Optimization (PAZ) Xingi Fan cxf2207> In the Facebook friendship network, there are 4039 nodes, V; 88234 edges For each node, except 0, we can construct a shortest path problem, where each node is the t, destination node. For each node V from 1 to 4038: Min 豆 Wij Xij (河) EE $\sum Xiy - \sum Xji = \begin{cases} 1 & \text{if } i=0 \\ -1 & \text{if } i=V \text{ (the t, destination)} \end{cases}$ 0 otherwise 0 \le \tij for each (inj) \in \text{E} the optimal obj = hav the exposure time for node v (user v) Then the average Exposerse time = \frac{1}{4038} \geq \text{\$\frac{4038}{2400}} \hv By solving 4079 4078 shortest path problems using Gurobi, and taking down each obj value, we can calculate average Exposure time then. E ▲ if using min-cost flow network. Min aj Xij Vaijse E St. Z Xij - Z Xki = bi = -1 #ieV ¥ (inj)∈E 0 < xij < kij = po Z bi = 0 Here: $b_0 = 4038$ bi = -1 (if exept for i=0) Cij=1 \$ 2.8294 (hours)

PYTHON CODES FOR PA 2 (Programming Assignment 2 CODES)

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
Created on Tue Nov 12 23:36:42 2019
@author: fanxiuqi
refer link:
https://www.gurobi.com/documentation/8.1/quickstart linux/py netflow py example.html
import sys
from gurobipy import *
#read the file
f = open('facebook_combined.txt', 'r')
# Loop over lines and extract variables of interest
for line in f:
                 line = line.strip()
                 #print(line)
                 nodes = line.split(' ')
                 G[(int(nodes[0]), int(nodes[1]))]=1
f.close()
print('Finished reading the file')
# Edges (only one direction in G)
edges1, weights = multidict(G)
# make it bidirectional
edges2 = [i[::-1] for i in edges1]
for i in edges2:
  G[i]=1
# Updated edges (bidirectional)
edges, capacity = multidict(G)
# Nodes
V = [i \text{ for } i \text{ in range}(4039)]
# Time cost
Cost = G
# Net supply
b 0 = 4038
b_ow = -1
# Facebook_friend network model
m1 = Model('fb')
# Create decision variables xij, 0<=xij<=1; bi:int
flow = m1.addVars(edges, vtype = GRB.CONTINUOUS, obj=Cost, name="flow")
m1.update()
```

The objective is to minimize

```
m1.modelSense = GRB.MINIMIZE
m1.update()
# Constraints
# outgoing - ingoing = bi
# Arc-capacity constraints are satisfied by Ib=0
m1.addConstr(
  (flow.sum(0,'*') - flow.sum('*',0) == 4038), "NodeNetflow")
m1.addConstrs(
  (flow.sum(i,'*') - flow.sum('*',i) == -1 for i in V[1:]), "NodeNetflow")
m1.update()
m1.optimize()
#print(m1.getAttr('x'))
print('\nOptimal Obj value:', m1.objVal)
print('Average exposure time over all users:', m1.objVal/4039)
runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')
Finished reading the file
Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros
Coefficient statistics:
 Matrix range [1e+00, 1e+00]
 Objective range [1e+00, 1e+00]
 Bounds range [0e+00, 0e+00]
 RHS range
            [1e+00, 4e+03]
Concurrent LP optimizer: dual simplex and barrier
Showing barrier log only...
Presolve removed 76 rows and 150 columns
Presolve time: 0.27s
Presolved: 3963 rows, 176318 columns, 352618 nonzeros
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 8.815e+04
Factor NZ: 2.964e+05 (roughly 70 MBytes of memory)
Factor Ops: 4.274e+07 (less than 1 second per iteration)
Threads:1
        Objective
                         Residual
                         Primal Dual Compl Time
Iter
      Primal
                 Dual
 0 1.51743348e+08 7.50000000e+01 1.01e-09 0.00e+00 1.47e+03
                                                               0s
 1 2.69241096e+07 8.36861626e+02 1.11e-09 6.66e-16 1.53e+02
                                                               0s
 2 1.51501970e+06 3.94523896e+03 1.41e-10 6.66e-16 8.57e+00
                                                               1s
 3 1.36653239e+05 6.62992755e+03 5.37e-09 8.88e-16 7.37e-01
                                                               1s
 4 3.77472853e+04 7.63850101e+03 3.20e-09 9.99e-16 1.71e-01
                                                              1s
 5 2.20931602e+04 9.45622299e+03 4.32e-09 8.88e-16 7.17e-02
                                                              15
 6 2.07741179e+04 9.95121524e+03 3.51e-09 1.11e-15 6.14e-02
                                                               1s
 7 1.49229362e+04 1.06246030e+04 2.43e-09 8.88e-16 2.44e-02
                                                               1s
```

8 1.32597050e+04 1.11334132e+04 2.00e-09 8.88e-16 1.21e-02

9 1.19052944e+04 1.13962000e+04 9.34e-09 8.88e-16 2.89e-03 1s 10 1.14323192e+04 1.14276074e+04 1.22e-08 1.11e-15 2.67e-05 1s 11 1.14280044e+04 1.14279996e+04 2.63e-07 1.11e-15 2.74e-08 1s 12 1.14280000e+04 1.14280000e+04 4.12e-10 8.88e-16 2.77e-14 1s

Barrier solved model in 12 iterations and 0.95 seconds Optimal objective 1.14280000e+04

Crossover log...

0 DPushes remaining with DInf 0.0000000e+00 1s

7932 PPushes remaining with PInf 0.0000000e+00 1s 0 PPushes remaining with PInf 0.0000000e+00 1s

Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00 1s

Iteration Objective Primal Inf. Dual Inf. Time 8243 1.1428000e+04 0.000000e+00 0.000000e+00 1s

Solved with barrier Solved in 8243 iterations and 1.05 seconds Optimal objective 1.142800000e+04 Optimal Obj value: 11428.0

Average exposure time over all users: 2.82941322109433

runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')

Finished reading the file

Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros

Coefficient statistics:

Matrix range [1e+00, 1e+00] Objective range [1e+00, 1e+00] Bounds range [0e+00, 0e+00] RHS range [1e+00, 4e+03]

Concurrent LP optimizer: dual simplex and barrier

Showing barrier log only...

Presolve removed 76 rows and 150 columns

Presolve time: 0.18s

Presolved: 3963 rows, 176318 columns, 352618 nonzeros

Ordering time: 0.00s

Barrier statistics: AA' NZ : 8.815e+04

Factor NZ: 2.964e+05 (roughly 70 MBytes of memory) Factor Ops: 4.274e+07 (less than 1 second per iteration)

Threads: 1

Objective Residual

Iter Primal Dual Primal Dual Compl Time

0 1.51743348e+08 7.50000000e+01 1.01e-09 0.00e+00 1.47e+03 0s
1 2.69241096e+07 8.36861626e+02 1.11e-09 6.66e-16 1.53e+02 0s
2 1.51501970e+06 3.94523896e+03 1.41e-10 6.66e-16 8.57e+00 0s
3 1.36653239e+05 6.62992755e+03 5.37e-09 8.88e-16 7.37e-01 0s

