

# Lecture II

Few recommendations about you

1. When you do not know something (for example what Kronecker product or the Pascal triangle are) search *first* in [www.wikipedia.org](http://www.wikipedia.org), *then* ask to your colleagues and just *finally* ask to me. When you will program most probably me and your colleagues won't be there to help you.
2. When you do not know how to use a function before asking write "help nameofthefunction" (for example help mean) in the command window, read carefully the instructions, and try to understand the basic features of the function and how to adapt it to your problem.
3. When you get an error do not get panic, read what Matlab is telling you: usually it will try to help you. If you do not understand immediately the mistake try to decompose your expression in simple smaller parts.
4. Learning how to program is a trial-and-error process do not get frustrated, it takes time to write a code.

And about the problem sets

5. Each problem set contains more exercises than you can face in 30minutes, you may do the rest by yourself at home.
6. To make the class useful for all of you, exercises have different degree of complexity. Some are trivial some are harder and of course some are wrong: you are supposed to understand why and correct them.
7. Even if an exercise looks trivial try to do it anyway, I'm sure it will not be as trivial as you thought [please search overconfidence and overestimation in [www.wikipedia.org](http://www.wikipedia.org) ]

### Problem Set I [Matrix Access]

1. Given  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$  access 1, 3, 7, 9 using row-column index. Write  $A(1)$ ,  $A(2)$ ,  $A(3)$ ,  $A(4)$  do you understand what's going on? If yes, Access 6 by linear indexing.

$A(1,1)$ ,  $A(1,3)$ ,  $A(3,1)$ ,  $A(3,3)$ . Writing  $A(1)$ ,  $A(2)$ ,  $A(3)$ ,  $A(4)$  you should get 1 4 7 2 .  $A(8)$

2. Find a function that tells you how many rows and how many columns has matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$   
 $[r \ c] = \text{size}(A)$

3. Find an instructions that tell you how many elements are in A. Can you get the same results knowing the number of columns and rows of A?

$\text{numel}(A)$ ,  $[r \ c] = \text{size}(A)$  then  $\text{numel}(A) = r * c$

4. Given  $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  access its

- first row
- last row
- first column,
- second column
- two rows
- last two columns
- rows 1 and 3
- columns 2 and 4
- columns 5
- rows 5

$B(1,:), B(\text{end},:)$  or  $B(4,:), B(:,1), B(:,2), B(1:2,:), B(:,3:4)$  or  $B(:, \text{end}-1:\text{end}), B([1 \ 3],:), B(:, [2 \ 4])$ ,  
 you can not access the 5<sup>th</sup> row or the 5<sup>th</sup> column of a 4x4 Matrix

5. Given B of point 4 obtain the submatrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   
 $B(1:2,1:2)$  or  $B(3:4,3:4)$

6. Given  $C = \begin{bmatrix} 1 & 5 & 6 & 7 \\ 2 & 1 & 5 & 6 \\ 3 & 2 & 1 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$  access the main diagonal (i.e. [1 1 1 1]), and the two minor diagonals: [5 5 5] and [2 2 2]

$\text{diag}(C), \text{diag}(C,1), \text{diag}(C,-1)$

7. Given C in previous point what happen if input  $\text{tril}(C)+\text{triu}(C)$ ? does it make sense to you? What about  $\text{triu}(C,1)+\text{tril}(C,-1)$ ? Find a way of getting exactly C using  $\text{triu}()$  and  $\text{tril}()$

$\text{tril}(C,-1)+\text{triu}(C)$  or  $\text{tril}(C)+\text{triu}(C,1)$

8. Given matrix  $A = \begin{bmatrix} 1 & 3 & 3 & 3 \\ 2 & 1 & 3 & 3 \\ 2 & 2 & 1 & 3 \\ 2 & 2 & 2 & 1 \end{bmatrix}$  using  $\text{tril}$  and  $\text{triu}$  create  $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\text{tril}(C,-1), \text{triu}(C,2)$

