Volatile Capital Flows and Financial Integration: The Role of Moral Hazard[1]

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Abstract. We study a model in which income and capital flows between countries are jointly determined in a world economy with integrated financial markets. In a setting that combines risky entrepreneurial activity with moral hazard, we find that a shift from autarky to financial integration leads to boom-bust cycles in capital flows, output and consumption. Moral hazard causes cycles because financial intermediaries incentivize effort by insisting entrepreneurs take an equity share in their own projects. The size of this stake rises with wealth, discouraging entrepreneurship and inhibiting capital formation. The reverse is true when wealth falls, generating cycles.

JEL Classifications: E2, E3, E4, F4

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1. Introduction

Since the 1990s the world economy has witnessed a striking increase in cross-border financial transactions associated with debt and equity markets.[2] At the same time, inflows and outflows have become increasingly volatile—an "unprecedented roller-coaster ride" according to the IMF (2011). One case in point is Brazil, where net inflows to the equity and debt securities market grew rapidly prior to the financial crisis of 2007–2008 and then plummeted as the crisis deepened. Inflows rebounded as economic activity began to pick up in 2009, only to fall again sharply in 2011. Figure 1 shows these fluctuations in terms of total net capital flows between 1990 and 2015.[3] Many other countries have also experienced large inflows of capital followed by large outflows in repeated "boom-bust" cycles.[4]

Inflows of capital typically coincide with expansion in domestic output, investment and consumption, while outflows are associated with contraction. The negative impacts of the contraction phase of these fluctuations have led policy leaders and some economists to call for policy changes that restrict or inhibit financial integration. From 1995 to 2010 at least 37 countries had capital controls in place. The International Monetary Fund, which for decades forcefully advocated free capital flow, now recommends capital controls in some cases in order to prevent financial crises (IMF (2012)).

Effective policy towards capital flows requires a clear understanding of the source of fluctuations. While popular media often points to poor fiscal discipline in the home country, Calvo et al. (2004), in a study of 32 developed and developing countries, found that fiscal deficits were frequently second order. They instead emphasized fluctuations in borrowing costs and the supply of credit from international sources. In a similar vein, Frankel and Rose (1996) studied emerging market volatility and boom-bust cycles for over 100 developing countries and found that both push (i.e., global) factors and pull

[2]For a historical overview see, for example, Eichengreen (2008).

[3]Data is quarterly and from International Financial Statistics published by the International Monetary Fund. Net inflows are the difference between gross inflows and gross outflows. Our calculations of net inflows use the methodology discussed in section 2.2 of Forbes and Warnock (2012). (Net outflows are recorded as negative values.)

[4]Case studies include Brixiova et al. (2010) and Lane (2013). More general discussion can be found in Broner et al. (2013), Ghilardi and Peiris (2014), Borio (2014), Evans and Hnatkovska (2014) and Müller-Plantenberg (2015).

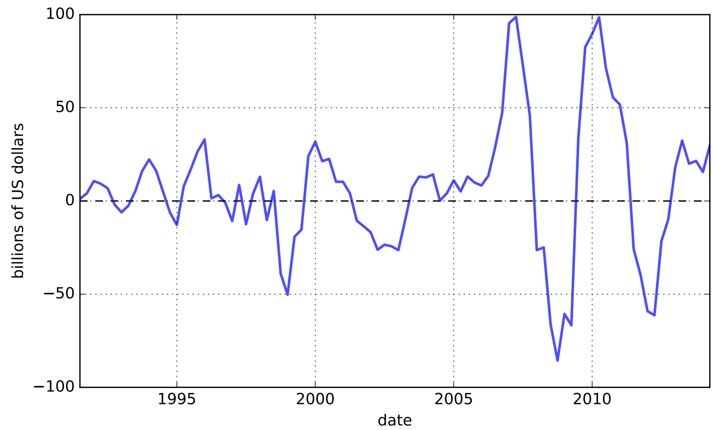

Figure 1. Net capital inflows for Brazil 1990–2014

(i.e., country-specific) factors were important. Fratzscher (2012) likewise emphasized push and pull factors related to international capital flows surrounding the global financial crisis in 2008. Large capital inflows were often accompanied by low borrowing costs and high domestic demand. Outflows coincided with higher interest rates and low demand.

Despite the large and growing literature on the dynamics of international credit markets, capital flows and their relationship to output and other real quantities, a number of modeling challenges remain. One is the joint determination of push and pull factors described above. While researchers including Arellano (2008), Aghion et al. (2001), Aghion et al. (2004), Caballé et al. (2006), Martin and Taddei (2013) and Kikuchi and Vachadze (2015) all build models that highlight certain aspects of fluctuations in cross-border capital flows, these models cannot fully address the push–pull nature of boom-bust cycles discussed above because they take the world interest rate as given.

A second modeling challenge is addressing the extent to which openness itself drives cyclical fluctuations in output and other quantities within individual countries. Empirical studies such as Calvo et al. (2004) find that greater openness increases vulnerability to crises in developing countries. The policy stance of the IMF vis-à-vis capital controls suggests a belief that openness either instigates or worsens crises. A key question then is whether integrated financial markets are themselves the causes of observed expansions and contractions in domestic wealth, investment and income, whether they simply propagate shocks, or whether they in fact mitigate crises.

In this paper we combine push and pull factors by constructing a multi-country model where the interest rate is determined endogenously through the interaction of supply and demand in an international credit market, and where cyclical capital flows are observed in equilibrium. We also directly address the issue of whether integration of financial markets causes boom-bust cycles. Under certain parameterizations we find that it does: Without financial integration the world economy converges to a unique, symmetric and stable steady state. With integration this steady state loses stability and cycles emerge.

The starting point of our analysis is a model of symmetry breaking and endogenous inequality across nations due to Matsuyama (2004). In a setting of overlapping generations, young agents in each country choose between entrepreneurial activity and investment in a safe asset. Entrepreneurs run indivisible projects that require a fixed investment and generate productive capital. They fund this investment from a combination of their own wealth and credit. Unlike Matsuyama (2004), the projects that entrepreneurs run are risky. Exposure to risk tends to weaken demand for funds, putting downward pressure on investment in domestic capital stock.

Risk exposure cannot be fully eliminated in our model due to a form of moral hazard. The source of this moral hazard is asymmetric information associated with entrepreneurial effort, which affects success probabilities while at the same time remaining unobservable. In particular, banks cannot write contracts that condition on the amount of effort that entrepreneurs exert. As a consequence, banks restrict lending in equilibrium in order to ensure that entrepreneurs invest at least some of their own wealth in their project. In this way, banks ensure that entrepreneurs bear some of the risk associated with their activities, and are therefore motivated to exert effort.

When countries exist in financial autarky, this financial friction has no impact on borrowing or entrepreneurship, since the domestic deposit rate adjusts at each point in time to equalize the aggregate demand for investment funds with aggregate supply. The world economy converges to a unique, symmetric, and stable steady state. The value of the steady state is determined only by productivity parameters.

Financial integration causes this symmetric steady state to lose stability. In other words, *symmetry breaking* occurs, as it does in Matsuyama (2004). Unlike Matsuyama (2004), however, the collapse of stability associated with integration causes cycles to emerge. The mechanism runs as follows: Countries with a large amount of capital have relatively wealthy domestic entrepreneurs. Due to the financial friction described above, wealthy entrepreneurs have a significant equity stake in their projects. This risk exposure leads in turn to relatively weak demand for funds, decreased investment in domestic capital stock, and capital outflow. Lower domestic investment leads to lower future wealth, which reduces the equity stake of entrepreneurs, and hence their risk exposure. This increases demand for funds, boosting investment and capital inflows. As a consequence, domestic capital stock rises and the cycle starts again.

The mechanism described above can lead to cycles in both small open economy and multi-country settings. In the latter case, push and pull factors both influence outcomes. For example, the world interest rate and hence the borrowing costs for entrepreneurs are always low relative to the no-moral-hazard case. In low wealth countries this fuels the boom, as looser credit encourages entrepreneurs to increase borrowing above the no-moral-hazard level. On the other hand, in a high wealth country, entrepreneurs bear significant risk and demand a high risk premium as compensation. Decreasing marginal product of capital implies that the fraction of entrepreneurs must decline in order to generate this premium. Credit flows to the low wealth country and this capital flow lays the seeds for the next cycle.

