MSOR

Yi-Ping Tseng

yt2690   yiping.t@columbia.edu

No.

Date

1. $z - (2x_1) - (-x_2) - (1 \cdot x_3)$ basis $= \{4, 5, 6\}$.

$$\begin{aligned}
2x_1 + x_2 + x_3 + x_4 &= 60 \\
x_1 - x_2 + 2x_3 + x_5 &= 10 \\
x_1 + x_2 - x_3 + x_6 &= 20.
\end{aligned}$$

Since $\bar{c}_1 = 2 \Rightarrow$ should entry the basis

$\bar{a}_{4,1} = 1$, $\bar{b}_4 = 60 \Rightarrow \frac{60}{1}$

$\bar{a}_{5,1} = 1$, $\bar{b}_5 = 10 \Rightarrow \frac{10}{1}$ ✓   change with x_5

$\bar{a}_{6,1} = 1$, $\bar{b}_6 = 20 \Rightarrow \frac{20}{1}$

$x_1 = 10 + x_2 - 2x_3 - x_5$

new obj: $z - 2(10 + x_2 - 2x_3 - x_5) - (-x_2) - (x_3)$

$\Rightarrow z - 20 - 2x_2 + 4x_3 + 2x_5 + x_2 - x_3$

$\Rightarrow z - x_2 + 3x_3 + 2x_5 = 20.$ basis $= \{1, 4, 6\}$.

$$\begin{aligned}
x_4 + 4x_2 - 5x_3 - 3x_5 &= 30 \\
x_1 - x_2 + 2x_3 + x_5 &= 10. \\
x_6 + 2x_2 - 3x_3 - x_5 &= 10
\end{aligned}$$

Since $\bar{c}_2 = 1 \Rightarrow$ should entry the basis

$\bar{a}_{4,2} = 1$ $\bar{b}_4 = 30$

$\bar{a}_{1,2} = 1$ $\bar{b}_1 = 10$

$\bar{a}_{6,2} = 1$ $\bar{b}_6 = 10$ ✓ → change with x_2

$x_2 = 5 - \frac{1}{2}x_6 + \frac{3}{2}x_3 + \frac{1}{2}x_5$

new obj: $z + \frac{3}{2}x_3 + \frac{3}{2}x_5 + \frac{1}{2}x_6 = 25$ basis $= \{1, 2, 4\}$.

$$\begin{aligned}
x_4 + 6x_3 + 2x_5 - 2x_6 &= 10 \\
x_1 + \tfrac{1}{2}x_3 + \tfrac{1}{2}x_5 + \tfrac{1}{2}x_6 &= 15 \\
2x_2 - 3x_3 - x_5 + x_6 &= 10
\end{aligned}$$

$opt^* = (15, 5, 0, 10, 0, 0)$ #.

No. 2

Date

adding slack & excess variables

$$\max \ 2x_1 + 3x_2$$

$$\begin{aligned}
\tfrac{1}{2}x_1 + \tfrac{1}{4}x_2 + s_1 &= 4 \\
x_1 + 3x_2 - e_2 &= 20 \\
x_1 + x_2 &= 4
\end{aligned}$$

adding artificial variables

$$\min \ a_2 + a_3 \Rightarrow \max \ -a_2 - a_3$$

$$\begin{aligned}
\tfrac{1}{2}x_1 + \tfrac{1}{4}x_2 + s_1 &= 4 \\
x_1 + 3x_2 - e_2 + a_2 &= 20 \\
x_1 + x_2 + a_3 &= 4
\end{aligned}$$

Chuyu culture

by adding slack & excess & artificial variables we can get this tableau.

$z \qquad\qquad a_2 + a_3 = 0.$

$\frac{1}{2}x_1 + \frac{1}{4}x_2 + s_1 = 4.$

$x_1 + 3x_2 - e_2 + a_2 = 20.$

$x_1 + x_2 + a_3 = 4$

basis $= \{s_1, a_2, a_3\}$. bfs $= (0, 0, 4, 0, 20, 4)$.

$a_2 = 20 - x_1 - 3x_2 + e_2 \qquad a_3 = 4 - x_1 - x_2$

$\Rightarrow z \quad -x_1 \quad -3x_2 \quad +e_2 \quad = -20.$

$+$

$-x_1 \quad -x_2 \quad -4.$

$z \quad -2x_1 \quad -4x_2 \quad +e_2 \quad = -24.$

$\frac{1}{2}x_1 + \frac{1}{4}x_2 + s_1 = 4.$

$x_1 + 3x_2 - e_2 + a_2 = 20.$

$x_1 + x_2 + a_3 = 4$

basis $= \{s_1, a_2, a_3\}$. bfs $= (0, 0, 4, 0, 20, 4)$.