9. Given  $A = \text{randi}([0 \ 10], 1, 10)$  do the following
- Input  $\text{find}(A > 5)$ . Do you get all the elements in A that are  $> 5$ ? Do you get a kind of “indexing”? If yes, which kind of indexing, “linear” or “row-column”?  
*no, just their linear index*
  - Using information in point a. retrieve the elements in A that are  $> 5$   
 $A(A > 5)$
  - Input  $\text{find}(A > 5 \ \& \ A < 8)$ . Can you explain the result? If yes obtain a vector that contains the elements in A that are between 5 and 8.  
 $A(\text{find}(A > 5 \ \& \ A < 8))$
  - What happens if you input  $\text{find}(A > 5 \ \& \ A < 4)$ ,  $\text{find}(A > 5 \ | \ A < 4)$  and  $\text{find}(A \geq 5 \ | \ A < 4)$ ?  
*empty matrix, there's no number that can be at the same time  $> 5$  and  $< 4$*   
*returns the index of all the number except 5*  
*returns the index of all the numbers*
  - Find the elements in A such that  $a \in [0, 2] \cup [8, 10]$   
 $A(\text{find}((A \geq 0 \ \& \ A \leq 2) \ | \ (A \geq 8 \ \& \ A \leq 10)))$
  - Find the elements in A such that  $a \in (0, 2] \cup [8, 10)$   
 $A(\text{find}((A > 0 \ \& \ A \leq 2) \ | \ (A \geq 8 \ \& \ A < 10)))$
  - Find the elements in A such that  $a \in [0, 2] \cap [8, 10]$   
 $A(\text{find}((A \geq 0 \ \& \ A \leq 2) \ \& \ (A \geq 8 \ \& \ A \leq 10)))$  *returns an empty matrix*
  - Find the elements in A that are less than 5 but different from 0  
 $A(\text{find}(A < 5 \ \& \ A \neq 0))$
  - Find the elements in A that are less than 5 or different from 0  
 $A(\text{find}(A < 5 \ | \ A \neq 0))$
10. Given matrix A in point 9, do the following
- Input  $A > 5$ . Can you understand the difference between  $\text{find}(A > 5)$  and  $A > 5$ ?  
*does not return an index but a logical vector where 1 stays for “true” and 0 for “false”*
  - Using information in point a retrieve the elements in A that are  $> 5$   
 $A(A > 5)$
11. Why the output of  $1 > 1$  is 0 and of  $1 \geq 1$  is 1?

*1 is not strictly bigger than 1 therefore  $1 > 1$  is false*

*1 is equal 1 therefore  $1 \geq 1$  is true (to be true just one of the two conditions ">" or "=" must be satisfied)*

12. Can you understand the output of  $1 \sim 1, 1 == 1, 1 \sim \sim 1, (1 \sim 1 \& 1 == 1), (1 \sim 1 \mid 1 == 1)$ ? Despite they might seem useless (they concerns Aristotle' Logic indeed) they can be very useful when you will program...and of course in case you want to read Aristotle' works.

*1 is equal 1 therefore "1 different from 1" is false and the output is 0!*

*1 is equal 1 therefore the output is 1 (true).*

*1 is different from not 1, therefore the output is 1 (true).*

*$1 \sim 1$  is false,  $1 == 1$  is true and the whole statement is false (to be true both must be true)*

*$1 \sim 1$  is false,  $1 == 1$  is true and the whole statement is true (to be true at least one statement must be true)*

13. Input  $B = \text{randi}([0 \ 1], 1, 10)$ , what happen if you write  $A(B)$ ? Why matlab does not allow this operation? Why it allows  $A(\text{logical}(B))$ ? [hint: input "help logical" in command window, then write  $C = A > 5$ , go to your workspace –see lecture 1- and compare what's written under the label "Value" For your B and C]

*it returns '??? Subscript indices must either be real positive integers or logicals.' Since 0 is not real positive Matlab does not allow the operation and you have to convert the numbers in B in logical statements.*

14. Given  $A = \text{randn}(5)$  set all the negative elements to 0 and all the positive ones to 1

$A(A < 0) = 0$

$A(A > 0) = 1$

15. Given matrix A in the previous point set all the negative elements *in the first row* to zero.

$A(1, \text{find}(A(1, :) < 0)) = 0$

16. Given A and B both equal to  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , can you understand the output  $\text{isequal}(A, B)$ ,  $\text{isequal}(A, B) == 1$  and  $\text{isequal}(A, B) == 0$ . What could you use instead of the function "isequal"?

