Lecture II

Few recommendations about you

1. When you do not know something (for example what Kronecker product or the Pascal triangle are) search *first* in www.wikipedia.org, *then* ask to your collegues and just *finally* ask to me. When you will program most probably me and your collegues won't be there to help you.
2. When you do not know how to use a function before asking write "help nameofthefunction" (for example help mean) in the command window, read carefully the instructions, and try to understand the basic features of the function and how to adapt it to your problem.
3. When you get an error do not get panic, read what Matlab is telling you: usually it will try to help you. If you do not understand immediately the mistake try to decompose your expression in simple smaller parts.
4. Learning how to program is a trial-and-error process do not get frustrated, it takes time to write a code.

And about the problem sets

5. Each problem set contains more exercises than you can face in 30minutes, you may do the rest by yourself at home.
6. To make the class useful for all of you, exercises have different degree of complexity. Some are trivial some are harder and of course some are wrong: you are supposed to understand why and correct them.
7. Even if an exercise looks trivial try to do it anyway, I'm sure it will not be as trivial as you though [please search overconfidence and overestimation in www.wikipedia.org]

Problem Set I [Matrix Access]

1. Given $A=\begin{matrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{matrix}$ access 1, 3, 7, 9 using row-column index. Write A(1), A(2), A(3), A(4) do you understand what's going on? If yes, Access 6 by linear indexing.

2. Find a function that tells you how many rows and how many columns has matrix $A=\begin{matrix}1 & 2\\ 3 & 4\\ 5 & 6\end{matrix}$

3. Find an instructions that tell you how many elements are in A. Can you get the same results knowing the number of columns and rows of A?

4. Given $B=\begin{matrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{matrix}$ access its
   a. first row
   b. last row
   c. first column,
   d. second column
   e. two rows
   f. last two columns
   g. rows 1 and 3
   h. columns 2 and 4
   i. columns 5
   j. rows 5

5. Given B of point 4 obtain the submatrix $\begin{matrix}1 & 0\\ 0 & 1\end{matrix}$

6. Given $C=\begin{matrix}1 & 5 & 6 & 7\\ 2 & 1 & 5 & 6\\ 3 & 2 & 1 & 5\\ 4 & 3 & 2 & 1\end{matrix}$ access the main diagonal (i.e. [1 1 1 1]),and the two minor diagonals: [5 5 5] and [2 2 2]

7. Given C in previous point what happen if input tril(C)+triu(C)? does it make sense to you? What about triu(C,1)+tril(C,-1)? Find a way of getting exactly C using triu()+tril()

8. Given matrix $A=\begin{matrix}1 & 3 & 3 & 3\\ 2 & 1 & 3 & 3\\ 2 & 2 & 1 & 3\\ 2 & 2 & 2 & 1\end{matrix}$ using tril and triu create $B=\begin{matrix}0 & 0 & 0 & 0\\ 2 & 0 & 0 & 0\\ 2 & 2 & 0 & 0\\ 2 & 2 & 2 & 0\end{matrix}$ and $C=\begin{matrix}0 & 0 & 3 & 3\\ 0 & 0 & 0 & 3\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{matrix}$

9. Given A=randi([0 10],1,10) do the following
    a. Input find(A>5). Do you get all the elements in A that are > 5? Do you get a kind of "indexing"? If yes, which kind of indexing, "linear" or "row-column"?
    b. Using information in point a. retrieve the elements in A that are >5
    c. Input find (A>5 & A<8). Can you explain the result? If yes obtain a vector that contains the elements in A that are between 5 and 8.
    d. What happends if you input find (A>5 & A<4), find (A>5 | A<4) and find (A>=5 | A<4)?
    e. Find the elements in A such that $a \in [0,2] \cup [8,10]$
    f. Find the elements in A such that $a \in (0,2] \cup [8,10)$
    g. Find the elements in A such that $a \in [0,2] \cap [8,10]$
    h. Find the elements in A that are less than 5 but different from 0
    i. Find the elements in A that are less than 5 or different from 0

10. Given matrix A in point 9, do the following
    a. Input A>5. Can you understand the difference between find(A>5) and A>5?
    b. Using information in point a retrieve the elements in A that are >5

11. Why the output of 1>1 is 0 and of 1>=1 is 1?

12. Can you understand the output of 1~=1,1==1, 1~=~1, (1~=1 & 1==1), (1~=1 | 1==1)? Despite they might seem useless (they concerns Aristotle' Logic indeed) they can be very useful when you will program…and of course in case you want to read Aristotle' works.

