Optimization (PA2)

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In the Facebook friendship network, there are 4039 nodes, V; 88234 edges

For each node, except 0, we can construct a shortest path problem, where each node is the t, destination node.

For each node V from 1 to 4038:

$$\text{Min} \sum w_{ij} x_{ij} \qquad (ij) \in E$$

$$\begin{cases} \sum_{i:(ij)\in E} x_{ij} - \sum_{j:(ji)\in E} x_{ji} = \begin{cases} 1 & \text{if } i=0 \\ -1 & \text{if } i=V \text{ (the t, destination)} \\ 0 & \text{otherwise} \end{cases} \\ 0 \le x_{ij} \qquad \text{for each } (ij) \in E \end{cases}$$

the optimal obj = h_V the exposure time for node V (user V)

Then the average Exposure time = $\frac{1}{4039} \sum_{V=0}^{4038} h_V$

By solving ~~4039~~ 4038 shortest path problems using Gurobi, and taking down each obj value, we can calculate average Exposure time then.

▲ if using min-cost flow network.

$$\text{Min} \quad c_{ij} x_{ij} \qquad \forall (ij) \in E$$

$$\text{s.t.} \quad \sum_{j \text{ outgoing}} x_{ij} - \sum_{k \text{ ingoing}} x_{ki} = b_i = -1 \qquad \forall i \in V$$

$$0 \le x_{ij} \le u_{ij} = \infty \qquad \forall (ij) \in E$$

$$\sum_{i \in V} b_i = 0$$

Here: $b_0 = 4038$ $b_i = -1$ ($i \in V$ except for $i=0$)

$c_{ij} = 1$

$u_{ij} = \infty$

≈ 2.8294 (hours)

Using Gurobi to construct model and optimize, the optimal solution for the min-cost problem is $z^* = 11428$ (total Exposure time), so avg Exposure time = $\frac{11428}{4039}$

PYTHON CODES FOR PA 2 (Programming Assignment 2 CODES)

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Tue Nov 12 23:36:42 2019

@author: fanxiuqi

refer link:
https://www.gurobi.com/documentation/8.1/quickstart_linux/py_netflow_py_example.html
"""

import sys
from gurobipy import *

#read the file
f = open('facebook_combined.txt', 'r')
G = {}
# Loop over lines and extract variables of interest
for line in f:
                        line = line.strip()
                        #print(line)
                        nodes = line.split(' ')
                        G[(int(nodes[0]), int(nodes[1]))]=1
f.close()
print('Finished reading the file')


# Edges (only one direction in G)
edges1, weights = multidict(G)
# make it bidirectional
edges2 = [i[::-1] for i in edges1]
for i in edges2:
   G[i]=1

# Updated edges (bidirectional)
edges, capacity = multidict(G)
# Nodes
V = [i for i in range(4039)]
# Time cost
Cost = G
# Net supply
b_0 = 4038
b_ow = -1


#  Facebook_friend network model
m1 = Model('fb')

# Create decision variables xij, 0<=xij<=1; bi:int
flow = m1.addVars(edges, vtype = GRB.CONTINUOUS, obj=Cost, name="flow")
m1.update()


# The objective is to minimize
```

```
m1.modelSense = GRB.MINIMIZE
m1.update()

# Constraints
# outgoing - ingoing = bi
# Arc-capacity constraints are satisfied by lb=0
m1.addConstr(
   (flow.sum(0,'*') - flow.sum('*',0) == 4038), "NodeNetflow")
m1.addConstrs(
   (flow.sum(i,'*') - flow.sum('*',i) == -1 for i in V[1:]), "NodeNetflow")
m1.update()


m1.optimize()
#print(m1.getAttr('x'))
print('\nOptimal Obj value:', m1.objVal)
print('Average exposure time over all users:', m1.objVal/4039)
```

==========OUTPUT==========

```
 runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')
Finished reading the file
Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 1e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+00, 4e+03]

Concurrent LP optimizer: dual simplex and barrier
Showing barrier log only...

Presolve removed 76 rows and 150 columns
Presolve time: 0.27s
Presolved: 3963 rows, 176318 columns, 352618 nonzeros

Ordering time: 0.00s

Barrier statistics:
 AA' NZ     : 8.815e+04
 Factor NZ  : 2.964e+05 (roughly 70 MBytes of memory)
 Factor Ops : 4.274e+07 (less than 1 second per iteration)
 Threads    : 1

             Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.51743348e+08  7.50000000e+01  1.01e-09 0.00e+00  1.47e+03     0s
   1   2.69241096e+07  8.36861626e+02  1.11e-09 6.66e-16  1.53e+02     0s
   2   1.51501970e+06  3.94523896e+03  1.41e-10 6.66e-16  8.57e+00     1s
   3   1.36653239e+05  6.62992755e+03  5.37e-09 8.88e-16  7.37e-01     1s
   4   3.77472853e+04  7.63850101e+03  3.20e-09 9.99e-16  1.71e-01     1s
   5   2.20931602e+04  9.45622299e+03  4.32e-09 8.88e-16  7.17e-02     1s
   6   2.07741179e+04  9.95121524e+03  3.51e-09 1.11e-15  6.14e-02     1s
   7   1.49229362e+04  1.06246030e+04  2.43e-09 8.88e-16  2.44e-02     1s
   8   1.32597050e+04  1.11334132e+04  2.00e-09 8.88e-16  1.21e-02     1s
```

