - x_{N,1} a_{2N} = 0$$

$$x_{1,2} a_1 + x_{2,2} a_2 + \dots + x_{N,2} a_N - x_{1,2} a_{N+2} - \dots - x_{N,2} a_{2N} = 0$$

⋮

$$a_1, \dots, a_N \text{ are non-negative}$$

$$x_{1,10} a_1 + x_{2,10} a_2 + \dots + x_{N,10} a_N - x_{1,10} a_{N+1} - x_{2,10} a_{N+2} - \dots - x_{N,10} a_{2N} = 0$$

$$a_1 + a_2 + \dots + a_N - a_{N+1} - a_{N+2} - \dots - a_{2N} = 0$$

objective value = 50

optimal solution = $a_1 = 0.109, a_2 = 0.2, a_3 = 0.19$, other wise $a_1 = 0, \dots, a_{2N} = 0$

running time = 0.25 sec

$$b) \text{ Dual} = \max y_1 a_1 + y_2 a_2 + \dots + y_N a_N - y_1 a_{N+1} - y_2 a_{N+2} - \dots - y_N a_{2N}$$

$$\text{s.t. } a_1 + a_{N+1} = 1$$

$$a_2 + a_{N+2} = 1$$

⋮

$$a_N + a_{2N} = 1$$

$$x_{1,1} a_1 + x_{2,1} a_2 + \dots + x_{N,1} a_N - x_{1,1} a_{N+1} - x_{2,1} a_{N+2} - \dots - x_{N,1} a_{2N} = 0$$

$$x_{1,2} a_1 + x_{2,2} a_2 + \dots + x_{N,2} a_N - x_{1,2} a_{N+2} - x_{2,2} a_{N+4} - \dots - x_{N,2} a_{2N} = 0$$

⋮

$$x_{1,10} a_1 + x_{2,10} a_2 + \dots + x_{N,10} a_N - x_{1,10} a_{N+1} - x_{2,10} a_{N+2} - \dots - x_{N,10} a_{2N} = 0$$

$$a_1 + a_2 + \dots + a_N - a_{N+1} - a_{N+2} - a_{N+4} - \dots - a_{2N} = 0$$

a_1, \dots, a_N are nonnegative

objective value = 1000

running time = 0.27

optimal solution = $a_1 = 1, a_2 = 1, a_3 = 1, \dots, a_9 = 1, a_{10} = 0, a_{11} = 1, a_{12} = 0$ (either one)

→ The objective value of dual is the same as the primal

→ The running time of dual is shorter than primal.

prob 1 - Min max difference

November 7, 2019

0.0.1 uni: my2638

0.0.2 name: Mingjia Yu

```
[ ]: import sys
import time
import numpy as np
import pandas as pd
from gurobipy import *
from myMatrixLpSolver import lp_optimize

data = pd.read_table('dataLR.txt', sep=',', header=None)
data.rename(
    columns={0: "y",1: "x1",2: "x2",3: "x3",4: "x4",5: "x5",6: "x6",7: "x7",8: "x8",9: "x9",10: "x10"},
    inplace=True
)
data['b'] = 1
data = pd.concat([data, data], axis=0).reset_index(drop=True)
print(data.head(5))

labellist = data["y"].values.tolist()
data["v"] = [1] * int(data.shape[0]/2) + [-1] * int(data.shape[0]/2)
dataList = data.drop("y", axis=1).values.tolist()

start = time.time()
lp_optimize(
    rows=data.shape[0],
    cols=12,
    c=[0]*11 + [1],
    obj_sense=GRB.MINIMIZE,
    A=dataList,
    sense=[GRB.GREATER_EQUAL] * int(data.shape[0]/2) + [GRB.LESS_EQUAL] *
    int(data.shape[0]/2),
```

```

    rhs=labelList,
    lb=[-GRB.INFINITY] * 12,
    ub=[GRB.INFINITY] * 12,
    vtype=[GRB.CONTINUOUS] * 12,
    solution=[0] * 12
)
end = time.time()
print(end - start)

# Dual
start = time.time()
A_transpose = np.array(dataList).T.tolist()
lp_optimize(
    rows=len(A_transpose),
    cols=len(labelList),
    c=labelList,
    obj_sense=GRB.MAXIMIZE,
    A=A_transpose,
    sense=[GRB.EQUAL] * len(A_transpose),
    rhs=[0]*11 + [1],
    lb=[0] * int(len(labelList) / 2) + [-GRB.INFINITY] * int(len(labelList) /
↪2),
    ub=[GRB.INFINITY] * int(len(labelList) / 2) + [0] * int(len(labelList) / 2),
    vtype=[GRB.CONTINUOUS] * len(labelList),
    solution=[0] * len(labelList)
)
end = time.time()
print(end - start)