```
4 3.77472853e+04 7.63850101e+03 3.20e-09 9.99e-16 1.71e-01 0s 5 2.20931602e+04 9.45622299e+03 4.32e-09 8.88e-16 7.17e-02 0s 6 2.07741179e+04 9.95121524e+03 3.51e-09 1.11e-15 6.14e-02 1s 7 1.49229362e+04 1.06246030e+04 2.43e-09 8.88e-16 2.44e-02 1s 8 1.32597050e+04 1.11334132e+04 2.00e-09 8.88e-16 1.21e-02 1s 9 1.19052944e+04 1.13962000e+04 9.34e-09 8.88e-16 2.89e-03 1s 1.14323192e+04 1.14276074e+04 1.22e-08 1.11e-15 2.67e-05 1s 1.14280004e+04 1.14279996e+04 2.63e-07 1.11e-15 2.74e-08 1s 1.14280000e+04 1.14280000e+04 4.12e-10 8.88e-16 2.77e-14 1s
```

Barrier solved model in 12 iterations and 0.75 seconds Optimal objective 1.14280000e+04

Crossover log...

0 DPushes remaining with DInf 0.0000000e+00 1s
7932 PPushes remaining with PInf 0.0000000e+00 1s
0 PPushes remaining with PInf 0.0000000e+00 1s

Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00 15

Iteration Objective Primal Inf. Dual Inf. Time 8243 1.1428000e+04 0.000000e+00 0.000000e+00 1s

Solved with barrier Solved in 8243 iterations and 0.84 seconds Optimal objective 1.142800000e+04

Optimal Obj value: 11428.0

Average exposure time over all users: 2.82941322109433

PYTHON CODES FOR PA 2 (Programming Assignment 2 CODES)___Screenshots version

