Volodya,

here is how I compute a linear finctional for windowed local adaptive inversion.

Best,

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Suppose we need to restore a vector x given noisy indirect observations

$$y = Ax + \xi \tag{1}$$

where A – a linear operator; ξ – zero-mean white Gaussian noise with unit standard deviation σ.

Consider estimation of a component (pixel) x_t of the vector x. Let D be a diagonal operator which defines a window around the pixel t, and $\bar{D}$ – complimentary window (for example, $\bar{D} = I - D$)

We are looking for a linear estimate

$$\begin{aligned} \hat{x}_t &= \langle w, y \rangle = \langle w, A(Dx + \bar{D}x) + \xi \rangle & (2)\\ &= \langle DA'w, x \rangle + \langle \bar{D}A'w, x \rangle + \langle w, \xi \rangle & (3) \end{aligned}$$

In this formula, the first two terms determine bias, and the last one – stochastic error. In order to build an adaptive procedure with nested windows D, we want to minimize the stochastic error $\langle w, \xi \rangle$ given the bias related to the term $\langle \bar{D}A'w, x \rangle$. Skipping rigorous motivation, I just write the optimization problem we solve in L_2 case:

$$\begin{aligned} \min_w \quad & ||\bar{D}A'w||_2^2 + \lambda ||w||_2^2 & (4)\\ \text{subject to} \quad & \mathbf{1}'DA'w = 1 & (5) \end{aligned}$$

Here $\mathbf{1}$ is a vector of ones, and λ controls tradeoff between bias and stochastic error. Using a quadratic penalty for the constraint, we can solve instead the following unconstrained problem

$$\min_w \{ f(w) = ||\bar{D}A'w||_2^2 + \lambda ||w||_2^2 + \mu(\mathbf{1}'DA'w - 1)^2 \} \tag{6}$$

with large enough penalty parameter μ. Denote

$$b' \equiv \mathbf{1}'DA'.$$

The optimality condition for (6) is

$$\nabla f = A\bar{D}^2A'w + \lambda w + \mu b(b'w - 1) = 0, \tag{7}$$

or

$$(A\bar{D}^2A' + \lambda I + \mu bb')w = \mu b \tag{8}$$

We will solve this equation with respect to w using, for example Conjugate Gradient method.