The fact that entrepreneurs retain some positive equity stake in their own projects, which is a key component of the cycles described above, is consistent with household level financial data,[5] while the underlying problem of asymmetric information between entrepreneurs and banks has been highlighted in many studies. For example, Leland and Pyle (1977) discuss how moral hazard prevents information transfer that could alleviate the obvious information asymmetries between potential entrepreneurs and creditors regarding the quality of projects entrepreneurs wish to run. At the same

[5] Vissing-Jørgensen and Moskowitz (2002) find that entrepreneurs typically invest a large share of their wealth in a single private company in which they have an active management interest, and that this lack of diversification persists despite the fact that private equity returns are on average no higher than the market return on publicly traded equity.

time, they show that information on project quality may be transferred if those with inside information are willing to invest in the project or firm.[6]

It is worthwhile to compare the implications of the moral hazard friction studied in this paper to that of the more traditional collateral-based restrictions found in much of the macroeconomic literature.[7] In both cases, entrepreneurs are borrowing constrained, but the nature of the constraint differs in one key respect. When banks demand collateral, the borrowing constraint tends to relax as wealth increases. In our model the converse is true. Financial intermediaries require an equity stake from entrepreneurs in order to induce effort on their part, and the required equity stake typically *rises* with wealth, since the intermediaries want the incentive to exert effort to continue to be non-trivial. Higher equity stakes are enforced by tightening the borrowing constraint.

The macroeconomic implications of these two kinds of borrowing constraints differ significantly. For example, in Matsuyama (2004) entrepreneurial investment involves a collateral-based borrowing constraint, which, combined with financial integration, leads to polarization of the world economy into rich and poor countries. The polarization is stable because initial differences in wealth are reinforced by the changing borrowing constraint, which tightens in poorer countries and loosens in richer countries. In contrast, in our model the relative tightness of the borrowing constraint moves in the opposite direction, generating cycles instead of polarization.[8]

[6]Darrough and Stoughton (1986) extend Leland and Pyle (1977), adding moral hazard. In their analysis, the fraction of equity retained by the entrepreneur is both a signal and an incentive device. Ghatak et al. (2001) consider a model where entrepreneurs self-finance due to credit market imperfections related to transaction costs. See also Holmstrom and Tirole (1997).

[7]See, for example, Kiyotaki and Moore (1997) or Bernanke and Gertler (1989).

[8]Standard collateral-based credit constraints can also generate cycles, as shown in Kikuchi and Stachurski (2009) and Agliari et al. (2015). In Kikuchi and Stachurski (2009), however, cycles are only exhibited in settings where countries are heterogeneous in terms of population size, and even then persistent differences reinforced through the collateral constraint are also observed, similar to those found in Matsuyama (2004). While Agliari et al. (2015) demonstrate the possibility of cycles even when countries are homogeneous, persistent differences are another possible outcome. Moreover, even when cycles emerge the symmetric steady state remains stable, due to the sub-critical nature of the bifurcation. It is arguably the case that cycles are a more inherent feature of the model presented in this paper, as a result of the countercyclical nature of the credit constraint. Here the only possible

While the links between the type of borrowing constraint we consider in this paper and dynamics of wealth, output and cross-border credit flows have not previously been studied in depth, the idea that asymmetric information exists between banks and entrepreneurs is prevalent, as discussed above. The notion that principals use financial incentives to induce unobservable effort on the part of agents is also entirely standard.[9] Overall, the idea that entrepreneurs face borrowing constraints arising from the desire of financial intermediaries to incentivize effort by requiring an equity stake is essentially plausible, compatible with observed patterns of entrepreneurial ownership, and consistent with many theoretical studies.

Regarding implications of the model, one of the objections to the endogenous cycle literature is that the cycles generated are often regular and periodic, as opposed to the irregular fluctuations that we tend to observe in prices and aggregate quantities. In section 5 we discuss an extension involving aggregate productivity shocks that brings the model's outputs closer to the data. We show how a damped cycle mechanism combined with productivity shocks can generate irregular boom-bust cycles and bursts of volatility.

Returning to the existing literature on financial instability, there are several useful multi-country models that treat global supply and demand for credit while discussing large capital flows or other closely related topics. Examples include Gertler and Rogoff (1990), Boyd and Smith (1997), Angeletos and Panousi (2011) and Bacchetta and Benhima (2015). These papers do not explicitly address boom and bust cycles, however, focusing instead on topics such as global credit imbalances.[10]

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equilibria for the financial integrated world economic are a stable symmetric steady state and a 2-cycle. Finally, in this paper the mechanism is substantially different, and the plausible nature of the alternative credit constraint merits investigation of its implications.

[9]See, for example, the efficiency wage theory of Shapiro and Stiglitz (1984) or, for an analysis related to entrepreneurs and asymmetric information, Tirole (1988), pp. 30–34.

[10]Financial instability has also been studied in the closed economy context, including the work of Azariadis and Smith (1998), Aghion et al. (1999), Reichlin and Siconolfi (2004), Myerson (2012), Favara (2012), Myerson (2014), Matsuyama et al. (2016) and Figueroa and Leukhina (2018). The mechanisms described in these papers are clearly significant in the present context because they demonstrate how entrepreneurs might prefer to invest less when rich than when poor, and their mechanisms generate cycles even without financial integration.

One multi-country model that does focus on fluctuations directly is found in Brunnermeier and Sannikov (2015). In a stochastic growth framework with incomplete markets, undercapitalized countries borrow excessively because firms fail to internalize the fact that increases in production capacity undermine their output price and worsen their terms of trade. In this setting, adverse technology shocks can cause abrupt stops. In a similar vein, Tille and Van Wincoop (2010) develop a two-country dynamic stochastic general equilibrium model to study the implications of portfolio choice for both gross and net international capital flows. Capital flows are driven by portfolio reallocation associated with time-varying expected returns and second moments.

While we also consider productivity shocks in this paper, that exercise is an extension aimed at investigating the interaction between the main mechanism and aggregate uncertainty. The core model has no stochastic component, and cycles arise with financial integration across many parameter values from almost all initial conditions. The cycles are driven by both push and pull factors, as low credit demand in one country fuels borrowing and investment in the other. Initially small fluctuations can grow in amplitude as these push and pull factors reinforce one another.

One paper that treats cycles in a multi-country environment via a fully endogenous mechanism is Matsuyama et al. (2015). In the model, cycles are driven by strategic complementarities in the timing of innovation. A major difference with our model in terms of equilibrium outcomes is that, in Matsuyama et al. (2015), cycles exist even in autarky. In our model, as mentioned above, the world economy under autarky is stable. Globalization of financial markets is itself a driver of fluctuations in income and wealth.

The remainder of the paper is structured as follows. Section 2 introduces the model without moral hazard, while section 3 inserts moral hazard. Section 4 studies dynamics. Section 5 introduces extensions, while section 6 concludes. Remaining proofs can be found in appendix A. Code for all simulations is posted at `https://github.com/jstac/cycles_moral_hazard`.

2. A Benchmark Model without Moral Hazard

In this section we introduce a simple version of our model without moral hazard. We will see that the banking sector fulfills its natural function of pooling assets with stochastic payoffs to mitigate individual credit risk. Later we will see how risk sharing is impeded by the introduction of moral hazard.

2.1. **Environment.** Consider an economy populated at any one time by two overlapping generations, each of which has unit mass. All agents are identical. Capital k and labor ℓ are used to produce a single consumption good via the production function $F(k, \ell)$. Here "capital" is best understood as all productive inputs supplied by the older generation, including both physical capital and technical and managerial expertise. Productive capital is immobile (in the sense of being non-tradable across countries) and depreciates fully in each period.

The young inelastically supply a single unit of labor. The resulting unit mass of labor from the young generation is combined at time t with the existing stock of capital k_t to produce current output $f(k_t) := F(k_t, 1)$. The function f is taken to be continuously differentiable, with $f''(k) < 0 < f'(k)$ for all $k > 0$, with $f(0) = 0$, $f'(0) = \infty$ and $f(k) < k$ for all sufficiently large k. We assume that factor markets are competitive, so that the wage of young agents is $w_t = \omega(k_t) := f(k_t) - k_t f'(k_t)$. Owners of productive capital receive the gross rental rate $f'(k_t)$.

Productive capital is generated by running projects. Each project takes one unit of the consumption good as input at time t and either succeeds, generating a positive quantity z of productive capital at time $t+1$, or fails, producing nothing. Outcomes are independent across time and agents. The success probability of any project is either q_0 or q_1, depending on entrepreneurial effort $e \in \{0, 1\}$. In particular, effort level e induces success probability q_e, and $0 < q_0 < q_1 < 1$. Since e is not observable, no contracts can be written that condition on e.[11]

[11]The restriction to only two possible outcomes, one producing no capital and the other with positive output, significantly simplifies our analysis. The assumption of zero revenue in the worst state is not implausible, however, and one can contemplate extensions where this zero revenue in the worst state is combined with a range of positive outcomes in better states. For example, suppose we reinterpret q_e as the probability of entering a "lottery" over a set of positive output states with some fixed distribution G. If $G = \delta_z$, then we recover the benchmark setting described above. If G has a sufficiently small support centered on z, then the model of production is nearly isomorphic to the current one and the key ideas are preserved (although the analysis is more complex). If G has a larger support, then the implications for the model depend on parameters.

