ASSET MANAGEMENT ASSIGNMENT #1

Mean Variance Asset Allocation

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PART I

From what we can see in the existing body of literature, mean variance optimization is often said to be a too powerful technique to be used on data of poor quality. What is meant by this is that the optimizer blindly trusts our estimations for expected returns and VarCov matrix, leading to portfolio compositions that are too imbalanced towards the best (in the data provider's mind) assets. There are a number of drawbacks stemming from the latter statement. An important role is played by correlations among assets: if they are low there is a chance that two assets which are not too different in terms of risk/return will be given not too dissimilar weights in the final portfolio, for diversification issues. Conversely, if correlations are high all the weight will be assigned to the best hypothetically performing asset, given that weighting the other one would just lower the return without sensibly improving the risk profile. What is humbly desired to say is that the disproportionate allocation to Japan is dangerous, no matter what the client is willing to risk, because all what is at hand is an estimate of future returns and risks. Figure 1 shows the disproportionate weight assigned to Japan from a certain level of risk onwards. In the appendix an efficient frontier and a representation of weights for Equity only is available.

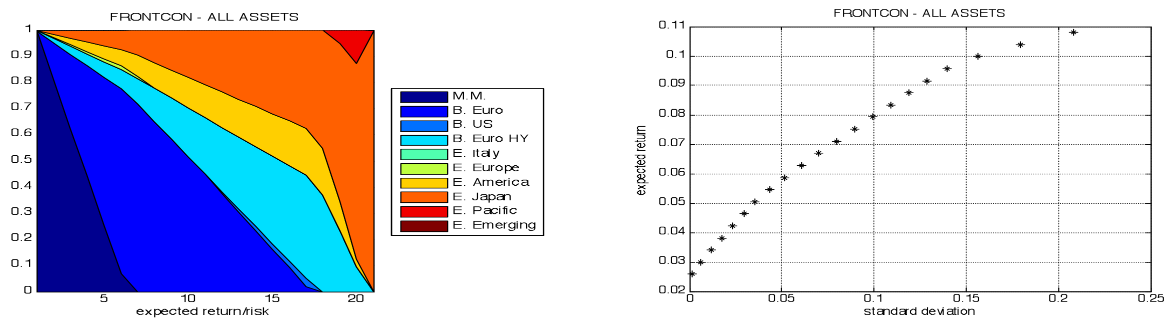

Figure 1

The fact that in the long (medium) run the uncertainty brought by volatility is going to fade leading to a more accurate definition of the average return implies additional problems, notably the fact that the client is probably not as much interested in average return as he/she is in total/final return. Another serious problem of mean variance optimization is that there seems not to be an economic rationale for the investment in a particular asset. For example if the optimizer chooses to invest in the US market one would expect that Canada would be also picked for correlation issues between the two markets, but this does not happen. The swings in weights following a small change in the inputs are not economically sound, not feasible because they go against well established market practices (i.e. completely ignoring EU Zone or the US in certain situations) and thus imply that the resulting portfolio is hardly marketable. All gets even worse if we allow for short sales.

In order to deal with these issues it was decided to perform a sensitivity analysis employing a DV01 kind of measure mixed with Value at Risk. The risk factor of choice is Japan's standard deviation. The procedure goes like this (it is just a selection from what is explained in the comments contained in the Matlab code, which are an important and complementary part of the report):

- Japan's and only Japan's volatility is increased by 100bp (this leads to a new VarCov matrix). The new risk configuration is then applied to the original weights (from initial data) in order to see the change in standard deviation but most of all in VaR. The sensitivity analysis was not performed on the whole frontier but on a specific portfolio, comparable to that which is being sold to the client

(Warning: comparable to the one proposed to the client along the expected return dimension, not risk. The value is around 9,9%) The figures are as follows:

Former PF volatility	0,152835728627879	(calculated from original data only)
New PF volatility	0,158274739946021	(same weights, but with Japan's sd increased)
Former PF VaR 99%	0,355549072370950	
New PF VaR 99%	0,368202104787792	

Delta VaR 0,012653032416842 From these figures it is possible to see that If the original estimates were wrong and by bad luck the most important asset's volatility were even just 1% above the forecasts, the consequence on a 100m $ would be a 1.265.303$ increase in VaR. VaR is more in the realm of conservatism in risk calculation, so the result seems to be significant.

The same reasoning will be applied in the second and third section as part of the explanation of the superiority of the alternative portfolio that it is going to be calculated.