*since A is equal to B then the function returns 1 (true)*

*since is true that A is equal to B then the output is 1 (true)*

*since is false that A is not equal to B the output is 0 (false)*

*$A == B$  can be used*

17. Given  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$  obtain  $B = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 6 & 7 \\ 0 & 8 & 9 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 0 \\ 7 & 0 & 0 \end{bmatrix}$  using function  $\text{pascal}(3)$  and the logical operator  $s == \sim$  [hint:  $A(\text{pascal}(3) \dots) = \dots$ ]

$A(\text{pascal}(3) == 1) = 0$   $A(\text{pascal}(3) \sim 1) = 0$

18. Given  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$  obtain  $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & 0 & 15 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 6 & 0 & 8 \\ 9 & 0 & 11 & 0 \\ 0 & 14 & 0 & 0 \end{bmatrix}$  using function `hankel(1:4,4:-1:1)` and logical operator “`~=`” [hint `A(hankel(1:4,4:-1:1)~=...)=0`]

$$A(\text{hankel}(1:4,4:-1:1) \sim= 2) = 0$$

$$A(\text{hankel}(1:4,4:-1:1) \sim= 3) = 0$$

19. Given A in point 17 what happen if you write `A([1 2 3; 2 3 2; 3 2 1])`. Why you get different results every time you write `A(randi([1 9],3))`? Why you might get an error if you write `A(randi([1 10],3))`? Why you will have almost surely an error writing `A(randi([1 20],3))`?  
*It extracts some entries of A, according to their linear indexing*  
*It extracts a random sample from A*  
*You might get an error because it might extract 10 but since the matrix is 3X3 there's no 10<sup>th</sup> element.*  
*Because most probably you will get a number higher than 9*

20. Given matrix A in point 18 substitutes the elements in its main diagonal (i.e. 1 6 11 16) with zeros [hint use `eye()`, the function `logical()`]  
 $A(\text{logical}(\text{eye}(\text{size}(A)))) = 0$

## Problem Set II [Single Matrix Manipulation]

1. Given a matrix  $M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  by adding a row and then a column obtain a matrix  $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$$M = [M; 1:2] \quad M = [M \ (1:3)']$$

2. From matrix M obtain a column vector  $v1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$M(:, 1:2) = []$$

3. From matrix M obtain a row vector  $v2 = [1 \ 2 \ 3]$

$$M(1:2, :) = []$$

4. From matrix M obtain a Matrix  $M2 = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$

$$M(:, 3) = []$$

5. From matrix M obtain a Matrix  $M3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

$$M(3, :) = []$$

6. Concatenate, if Matlab allows it, horizontally and vertically these couples of matrices

$$A1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } A2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } B2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$M = [A1 \ A2]$  is possible  $M = [A1; A2]$  is not they do not have the same numb of cols

$M = [B1 \ B3]$  is not possible  $M = [B1; B3]$  it is

7. From  $C1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  obtain a column vector and a row vector using reshape function

$$M = \text{reshape}(C1, 1, 6)$$

$$M = \text{reshape}(C1, 6, 1)$$

8. From  $C2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$  obtain a matrix  $C3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\text{reshape}(C2, 2, 4)$$

9. What's the difference between `transpose(1:10)`, `(1:10)'` and `1:10'`? why `transpose (1:10)'` is equal to `1:10'`?

*`transpose(1:10)` is the same of `(1:10)'`. `1:10'` is different, i.e. is still a row vector because the transpose operator acts only on 10, and being a scalar we know that  $n'=n$  because  $(x')'=x$*

10. Apply to a matrix 4x4 of uniform random integer numbers between 1 and 10 the functions `sort` and `flipud`, what's the difference?

*function "sort" sorts the value row by row function `flipud` just swaps the rows*

11. Given the previous matrix, sort the numbers by row and not by column. Would you get the same result using function `fliplr`?