Agents work only when young and consume only when old. The lifetime utility of an agent born at t is given by $\ln c_{t+1} - v(e_t)$, where c_{t+1} is old age consumption and e_t is effort. The second term represents disutility of effort and we assume that $0 = v(0) \leq v(1)$. In this section, we set $v(1) = v(0) = 0$, so the incentive to avoid effort is removed. Hence all projects succeed with probability q_1. In the next section we will set $v(1) > 0$, introducing moral hazard.[12]

Evidently young agents wish to transfer all their labor income to the second period of their lives. We assume that the consumption good is non-storable, leaving two options for transferring wealth: First, they can become *passive investors*, lending all w_t units of their wealth at time t. Second, they can become *entrepreneurs*, running a single project of the type described above. Below ϕ_t denotes the fraction of agents who choose to become entrepreneurs at the end of time t.

The credit market consists of financial intermediaries referred to below as banks. Passive investors deposit their entire wealth w_t with these banks and receive $r_{t+1}w_t$ units of the consumption good at time $t+1$. Agents who choose to become entrepreneurs borrow an amount b_t from the banks, so that their time t assets are b_t plus their wage income w_t. The residual $b_t + w_t - 1$ after paying the unit cost of a project can invested at the deposit rate r_{t+1}.

Entrepreneurs enjoy limited liability, so that second period consumption when the project fails is their return on residual assets, written as

$$c^{\ell}_{t+1} = (b_t + w_t - 1)r_{t+1}. \tag{1}$$

When their project succeeds, their consumption is

$$c^{h}_{t+1} = (b_t + w_t - 1)r_{t+1} + zf'(k_{t+1}) - b_t r^{e}_{t+1}. \tag{2}$$

Here r^{e}_{t+1} is the borrowing rate charged to entrepreneurs.

Shocks faced by entrepreneurs are idiosyncratic, so that any positive mass ϕ of projects produces $R\phi$ units of physical capital with probability one, where $R := q_1 z$.

[12] To eliminate moral hazard one could retain positive disutility of effort while instead setting $q_0 = q_1$. Both scenarios yield the same outcomes. The other alternative for shutting down moral hazard is to make e observable and allow banks to condition on it in their contracts. Doing so complicates the exposition, however, without bringing us closer to observed banking behavior.

2.2. **Equilibrium.** The banking sector is competitive. Each active bank is assumed to make loans to a positive mass of entrepreneurs. Of these entrepreneurs, a fraction $1-q_1$ default. Hence banks who lend to entrepreneurs at the deposit rate will become insolvent with probability one. To pin down the rate r^e_{t+1} at which entrepreneurs can borrow, we assume free entry and hence zero profits. This implies that the expected return on loans equals their cost, or $q_1 r^e_{t+1} = r_{t+1}$.

Faced with this borrowing rate, entrepreneurs choose b_t to maximize

$$U(b_t) := q_1 \ln c^h_{t+1} + (1-q_1)\ln c^\ell_{t+1} - v(1), \tag{3}$$

where c^ℓ_{t+1} and c^h_{t+1} are as given in (1) and (2), and understood to be functions of b_t. The derivative can be written as

$$U'(b_t) = (1-q_1) r_{t+1} \left(\frac{1}{c^\ell_{t+1}} - \frac{1}{c^h_{t+1}} \right). \tag{4}$$

Thus, expected utility strictly increases with borrowing whenever $c^h_{t+1} > c^\ell_{t+1}$ and strictly decreases when $c^h_{t+1} < c^\ell_{t+1}$. The optimal choice is to set b_t such that $c^h_{t+1} = c^\ell_{t+1}$. This in turn gives the maximizing value

$$b^*_t = \frac{Rf'(k_{t+1})}{r_{t+1}}. \tag{5}$$

Agents are willing to start projects whenever

$$Rf'(k_{t+1}) \geq r_{t+1}. \tag{6}$$

Apart from boundary cases, (6) holds with equality, equalizing the rate of return for entrepreneurs and passive investors. Equality in turn gives $b^*_t = 1$. Thus, in equilibrium, entrepreneurs borrow sufficient funds to finance the entire project, eliminating all risk.[13] All agents have fixed second period consumption

$$c_{t+1} = w_t r_{t+1} = w_t R f'(k_{t+1}). \tag{7}$$

Banks serve their natural portfolio diversification role, securitizing the obligations of the entrepreneurs, pooling the returns from their projects and selling the consolidated debt at a rate that equals expected returns from investment.

[13] In (6) we are ignoring the boundary cases $\phi_t = 0$ and $\phi_t = 1$, where the equality becomes an inequality. We return to these corner solutions below.

3. Moral Hazard

Next we investigate the setting where $v(1) > 0$, so effort generates positive disutility. In doing so we introduce moral hazard into financial markets. As we will see, this causes banks to restrict lending to entrepreneurs, forcing them to take an equity stake in their projects in order to induce effort.

3.1. **Equilibrium Lending.** As before we write c^{ℓ}_{t+1} for consumption of entrepreneurs when the project fails and c^h_{t+1} for consumption when it succeeds. Given their lifetime utility specification, entrepreneurs exert effort whenever

$$q_1 \ln c^h_{t+1} + (1 - q_1) \ln c^{\ell}_{t+1} - v(1) \geq q_0 \ln c^h_{t+1} + (1 - q_0) \ln c^{\ell}_{t+1}. \tag{8}$$

This inequality can be expressed in terms of consumption as

$$c^h_{t+1} \geq \eta \, c^{\ell}_{t+1} \quad \text{where} \quad \eta := \exp\left\{ \frac{v(1)}{q_1 - q_0} \right\}. \tag{9}$$

Note that $\eta > 1$ because $v(1) > 0$ and $q_1 > q_0$. Inequality (9) says that entrepreneurs will exert effort when the benefit in terms of relative consumption difference exceeds the cost. By using the definitions of c^{ℓ}_{t+1} and c^h_{t+1}, inequality (8) can also be expressed as $b_t \leq \hat{b}_t$ where

$$\hat{b}_t := \frac{z f'(k_{t+1}) + (1 - w_t)(\eta - 1) r_{t+1}}{(\eta - 1) r_{t+1} + r^e_{t+1}}. \tag{10}$$

Thus, agents exert effort when their liabilities to the bank are sufficiently low (i.e., $b_t \leq \hat{b}_t$). The intuition is that borrowing allows entrepreneurs to reduce risk, as seen in section 2. When consumption differs little across outcomes, the motivation for exerting effort towards success is diminished.

From the perspective of the financial intermediaries, the implication of the preceding analysis is that they can induce entrepreneurs to exert effort ex-post by limiting loan size. In particular, if $b_t \in [0, \hat{b}_t]$ then entrepreneurs exert effort and the profit banks earn per unit of lending is $q_1 \, r^e_{t+1} - r_{t+1}$. Free entry into banking implies that $q_1 r^e_{t+1} \leq r_{t+1}$. From this equation we see that financial intermediaries never lend more than $\hat{b}_t$, since, if they do, then entrepreneurs fail to exert effort, the success probability drops to q_0, and profit per unit of lending is $q_0 \, r^e_{t+1} - r_{t+1} < q_1 \, r^e_{t+1} - r_{t+1} \leq 0$. Thus, in equilibrium we have $b_t \leq \hat{b}_t$, entrepreneurs exert effort, and, with zero profits in the banking sector,

$$q_1 \, r^e_{t+1} = r_{t+1}. \tag{11}$$

3.2. **Agent Choices.** We have seen that banks restrict loans to a level that induces entrepreneurial effort. Next we claim that entrepreneurs choose to set $b_t = \hat{b}_t$ in equilibrium. The reasoning is straightforward: Banks are indifferent between all b_t such that $0 \leq b_t \leq \hat{b}_t$, since in every case they receive zero profit. As for entrepreneurs, recall that (9) and (10) are equivalent, so that, at any $b_t \leq \hat{b}_t$, we have $c^h_{t+1} \geq \eta c^\ell_{t+1} > c^\ell_{t+1}$. By (4) this implies that $U'(b_t) > 0$, so the entrepreneur prefers to strictly increase borrowing. Only at $\hat{b}_t$, when no further loans are forthcoming, is the entrepreneur content not to deviate.

It remains to determine the fraction ϕ_t of agents in the economy that choose to start projects, and hence the supply of productive capital at time $t + 1$. As a first step, observe that, using (11), the expression for $\hat{b}_t$ can be written as

$$\hat{b}_t := \frac{1}{1 + q_1(\eta - 1)} \left(q_1(\eta - 1)(1 - w_t) + \frac{Rf'(k_{t+1})}{r_{t+1}} \right). \tag{12}$$

Agents are willing to become entrepreneurs whenever $U(\hat{b}_t)$ is no lower than $\ln(w_t r_{t+1})$, the lifetime utility of a passive investor. Using (3) and (12), one can show that this statement is equivalent to

$$Rf'(k_{t+1}) \geq (1 + \theta w_t) r_{t+1} \tag{13}$$

where

$$\theta := \frac{(\eta - 1)q_1 + 1}{\eta^{q_0}} - 1. \tag{14}$$

As we are assuming that $v(1) > 0$ and $q_0 < q_1$, it follows that $\eta > 1$ and hence θ is strictly positive.[14] The parameter θ captures the severity of moral hazard, with higher values indicating greater intensity.