PART II

If it is allowed to suggest a way to proceed, two alternatives could pop up in mind to enhance portfolio risk spreading and performance to various extents.

The first methodology could be resampling. The technique aims at mitigating the most undesirable results of mean variance optimization, namely the abrupt jumps in portfolio weights that entail very little differences in risk/return, and the absence of diversification. The procedure is detailed in the Matlab files. Here all that it is wanted to say is to clarify some steps in the methodology. The data we were provided with are yearly returns and standard deviations, and it was not transformed/scaled down for example into a monthly basis. When it comes to building the multivariate random distribution (the code is `% r=mvnrnd(ret,cova,100)%`), the third input should theoretically be the length of the investment period, which should be five years. What was done instead was to formulate way more possible realizations per asset (the r matrix is a 100x10) because of the technical problem of the impossibility to put the number "5" as third input. The error that Matlab returned concerns the impossibility to assure that the VarCov matrix is psd, probably for linear dependence issues.
For what concerns the results it can be seen that resampling seems to work flawlessly in that now the weights plot is smoother, Japan's weight is still high but significantly less than in the first attempt (around 10% less)(Figure 2).

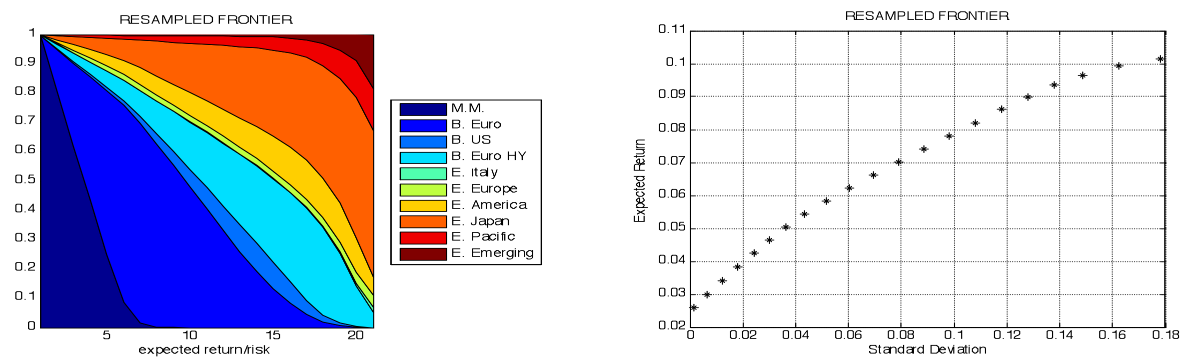

Figure 2

Resampling entails generating a multivariate distribution out of the initial expected returns vector and VarCov matrix for many times (10.000 in this specific case). At every loop the code stores the results in a 100x10 matrix of which it calculates the columnwise (assetwise) mean and covariance matrix. From these latter results a simple mean variance optimization is ran, and the weights are stored in an ad hoc matrix. The final step is to divide the weights matrix by the number of iterations. What can be disposed now by the user is a set of automatically/"optimally" constrained weights that discount the uncertainty effect. If Japan's expected return is really high, it might not be so lucky in all the 10 thousands iterations. The outcome is a reduced importance for it in the final allocation. Resampling could also have the undesirable property to allow a not particularly interesting asset into the final allocation just because of a few lucky draws that make its weight not negligible, but this fact could be indulgently forgiven if confronted with the benefit of added diversification. Resampling does not however constrain weights in the lower end of the risk spectrum because of the modest volatility that asset of the money market kind feature. This last finding is of great importance because one of the difficulties in mean variance optimization is how to calibrate assets' constraints in a not arbitrary/result affecting way. Resampling solves the problem constraining when it is the moment to do it, but suffers from poor theoretical backing in that it is not crystal clear why an average set of weights should be an optimal set of weights. It has also to be said that once the original efficient frontier is plotted against the resampled one, the latter inevitably plots below the former. This makes sense because given the initial data and the way the optimizer operates. The efficient frontier from resampling is also shorter than the original one (just on the right side) because of the previous discussion on weight constraints: it is made impossible for the riskiest/most rewarding assets to get a significant weight because of their inherent instability. The same line of reasoning is compatible with what happens to the left end, in which volatility is not the issue, so that it does not prevent weights to get as big as needed. Figure 3 shows the latter passages.