13. Input B=randi([0 1],1,10), what happen if you write A(B)? Why matlab does not allow this operation? Why it allows A(logical(B))? [hint: input "help logical" in command window, then write C=A>5, go to your workspace –see lecture 1- and compare what's written under the label "Value" For your B and C]

14. Given A=randn(5) set all the negative elements to 0 and all the positive ones to 1

15. Given matrix A in the previous point set all the negative elements *in the first row* to zero.

16. Given A and B both equal to $\begin{matrix}1 & 2\\ 3 & 4\end{matrix}$, can you understand the output isequal(A,B), isequal(A,B)==1 and isequal(A,B)==0. What could you use instead of the function "isequal"?

17. Given A=$\begin{matrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{matrix}$ obtain B=$\begin{matrix}0 & 0 & 0\\ 0 & 5 & 6\\ 0 & 8 & 9\end{matrix}$ and C=$\begin{matrix}1 & 2 & 3\\ 4 & 0 & 0\\ 7 & 0 & 0\end{matrix}$ using function pascal(3) and the logical operator s "==" "~=" `[hint: A(pascal(3)…)=…]`

18. Given A= $\begin{matrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{matrix}$ obtain A= $\begin{matrix} 0 & 2 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & 0 & 15 & 0 \end{matrix}$ and B= $\begin{matrix} 0 & 0 & 3 & 0 \\ 0 & 6 & 0 & 8 \\ 9 & 0 & 11 & 0 \\ 0 & 14 & 0 & 0 \end{matrix}$ using function hankel(1:4,4:-1:1) and logical operator “~=” [hint A(hankel(1:4,4:-1:1) ~=…)=0]

19. Given A in point 17 what happen if you write A([1 2 3; 2 3 2; 3 2 1]). Why you get different results every time you write A(randi([1 9],3))? Why you might get an error if you write A(randi([1 10],3))? Why you will have almost surely an error writing A(randi([1 20],3))?

20. Given matrix A in point 18 substitutes the elements in its main diagonal (i.e. 1 6 11 16) with zeros [hint use eye(), the function logical()]

Problem Set II [Single Matrix Manipulation]

1. Givena a matrix M=$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$ by adding a row and then a column obtain a matrix M=$\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 3 \end{matrix}$

2. From matrix M obtain a column vector v1=$\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$

3. From matrix M obtain a row vector v2=1 2 3

4. From matrix M obtain a Matrix M2=$\begin{matrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \end{matrix}$

5. From matrix M obtain a Matrix M3= $\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 2 \end{matrix}$

6. Concatenate, if Matlab allows it, horizontally and vertically these couples of matrices

   A1=$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$ and A2=$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$

   B1=$\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$ and B2=$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$

7. From C1=$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$ obtain a column vector and a row vector using reshape function

8. From C2=$\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}$ obtain a matrix C3=$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

9. What's the difference between transpose(1:10), (1:10)' and 1:10'? why transpose (1:10)' is equal to 1:10?

10. Apply to a matrix 4x4 of uniform random integer numbers between 1 and 10 the functions sort and flipud, what's the difference?

11. Given the previous matrix, sort the numbers by row and not by column. Would you get the same result using function fliplr?