One way to understand (13) is to compare it with the no-moral-hazard equivalent (6). The difference is the term $1+\theta w_t$, which exceeds unity whenever $\theta > 0$ and $w_t > 0$. We can view this term as the risk premium agents require to become entrepreneurs. When $\theta > 0$, moral hazard is present, lending is restricted and agents who wish to become entrepreneurs must stake some equity in the project. Being risk averse, they require a positive risk premium in order to induce them to do so. Positivity of θw_t equates to positive risk and a positive equity stake.

[14] For $\eta > 1$ and $q_0 \in (0, 1)$, $\frac{\eta^{q_0} - 1}{\eta - 1} < \lim_{\eta \downarrow 1} \frac{\eta^{q_0} - 1}{\eta - 1} = q_0$. Since $q_0 < q_1$, it follows that $\theta > 0$.

A second observation regarding the risk premium visible in (13) is that, not only is it positive when $\theta > 0$, it is also increasing in w_t. The increase in the premium demanded by entrepreneurs is due to the fact that the borrowing constraint *tightens* as w_t increases, as is evident in (12), forcing them to risk more of their own wealth. Intuitively, higher wealth brings the ratio of consumption for successful and unsuccessful entrepreneurs closer to one, reducing the incentive for entrepreneurs to exert effort toward raising the probability of success. Banks respond to this change in incentives by further restricting borrowing.[15]

The incentives embedded in (13) contrast directly with the corresponding inequality in Matsuyama (2004), which is given by $Rf'(k_{t+1}) \geq ((1 - w_t)/\lambda)r_{t+1}$. In that setting, higher wages loosen the borrowing constraint, since they boost available collateral. This drives permanent reinforcement of initial differences in wealth when financial markets are integrated, as opposed to the cycles observed in our model.

Returning to the problem of determining ϕ_t, observe that the stock of productive capital at $t + 1$ equals expected output per entrepreneur times the mass of entrepreneurs, or

$$k_{t+1} = R\phi_t. \tag{15}$$

Second, since the marginal product of capital is assumed to be infinite at $k = 0$, inequality (13) implies that at least some agents start projects in equilibrium, and hence $\phi_t = 0$ is never observed. On the other hand, the alternate boundary case $\phi_t = 1$ cannot be ruled out when financial markets are integrated. In this scenario, (13) need not bind. Finally, if $\phi_t \in (0, 1)$, then (13) holds with equality. We can summarize this discussion by combining (13) and (15) to obtain

$$\phi_t = \phi(w_t, r_{t+1}) \quad \text{when} \quad \phi(w, r) := \min \left\{ \frac{1}{R} (f')^{-1} \left[\frac{(1 + \theta w)r}{R} \right], \ 1 \right\}. \tag{16}$$

[15] One possible modification of our model is to allow banks to impose a collateral requirement on entrepreneurs. If banks impose collateral requirement $m \in [0, w]$, then an entrepreneur's consumption in the low and high states become $c^\ell_{t+1} = (b_t + w_t - 1 - m)r_{t+1}$ and $c^h_{t+1} = (b_t + w_t - 1)r_{t+1} + zf'(k_{t+1}) - b_t r^e_{t+1}$ respectively. The high effort condition is again $c^h_{t+1} \geq \eta \, c^\ell_{t+1}$. Working through the logic of equations (3)–(12) with these modified entrepreneurial consumption outcomes and a correspondingly modified profit condition for banks shows that collateral does not affect the equilibrium relationship between capital, consumption and wages. Intuitively, when banks require collateral the maximum loan size $\hat{b}$ increases by an equal and offsetting amount. The net effect is that banks continue to make zero profits and consumption is unchanged across good and bad states in equilibrium.

Thus, $\phi(w, r)$ is the equilibrium fraction of entrepreneurs in the economy when the wage is w and the deposit rate is r. Since f is strictly concave, $(f')^{-1}$ is strictly decreasing. Hence, recalling that θ is positive whenever $v(1) > 0$, the value $\phi(w, r)$ is strictly decreasing in both w and r apart from the boundary case $\phi_t = 1$.

4. Dynamics

We now turn to dynamics, beginning with autarky and the small open economy case, and then turning to endogenously determined interest rates when multiple economies integrate financial markets. To simplify our discussion, we focus on the case $f(k) = k^\alpha$. Moreover, we assume throughout that

$$\omega(R) = (1 - \alpha)R^\alpha < 1. \tag{17}$$

Since $w_t = \omega(k_t) = \omega(R\phi_t) \leq \omega(R)$, the restriction (17) yields $w_t < 1$. Below we use w_t as the state variable. Since $w_t = \omega(k_t)$ and the map ω is continuous and strictly increasing, the dynamics of capital are identical up to a homeomorphic transformation. The same is true for output $f(k_t)$.

4.1. **Autarky.** Consider first an economy with no ties to international financial markets. Aggregate demand for funds to start projects, which is equal at time t to the fraction ϕ_t of entrepreneurs, must then be equal to domestic credit supply. Thus, $\phi_t = w_t$. Applying ω to both sides of (15) and then using $\phi_t = w_t$ gives

$$w_{t+1} = \omega(Rw_t) = (1 - \alpha)(Rw_t)^\alpha. \tag{18}$$

The system has a unique, globally stable steady state given by

$$w^* := \{(1 - \alpha)R^\alpha\}^{1/(1-\alpha)}. \tag{19}$$

The value w^* is referred to below as the *autarkic steady state*. It is the steady state value of wages in each country when financial markets are not integrated. From every initial condition $w_0 > 0$ we have $w_t \to w^*$ as $t \to \infty$. Notice that dynamics and the long run steady state w^* are determined by productivity parameters. If we shut down moral hazard by setting $v(1) = 0$, the time path of output, capital and income is entirely unaffected.

In fact, moral hazard shows up only in interest rates. To see this, recall that, since $w_t < 1$ for all t by (17), the boundary case $\phi_t = 1$ can be excluded in autarky because entrepreneurs cannot function without investors. Thus (13) holds with equality, and hence $r_{t+1} = Rf'(k_{t+1})/(1+\theta w_t)$. If we set $v(1) = 0$, then $\theta = 0$, and the last expression becomes $r_{t+1} = Rf'(k_{t+1})$. We see that the interest rate is unambiguously lower with moral hazard, a point that we return to in section 4.4. Nevertheless, moral hazard has no impact on dynamics or the long run steady state w^* because, in autarky, the interest rate adjusts to equalize supply and demand for credit.

4.2. **A Small Open Economy.** Next we turn to the small open economy setting, where borrowing and lending is possible on international credit markets and the deposit rate r_{t+1} is fixed at some exogenously given value r^*. Studying this case lays the ground for our later analysis of integrated financial markets. We will show that moral hazard leads the economy to experience either permanent cycles or damped oscillations for wages and output.

To begin, observe that, while the autarkic equilibrium condition $\phi_t = w_t$ no longer holds (since aggregate supply and demand for funds need not be equal), the capital accumulation identity (15) still holds, implying that $w_{t+1} = \omega(R\phi_t)$. Hence, with ϕ as defined in (16),

$$w_{t+1} = h(w_t) \quad \text{where} \quad h(w) := \omega(R\phi(w_t, r^*)). \tag{20}$$

In view of (16), the function h is strictly decreasing in w for any positive value of r^* when the resource constraint is not binding and $\theta > 0$.

Since h is decreasing and bounded, there are two possible dynamics: damped oscillations or permanent 2-cycles.[16] Figure 2 shows both h and $h^2 := h \circ h$ under one parameterization in separate 45 diagrams. The parameter values are $z = 5$, $v(1) = 1.5$, $q_0 = 0.5$, $q_1 = 0.75$, $r^* = 1.05$ and $\alpha = 0.65$. This parameter set induces a stable 2-cycle, as can be seen from the right panel of figure 2. The unique fixed point of h is unstable.

[16] A pair of distinct points a, b in $[0, 1)$ is called a *2-cycle* of the dynamical system induced by h if $h(a) = b$ and $h(b) = a$. In particular, both a and b are fixed points of h^2, while neither is a fixed point of h. The two-cycle is called stable if both a and b are attractors for h^2.