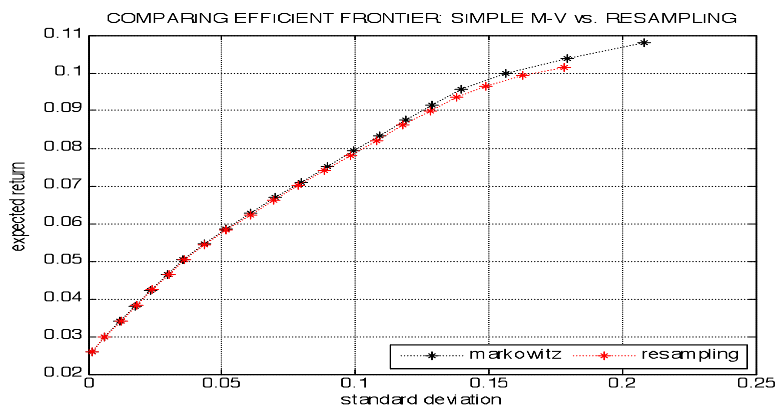

Figure 3

The following section continues the sensitivity analysis started in Part I applied to resampled portfolios. The one labeled "former" is just the original resampled portfolio while the second one has the same weights but Japan's volatility is increased. Ex ante we would expect a smaller difference in VaR given that the resampled portfolio should weigh less Japan. Warning: figures with several digits are shown. Resampling however changes returns at every set of iterations, implying differences in weights and thus in risk, so it may be sensible to consider just the first 4 or 5 digits)

Former PF volatility 0,161699023358113

New PF volatility	0,165829338712438
Former PF VaR 99%	0,376168179223627
New PF VaR 99%	0,385776729567280
Delta VaR	0,009608550343653

As said before the resampled portfolio's risk is higher than in the original case because the lower efficient frontier (Figure 4 above). As can be seen the assumptions are confirmed by the figures. The result is actually even stronger because resampling forced taking a riskier portfolio than in the original case to reach the same expected return.
A new resampling has been eventually performed with the frontcon generated with the modified standard deviation. The plot would be useless given the imperceptible (as expected) differences so it has been deleted from the code.

The second device of choice in order to improve the portfolio's performance is a Black and Litterman allocation. The reasons behind such a decision are the same that led to the implementation of a resampling exercise: the fear of the wild picks entailed by the mean variance optimization. The starting point consisted in finding a reference point upon which it should be possible to express one personal opinions regarding stock markets' evolution. This reference point can be devised in a sort of market portfolio that captures the idea of a CAPM like equilibrium. To this end the iter went on as follows: a reverse optimization was ran in order to find the equilibrium returns. The weights employed for the reverse optimization were the ones found in the report, which should represent the actual financial markets' importance. The formula is detailed in the Matlab code but one detail is worth mentioning in this section: the risk aversion parameter chosen was a standard one if the utility function that is thought to rule is of a quadratic kind. Talking about numbers, the value is 2. Once obtained the expected returns it was possible to express views on the future behavior of the market. The idea was trying to get a set of returns resembling the one given in the report. Once views are expressed and expected returns obtained the final step is to perform a mean variance optimization and check the portfolio composition of interest among the multiple ones residing in the efficient frontier. The difference in doing so instead of just giving the raw data to the optimizer directly are significant. With a B&L kind of allocation the starting point is a sort of equilibrium, thus the weights are balanced and lend themselves to a sound economic explanation (if not perfectly sound, at least they are plausible). When one moves from the equilibrium situation weights change but in a less dramatic/more predictable way than before. Talking about predictability, if Japan is seen as a nice investment, unless the views are of a relative kind and against its nearby countries, even the pacific area should benefit from an enhanced expected return, for correlation issues. Coming to the implementation that was performed, two ways have been followed.
The first concerned expressing as many views (absolute) as the number of equity markets. Every view is the expected value given in the initial report, with various degrees of uncertainty.
In the second case three views were expressed:

1) Absolute view on Japan: more than 5% jump compared to its equilibrium expected retrun;
2) Relative view on Japan (long) as opposed to western Countries (short) with a differential that varied in the several attempts in the range 2, 3%;
3) Relative view on European markets: just to adjust expected returns to make them more similar to those in the report in which Italy was awarded the worst result, as opposed to what happened in the equilibrium situation.