12. Using Kronecker product, from a matrix A= $\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$ obtain the following matrices

a) $\begin{matrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{matrix}$

b) $\begin{matrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{matrix}$

c) $\begin{matrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 3 & 3 & 4 & 4 \\ 3 & 3 & 4 & 4 \end{matrix}$

d) $\begin{matrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{matrix}$

13. Obtain matrix d) in previous point using repmat() and matrix a) using function blkdiag()

14. Read the documentation for the function circshift and from $A=\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$ obtain $B=\begin{matrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{matrix}$

$C=\begin{matrix} 6 & 4 & 5 \\ 9 & 7 & 8 \\ 3 & 1 & 2 \end{matrix}$

15. Given matrix A in previous point, can you see the difference between rot90(A) and A’ ? what happen if you write rot90(A,1) rot90(A,2) rot90(A,3) rot90(A,4)? what if rot90(A,5)?

16. From matrix $A=\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$ obtain a column vector a=[1 2 3 4 . . . .9]’. Applying reshape(A,9,1) is not enough…

17. Generate a population of 20 random variables from a N(1,2) and pick randomly from it a sub sample of 5 observations (do not worry if the same variable appear more than once i.e. sample with replacement)

18. Generate a matrix of ones of random rows and columns dimensions (no bigger than 10x10).

19. Given A=[(1:4)' 2*(1:4) '] Can you explain why rank(A)=1 while rank(B) =2 where B=[(1:4)' (1:2:8)']

20. Given A=eye(5), why trace(A)=sum(diag(A))=5?

21. Given A=randi([10 20],10), why A*inv(A) is equal to eye(size(A)))?

22. Why L*U=A where L and U are obtained by [L U]=lu(A).

Problem Set III [Multiple Matrices Manipulation]

1. Compute $\sqrt{2}$ $\frac{1}{\sqrt{2}}$ $\sqrt[4]{2}$ $\frac{1}{\sqrt[4]{2}}$ $\sqrt{\frac{1}{\sqrt[3]{2}}}$ $\sqrt{\frac{1}{\sqrt[5]{2^3}}}$

2. Compute[1] $\frac{4}{\log(1)}$ $\frac{4}{0}$ $\frac{0}{0}$

3. Compute $e^{\log(5)}$ log^{e^5}

4. What's the difference for matlab between 4/2 and 4\2?

5. Compute $\frac{(\sqrt{(loge \wedge 4)/2})}{2\pi}$

6. Add v1=$\begin{matrix}1\\2\\3\end{matrix}$ to v2=$\begin{matrix}1\\2\\3\\4\end{matrix}$; Subtract v1=$\begin{matrix}1\\2\\3\end{matrix}$ from v2=1 2 3. If Matlab gives you an error make some changes in order to make the operations possible.

7. Multiply v1=$\begin{matrix}1\\2\\3\\4\end{matrix}$ with v2=1 2; do the reverse, multiply v2 with v1. Without inputing the two operations in the command window can you "predict" which is possible (and why)?

8. Multiply v1= $\begin{matrix}1\\1\\1\end{matrix}$ with v2=1 0 1; do the reverse multiply v2 with v1

9. Given v1=[1 2 3] and v2=[2 4 6]. What's the difference between v1./v2 and v2.\v1?

10. Multiply vector v1=$\begin{matrix}1\\2\\3\end{matrix}$ and v2=$\begin{matrix}2\\4\\6\end{matrix}$ in such a way to get v3= $\begin{matrix}2\\8\\12\end{matrix}$

11. Given three stocks with the following returns r_1=0.02 $r_2 = -0.03$ $r_3 = 0.05$ Compute the return of a portfolio with the following weights ω_1=0.3 $\omega_2 = 0.6$ $\omega_3 = 0.1$. [hint. $R^{port} = \sum_{i=1}^{N} \omega_i \; R_i$]

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[1] Use the helper to find how to compute natural logarithm and exponential

12. Add M1=$\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix}$ to M2=$\begin{matrix}1&1\\1&1\\1&1\end{matrix}$; Subtract M1=$\begin{matrix}1&2&3\\1&2&3\end{matrix}$ from M2=$\begin{matrix}1&1\\2&2\\3&3\end{matrix}$.

13. Multiplty M1=$\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix}$ with M2=$\begin{matrix}1&1\\1&1\\1&1\end{matrix}$; do the reverse multiply M2 with M1. Without inputing the two operations in the command window can you "predict" which of the two operations is impossible (and why)?