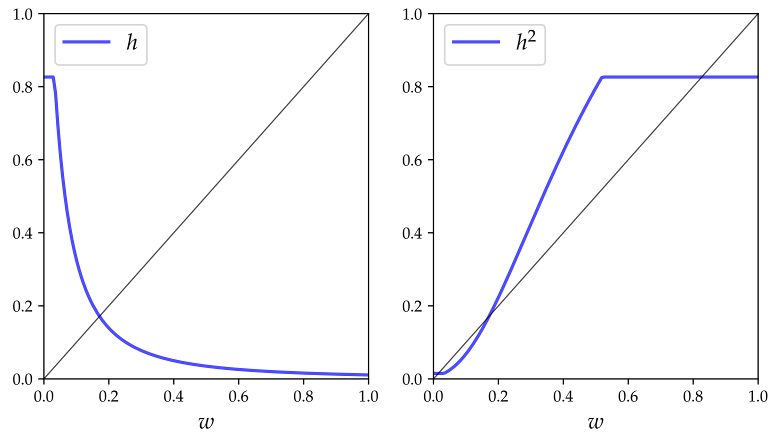

FIGURE 2. Time one and time two maps for the small open economy

Figure 3 illustrates the case of damped cycles, this time via time series. The interest rate is now $r^* = 1.10$ and $\alpha = 0.6$. The figure shows the impulse response of wages and output to a one-off productivity shock that increases the output of all entrepreneurial projects in the domestic economy by 10% and then reverts to the original value. The horizontal axis measures time. Values of the shock, wage and output are all normalized so that the initial value is 1. The one period drop in output from the peak of the boom in period 3 to the trough in period 4 is approximately 28%.

At the peak of the boom, entrepreneurs have relatively high wealth. The desire on the part of banks to incentivize effort leads them to insist on a correspondingly high equity stake, which in turn increases the risk premium demanded by entrepreneurs. At the same time, the marginal product of capital declines. Together, these forces lead to relatively weak demand for funds, depressing formation of domestic capital stock. In the next period wealth is correspondingly lower and the opposite mechanism takes hold.

Moral hazard is essential to these oscillations. In particular, if we switch off moral hazard by setting $v(1) = 0$, then $\theta = 0$. In view of (16), this means that ϕ and hence h in (20) are constant in w. Thus, w_t moves immediately to and remains at the steady state value. No fluctuations are observed.

The values of α chosen in the preceding simulations were high relative to standard calibrations. Higher values of α tend to be associated with larger damped oscillations or permanent cycles. Such values are not unreasonable if we view k as embodying all

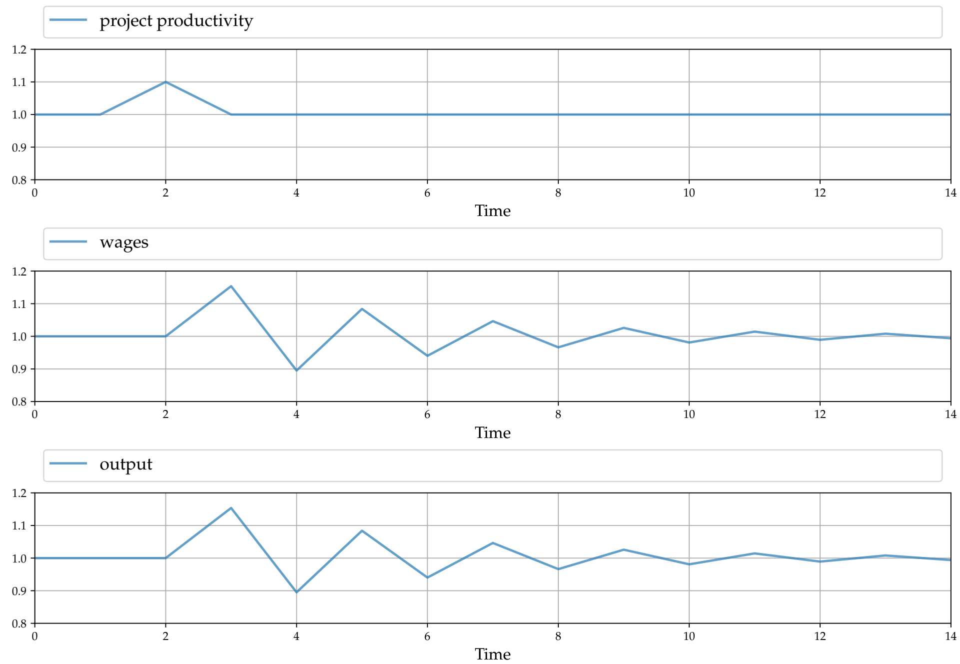

FIGURE 3. Impulse response for wages and output, small open economy

inputs to production supplied by the older generation, including technical and managerial skills, as well as the physical capital generated by running projects. The "capital share" α is the share of income accruing to all such inputs. Analysis of compensation in the Current Population Survey suggests that values of α in the range 0.5–0.7 are not unreasonable.[17]

4.3. **A Two-Country Model.** We now proceed to analysis of financial integration in a two country setting. To this end, consider two countries labeled X and Y that are identical in all ways except for the current state, which we represent by the respective

[17]In the Current Population Survey of the US Bureau of Labor Statistics, the share of total labor earnings paid to "management, professional, and related occupations" has risen from 0.47 in 2002 to 0.59 in 2015. The average value for the years 2002–2015 is 0.54. Taking this average and assuming that the total share of all income retained by labor is 0.66, the share of all income accruing to non-managerial positions is 0.304. Hence, if we identify labor compensation for the older generation in our model with management, professional and related occupations, the corresponding value for α is approximately 0.7. This number is likely to exaggerate the appropriate share of compensation to the older generation in our model because the managerial earnings category in the Current Population Survey is relatively broad. If we halve the labor income share of management from 0.54 to 0.26, then the corresponding value for α becomes 0.52.

wage pair (w_t^X, w_t^Y). We assume that a global competitive market for credit exists, with international financial intermediaries taking deposits from investors and making loans to entrepreneurs in both countries. Equilibrium in the credit market requires that the international demand for credit equals international supply. Thus, in equilibrium, the world deposit rate r_{t+1} is the r that solves

$$\phi(w_t^X, r) + \phi(w_t^Y, r) = w_t^X + w_t^Y. \tag{21}$$

Here ϕ is as defined in (16). As the state space for the two-country model we take all pairs (w^X, w^Y) in

$$S := (0, \bar{w}]^2 \setminus (\bar{w}, \bar{w}) \tag{22}$$

where $\bar{w} := \omega(R)$ is the wage obtained when all domestic agents are entrepreneurs.[18] Continuity and monotonicity imply that, for each $(w^X, w^Y) \in S$, there exists a unique rate $r \in (\hat{r}, \infty)$ that solves (21), where $\hat{r} := \alpha R^\alpha / (1 + \theta\omega(R))$. Let $r(w^X, w^Y)$ be this value. Using the identities $k_{t+1} = R\phi_t$ and $w_t = \omega(k_t)$ applied to each country, we obtain

$$w_{t+1}^X = \omega(R\phi_t^X) \quad \text{and} \quad w_{t+1}^Y = \omega(R\phi_t^Y) \tag{23}$$

where $\phi_t^X := \phi(w_t^X, r(w_t^X, w_t^Y))$ and $\phi_t^Y := \phi(w_t^Y, r(w_t^X, w_t^Y))$ are the proportion of agents that become entrepreneurs in countries X and Y respectively. Since r is symmetric, in the sense that $r(x, y) = r(y, x)$ for any pair $(x, y) \in S$, by setting

$$\Phi(x, y) := \omega(R\phi(x, r(x, y)))$$

we can write the system for evolution of wages in the two country model as

$$w_{t+1}^X = \Phi(w_t^X, w_t^Y) \quad \text{and} \quad w_{t+1}^Y = \Phi(w_t^Y, w_t^X). \tag{24}$$

In analyzing (24) we begin with some theoretical results, the implications of which will be explored below. To this end, let T be the map sending (w_t^X, w_t^Y) into (w_{t+1}^X, w_{t+1}^Y) defined in (24). The function T maps S into itself, where S is the state space defined by (22). Hence (S, T) is a two dimensional dynamical system. In the next proposition, w^* is the autarkic steady state given in (19).

[18]We remove the point $(\bar{w}, \bar{w})$ from the state space S because such an outcome is unfeasible (since projects cannot be funded without any investors in the global economy) and no interest rate can clear the credit market in this state.

Proposition 4.1. *The point (w^*, w^*) is the unique fixed point of T in S. Moreover, (w^*, w^*) is locally stable for T if and only if*

$$\frac{2\alpha - 1}{1 - \alpha}\theta w^* < 1. \tag{25}$$

In the proof we show that the eigenvalues of the Jacobian of T evaluated at (w^*, w^*) are $\mu^1 := \alpha$ and $\mu^2 := -\alpha\theta w^*/((1-\alpha)(1+\theta w^*))$. Note that $\mu^2 < 0$ when $\theta > 0$. The fact that the eigenvalues have opposite signs means that the system will exhibit either damped or permanent cycles, analogous to the case for the small open economy. Equation (25) is just a rearrangement of the statement $\mu^2 > -1$.

Now we turn to the case where condition (25) fails.

Proposition 4.2. *The dynamical system (S, T) has a 2-cycle if and only if*

$$\frac{2\alpha - 1}{1 - \alpha}\theta w^* > 1. \tag{26}$$

The 2-cycle that exists under (26) is locally stable in a neighborhood of the bifurcation occuring at $\frac{2\alpha-1}{1-\alpha}\theta w^* = 1$.