```
1 #!/usr/bin/env python3
 2 # -*- coding: utf-8 -*-
 3 """
 4 Created on Tue Nov 12 23:36:42 2019
 6 @author: fanxiuqi
 8 refer link:
 9 https://www.gurobi.com/documentation/8.1/quickstart_linux/py_netflow_py_example.html
10
11
12 import sys
13 from gurobipy import *
15 #read the file
16 f = open('facebook_combined.txt', 'r')
17 G = \{\}
18 # Loop over lines and extract variables of interest
19 for line in f:
20
           line = line.strip()
21
           #print(line)
22
           nodes = line.split(' ')
23
           G[(int(nodes[0]), int(nodes[1]))]=1
24 f.close()
25 print('Finished reading the file')
26
27
28 # Edges (only one direction in G)
29 edges1, weights = multidict(G)
30 # make it bidirectional
31 \text{ edges2} = [i[::-1] \text{ for } i \text{ in edges1}]
32 for i in edges2:
33
       G[i]=1
34
35 # Updated edges (bidirectional)
36 edges, capacity = multidict(G)
37 # Nodes
38 V = [i for i in range(4039)]
39 # Time cost
40 \text{ Cost} = G
41 # Net supply
42 b_0 = 4038
43 b_ow = -1
44
45
46 # Facebook_friend network model
47 m1 = Model('fb')
48
49 # Create decision variables xij, 0<=xij<=1; bi:int
50 flow = m1.addVars(edges, vtype = GRB.CONTINUOUS, obj=Cost, name="flow")
51 ml.update()
52
53
54 # The objective is to minimize
55 m1.modelSense = GRB.MINIMIZE
56 m1.update()
57
58 # Constraints
59 # outgoing - ingoing = bi
60 # Arc-capacity constraints are satisfied by lb=0
61 m1.addConstr(
62
       (flow.sum(0, '*') - flow.sum('*', 0) == 4038), "NodeNetflow")
63 m1.addConstrs(
64
       (flow.sum(i,'*') - flow.sum('*',i) == -1 for i in V[1:]), "NodeNetflow")
65 m1.update()
66
67
68 m1.optimize()
69 #print(m1.getAttr('x'))
70 print('\nOptimal Obj value:', m1.objVal)
71 print('Average exposure time over all users:', m1.objVal/4039)
```

```
In [24]: runfile('/Users/fanxiuqi/Desktop/PA2/PA2.py', wdir='/Users/fanxiuqi/Desktop/PA2')
Finished reading the file
Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros
Coefficient statistics:
                   [1e+00, 1e+00]
 Matrix range
                   [1e+00, 1e+00]
[0e+00, 0e+00]
 Objective range
 Bounds range
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                   [1e+00, 4e+03]
Concurrent LP optimizer: dual simplex and barrier
Showing barrier log only...
Presolve removed 76 rows and 150 columns
Presolve time: 0.18s
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Barrier statistics:
AA' NZ
           : 8.815e+04
Factor NZ : 2.964e+05 (roughly 70 MBytes of memory)
Factor Ops: 4.274e+07 (less than 1 second per iteration)
Threads
            : 1
                                           Residual
                  Objective
Iter
           Primal
                           Dual
                                        Primal
                                                  Dual
                                                            Compl
                                                                      Time
      1.51743348e+08
                       7.50000000e+01 1.01e-09 0.00e+00
                                                          1.47e+03
  0
                                                                        0s
  1
      2.69241096e+07
                       8.36861626e+02 1.11e-09 6.66e-16
                                                          1.53e+02
                                                                        0s
      1.51501970e+06 3.94523896e+03
                                       1.41e-10 6.66e-16
                                                          8.57e+00
                                                                        0s
      1.36653239e+05 6.62992755e+03
                                       5.37e-09 8.88e-16
                                                          7.37e-01
                                                                        0s
      3.77472853e+04 7.63850101e+03 3.20e-09 9.99e-16
                                                          1.71e-01
                                                                        0s
       2.20931602e+04 9.45622299e+03
                                       4.32e-09 8.88e-16
                                                                        0s
                                                          7.17e-02
      2.07741179e+04 9.95121524e+03
                                       3.51e-09 1.11e-15
                                                          6.14e-02
                                                                        1s
      1.49229362e+04 1.06246030e+04
                                       2.43e-09 8.88e-16
                                                          2.44e-02
                                                                        1s
      1.32597050e+04 1.11334132e+04
                                       2.00e-09 8.88e-16
  8
                                                          1.21e-02
                                                                        1s
  q
       1.19052944e+04
                       1.13962000e+04
                                       9.34e-09 8.88e-16
                                                          2.89e-03
                                                                        1s
 10
      1.14323192e+04 1.14276074e+04
                                       1.22e-08 1.11e-15
                                                          2.67e-05
                                                                        1s
                                       2.63e-07 1.11e-15
       1.14280044e+04 1.14279996e+04
  11
                                                          2.74e-08
                                                                        1s
       1.14280000e+04 1.14280000e+04
                                       4.12e-10 8.88e-16 2.77e-14
Barrier solved model in 12 iterations and 0.75 seconds
Optimal objective 1.14280000e+04
Crossover log...
       0 DPushes remaining with DInf 0.0000000e+00
                                                                   1s
   7932 PPushes remaining with PInf 0.0000000e+00
                                                                   1s
       0 PPushes remaining with PInf 0.0000000e+00
                                                                   15
  Push phase complete: Pinf 0.0000000e+00, Dinf 0.0000000e+00
                                                                   1s
Iteration
            Objective
                             Primal Inf.
                                            Dual Inf.
                                                           Time
            1.1428000e+04
                                           0.000000e+00
   8243
                            0.000000e+00
                                                             1s
Solved with barrier
Solved in 8243 iterations and 0.84 seconds
Optimal objective 1.142800000e+04
Optimal Obj value: 11428.0
Average exposure time over all users: 2.82941322109433
In [25]:
```