B&L features the possibility of expressing one's confidence in what is to be forecasted: for every view the user has to specify the degree of uncertainty surrounding his/her guess. This was done in practice employing Drobetz's solution which translates in a volatility measure what was in the beginning expressed as a confidence interval. One can say "I am pretty sure that Japan's expected return will lie inside a -/+1% band with 90% of probability" and then calculate the appropriate standard deviation, which is eventually put in its designed slot in a diagonal matrix. The exact procedure can be found in the B&L Matlab code. B&L allows the user to freely specify his/her views, but this is not all, because there exists a term τ that can be employed in order to dampen/smooth views. τ should theoretically correspond to 1/(sqrt(n)) with n equal to the number of years, but it can also be employed in the just mentioned way.
A discrepancy emerged comparing B&L and the initial report: the allocation provided at the beginning is said to be a medium term forecast, which is not exactly what can be termed "tactical", while B&L is often thought as a tool to express and exploit in a rational way a temporary intuition on a market's performance while keeping on the "market portfolio" when a better idea is not at disposal. It can be said that the two paths could be reconciled updating B&L from time to time in order to reflect and react to new information as it becomes available. There exist other small technicalities regarding the weighting scheme in the relative views' implementation; their explanation is left to the Matlab file for ease of exposition. What suffices to tell here is that whenever a relative view is expressed, there are a long and a short side involved. Every side can be populated by the desired number of assets. The whole amount of a side is set to 1, and it constituents share proportionally the weight based on their market capitalizations.

The neutral expected returns were

Equity Italy	0,0750959160142620
Equity EU ex Italy	0,0772447090950173
Equity America	0,0712151517315265
Equity Japan	0,0440947024263289
Equity Pacific Ex Japan	0,0630096503264546
Equity Emerging Mkts	0,0718720919821651

From what we can see Japan now gets the lowest expected return, but it is not alone, every market's performance gets resized, as the following Figure 4 shows

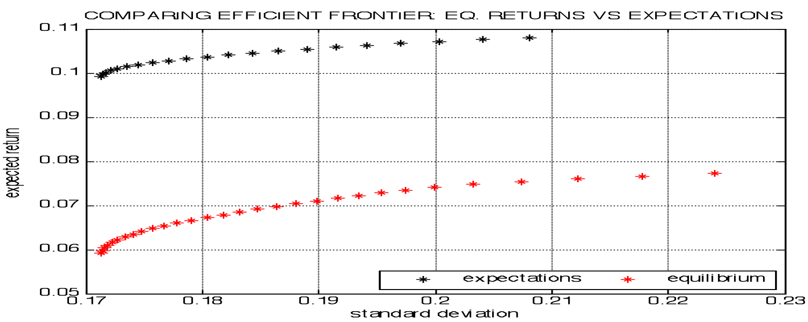

Figure 4

In Figure 5 is shown how the weight distribution among assets changes when the optimizer works on the initial report's data and when it is used to build a frontier with the equilibrium returns (only equity in both the graphs). As can be seen a diversification effect takes place because the relation between return and risk is milder as it is implied in the relative market capitalization. Another interesting effect arises: US and EU now receive a considerable weight as it could intuitively be argued.

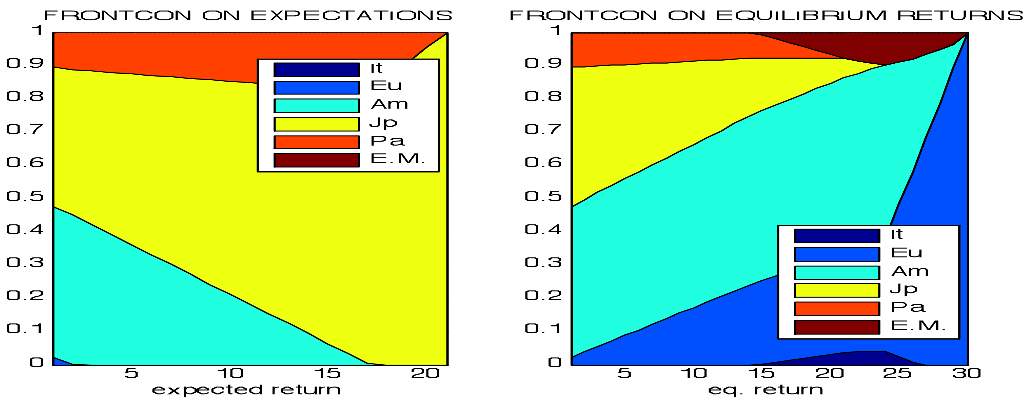

Figure 5

Turning now to the first methodology in which B&L was declined in this report (absolute views corresponding to the initial data), the following Figure 6 shows how the composition of the portfolio has changed running a MV optimization. The outcome is reasonable: there's a high weight on Emerging Markets because they have a high risk but also a high return (in that even if their implied view is not more optimistic than the Japanese, they start from a higher equilibrium return. To put it in another way, the returns that stem from the first attempt in BL are more proportional to their standard deviation than the original ones, as the next table testifies).