Together propositions 4.1 and 4.2 show that a supercritical flip bifurcation occurs at $\frac{2\alpha-1}{1-\alpha}\theta w^* = 1$. In particular, as the stability coefficient $\frac{2\alpha-1}{1-\alpha}\theta w^*$ varies from less to greater than unity, the unique stable steady state (w^*, w^*) loses stability and an attracting 2-cycle emerges.

Figure 4 illustrates propositions 4.1–4.2 by showing the vector field of the time two map T^2 under both stable and unstable configurations. Arrows indicate direction of movement under T^2. Each panel contains two curves, which trace the sets

$$\Lambda_i := \{(w^x, w^y) \in S : \pi_i(w^x, w^y) = \pi_i(T^2(w^x, w^y))\}$$

for $i \in \{1, 2\}$, where π_i is the projection of $\mathbb{R}^2$ onto its i-th coordinate (i.e., $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$). Pairs (w^x, w^y) at which these curves intersect are fixed points of T^2, and hence either steady states of T or one component of a 2-cycle.

In all panels $\alpha = 0.55$ and $z = 3$, while the disutility of effort $v(1)$ increases from 1.5 in the top left panel to 2.25 in the bottom right. Greater disutility of effort strengthens the moral hazard effect, tightens the borrowing constraint for a given level of wages and leads to a rise in θ. This in turn boosts the term on the left hand side of (26),

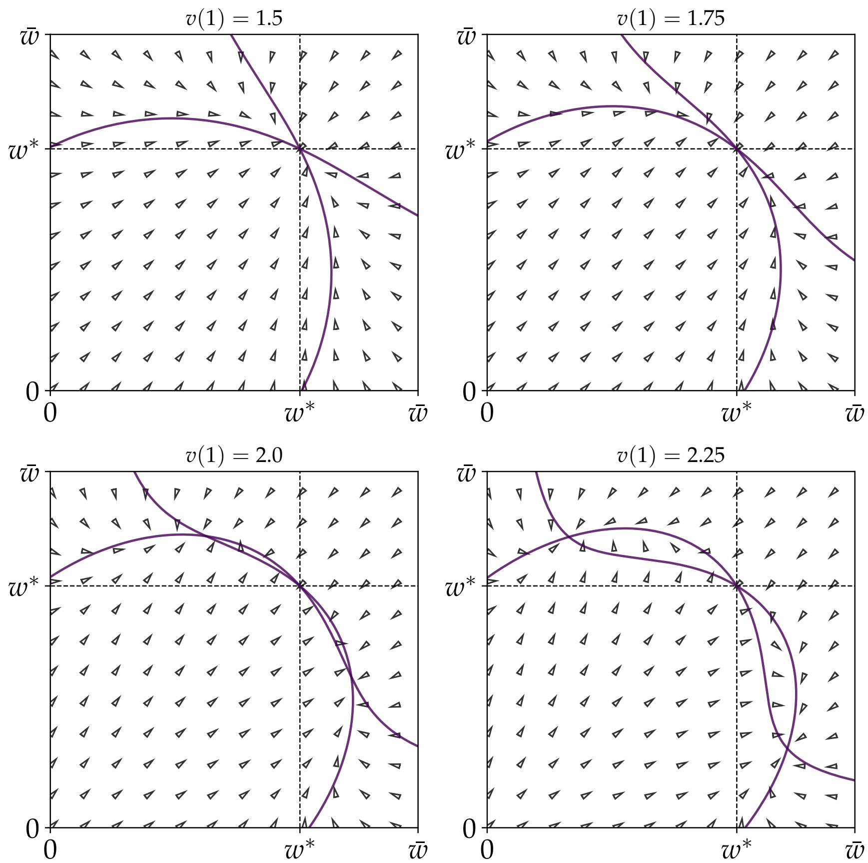

FIGURE 4. Vector fields for T^2 with increasing moral hazard

from 0.55 in the top left panel to 0.81 in the top right panel, followed by 1.17 in the bottom left panel and 1.66 in the bottom right.

In line with the results in propositions 4.1–4.2, the figure shows that a sufficiently large increase in moral hazard causes the symmetric steady state to lose stability. At the level of moral hazard where the left hand side of (26) crosses unity a stable 2-cycle emerges. This process is related to the symmetry breaking effect studied in Matsuyama (2004), although the cause here is not financial integration (since financial integration holds across all four parameterizations) but rather the rise in moral hazard.

Symmetry breaking also occurs with a shift from autarky to financial integration, similar to the effect studied in Matsuyama (2004), whenever inequality (26) holds. In this setting, integration causes the symmetric steady state (w^*, w^*), which governs long run outcomes in autarky, to lose stability. At the same time a stable 2-cycle emerges. As Figure 5 illustrates this process using a time series view of symmetry breaking via financial integration when the conditions of proposition 4.2 hold. The horizontal axis

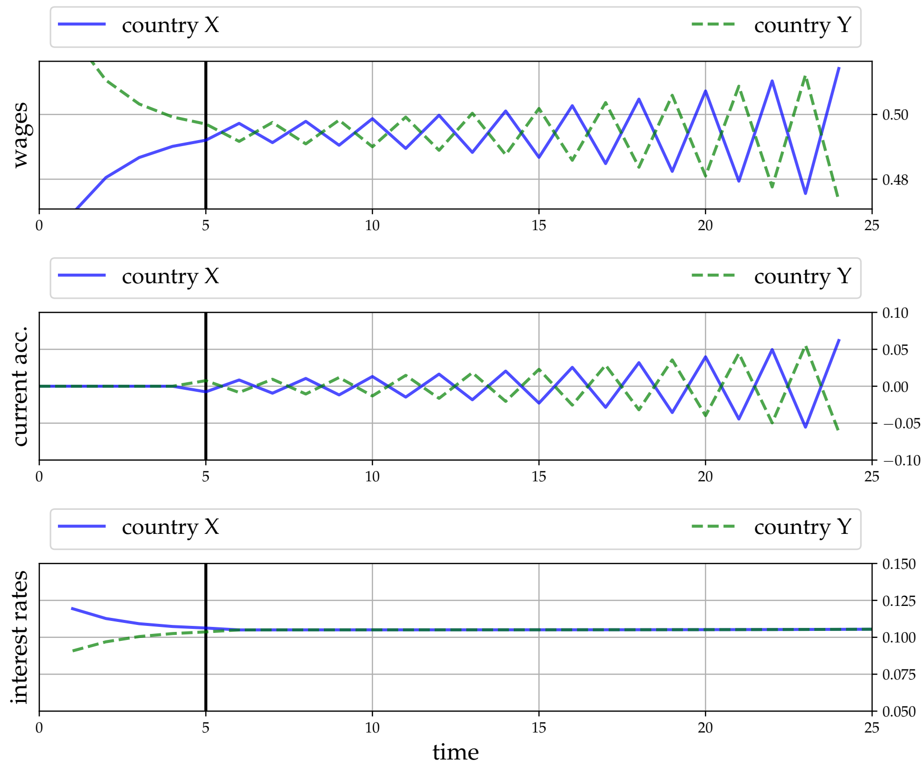

FIGURE 5. Integration and cycles

is time. For the first five periods the two countries are assumed to exist in autarky. During this period, wages in both countries converge towards w^*. Integration of capital markets takes places at $t = 5$, leading to the onset of cyclical fluctuations. A build up of wages corresponds to a build up of physical capital. This build up is preceded at each step by a current account deficit, with foreign credit fueling the growth of local productive capital.

As emphasized in the introduction, the symmetry breaking observed in Matsuyama (2004) is associated with a permanent amplifying effect on initial inequality, unlike the cyclical behavior observed here. The different outcomes are due to the different nature of the financial frictions. The collateral-based borrowing constraint in Matsuyama (2004) loosens with higher wealth, reinforcing initial differences. In contrast, the model presented here is such that banks require entrepreneurs to increase their equity stake as wealth rises, disincentivizing additional capital formation. The reverse is true when wealth falls, generating cycles.

4.4. **Relative Interest Rates.** Consider the no-moral-hazard case, where $\theta = 0$. The risk premium in (13) then disappears and, assuming an interior solution, we have

$$r^n_{t+1} = Rf'(k^n_{t+1}). \tag{27}$$

Here and below, superscript n indicates no moral hazard. If we match global supply and demand for funds we can obtain the equilibrium rate

$$r^n_{t+1} = \alpha R^{\alpha} \left(\frac{2}{w^{X,n}_t + w^{Y,n}_t} \right)^{1-\alpha}. \tag{28}$$

The no-moral-hazard law of motion for wages in each country is

$$w^{X,n}_{t+1} = w^{Y,n}_{t+1} = \omega \left[\frac{R(w^{X,n}_t + w^{Y,n}_t)}{2} \right].$$

Wages in the two countries are equalized after one period, and the common law of motion for each country becomes $w^n_{t+1} = \omega(Rw^n_t)$.