```
   9   1.19052944e+04  1.13962000e+04  9.34e-09 8.88e-16  2.89e-03     1s
  10   1.14323192e+04  1.14276074e+04  1.22e-08 1.11e-15  2.67e-05     1s
  11   1.14280044e+04  1.14279996e+04  2.63e-07 1.11e-15  2.74e-08     1s
  12   1.14280000e+04  1.14280000e+04  4.12e-10 8.88e-16  2.77e-14     1s

Barrier solved model in 12 iterations and 0.95 seconds
Optimal objective 1.14280000e+04

Crossover log...

      0 DPushes remaining with DInf 0.0000000e+00                 1s

   7932 PPushes remaining with PInf 0.0000000e+00                 1s
      0 PPushes remaining with PInf 0.0000000e+00                 1s

  Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00      1s

Iteration    Objective       Primal Inf.    Dual Inf.      Time
   8243    1.1428000e+04   0.000000e+00   0.000000e+00      1s

Solved with barrier
Solved in 8243 iterations and 1.05 seconds
Optimal objective  1.142800000e+04
Optimal Obj value: 11428.0
Average exposure time over all users: 2.82941322109433

runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')
Finished reading the file
Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 1e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+00, 4e+03]

Concurrent LP optimizer: dual simplex and barrier
Showing barrier log only...

Presolve removed 76 rows and 150 columns
Presolve time: 0.18s
Presolved: 3963 rows, 176318 columns, 352618 nonzeros

Ordering time: 0.00s

Barrier statistics:
 AA' NZ     : 8.815e+04
 Factor NZ  : 2.964e+05 (roughly 70 MBytes of memory)
 Factor Ops : 4.274e+07 (less than 1 second per iteration)
 Threads    : 1

             Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.51743348e+08  7.50000000e+01  1.01e-09 0.00e+00  1.47e+03     0s
   1   2.69241096e+07  8.36861626e+02  1.11e-09 6.66e-16  1.53e+02     0s
   2   1.51501970e+06  3.94523896e+03  1.41e-10 6.66e-16  8.57e+00     0s
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```

```
   4   3.77472853e+04  7.63850101e+03  3.20e-09 9.99e-16  1.71e-01    0s
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  12   1.14280000e+04  1.14280000e+04  4.12e-10 8.88e-16  2.77e-14    1s

Barrier solved model in 12 iterations and 0.75 seconds
Optimal objective 1.14280000e+04

Crossover log...

      0 DPushes remaining with DInf 0.0000000e+00                1s

   7932 PPushes remaining with PInf 0.0000000e+00                1s
      0 PPushes remaining with PInf 0.0000000e+00                1s

  Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00      1s

Iteration    Objective       Primal Inf.    Dual Inf.      Time
   8243    1.1428000e+04   0.000000e+00   0.000000e+00      1s

Solved with barrier
Solved in 8243 iterations and 0.84 seconds
Optimal objective  1.142800000e+04

Optimal Obj value: 11428.0
Average exposure time over all users: 2.82941322109433
```

PYTHON CODES FOR PA 2 (Programming Assignment 2 CODES)___*Screenshots version*

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# -*- coding: utf-8 -*-
"""
Created on Tue Nov 12 23:36:42 2019

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m1.optimize()
#print(m1.getAttr('x'))
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print('Average exposure time over all users:', m1.objVal/4039)
```

==========OUTPUT==========

```
In [24]: runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')
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Showing barrier log only...

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   7   1.49229362e+04  1.06246030e+04  2.43e-09 8.88e-16  2.44e-02     1s
   8   1.32597050e+04  1.11334132e+04  2.00e-09 8.88e-16  1.21e-02     1s
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  10   1.14323192e+04  1.14276074e+04  1.22e-08 1.11e-15  2.67e-05     1s
  11   1.14280044e+04  1.14279996e+04  2.63e-07 1.11e-15  2.74e-08     1s
  12   1.14280000e+04  1.14280000e+04  4.12e-10 8.88e-16  2.77e-14     1s


Barrier solved model in 12 iterations and 0.75 seconds
Optimal objective 1.14280000e+04

Crossover log...

       0 DPushes remaining with DInf 0.0000000e+00                 1s

    7932 PPushes remaining with PInf 0.0000000e+00                 1s
       0 PPushes remaining with PInf 0.0000000e+00                 1s

  Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00      1s

Iteration    Objective       Primal Inf.    Dual Inf.      Time
    8243    1.1428000e+04   0.000000e+00   0.000000e+00      1s

Solved with barrier
Solved in 8243 iterations and 0.84 seconds
Optimal objective  1.142800000e+04

Optimal Obj value: 11428.0
Average exposure time over all users: 2.82941322109433

In [25]:
```