	mBL	st.dev
It	0.090350093961469	24,30%
EU ex It	0.090350623346117	22,40%
Ame	0.088468622543346	19,80%
Jap	0.086622743767085	20,81%
Pacific ex Jap	0.089483111611938	21,70%
Emerging	0.100077637033051	24,80%

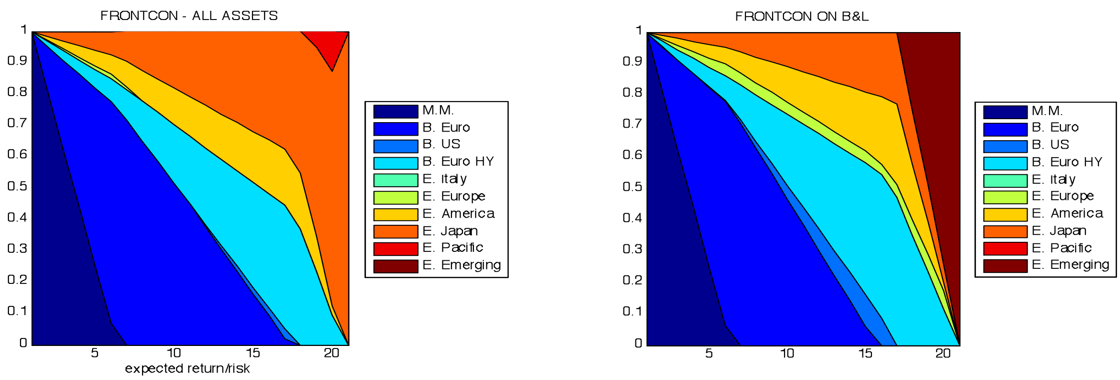

Figure 6

it is now possible to compute a measure of risk for the portfolio destined to the client, in much the same way as before, incrementing Japan's standard deviation by 100bp, in order to rank all the portfolios computed and to choose a better solution than the simple MV.

Former PF volatility 0,237492552976313

New PF volatility	0,237567340993432
Former PF VaR 99%	0,552490295716978
New PF VaR 99%	0,552664278661607
Delta VaR	0,000173982944629

The result of the first BL attempt is really unsatisfactory and won't be investigated further. The weights move from the equilibrium solution to this one in an economically sound way but the extreme risk make the portfolio a really bad choice. Choosing as risk factor Japan's volatility leads to very similar VaR, and this is completely predictable since its share in the chosen portfolio is negligible. A portfolio composition like the one surrounding pf 15 in Figure 7 (right) would have been very well diversified, but suffers from poor return.

From the new set of returns it has been decided to mix BL with resampling, thus to resample the BL output. What emerged is depicted in Figure 7

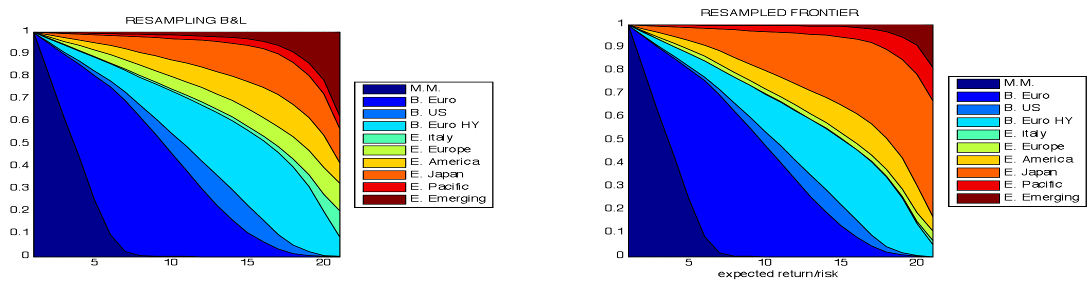

Figure 7

The two resampled weight schemes show some similarities, but are evidently different when it comes to Japan in that its low return compared to the former situation prevents it to enter the efficient frontier more prominently. It is not possible to calculate a risk measure consistent with the other ones proposed before in this new environment: the max expected return for the portfolio is around 9,2%, so roughly 70bp lower than the target one which is to be proposed to the client.