Continuing with the interior case and assuming that (13) holds with equality, the presence of moral hazard and hence positive θ gives $Rf'(k_{t+1}) = (1 + \theta w_t)r_{t+1}$, where, as before, r_{t+1} is the deposit rate under moral hazard. With interiority we can solve (21) for r_{t+1} explicitly, obtaining

$$r_{t+1} = \alpha R^{\alpha} \left[\left(\frac{1}{1 + \theta w^X_t} \right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{1 + \theta w^Y_t} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} \left(\frac{1}{w^X_t + w^Y_t} \right)^{1-\alpha}. \tag{29}$$

Comparing (29) with (28), we see that $r_{t+1} < r^n_{t+1}$ always holds. The no-moral-hazard interest rate is higher because the ability to insure against risk by borrowing increases the incentive to start projects, and hence the aggregate demand for funds.

5. Extensions

The boom-bust cycles observed in the preceding sections are preserved under various modifications, such as the introduction of aggregate uncertainty or additional countries. This section provides illustrations.

5.1. **Aggregate Shocks.** The model can be shifted closer to the data by complementing the existing idiosyncratic shocks with aggregate level productivity shocks. As we show, boom-bust cycles continue to occur under many parameterizations. In fact one could argue that they occur under a larger range of parameters, since convergence to a periodic attractor is not necessary for repeated cycles. For example, when combined with a mechanism that produces damped cycles, productivity shocks produce bursts of cyclic volatility consistent with observed fluctuations (i.e., occasional crises follows by periods of relative stability).

Regarding the nature of the shocks, we suppose that the value z (output of a successful project) is chosen randomly at the start of each period. This implies a random choice of parameter R. To simplify the equilibrium, we suppose that its value is previsible, in the sense that its current value is visible when agents decide whether to become investors or entrepreneurs. As a result, the equilibrium choices can be determined in the same manner as in section 3.

In our simulation, we assume that the two countries share parameters $\alpha = 0.48$, $v(1) = 4$, $q_0 = 0.2$ and $q_1 = 0.8$. In country Y, productivity is steady, with $z = 1$. In country X, productivity fluctuates around $z = 1$, as shown in figure 6. For comparison, we also show the no-moral-hazard case, where $v(1) = 4$ is replaced with $v(1) = 0$. Without moral hazard the shocks have no persistence. The two economies return immediately to equilibrium. With moral hazard, however, damped fluctuations imply that volatility persists for several periods after the shock has hit. Moreover, significant volatility is transmitted to country Y, despite the fact that its productivity is constant.

5.2. **A Model with More Countries.** Extending the model to an N-country setting is straightforward. Returning to the case without aggregate shocks and assuming that all countries are structurally identical in parameters, the world deposit rate r_{t+1} becomes the r that solves

$$\sum_{i=1}^{N} \phi(w_t^i, r) = \sum_{i=1}^{N} w_t^i.$$

Let $N = 200$, $\alpha = 0.6$, $z = 20$, $q_1 = 0.15$, $q_0 = 0.02$, and $v(1) = 0.3622$, implying that $R = 3$ and $\theta = 16.21$. The countries differ only in their initial wage, which we assume to be $w_0^i = \frac{i}{200}$ for $i = 1, 2, 3, ..., 200$. A straightforward numerical simulation shows that the world economy in this case converges to a stable 2-cycle with the deposit rate

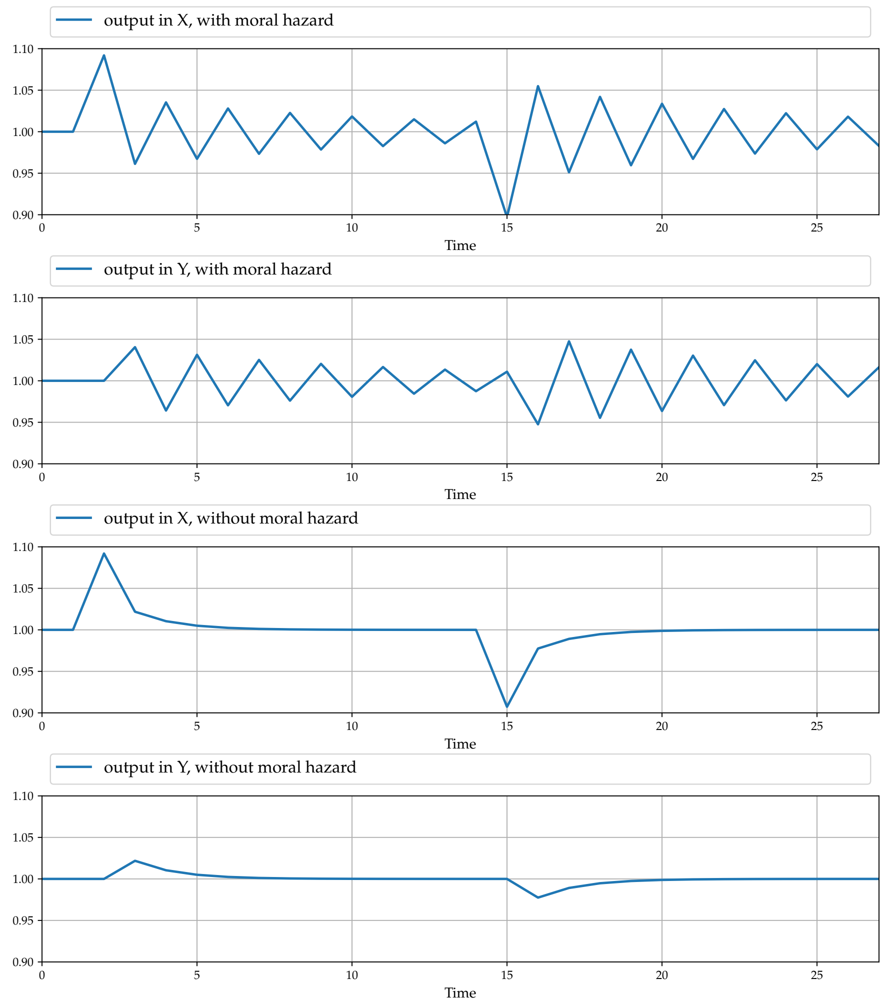

FIGURE 6. Aggregate shocks and cycles

fluctuating between 0.926 and 0.929. The world economy is divided into two groups, $n = 1, 2, ..., 98$ and $n = 99, 100, ..., 200$, whose wage switches between 0.032 and 0.573.

6. CONCLUSION

This paper shows how integration of financial markets can introduce volatility and boom-bust cycles into a previously stable world economy. We provide conditions under which damped and permanent cycles emerge as a direct consequence of financial integration, with both push and pull factors playing a role. Push factors correspond to endogenously generated variations in the world interest rate. Pull factors correspond to changes in domestic demand for capital associated with moral hazard and the corresponding borrowing constraints imposed by financial intermediaries. Cycles are eliminated when moral hazard is not present.

The paper connects aggregate dynamics in an international setting with the micro-financial literature on informational asymmetries between entrepreneurs and financial intermediaries. These asymmetries cause creditors to insist that entrepreneurs take an equity stake in their own project. In the case of our model, the borrowing constraint tightens as wealth increases, forcing entrepreneurs to increase their equity stake. Since entrepreneurs are risk averse, this effect puts downward pressure on investment. Testable implications of our model include the expansion and contraction of leverage accompanied by the adjustment of entrepreneurial investment at the extensive margin along the boom-bust cycle.

Appendix A. Proofs

Proof of Proposition 4.1. For a given parameter pair (α, θ), we can define three disjoint sets

$$\Omega_0 := \{(x, y) \mid G(x, y) \leq 1 \text{ and } G(y, x) \leq 1\}$$
$$\Omega_1 := \{(x, y) \mid G(x, y) > 1 \text{ and } G(y, x) \leq 1\}$$
$$\Omega_2 := \{(x, y) \mid G(x, y) \leq 1 \text{ and } G(y, x) > 1\}$$

where

$$G(x, y) := \left(\frac{1 + \theta y}{1 + \theta x}\right)^{\frac{1}{1-\alpha}} (x + y - 1). \tag{30}$$

For any $(x, y) \in S$, the solution of (21) is

$$r(x, y) = \begin{cases} \frac{\alpha R^\alpha}{(x+y)^{1-\alpha}} \left[\left(\frac{1}{1+\theta x}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{1+\theta y}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} & \text{if} \quad (x, y) \in \Omega_0 \\ \frac{\alpha R^\alpha}{(x+y-1)^{1-\alpha}} \frac{1}{1+\theta y} & \text{if} \quad (x, y) \in \Omega_1 \\ \frac{\alpha R^\alpha}{(x+y-1)^{1-\alpha}} \frac{1}{1+\theta x} & \text{if} \quad (x, y) \in \Omega_2. \end{cases}$$

This implies that

$$\phi(x, r(x,y)) = \begin{cases} \frac{(1+\theta y)^{\frac{1}{1-\alpha}}}{(1+\theta x)^{\frac{1}{1-\alpha}}+(1+\theta y)^{\frac{1}{1-\alpha}}}(x+y) & \text{if} \quad (x,y) \in \Omega_0 \\ 1 & \text{if} \quad (x,y) \in \Omega_1 \\ x+y-1 & \text{if} \quad (x,y) \in \Omega_2. \end{cases} \tag{31}$$