x_2 enter a_3 leave

$x_2 = 4 - x_1 - a_3.$

$+2x_1$

$z \quad -2x_1 + 4x_1 \quad + e_2 \quad + 4a_3 = -8.$

$\frac{1}{2}x_1 - \frac{1}{4}x_1 \;\; \frac{1}{4}x_1 \quad + s_1 \quad - \frac{1}{4}a_3. = 3.$

$-2x_1$

$x_1 - 3x_1 \quad - e_2 + a_2 \quad - 3a_3 = 8$

$x_1 + x_2 + a_3 = 4$

$\Rightarrow$ this is optimal for Phase I LP., but we cannot let artifical variables set to 0 $\Rightarrow$ this mean the original LP is infeasible #

Chuyu culture

7

No.
Date

1. no basic variables appear in object function
so x_1 cannot be basic variable
→ this statement is wrong

2. each basic variable appears in exactly one constraint,
since constraint one only have x_5 (x_6 is non-basic),
so x_5 must be basic variable
→ this statement missed x_5
→ this statement is wrong

3. the possible basic variable combination are (x_2, x_3, x_5),
(x_2, x_4, x_5), and (x_3, x_4, x_5)
if the basis is $\{3,4,5\}$, and the tableau is as follow.

$$
\begin{array}{cccccccc}
z & -x_1 & +x_2 & & & & -x_6 & = 7 \\
\hline
 & & & & & x_5 & + x_6 & = 1 \\
 & x_1 & + x_2 & + x_3 & & & + 2x_6 & = 3 \\
 & x_1 & + x_2 & & + x_4 & & 0 & = 4
\end{array}
$$

in this condition x_6 can still be larger to get a larger z.
⇒ so this $\bar{x}$ solution is only feasible but not optimal.
⇒ this statement is right !!

4. as ③, this $\bar{x}$ is only feasible solution, not optimal ⇒ this statement is wrong.

Chnyu culture

5. by ⑦ , $\bar{x}$ is only a feasible solution not an optimal Solution , and we cannt make sure whether the tableau is has unique optimal solution or not ,
so this statement is also wrong

6. if any constraints are not satisfied , than this is an infeasible solution , say basis is {3, 4, 5}.

$z \quad -x_1 \quad +x_2 \qquad\qquad\qquad +x_6 = 7.$

$\qquad\qquad\qquad\qquad\qquad x_5 \quad +x_6 = -1$

$\quad * \quad * \quad x_3 \quad * \quad * \quad +2x_6 = 5$

$\quad x_1 \quad * \quad * \quad * \quad * \qquad - \;*$

when this condition happens , it means $\bar{x} = (0, 0, a, b, -1, 0)$
it violates the x_i's are nonnegative constraints
⇒ $\bar{x}$ is infeasible

we can conclude this statement might be right.

(4).

No.
Date : :

4. in this condition, we have to rewrite our LP. into

$$\max \quad c^T x - \sum_{i=1}^{m} d_i \lambda_i \; : \; Ax \le b + \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_i \end{pmatrix} \qquad \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_i \end{pmatrix} \le \begin{pmatrix} k_1 \\ \vdots \\ k_i \end{pmatrix}$$

$$x \ge 0, \; \lambda_i \ge 0.$$

5. if we add primary function as secondary function's constraint that we can maximize secondary function and reach the maximum value of primary function at the same time.

if we already know the maximum value of primary function is

$$c^T x = z$$

we can than apply z as our constraint value, say

$$\max \; (d^T x) \quad \text{where} \quad \begin{cases} Ax = b. \\ c^T x = z \end{cases}$$

→ so we can maximize primary and secondary function at the same time.

6. ① prove $\max c^T x : Ax = b, \; x \ge 0$ can transform into $\max c^T x : Ax \le b$

we know $Ax = b$ can be converted as $\begin{cases} Ax \le b \\ Ax \ge b. \Rightarrow -Ax \le -b. \end{cases}$

⇒ this prove that we can get $Ax \le b$ by transforming $Ax = b, \; x \ge 0$

② prove $\max c^T x : Ax \le b$ can transform into $\max c^T x : Ax = b, \; x \ge 0$.

as long as we add slack variables and adding constraints, we can

⇒ then get $\max c^T x : Ax = b, \; x \ge 0$.

Chrys culture

6. (continue.)

③ $AX = b$, $X \geq 0$ can not directly remove $X \geq 0$,

④ $AX \leq b$ if we add things into AX and force it become $AX = b$ without adding constraints, this might violate the equality.

⇒ by ①, ②, ③, ④ we know we can transform LP into
$\max c^T x : Ax = b, x \geq 0$ or $\max c^T x : Ax \leq b$

but not $Ax = b$ without any constraints.