```

```

python3 zac.py
    y  x1  x2  x3  x4  x5  x6  x7  x8  x9  x10  b
0    36   5   8 -10  30 -17 -29  11  41 -42 -26  1
1 -4077  -3   0  -8 -37  30 -35  37  46  -7 -58  1
2   8176   0 -14  18 -25 -41 -33 -22  66 -34  56  1
3   2190   2   3  20   7 -43 -52  46  26 -38   1  1
4 -10938   4 -10  23 -14  40 -50  19   6 -29 -88  1
Academic license - for non-commercial use only
Optimize a model with 20000 rows, 12 columns and 237130 nonzeros
Coefficient statistics:
  Matrix range      [1e+00, 1e+02]
  Objective range   [1e+00, 1e+00]
  Bounds range      [0e+00, 0e+00]
  RHS range         [6e+00, 2e+04]
Presolve time: 0.11s
Presolved: 12 rows, 20000 columns, 237130 nonzeros

Iteration    Objective          Primal Inf.    Dual Inf.      Time
     0      -0.00000000e+00    4.0000000e+00    6.713220e+09     0s
    242      5.00000000e+01    0.0000000e+00    0.0000000e+00     0s

Solved in 242 iterations and 0.16 seconds
Optimal objective  5.0000000000e+01
Obj Value:  49.99999999999997
C0 -8.9999999999999716

C1 -1.8430868428878497e-13

C2 -12.0000000000000059

C3 27.999999999999947

C4 -24.99999999999996

C5 5.9999999999999854

C6 20.999999999999975

C7 63.999999999999964

C8 36.000000000000005

C9 100.0

C10 76.999999999999562

C11 49.99999999999997

1.0383250713348389

```

Optimize a model with 12 rows, 20000 columns and 237130 nonzeros

Coefficient statistics:

Matrix range [1e+00, 1e+02]
 Objective range [6e+00, 2e+04]
 Bounds range [0e+00, 0e+00]
 RHS range [1e+00, 1e+00]

Concurrent LP optimizer: dual simplex and barrier

Showing barrier log only...

Presolve time: 0.15s

Presolved: 12 rows, 20000 columns, 237130 nonzeros

Ordering time: 0.00s

Barrier statistics:

AA' NZ : 6.600e+01
 Factor NZ : 7.800e+01 (roughly 8 MBytes of memory)
 Factor Ops : 6.500e+02 (less than 1 second per iteration)
 Threads : 1

Iter	Objective		Residual		Compl	Time
	Primal	Dual	Primal	Dual		
0	2.80806489e-11	-0.000000000e+00	6.71e+02	1.25e+01	2.24e+01	0s
1	-9.72032577e-02	2.69590365e+03	5.75e+01	9.09e-13	2.07e+00	0s
2	-3.46321766e-03	2.35712357e+03	5.68e-14	9.09e-13	1.18e-01	0s
3	4.16862741e-03	1.59430584e+02	4.75e-14	9.09e-13	7.97e-03	0s
4	5.97110909e-01	1.02042814e+02	7.55e-15	6.82e-13	5.07e-03	0s
5	2.07055043e+01	1.02794489e+02	1.99e-14	6.82e-13	4.10e-03	0s

