

An analysis of ϵ -lexicase selection for large-scale many-objective optimization

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ABSTRACT

In this paper we adapt ϵ -lexicase selection, a parent selection strategy designed for genetic programming, to solve many-objective optimization problems. ϵ -lexicase selection has been shown to perform well in regression due to its use of full program semantics for conducting selection. A recent theoretical analysis showed that the behavior of this selection strategy is to preserve individuals located near the boundaries of Pareto front in semantic space. We hypothesize that this strategy, of biasing search to extreme positions in objective space may be beneficial as the number of objectives in problems increases. Here, we replace program semantics with objective fitness to conduct selection for many-objective optimization. We derive the probabilities of selection under lexicase selection to illustrate its behavior. We then compare this method to multi-objective optimization methods from literature on problems ranging from 5 to 100 objectives. We find that ϵ -lexicase selection outperforms state-of-the-art optimization algorithms in terms of convergence to the Pareto front, spread of solutions, and wallclock runtime across problems and objectives.

CCS CONCEPTS

• **Computer systems organization** → **Embedded systems**; **Redundancy**; **Robotics**; • **Networks** → **Network reliability**;

KEYWORDS

ACM proceedings, \LaTeX , text tagging

ACM Reference Format:

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1 INTRODUCTION

The *proceedings* are the records of a conference.¹ ACM seeks to give these conference by-products a uniform, high-quality appearance. To do this, ACM has some rigid requirements for the format of the proceedings documents: there is a specified format (balanced double columns), a specified set of fonts (Arial or Helvetica and Times

¹This is a footnote

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Roman) in certain specified sizes, a specified live area, centered on the page, specified size of margins, specified column width and gutter size.

2 THE BODY OF THE PAPER

Typically, the body of a paper is organized into a hierarchical structure, with numbered or unnumbered headings for sections, subsections, sub-subsections, and even smaller sections. The command `\section` that precedes this paragraph is part of such a hierarchy.² \LaTeX handles the numbering and placement of these headings for you, when you use the appropriate heading commands around the titles of the headings. If you want a sub-subsection or smaller part to be unnumbered in your output, simply append an asterisk to the command name. Examples of both numbered and unnumbered headings will appear throughout the balance of this sample document.

Because the entire article is contained in the **document** environment, you can indicate the start of a new paragraph with a blank line in your input file; that is why this sentence forms a separate paragraph.

2.1 Type Changes and *Special Characters*

We have already seen several typeface changes in this sample. You can indicate italicized words or phrases in your text with the command `\textit`; emboldening with the command `\textbf` and typewriter-style (for instance, for computer code) with `\texttt`. But remember, you do not have to indicate typestyle changes when such changes are part of the *structural* elements of your article; for instance, the heading of this subsection will be in a sans serif³ typeface, but that is handled by the document class file. Take care with the use of⁴ the curly braces in typeface changes; they mark the beginning and end of the text that is to be in the different typeface.

You can use whatever symbols, accented characters, or non-English characters you need anywhere in your document; you can find a complete list of what is available in the *\LaTeX User's Guide* [?].

2.2 Math Equations

You may want to display math equations in three distinct styles: inline, numbered or non-numbered display. Each of the three are discussed in the next sections.

2.2.1 Inline (In-text) Equations. A formula that appears in the running text is called an inline or in-text formula. It is produced

²This is a footnote.

³Another footnote here. Let's make this a rather long one to see how it looks.

⁴Another footnote.

by the `math` environment, which can be invoked with the usual `\begin . . . \end` construction or with the short form `$. . . $`. You can use any of the symbols and structures, from α to ω , available in \LaTeX [?]; this section will simply show a few examples of in-text equations in context. Notice how this equation: $\lim_{n \rightarrow \infty} x = 0$, set here in in-line math style, looks slightly different when set in display style. (See next section).

2.2.2 Display Equations. A numbered display equation—one set off by vertical space from the text and centered horizontally—is produced by the `equation` environment. An unnumbered display equation is produced by the `displaymath` environment.