*`sort(X,2)`, you will not get the same result*

12. Using Kronecker product, from a matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  obtain the following matrices

a) 
$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{bmatrix}$$
*`kron(eye(2),A)`*

b) 
$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{bmatrix}$$
*`kron(A,eye(2))`*

c) 
$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 3 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 \end{bmatrix}$$
*`kron(A,ones(2))`*

d) 
$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{bmatrix}$$
*`kron(ones(2),A)`*

13. Obtain matrix d) in previous point using `repmat()` and matrix a) using function `blkdiag()`

*`repmat(A,2,2)`  
`blkdiag(A,A)`*

14. Read the documentation for the function `circshift` and from
- |    |   |   |   |           |   |   |   |
|----|---|---|---|-----------|---|---|---|
|    | 1 | 2 | 3 |           | 4 | 5 | 6 |
| A= | 4 | 5 | 6 | obtain B= | 7 | 8 | 9 |
|    | 7 | 8 | 9 |           | 1 | 2 | 3 |

```

6 4 5
C=9 7 8
3 1 2

```

```

B=circshift(A,[2])
C=circshift(A,[2 1])

```

15. Given matrix A in previous point, can you see the difference between `rot90(A)` and `A'` ? what happen if you write `rot90(A,1)` `rot90(A,2)` `rot90(A,3)` `rot90(A,4)`? what if `rot90(A,5)`?

*rot90(A) is different from A', rot90(A,5) is equal to rot90(A,1)*

16. From matrix
- |    |   |   |   |
|----|---|---|---|
|    | 1 | 2 | 3 |
| A= | 4 | 5 | 6 |
|    | 7 | 8 | 9 |
- obtain a column vector `a=[1 2 3 4 . . . 9]'`. Applying `reshape(A,9,1)` is not enough...

```
reshape(A',9,1)
```

17. Generate a population of 20 random variables from a  $N(1,2)$  and pick randomly from it a sub sample of 5 observations (do not worry if the same variable appear more than once i.e. sample with replacement)

```
R=random('norm',1,2,20,1) R(randi([1 20],1,5))
```

18. Generate a matrix of ones of random rows and columns dimensions (no bigger than 10x10).

```
ones(ceil(rand*10),ceil(rand*10))
```

19. Given `A=[(1:4)' 2*(1:4)']` Can you explain why `rank(A)=1` while `rank(B)=2` where `B=[(1:4)' (1:2:8)']`

*in matrix A the second column is the double of the first therefore there is just one linear independent column*

20. Given `A=eye(5)`, why `trace(A)=sum(diag(A))=5`?

*By definition of trace: the sum of the elements in the diagonal*

21. Given `A=randi([10 20],10)`, why `A*inv(A)` is equal to `eye(size(A))`?

*by definition of inverse matrix,  $A * A^{-1} = I$*

22. Why `L*U=A` where L and U are obtained by `[L U]=lu(A)`.

*read the output of help lu*



### Problem Set III [Multiple Matrices Manipulation]

1. Compute  $\sqrt{2}$   $\frac{1}{\sqrt{2}}$   $\sqrt[4]{2}$   $\frac{1}{\sqrt[4]{2}}$   $\sqrt{\frac{1}{\sqrt[3]{2}}}$   $\sqrt{\frac{1}{\sqrt[5]{2^3}}}$

$2^{(1/2)}$  or  $\text{sqrt}(2)$

$1/\text{sqrt}(2)$

$2^{1/4}$

$1/(2^{(1/4)})$

$\text{sqrt}(1/2^{(1/3)})$

$\text{sqrt}(1/2^{(3/5)})$

2. Compute  $\frac{4}{\log(1)}$   $\frac{4}{0}$   $\frac{0}{0}$

$4/\log(1)$ ,  $4/0$ ,  $0/0$

3. Compute  $e^{\log(5)}$   $\log e^5$

$\exp(\log(5))$ ,  $\log(\exp(5))$

4. What's the difference for matlab between  $4/2$  and  $4\backslash 2$ ?

$\frac{4}{2}$  and  $\frac{2}{4}$

5. Compute  $\frac{\sqrt{(\log e^4)/2}}{2\pi}$

$(\text{sqrt}(\log(\exp(4)))/2)/2*\pi$

6. Add  $v1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  to  $v2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ; Subtract  $v1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  from  $v2 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ . If Matlab gives you an error make some changes in order to make the operations possible.

$v1-v1$  is not possible since they do not have the same size

$v2-v1$  is not possible since  $v1$  is a column vector while  $v2$  is a row vector

7. Multiply  $v1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  with  $v2 = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$ ; do the reverse, multiply  $v2$  with  $v1$ . Without inputting the two

operations in the command window can you "predict" which is possible (and why)?