Barrier performed 5 iterations in 0.25 seconds

Barrier solve interrupted - model solved by another algorithm

Solved with dual simplex

Solved in 49 iterations and 0.25 seconds

Optimal objective 5.000000000e+01

Obj Value: 50.00000000000073

C0 0.10976719926660782

C1 0.19978884547085207

C2 0.0

C3 0.0

C4 0.0

C5 0.0

C6 0.0

C7 0.19044395526254007

C8 0.0

C9 0.0

C10 0.0

```
Solved with dual simplex
Solved in 49 iterations and 0.25 seconds
Optimal objective 5.000000000e+01
Obj Value: 50.00000000000073
C0 0.10976719926660782

C1 0.19978884547085207

C2 0.0

C3 0.0

C4 0.0

C5 0.0

C6 0.0

C7 0.19044395526254007

C8 0.0

C9 0.0

C10 0.0

C11 0.0

C12 0.0

C13 0.0

C14 0.0

C15 0.0

C16 0.0

C17 0.0

C18 0.0

C19 0.0

C20 0.0

C21 0.0

C22 0.0

C23 0.0

C24 0.0

C25 0.0

C26 0.0

C27 0.0
```

prob 2 - Min sum of difference

November 7, 2019

0.0.1 uni: my2638

0.0.2 name: Mingjia Yu

```
[ ]: import os
import sys
import time
import numpy as np
import pandas as pd
from gurobipy import *
from myMatrixLpSolver import lp_optimize

data = pd.read_table('dataLR.txt', sep=',', header=None)
data.rename(
    columns={0: "y",1: "x1",2: "x2",3: "x3",4: "x4",5: "x5",6: "x6",7: "x7",8: "x8",9: "x9",10: "x10"},
    inplace=True
)
N = data.shape[0]

data['b'] = 1
data = pd.concat([data, data], axis=0).reset_index(drop=True)
labelList = data["y"]

data.drop("y", axis=1, inplace=True)
data = data.values
I = np.identity(n=N)
II = np.concatenate((I, np.negative(I)), axis=0)
data = np.concatenate((data, II), axis=1)

start = time.time()
lp_optimize(
    rows=data.shape[0],
```



```

cols=data.shape[1],
c=[0]*11 + [1]*N,
obj_sense=GRB.MINIMIZE,
A=data.tolist(),
sense=[GRB.GREATER_EQUAL] * int(data.shape[0]/2) + [GRB.LESS_EQUAL] *
↪int(data.shape[0]/2),
rhs=labelList,
lb=[-GRB.INFINITY] * (11 + N),
ub=[GRB.INFINITY] * (11 + N),
vtype=[GRB.CONTINUOUS] * (11 + N),
solution=[0]*(11 + N)
)
end = time.time()
print(end - start)

# Dual
start = time.time()
A_transpose = data.T.tolist()
lp_optimize(
    rows=len(A_transpose),
    cols=len(labelList),
    c=labelList,
    obj_sense=GRB.MAXIMIZE,
    A=A_transpose,
    sense=[GRB.EQUAL] * len(A_transpose),
    rhs=[0]*11 + [1]*N,
    lb=[0] * int(len(labelList) / 2) + [-GRB.INFINITY] * int(len(labelList) /
↪2),
    ub=[GRB.INFINITY] * int(len(labelList) / 2) + [0] * int(len(labelList) / 2),
    vtype=[GRB.CONTINUOUS] * len(labelList),
    solution=[0] * len(labelList)
)
end = time.time()
print(end - start)

```

Optimize a model with 20000 rows, 10011 columns and 237130 nonzeros

Coefficient statistics:

Matrix range [1e+00, 1e+02]
 Objective range [1e+00, 1e+00]
 Bounds range [0e+00, 0e+00]
 RHS range [6e+00, 2e+04]

Concurrent LP optimizer: dual simplex and barrier

Showing barrier log only...