Turning now to the second way in which BL has been implemented, the same path will be followed. We can start with the comparison between the initial data weights and the BL(II) ones in Figure 8

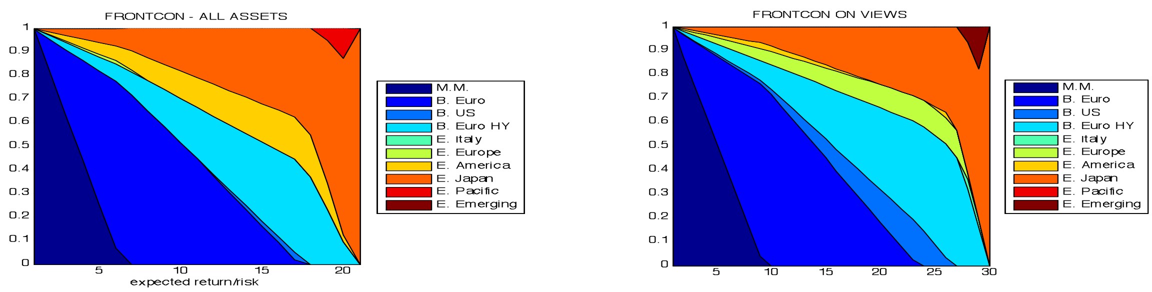

Figure 8

The outcome is not too dissimilar from the initial one. In some trials done during the preparation, with stronger confidence in the views, the picture resembled more to the original one. The last picture shows the difference between resampled frontiers, which are fairly similar indeed.It could be argued this last declination of BL and the initial situation are quite similar. But they are not

Sensitivity analysis

Former PF volatility	0.175864164999335
New PF volatility	0.181973943205704
Former PF VaR 99%	0.409121226366170
New PF VaR 99%	0.423334695907418
Delta VaR	0.014213469541248

As before it has little meaning to calculate a resampled frontier and a VaR for the last declination of BL since a sufficient expected return is not aviable.

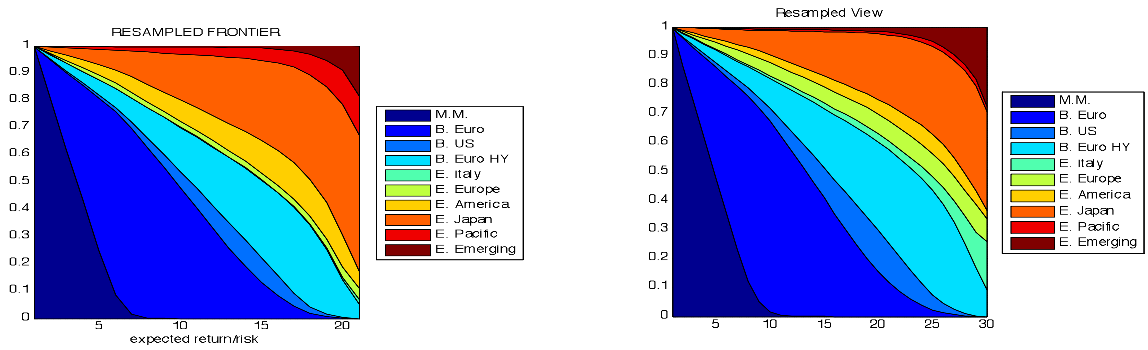

Figure 9

CONCLUSION

From what we have seen both BL and resampling are viable alternatives to a simple MV optimization. From the numerical computation of risk it emerged though that probably the best solution is the first resampling. It allows to limit the risk of an erroneous forecast on the most important asset (Jap stocks) and at the same time offers a standard deviation which is just slightly above the original MV one (which by the way is so low for the reasons explained above. We have seen indeed what happens if Japan’s volatility was mistakenly undervalued by 100bp, but much worse scenarios could happen). In the end what is recommended is enough diversification to smooth the effects of wrong estimates both in risk and in return, a thing that a standard MV optimization tend to forget way too often.

APPENDIX

The following figure shows the result of a simulation exercise (that can be found in the Matlab files) in which the one year forward returns have been simulated, in a stepwise procedure for five years. The result can be explained on the basis of the fact that the resampled frontier plots under the simple MV one, leading to a higher VaR. this is not particularly intuitive given that we should expect less risk for the same amount of return in a diversified portfolio. Even if risk is higher one should prefer the more diversified portfolio for the reasons outlined above.