$v1*v2$  equals a matrix

$v2*v1$  is not possible

8. Multiply  $v1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  with  $v2 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ ; do the reverse multiply  $v2$  with  $v1$

<sup>1</sup> Use the helper to find how to compute natural logarithm and exponential

$v1*v2$  is a matrix (3x3)

$v2*v1$  is a scalar

9. Given  $v1=[1 \ 2 \ 3]$  and  $v2=[2 \ 4 \ 6]$ . What's the difference between  $v1./v2$  and  $v2.\backslash v1$ ?

$v1./v2$  is equal  $\frac{1}{2} \frac{2}{4} \frac{3}{6}$

$v2.\backslash v1$  is equal  $\frac{2}{1} \frac{4}{2} \frac{6}{3}$

10. Multiply vector  $v1=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $v2=\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$  in such a way to get  $v3=\begin{bmatrix} 8 \\ 12 \end{bmatrix}$

$v3=v1.*v2$

11. Given three stocks with the following returns  $r_1=0.02$   $r_2=-0.03$   $r_3=0.05$  Compute the return of a portfolio with the following weights  $\omega_1=0.3$   $\omega_2=0.6$   $\omega_3=0.1$ . [hint.

$$R^{port} = \sum_{i=1}^N \omega_i R_i ]$$

$w=[0.3 \ 0.6 \ 0.1]$ ;  $R=[0.02 \ -0.03 \ 0.05]$ ;

$Rp=w*R'$

$Rp=\text{sum}(w.*R)$

12. Add  $M1=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  to  $M2=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$  ; Subtract  $M1=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$  from  $M2=\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}$  .

*None of them is possible since M1 and M2 do not have the same size*

13. Multiply  $M1=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  with  $M2=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$  ; do the reverse multiply M2 with M1. Without

inputting the two operations in the command window can you "predict" which of the two operations is impossible (and why)?

*M1\*M2 is possible since the number of columns of M1 is equal to the number of rows of M2*

*M2\*M1 is not possible the commutative property does not hold for matrices*

14. Given  $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B=\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$  compute  $C=A*B$  and  $D=B*A$ . Are C and D equal? If yes, why?

*they are different*

15. Given  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ,  $B = \text{eye}(3)$ ,  $C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ ,  $D = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ ,  $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ,  $F = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . Compute  $B*A$ ,  $C*A$ ,  $D*A$ ,  $E*A$ ,  $F*A$ . Can you explain what is going on here?

$B*A=A$ , the other 4 operations make some changes in the order of the rows of  $A$

16. Given three stocks with variance  $\sigma_1^2 = 0.6$ ,  $\sigma_2^2 = -0.4$ ,  $\sigma_3^2 = 0.1$  and covariance  $\sigma_{12} = 0.9$ ,  $\sigma_{13} = -0.4$ ,  $\sigma_{23} = 0.3$ , compute the variance of a portfolio with weights  $\omega_1 = 0.4$ ,  $\omega_2 = 0.2$ ,  $\omega_3 = 1 - \omega_1 - \omega_2$  using the usual formula  $\sigma_p^2 = \omega' S \omega$ . Is there anything strange in the inputs?

$$w = [0.4 \ 0.2 \ 0.4]'; S = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.9 & -0.4 \\ 0.9 & -0.4 & 0.3 \\ -0.4 & 0.3 & 0.1 \end{bmatrix}$$

$$\text{PortVariance} = w' * S * w$$

Variance can not be negative by definition therefore  $\sigma_2^2 = -0.4$  is wrong

17. Given  $v_1 = [1 \ 2 \ 3]$  and  $M_1 = \text{ones}(3)$  compute  $v_2 = v_1^2$ ,  $M_2 = M_1^3$  and  $M_3 = M_1.^3$ . Can you explain why you get such results?

$$v_2 = [1^2 \ 2^2 \ 3^2]$$

$$M_2 = M_1 * M_1 * M_1$$

$M_3$  is a matrix where each elements appears at power of 3

18. Given  $a = [1 \ 1 \ 1]$  and  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$  what happen if you do the following  $a*A$ ,  $A*a$ ,  $a'*A$ ,  $a'*A$ ? Are you able to "predict" which operation is possible which not?

