IEORE4004: Optimization Models and Methods - Prof. Christian Kroer 10/15/2019

Assignment 3

Deadline: 10/24/2019 at 11:40am

- This assignment sheet has 7 exercises. You must submit your solution via Canvas before the deadline. The time constraint is strict. Late submissions will not be accepted.
- You are allowed to discuss the assignment with others but the write-up must be individual work. Please mention in your write-up all the people you have discussed the solution with.
- Unless otherwise specified, you should NOT use Gurobi or other softwares to find the optimal solution. For solutions obtain from these softwares, you should include your code as well as the output in the submission.

Problem 1: Consider the following LP

$$
\begin{array}{llllll}
\max & 2x_1 & -x_2 & +x_3 & & \\
\text{s.t.} & & & & & \\
 & 3x_1 & +x_2 & +x_3 & \geq & 40 \\
 & x_1 & -x_2 & +2x_3 & = & 15 \\
 & x_1 & +x_2 & -x_3 & \leq & 30 \\
 & x_1 & & & \geq & 0 \\
 & x_2 & & & \leq & 0
\end{array}
$$

1. Write the dual of the previous problem.
2. Using complementary slackness, check if any of the following solutions is optimal for the primal problem: $(10, 10, 10)^T$, $(30, -5, 0)^T$, $(25, 0, 5)^T$. Provide an optimal solution of the dual.

Problem 2: [1] Consider the following LP and its optimal tableau:

$$
\begin{array}{rl}
\max & z = 3x_1 + 2x_2 \\
\text{s.t.} & 2x_1 + 5x_2 \leq 8 \\
 & 3x_1 + 7x_2 \leq 10 \\
 & x_1, x_2 \geq 0
\end{array}
$$

z	x_1	x_2	s_1	s_2	rhs
1	0	0	0	1	10
0	0	1/3	1	$-2/3$	4/3
0	1	7/3	0	1/3	10/3

a. Find the dual of this LP and its optimal solution

b. Find the range of values of $b2$ for which the current primal basis remains optimal. Also find the new optimal solution if $b2 = 5$.

c. Change the objective function of the given (primal) LP so that its dual becomes infeasible, orargue why this cannot be done.

[1] This is based on Problem 10 of the review problems in Section 6 of the textbook by Winston and Venkataraman.

Problem 3: [2] Consider the following LP and its optimal tableau:

$$\begin{array}{rl} \max & z = 4x_1 + x_2 + 2x_3 \\ \text{s.t.} & 8x_1 + 3x_2 + x_3 \leq 2 \\ & 6x_1 + x_2 + x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

z	x_1	x_2	x_3	s_1	s_2	rhs
1	12	5	0	2	0	4
0	8	3	1	1	0	2
0	-2	-2	0	-1	1	6

a. Find the dual of this LP and its optimal solution

b. Find the range of values of of the objective function coefficient for x_3 for which the current basis remains optimal.

c. Find the range of values of of the objective function coefficient for x_1 for which the current basis remains optimal.

Problem 4: Consider the Rock-Paper-Scissors game seen in class.

a. Compute, using your favorite software, the optimal strategy of each player.

b. Suppose the probability that the row player plays rock (resp. scissors, paper) is 2/5 (resp. 1/10, 1/2). Compute the optimal strategy for the column player and the expected outcome, without using software.

Problem 5: Consider the following payoff matrix for a zero-sum game. Compute, using your favorite software, the optimal strategy of each player, and the expected payoff.

$$A = \begin{pmatrix} 2 & -1 & 3 & -2 \\ 1 & 4 & -3 & 0 \\ 0 & -2 & -1 & 3 \end{pmatrix}$$

Problem 6: Can you modify the label-correction algorithm for shortest paths in networks so that it recognizes the existence of a cycle of negative cost[3]? Think about a modification of the algorithm that detects this cycle as quickly as possible (i.e. an algorithm that detects it after $|V|$ scans of the full edge list is better than an algorithm that detects it after $|V|^2$ scans).

Problem 7: Consider the currency trade problem modelled as a longest path problem seen in class. Develop an algorithm that detects if you can trade currency starting with 100$ and ending up with more than 100$.

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[2] This is based on Problem 13 of the review problems in Section 6 of the textbook by Winston and Venkataraman.
[3] Note that your algorithm does not have to solve the shortest path problem in the network.