It follows from (23) that the steady state pair (x, y) must satisfy

$$\phi(x, r(x,y)) = \tfrac{\omega^{-1}(x)}{R} \quad \text{and} \quad \phi(y, r(x,y)) = \tfrac{\omega^{-1}(y)}{R}. \tag{32}$$

To show existence and uniqueness of the steady state (w^*, w^*), we consider three cases separately and show that there exists a unique steady state in Ω_0, while there are no steady states in Ω_1 and Ω_2.

a) *Steady states in* Ω_0. From (21), (31) and (32), the steady state (x, y) must satisfy

$$x + y = \tfrac{\omega^{-1}(x)}{R} + \tfrac{\omega^{-1}(x)}{R} \quad \text{and} \quad \left(\tfrac{1+\theta y}{1+\theta x}\right)^{\frac{1}{1-\alpha}} = \left(\tfrac{x}{y}\right)^{\frac{1}{\alpha}}. \tag{33}$$

The second equation in (33) can be rewritten as $y^{\frac{1-\alpha}{\alpha}}(1+\theta y) = x^{\frac{1-\alpha}{\alpha}}(1+\theta x)$ which has a unique solution $y = x$ because $x \mapsto x^{\frac{1-\alpha}{\alpha}}(1+\theta x)$ is strictly increasing function. This with the first equation in (33) implies that the steady state x must satisfy $x = \frac{\omega^{-1}(x)}{R}$ which has a unique solution $x = w^*$. Hence, there exists a unique steady state $(w^*, w^*) \in \Omega_0$.

b) *Steady states in* Ω_1. From (21), (31) and (32) that the steady state $(x, y) \in \Omega_1$ must satisfy

$$1 = \tfrac{\omega^{-1}(x)}{R} \quad \text{and} \quad x + y - 1 = \tfrac{\omega^{-1}(y)}{R}. \tag{34}$$

This implies that $x = \bar{w}$ and y solves $y = \omega[R(\bar{w} + y - 1)]$. This with (30) implies

$$G(\bar{w}, y) = \left(\frac{1+\theta y}{1+\theta \bar{w}}\right)^{\frac{1}{1-\alpha}} \frac{\omega^{-1}(y)}{\omega^{-1}(\bar{w})} < 1$$

because $y \in (0, \bar{w}) \iff y^{\frac{1-\alpha}{\alpha}}(1+\theta y) < \bar{w}^{\frac{1-\alpha}{\alpha}}(1+\theta \bar{w})$. $G(\bar{w}, y) < 1$, however, implies that $(\bar{w}, y)$ does not belong to Ω_1. We conclude that there is no steady state in Ω_1.

c) *Steady states in* Ω_2. We can use exactly the same logic as in (b) to show that there is no steady state in Ω_2.

Regarding stability of the symmetric steady state (w^*, w^*), let

$$J(x,y) = \begin{bmatrix} \Phi_x(x,y) & \Phi_y(x,y) \\ \Phi_x(y,x) & \Phi_y(y,x) \end{bmatrix} \tag{35}$$

be the Jacobian associated with the dynamical system (24). In order to assess stability, we wish to evaluate the eigenvalues of this matrix at $x = y = w^*$. Observe that J is a symmetric matrix. In this case, eigenvalues of J are $\mu_1 = \Phi_x(w^*, w^*) + \Phi_y(w^*, w^*)$ and $\mu_2 = \Phi_x(w^*, w^*) - \Phi_y(w^*, w^*)$. It follows from (23) and (24) that

$$\begin{aligned} \Phi_x(w^*, w^*) &= R\omega'(Rw^*)\left(\phi_1(w^*, r^*) + \phi_2(w^*, r^*) r_1(w^*, w^*)\right) \\ \Phi_y(w^*, w^*) &= R\omega'(Rw^*)\phi_2(w^*, r^*) r_2(w^*, w^*) \end{aligned} \tag{36}$$

while it follows from (21) that

$$r_1(w^*, w^*) = r_2(w^*, w^*) = \frac{1 - \phi_1(w^*, r^*)}{2\phi_2(w^*, r^*)}. \tag{37}$$

This with (36) implies that

$$\mu_1 = R\omega'(Rw^*) = \alpha \quad \text{and} \quad \mu_2 = R\omega'(Rw^*)\phi_1(w^*, r^*) = \alpha\phi_1(w^*, r^*), \tag{38}$$

because $w^* = \omega(Rw^*)$. Since $\alpha \in (0,1)$, the local stability of (w^*, w^*) depends on the value of μ_2. Taking the natural logarithm of $\phi(w,r) = \left(\frac{\alpha R^\alpha}{r(1+\theta w)}\right)^{\frac{1}{1-\alpha}}$ and differentiating it with respect to its first argument, we obtain[19]

$$\phi_1(w^*, r^*) = -\frac{1}{1-\alpha}\frac{\theta w^*}{1+\theta w^*} < 0.$$

This with (38) implies that $\mu_2 = -\frac{\alpha}{1-\alpha}\frac{\theta w^*}{1+\theta w^*}$. Thus

$$\mu_2 \in (-1, 0) \iff \tfrac{2\alpha - 1}{1-\alpha}\theta w^* < 1. \tag{39}$$

□

Proof of Proposition 4.2. The map T^2 can be represented by

$$\begin{aligned} w_{t+2}^X &= \Phi(\Phi(w_t^X, w_t^Y), \Phi(w_t^Y, w_t^X)) \\ w_{t+2}^Y &= \Phi(\Phi(w_t^Y, w_t^X), \Phi(w_t^X, w_t^Y)) \end{aligned} . \tag{40}$$

It follows from (40) that the pair (x,y) is a 2-cycle if it satisfies the system of equations

$$y = \Phi(x,y) \quad \text{and} \quad x = \Phi(y,x)$$

[19] We know that $\phi(w^*, r^*) < 1$.

which, using (32), can be rewritten as

$$\phi(x, r(x,y)) = \tfrac{\omega^{-1}(y)}{R} \quad \text{and} \quad \phi(y, r(x,y)) = \tfrac{\omega^{-1}(x)}{R}. \tag{41}$$

From (21), (31) and (41), the pair (x, y) must satisfy

$$x + y = \tfrac{\omega^{-1}(x)}{R} + \tfrac{\omega^{-1}(x)}{R} \quad \text{and} \quad \left(\tfrac{1+\theta y}{1+\theta x}\right)^{\frac{1}{1-\alpha}} = \left(\tfrac{y}{x}\right)^{\frac{1}{\alpha}}. \tag{42}$$

By introducing a variable transformation $y = \tau x$, (42) can be rewritten as

$$x = w^* \left(\tfrac{1+\tau}{1+\tau^{\frac{1}{\alpha}}}\right)^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad x = \tfrac{1}{\theta}\frac{1-\tau^{\frac{1-\alpha}{\alpha}}}{\tau^{\frac{1-\alpha}{\alpha}} - \tau}. \tag{43}$$

To show existence and uniqueness of a 2-cycle we now combine the two equations in (43) and find a τ which solves

$$w^* \left(\frac{1+\tau}{1+\tau^{\frac{1}{\alpha}}}\right)^{\frac{\alpha}{1-\alpha}} = \frac{1}{\theta}\frac{1-\tau^{\frac{1-\alpha}{\alpha}}}{\tau^{\frac{1-\alpha}{\alpha}} - \tau},$$

which can be rewritten as

$$\frac{2\alpha - 1}{1-\alpha}\theta w^* = \Delta(\tau) := \frac{2\alpha-1}{1-\alpha}\left(\frac{1+\tau^{\frac{1}{\alpha}}}{1+\tau}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-\tau^{\frac{1-\alpha}{\alpha}}}{\tau^{\frac{1-\alpha}{\alpha}} - \tau}. \tag{44}$$

We can show that Δ has a U-shape with the global minimum at $\tau = 1$ and $\Delta(\tau) = \Delta\left(\frac{1}{\tau}\right)$. In addition, Δ satisfies the boundary conditions $\lim_{\tau\downarrow 0} \Delta(\tau) = \lim_{\tau\uparrow\infty} \Delta(\tau) = \infty$. Moreover, using the L'Hospital's rule, we obtain $\lim_{\tau\to 1} \Delta(\tau) = 1$. The properties of Δ imply that (44) has two solutions $\tau_1 \in (0,1)$ and $\tau_2 = \frac{1}{\tau_1} \in (1,\infty)$ if and only if (26) is satisfied. The corresponding pair (x,y) of a 2-cycle can be obtain by (43) and $y = \tau x$.

The stability claim in proposition 4.2 follows from standard results on period doubling bifurcations combined with the expressions for the eigenvalues of the Jacobian J obtained in (38) and (39). See, for example, Azariadis (1993), theorem 8.4. □

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