EECS 592
FOUNDATIONS OF ARTIFICIAL INTELLIGENCE

Issued: Sun., Mar. 25, 2018
Due: Fri., Apr. 6, 2018 (11:59pm to Canvas)

PROJECT #4: UNCERTAIN REASONING

Please take care to complete all parts of this project independently. Do not use any solutions found online or written by other students. If you are struggling please go to office hours, post to Piazza, and/or send us email. Please submit a single PDF file `project4.pdf` to Canvas for all handwritten problems (1-7). The PDF file can contain a scan of handwritten or typewritten/typeset solutions, and any supporting graphics. Please also submit a single code file `project4.tar.gz` to Canvas containing your Python or C++ code solution to Problem #8.

1. **R&N Problem 13.7:** (10 points)
   Consider the set of all possible five-card poker hands dealt fairly from a standard deck of fifty-two cards.
   a. How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)?
   b. What is the probability of each atomic event?
   c. What is the probability of being dealt a royal straight flush? Four of a kind?

2. **R&N Problem 13.18:** (10 points)
   Suppose you are given a bag containing n unbiased coins. You are told that $n - 1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.
   a. Suppose you reach into the bag, pick out a coin at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?
   b. Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?
   c. Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns fake if all k flips come up heads; otherwise it returns normal. What is the (unconditional) probability that this procedure makes an error?

3. **R&N Problem 14.1:** (10 points)
   We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.
   a. Draw the Bayesian network corresponding to this setup and define the necessary CPTs (conditional probability tables).
   b. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

4. **Car Diagnosis Bayes Net (Modified from R&N Problem 14.8):** (10 points)
   Consider the network for car diagnosis shown in Figure 14.21 (reproduced below).
   a. Extend the network with Boolean variables *IcyWeather* and *StarterMotor*. Sketch the updated Bayesian network with eight nodes.
   b. How many independent values are contained in the full joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?
   c. How many independent probability values do your Bayesian network tables contain?
   d. What nodes (if any) are conditionally independent of *Starts* given *Moves* and *Battery*?

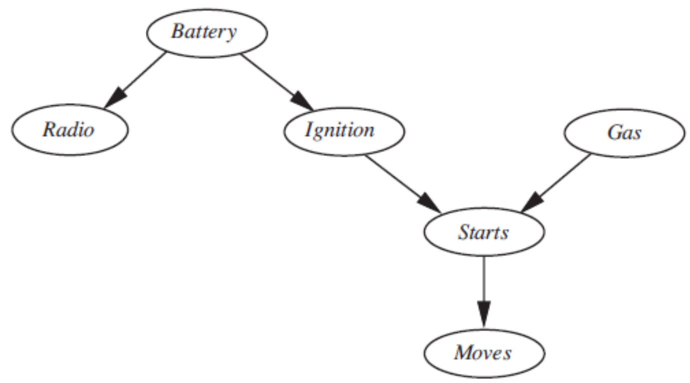
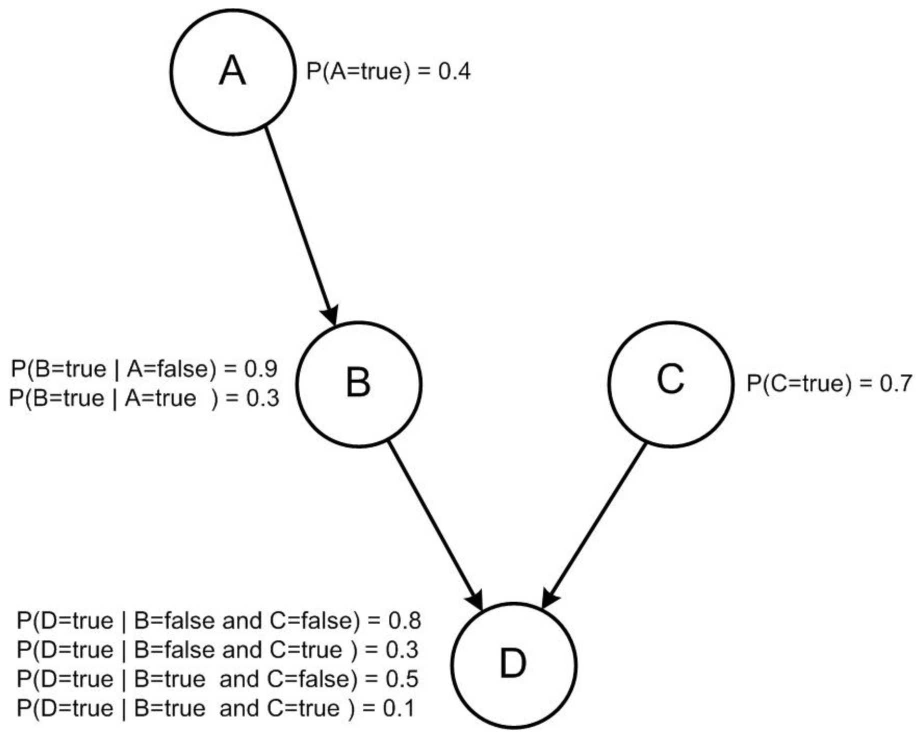


Figure 14.21 A Bayesian network describing some features of a car's electrical system and engine. Each variable is Boolean, and the *true* value indicates that the corresponding aspect of the vehicle is in working order.