$a*A$  is a 1x3 row vector

$A*a$  and  $a'*A$  are not possible

$a'*A$  is a 3x1 column vector

19. Build a two Nobsx1 vectors  $k$  and  $y$  where each entry of  $y$  is  $y_t = 3 + 0.5 * X_{1t} + 0.9 * X_{2t} + \varepsilon_t$  where  $X_{it} \sim N(i, 2)$  and  $\varepsilon_t \sim N(0, 1)$ ; each entry of  $k$  is  $k_t = 3 + 0.5 * X_{1t} + 0.9 * X_{2t}$  where  $X_{it} \sim N(i, 2)$ . Using the standard OLS formula estimate  $\hat{\beta} = (X'X)^{-1}X'y$  where  $X$  is your data

matrix, in this case:  $\begin{bmatrix} 1 & X_{1,1} & X_{2,1} \\ \vdots & \vdots & \vdots \\ 1 & X_{1,50} & X_{2,50} \end{bmatrix}$ . Do it for Nobs=50, Nobs=1000 and Nobs=5000. Which results

do you expect for the betas computed using  $y$  and  $k$ ?

Since we simulated the data with an error term, we expect that Betas for  $y$  are respectively around 3, 0.5 and 0.9, with the estimates getting closer to the real values as Nobs increase.

Since we simulated the data with no error term, we expect that Betas for  $k$  are respectively exactly 3 0.5 and 0.9 independently on the Nobs.

```
Nobs=5000
X1=normrnd(1,2,Nobs,1)
X2=normrnd(2,2,Nobs,1)
y=3*ones(Nobs,1)+0.5*X1+0.9*X2+randn(Nobs,1)
X=[ones(Nobs,1) X1 X2]
Beta=[inv(X'*X)*X'*y]
```

```
Nobs=100
X1=normrnd(1,2,Nobs,1)
X2=normrnd(2,2,Nobs,1)
k=3*ones(Nobs,1)+0.5*X1+0.9*X2
X=[ones(Nobs,1) X1 X2]
Beta=[inv(X'*X)*X'*k]
```

20. Given  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  compute  $B = \text{inv}(A)$  and  $C = 1./A$ . Can you explain the results?

$$B = A^{-1} \text{ while } C = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$$

21. Using the product of two vectors write the multiplication table

$$(1:10)' * (1:10)$$

22. Given  $a = \text{randi}([0 \ 1], 10, 1)$  create vector  $b$  such that has the same length of  $a$  but where  $a$  has a 1  $b$  has 0 and viceversa when  $a$  has 0  $b$  has 1

$$\text{abs}(a-1)$$

23. Using product vector compute the sum of the elements in  $v1 = [1 \ 2 \ 3 \ 4 \ 5]$

$$\text{ones}(5,1) * v1'$$

24. Using product vector write 10

$$\text{ones}(1,10) * \text{ones}(10,1)$$

25. Using product vector replicate the result of  $X = \text{repmat}((1:3)', 1, 5)$

$$(1:3)' * \text{ones}(1,5)$$

26. Using product vector replicate the result of  $X = \text{repmat}((1:3), 5, 1)$

$$\text{ones}(5,1) * (1:3)$$

27. Write these two series: 2 4 8 16 32 64 128 256 512... and 1 4 9 16 25 36 49 64 81 100 using the power operator

$2.^{[1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10]}$   
 $[1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10].^2$

28. Given matrix A of exercise 14 build a matrix B whose first row is the sum of row 1 and 2 of matrix A, whose second row is the sum of row 2 and 3 whose last row is the sum of row 1,2 and 3 of matrix A.

$$\begin{matrix} & 1 & 1 & 0 \\ \text{Use matrix } G= & 0 & 1 & 1 \\ & 1 & 1 & 1 \end{matrix}$$

29. Given a 10x2 matrix of r.v I.I.N(1,2), build the corresponding demeaned values

$X=\text{Random}('norm',1,2,10,2), \text{Demeaned}=X-\text{ones}(10,1)*\text{mean}(X)$

30. Given the demenead values, standardize them [recall that if  $x \sim N(\mu, \sigma^2)$  then  $z = \frac{x-\mu}{\sigma}$  with  $z \sim N(0,1)$ ]

$\text{Standardized}=\text{Demeaned}./(\text{ones}(10,1)*\text{sqrt}(\text{var}(X)))$