Again, in either environment, you can use any of the symbols and structures available in \LaTeX ; this section will just give a couple of examples of display equations in context. First, consider the equation, shown as an inline equation above:

$$\lim_{n \rightarrow \infty} x = 0 \tag{1}$$

Notice how it is formatted somewhat differently in the `displaymath` environment. Now, we'll enter an unnumbered equation:

$$\sum_{i=0}^{\infty} x + 1$$

and follow it with another numbered equation:

$$\sum_{i=0}^{\infty} x_i = \int_0^{\pi+2} f \tag{2}$$

just to demonstrate \LaTeX 's able handling of numbering.

2.3 Citations

Citations to articles [? ? ? ?], conference proceedings [?] or maybe books [? ?] listed in the Bibliography section of your article will occur throughout the text of your article. You should use BibTeX to automatically produce this bibliography; you simply need to insert one of several citation commands with a key of the item cited in the proper location in the `.tex` file [?]. The key is a short reference you invent to uniquely identify each work; in this sample document, the key is the first author's surname and a word from the title. This identifying key is included with each item in the `.bib` file for your article.

The details of the construction of the `.bib` file are beyond the scope of this sample document, but more information can be found in the *Author's Guide*, and exhaustive details in the *\LaTeX User's Guide* by L^ampo^rt [?].

This article shows only the plainest form of the citation command, using `\cite`.

Some examples. A paginated journal article [?], an enumerated journal article [?], a reference to an entire issue [?], a monograph (whole book) [?], a monograph/whole book in a series (see 2a in spec. document) [?], a divisible-book such as an anthology or compilation [?] followed by the same example, however we only output the series if the volume number is given [?] (so Editor00a's series should NOT be present since it has no vol. no.), a chapter in a divisible book [?], a chapter in a divisible book in a series [?], a multi-volume work as book [?], an article in a proceedings (of a conference, symposium, workshop for example) (paginated proceedings article) [?], a proceedings article with all possible

Table 1: Frequency of Special Characters

Non-English or Math	Frequency	Comments
∅	1 in 1,000	For Swedish names
π	1 in 5	Common in math
\$	4 in 5	Used in business
Ψ ₁ ²	1 in 40,000	Unexplained usage

elements [?], an example of an enumerated proceedings article [?], an informally published work [?], a doctoral dissertation [?], a master's thesis: [?], an online document / world wide web resource [? ? ?], a video game (Case 1) [?] and (Case 2) [?] and [?] and (Case 3) a patent [?], work accepted for publication [?], 'YYYYb'-test for prolific author [?] and [?]. Other cites might contain 'duplicate' DOI and URLs (some SIAM articles) [?]. Boris / Barbara Beeton: multi-volume works as books [?] and [?].

A couple of citations with DOIs: [? ?].

Online citations: [? ? ?].

2.4 Tables

Because tables cannot be split across pages, the best placement for them is typically the top of the page nearest their initial cite. To ensure this proper "floating" placement of tables, use the environment `table` to enclose the table's contents and the table caption. The contents of the table itself must go in the `tabular` environment, to be aligned properly in rows and columns, with the desired horizontal and vertical rules. Again, detailed instructions on `tabular` material are found in the *\LaTeX User's Guide*.

Immediately following this sentence is the point at which Table 1 is included in the input file; compare the placement of the table here with the table in the printed output of this document.

To set a wider table, which takes up the whole width of the page's live area, use the environment `table*` to enclose the table's contents and the table caption. As with a single-column table, this wide table will "float" to a location deemed more desirable. Immediately following this sentence is the point at which Table 2 is included in the input file; again, it is instructive to compare the placement of the table here with the table in the printed output of this document.

It is strongly recommended to use the package `booktabs` [?] and follow its main principles of typography with respect to tables:

- (1) Never, ever use vertical rules.
- (2) Never use double rules.

It is also a good idea not to overuse horizontal rules.