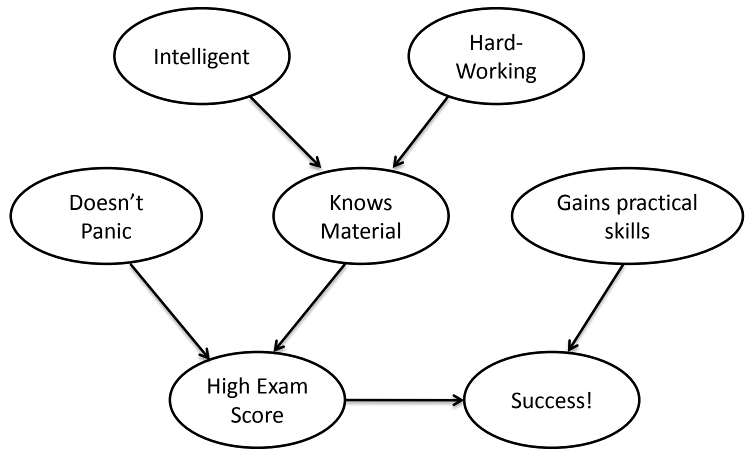
5. **Simple Bayes Computations:** Consider the Bayesian network drawn below. (10 points)



Show your work for the following calculations.

a. Compute *P(A = true and B = false and C = true and D = false).*
b. Compute *P(D = true | A = false and B = true and C = false).*
c. Compute *P(A = true | B = false and C = true and D = false*).
d. Compute *P(B = false | A = true and C = false*).
e. Compute *P(B = false).*

6. **Exam Bayes Net (15 points):** Consider a problem in which a professor wants to determine whether a student has understood material based on an exam score. The below figure illustrates a possible Bayes net that can help answer this question, along with symbol definitions. Associated probabilities are listed below as well.



Intelligent = I, Hard working = H, Doesn't panic = DP, High Exam Score = Ex, Knows material = KM, Gains practical skill = PS, Success! =S

Probability data:
P(I) = 0.7; P(H) = 0.6; P(DP)=0.5; P(PS)=0.7
P(KM|I,H)=1.0; P(KM|I,~H)=0.4; P(KM|~I,H)=0.6; P(KM|~I,~H)=0.05
P(S|PS,Ex)=0.8, P(S|~PS,Ex)=0.7, P(S|PS,~Ex)=0.7, P(S|~PS,~Ex)=0.3
P(Ex|DP,KM)=0.85; P(Ex|~DP,KM)=0.7; P(Ex|DP,~KM)=0.2; P(Ex|~DP,~KM)=0.1

a. Given a high exam score (Ex), which variables are conditionally independent of intelligent (I)?
b. Given knows material (KM) as evidence, which node(s) are conditionally independent of success (S)?
c. Given success (S), which node(s) are conditionally independent of high exam score (Ex)?
d. Given no evidence, compute P(KM).
e. Compute P(S | KM), the probability of Success given Knows Material.
f. Compute P(PS | S), the probability of gaining Practical Skill given exam Success.
g. Compute P(KM | S), the probability of Knows Material given exam Success.

7. **Bayes Net Exact Inference -- Coding** (35 points)

Implement the `ENUMERATION-ASK(X, E, BN)` function shown in R&N Figure 14.9, reproduced on the next page, within a program or script that manages a single test case and prints the result to the screen. To simplify code for this problem, assume all nodes are labeled by single characters as in Problem 5 above, and assume all variables have Boolean values T (True), F (False). The Bayes Net (BN) will be defined by file `bn.txt`. The specific test case query will be defined in file `input.txt`. Output `Q(X)` for X=T should be printed to the screen. Examples of each formatted input file are provided below for the Problem 5 Bayes Net. The grader will use one easy Bayes Net and one larger Bayes Net. We will only test polytree cases that run successfully given a correct implementation.

function ENUMERATION-ASK(X, $\mathbf{e}$, bn) **returns** a distribution over X
  inputs: X, the query variable
    $\mathbf{e}$, observed values for variables $\mathbf{E}$
    bn, a Bayes net with variables $\{X\} \cup \mathbf{E} \cup \mathbf{Y}$ /* $\mathbf{Y}$ = *hidden variables* */

  $\mathbf{Q}(X) \leftarrow$ a distribution over X, initially empty
  for each value x_i of X **do**
    $\mathbf{Q}(x_i) \leftarrow$ ENUMERATE-ALL(bn.VARS, $\mathbf{e}_{x_i}$)
      where $\mathbf{e}_{x_i}$ is $\mathbf{e}$ extended with $X = x_i$
  return NORMALIZE($\mathbf{Q}(X)$)

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function ENUMERATE-ALL($vars$, $\mathbf{e}$) **returns** a real number
  if EMPTY?($vars$) **then return** 1.0
  $Y \leftarrow$ FIRST($vars$)
  if Y has value y in $\mathbf{e}$
    then return $P(y \mid parents(Y)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}$)
    else return $\sum_y P(y \mid parents(Y)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}_y$)
      where $\mathbf{e}_y$ is $\mathbf{e}$ extended with $Y = y$

Figure 14.9 The enumeration algorithm for answering queries on Bayesian networks.

Example `bn.txt` file:

```
% Random Variables
A, B, C, D
% Graph Edges (From, To)
A, B
B, D
C, D
% Probability values
P(A=T)=0.4
P(C=T)=0.7
P(B=T|A=F)=0.9
P(B=T|A=T)=0.3
P(D=T|B=F,C=F)=0.8
P(D=T|B=F,C=T)=0.3
P(D=T|B=T,C=F)=0.5
P(D=T|B=T,C=T)=0.1
```

An example `input.txt` file for *P(D|A=T,C=F)*:

```
% Query random variable (print the probability X=T)
D
% Evidence vector (insert blank line when no evidence is given)
A=T, C=F
% End of input
```