14. Given A=$\begin{matrix}1&2\\3&4\end{matrix}$ and B=$\begin{matrix}4&3\\2&1\end{matrix}$ compute C=A*B and D=B*A. Are C and D equal? If yes, why?

15. Given A=$\begin{matrix}1&2&3\\4&5&6\\7&8&9\end{matrix}$, B=eye(3), C=$\begin{matrix}0&0&1\\0&1&0\\1&0&0\end{matrix}$, D=$\begin{matrix}0&0&1\\0&1&0\\0&0&1\end{matrix}$, E=$\begin{matrix}0&1&0\\1&0&0\\0&1&0\end{matrix}$, F=$\begin{matrix}1&1&0\\0&1&0\\0&0&1\end{matrix}$. Compute B*A, C*A, D*A , E*A , F*A. Can you explain what is going on here?

16. Given three stocks with variance $\sigma_1^2 = 0.6 \quad \sigma_2^2 = -0.4 \quad \sigma_3^2 = 0.1$ and covariance $\sigma_{12} = 0.9 \quad \sigma_{13} = -0.4 \quad \sigma_{23} = 0.3$, compute the variance of a portfolio with weights $\omega_1 = 0.4 \ \ \omega_2 = 0.2 \quad \omega_3 = 1 - \omega_1 - \omega_2$ using the usual formula $\sigma_p^2 = \omega' S \omega$. Is there anything strange in the inputs?

17. Given v1=1 2 3 and M1=ones(3) compute v2=v1^2 , M2=M1^3 andM3= M.^3. Can you explain why you get such results?

18. Given a=[1 1 1] and A=$\begin{matrix}1&2&3\\4&5&6\\7&8&9\end{matrix}$ what happen if you do the following a*A, A*a, a'*A,a'*A? Are you able to "predict" which operation is possible which not?

19. Build a two Nobsx1 vectors k and y where each entry of y is $y_t = 3 + 0.5 * X_{1t} + 0.9 * X_{2t} + \varepsilon_t$ where $X_{it} \sim N(i, 2)$ and $\varepsilon_t \sim N(0,1)$; each entry of k is $k_t = 3 + 0.5 * X_{1t} + 0.9 * X_{2t}$ where $X_{it} \sim N(i, 2)$. Using the standard OLS formula estimate $\hat{\beta} = (X'X)^{-1}X'y$ where X is your data matrix, in this case: $\begin{matrix}1 & X_{1,1} & X_{2,1}\\ \vdots & \vdots & \vdots \\ 1 & X_{1,50} & X_{2,50}\end{matrix}$. Do it for Nobs=50, Nobs=1000 and Nobs=5000. Which results do you expect for the betas computed using y and k?

20. Given A=$\begin{matrix}1&2\\3&4\end{matrix}$ compute B=inv(A) and C=1./A. Can you explain the results?

21. Using the product of two vectors write the multiplication table

22. Given a=randi([0 1],10,1) create vector b such that has the same length of a but where a has a 1 b has 0 and viceversa when a has 0 b has 1

23. Using product vector compute the sum of the elements in v1=[1 2 3 4 5]

24. Using product vector write 10

25. Using product vector replicate the result of X= repmat((1:3)',1,5)

26. Using product vector replicate the result of X=repmat((1:3),5,1)

27. Write these two series: 2 4 8 16 32 64 128 256 512… and 1 4 9 16 25 36 49 64 81 100 using the power operator

28. Given matrix A of exercise 14 build a matrix B whose first row is the sum of row 1 and 2 of matrix A, whose second row is the sum of row 2 and 3 whose last row is the sum of row 1,2 and 3 of matrix A.

29. Given a 10x2 matrix of r.v I.I.N(1,2), build the corresponding demaned values

30. Given the demenead values, standardize them [recall that if $x \sim N(\mu, \sigma^2)\ then\ z = \frac{x-\mu}{\sigma}\ with\ \ z \sim N(0,1)$]