

PA2 Mingjia Yu (my2638)

- ① To find the average exposure time over all users, we have to solve a individual shortest path problem.

4039

$\sum_{i=0}^{4039}$ From i to the remaining users $0 \rightarrow 4039$ w/o i

Objective Function = $\text{Min } \sum W_{ij} X_{ij}$

s.t outflow - inflow for each shortest path problem:

$$\sum X_{ij} - \sum X_{ji} = 1, \text{ if } i \text{ is the starting pt.}$$

$$\sum X_{ij} - \sum X_{ji} = -1, \text{ if } i \text{ is the destination}$$

$$\sum X_{ij} - \sum X_{ji} = 0, \text{ otherwise}$$

All $X_{ij} \geq 0$, nonnegative

The objective value is the total exposure time.

$$\therefore \text{the average exposure time} = \frac{\text{Total exposure time}}{\# \text{ of users}}$$

$$= \frac{\sum_{i=0}^{4039} \sum_{j \in E} W_{ij} X_{ij}}{4039}$$

Minimum Cost flow:

$$\text{min } \sum_{(i,j) \in E} C_{ij} X_{ij}$$

$$\sum_{(i,j) \in E} X_{ij} - \sum_{(j,i) \in E} X_{ji} = b_i$$

flow out of i flow into i net supply

$$0 \leq X_{ij} \leq U_{ij} \text{ for all } (i,j) \in E$$

$$\sum b_i = 0 \quad \downarrow \quad \boxed{\infty}$$

$$\min \sum_{(i,j) \in E} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \text{outflow} - \text{inflow} = \text{net supply}$$

$$\sum x_{ij} - \sum x_{ji} = -1 \quad \text{for each node}$$

$$0 \leq x_{ij} \leq \infty, \text{ non-negative.}$$

↑
no capacity constraint

$$c_{ij} = 1 \quad \forall i, j$$

$$b_0 = 4038 \quad (\text{Total \# of flow})$$

$$\therefore \text{Average exposure} = \frac{\sum_{(i,j) \in E} c_{ij} x_{ij}}{4039} = \frac{11428}{4039} = \boxed{2.83}$$

PA2

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0.1 PA2

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```
[ ]: import pandas as pd
      from gurobipy import *

n = 4039
data = pd.read_table('facebook_combined.txt', header=None)
data.columns = ['Edge']
data['weight'] = 1
data['Edge'] = data['Edge'].apply(lambda x: (int(x.split(' ')[0]), int(x.
    ↪split(' ')[1])))
data.head()

bidirect = data.copy(deep=True)
bidirect['Edge'] = bidirect['Edge'].apply(lambda t: (t[1], t[0]))
bidirect.head()

data = pd.concat([data, bidirect], axis=0)
graph = data.set_index('Edge').to_dict()['weight']
Vertex = [i for i in range(n)]

m = Model()
nw = m.addVars(data['Edge'], vtype=GRB.CONTINUOUS, obj=graph, name="nw", lb=0)
m.modelSense = GRB.MINIMIZE
m.addConstr((nw.sum(0, '*') - nw.sum('*', 0) == n - 1), "net supply")
m.addConstrs((nw.sum(i, '*') - nw.sum('*', i) == -1 for i in Vertex[1:]), "net_
    ↪supply")
m.update()
```

```
m.optimize()  
print('Obj value:', m.objVal)  
print('Average exposure time:', m.objVal / n)
```


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 Optimize a model with 4039 rows, 176468 columns and 352936 nonzeros
 Coefficient statistics:
 Matrix range [1e+00, 1e+00]
 Objective range [1e+00, 1e+00]
 Bounds range [0e+00, 0e+00]
 RHS range [1e+00, 4e+03]

Concurrent LP optimizer: dual simplex and barrier
 Showing barrier log only...

Presolve removed 76 rows and 150 columns
 Presolve time: 0.35s
 Presolved: 3963 rows, 176318 columns, 352618 nonzeros

Ordering time: 0.00s

Barrier statistics:
 AA' NZ : 8.815e+04
 Factor NZ : 2.964e+05 (roughly 70 MBytes of memory)
 Factor Ops : 4.274e+07 (less than 1 second per iteration)
 Threads : 1

Iter	Objective		Residual		Compl	Time
	Primal	Dual	Primal	Dual		
0	1.51743348e+08	7.50000000e+01	1.01e-09	0.00e+00	1.47e+03	1s
1	2.69241096e+07	8.36861626e+02	1.11e-09	6.66e-16	1.53e+02	1s
2	1.51501970e+06	3.94523896e+03	1.41e-10	6.66e-16	8.57e+00	1s
3	1.36653239e+05	6.62992755e+03	5.37e-09	8.88e-16	7.37e-01	1s
4	3.77472853e+04	7.63850101e+03	3.20e-09	9.99e-16	1.71e-01	1s
5	2.20931602e+04	9.45622299e+03	4.32e-09	8.88e-16	7.17e-02	1s
6	2.07741179e+04	9.95121524e+03	3.51e-09	1.11e-15	6.14e-02	1s
7	1.49229362e+04	1.06246030e+04	2.43e-09	8.88e-16	2.44e-02	1s
8	1.32597050e+04	1.11334132e+04	2.00e-09	8.88e-16	1.21e-02	1s
9	1.19052944e+04	1.13962000e+04	9.34e-09	8.88e-16	2.89e-03	1s
10	1.14323192e+04	1.14276074e+04	1.22e-08	1.11e-15	2.67e-05	2s
11	1.14280044e+04	1.14279996e+04	2.63e-07	1.11e-15	2.74e-08	2s
12	1.14280000e+04	1.14280000e+04	4.12e-10	8.88e-16	2.77e-14	2s

Barrier solved model in 12 iterations and 1.67 seconds
 Optimal objective 1.14280000e+04

Crossover log...

0 DPushes remaining with DInf	0.0000000e+00	2s		
7932 PPushes remaining with PInf	0.0000000e+00	2s		
0 PPushes remaining with PInf	0.0000000e+00	2s		
Push phase complete: Pinf	0.0000000e+00, Dinf 0.0000000e+00	2s		
Iteration	Objective	Primal Inf.	Dual Inf.	Time
8243	1.1428000e+04	0.0000000e+00	0.0000000e+00	2s

Solved with barrier
 Solved in 8243 iterations and 1.88 seconds
 Optimal objective 1.14280000e+04
 Obj value: 11428.0
 Average exposure time: 2.82941322109433