Presolve removed 10000 rows and 0 columns

Presolve time: 0.30s

Presolved: 10000 rows, 10011 columns, 118565 nonzeros

Ordering time: 0.00s

Barrier statistics:

Dense cols : 11
 Free vars : 11
 AA' NZ : 1.086e+05
 Factor NZ : 1.186e+05 (roughly 10 MBytes of memory)
 Factor Ops : 1.408e+06 (less than 1 second per iteration)
 Threads : 1

Iter	Objective		Residual		Compl	Time
	Primal	Dual	Primal	Dual		
0	5.31670428e+08	9.20933000e+05	0.00e+00	2.00e+04	9.96e+05	0s
1	5.01525896e+08	2.55734794e+01	0.00e+00	1.42e-10	2.51e+04	0s
2	1.12558303e+06	3.17766341e+01	0.00e+00	1.02e-10	5.63e+01	0s
3	2.98823552e+04	9.12312611e+02	0.00e+00	5.09e-11	1.45e+00	0s
4	1.03099481e+03	9.99891159e+02	0.00e+00	2.91e-11	1.56e-03	0s
5	1.00003099e+03	9.9999891e+02	0.00e+00	5.82e-11	1.56e-06	0s
6	1.00000003e+03	1.00000000e+03	0.00e+00	9.82e-11	1.56e-09	0s

Barrier solved model in 6 iterations and 0.46 seconds

Optimal objective 1.00000003e+03

Crossover log...

0 DPushes remaining with DInf 1.7337243e-12 3s

0 PPushes remaining with PInf 0.0000000e+00 3s

Push phase complete: Pinf 0.0000000e+00, Dinf 1.7337243e-12 3s

Iteration	Objective	Primal Inf.	Dual Inf.	Time
8934	1.0000000e+03	0.000000e+00	0.000000e+00	3s

Crossover log...

0 DPushes remaining with DInf 1.7337243e-12 3s

0 PPushes remaining with PInf 0.0000000e+00 3s

Push phase complete: Pinf 0.0000000e+00, Dinf 1.7337243e-12 3s

Iteration	Objective	Primal Inf.	Dual Inf.	Time
8934	1.0000000e+03	0.0000000e+00	0.0000000e+00	3s

Solved with barrier

Solved in 8934 iterations and 3.27 seconds

Optimal objective 1.000000000e+03

Obj Value: 999.9999999999945

C0 -9.0000000000000455

C1 0.0

C2 -12.0

C3 28.000000000000018

C4 -25.000000000000014

C5 6.000000000000018

C6 21.00000000000001

C7 64.00000000000003

C8 36.00000000000003

C9 99.9999999999996

C10 27.000000000001933

C11 99.99999999999864

C12 99.999999999995

C13 100.00000000000091

C14 99.9999999999902

C15 99.99999999999818

C16 99.999999999995

Iter	Objective		Residual		Compl	Time
	Primal	Dual	Primal	Dual		
0	4.38774667e+07	4.54461388e+07	2.13e+06	2.27e-13	1.19e+05	0s
1	2.34772123e+01	4.36838665e+07	1.46e-07	1.14e-12	2.18e+03	0s
2	8.36102686e+01	7.46213964e+05	1.46e-11	1.36e-12	3.73e+01	0s
3	9.49403501e+02	1.44621193e+04	1.60e-10	1.36e-12	6.76e-01	0s
4	9.99951643e+02	1.01349144e+03	2.76e-10	1.36e-12	6.77e-04	0s
5	9.99999952e+02	1.00001349e+03	2.47e-10	1.36e-12	6.77e-07	0s
6	1.00000000e+03	1.00000001e+03	9.46e-11	1.14e-12	6.77e-10	0s

Barrier solved model in 6 iterations and 0.28 seconds

Optimal objective 1.00000000e+03

Crossover log...

1 DPushes remaining with DInf 0.00000000e+00 0s

0 DPushes remaining with DInf 3.1254999e-12 0s

9979 PPushes remaining with PInf 0.00000000e+00 0s

0 PPushes remaining with PInf 0.00000000e+00 0s

Push phase complete: Pinf 0.00000000e+00, Dinf 2.4300562e-12 0s

Iteration	Objective	Primal Inf.	Dual Inf.	Time
9983	1.00000000e+03	0.0000000e+00	0.0000000e+00	0s

Solved with dual simplex

Solved in 106 iterations and 0.37 seconds

Optimal objective 1.000000000e+03

Obj Value: 999.999999998399

C0 1.0

C1 1.0

C2 1.0

C3 1.0

C4 1.0

C5 1.0

C6 1.0

C7 1.0

C8 1.0