2.5 Figures

Like tables, figures cannot be split across pages; the best placement for them is typically the top or the bottom of the page nearest their initial cite. To ensure this proper "floating" placement of figures, use the environment `figure` to enclose the figure and its caption.

This sample document contains examples of `.eps` files to be displayable with \LaTeX . If you work with pdf \LaTeX , use files in the `.pdf` format. Note that most modern \TeX systems will convert `.eps` to `.pdf` for you on the fly. More details on each of these are found in the *Author's Guide*.

Table 2: Some Typical Commands

Command	A Number	Comments
<code>\author</code>	100	Author
<code>\table</code>	300	For tables
<code>\table*</code>	400	For wider tables

Figure 1: A sample black and white graphic.

Figure 2: A sample black and white graphic that has been resized with the `includegraphics` command.

As was the case with tables, you may want a figure that spans two columns. To do this, and still to ensure proper “floating” placement of tables, use the environment `figure*` to enclose the figure and its caption. And don’t forget to end the environment with `figure*`, not `figure`!

2.6 Theorem-like Constructs

Other common constructs that may occur in your article are the forms for logical constructs like theorems, axioms, corollaries and proofs. ACM uses two types of these constructs: theorem-like and definition-like.

Here is a theorem:

THEOREM 2.1. *Let f be continuous on $[a, b]$. If G is an antiderivative for f on $[a, b]$, then*

$$\int_a^b f(t) dt = G(b) - G(a).$$

Here is a definition:

Definition 2.2. If z is irrational, then by e^z we mean the unique number that has logarithm z :

$$\log e^z = z.$$

The pre-defined theorem-like constructs are **theorem**, **conjecture**, **proposition**, **lemma** and **corollary**. The pre-defined definition-like constructs are **example** and **definition**. You can add your own constructs using the `amsthm` interface [?]. The styles used in the `\theoremstyle` command are `acmplain` and `acmdefinition`.

Another construct is **proof**, for example,

PROOF. Suppose on the contrary there exists a real number L such that

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = L.$$

Then

$$l = \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \left[g(x) \cdot \frac{f(x)}{g(x)} \right] = \lim_{x \rightarrow c} g(x) \cdot \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = 0 \cdot L = 0,$$

which contradicts our assumption that $l \neq 0$. \square

3 CONCLUSIONS

This paragraph will end the body of this sample document. Remember that you might still have Acknowledgments or Appendices; brief samples of these follow. There is still the Bibliography to deal with; and we will make a disclaimer about that here: with the exception of the reference to the \LaTeX book, the citations in this paper are to articles which have nothing to do with the present subject and are used as examples only.

A HEADINGS IN APPENDICES

The rules about hierarchical headings discussed above for the body of the article are different in the appendices. In the **appendix** environment, the command `section` is used to indicate the start of each Appendix, with alphabetic order designation (i.e., the first is A, the second B, etc.) and a title (if you include one). So, if you need hierarchical structure *within* an Appendix, start with **subsection** as the highest level. Here is an outline of the body of this document in Appendix-appropriate form:

A.1 Introduction

A.2 The Body of the Paper

A.2.1 Type Changes and Special Characters.

A.2.2 Math Equations.

Inline (In-text) Equations.

Display Equations.

A.2.3 Citations.

A.2.4 Tables.

A.2.5 Figures.

A.2.6 Theorem-like Constructs.

A Caveat for the \TeX Expert.

A.3 Conclusions

A.4 References

Generated by bibtex from your `.bib` file. Run latex, then bibtex, then latex twice (to resolve references) to create the `.bbl` file. Insert that `.bbl` file into the `.tex` source file and comment out the command `\thebibliography`.

B MORE HELP FOR THE HARDY

Of course, reading the source code is always useful. The file `acmart.pdf` contains both the user guide and the commented code.

Figure 3: A sample black and white graphic that needs to span two columns of text.

Figure 4: A sample black and white graphic that has been resized with the `includegraphics` command.

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