Fixed Income Assignment

Master in Science

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1 Rules

- Groupwork (no more than 4 persons per group. Please do not ask to have more than 4 people in the group. It is 4!) is allowed for the take home examination. The group composition has to be sent to me by 19 of December 2010.
- The deadline for the take home examination is the **15th of January 2011**. Late deliveries will be penalised (-3 points for day, starting from 36/30).
- You need to upload your project, in a **zipped file**, using the web-learning system.

- The project consists of **one** Excel file and **one** short report (pdf and .doc versions, no more than 20 pages including Tables and Figures) where you state your procedure, your assumptions and your results. Please name the Excel and the Word files with the surnames of the components of the group.

- Do not send reports by e-mail: it is very likely that they will go lost or that they will be considered as spam.

- The Excel/Matlab file and your report should be easily read from a non-expert.

2 Assignment

You have been provided with a zipped file (DatiMercato.zip) containing relevant information concerning LIBOR and SWAP rates, the volatility surface in the cap market, credit default swap spread (as proxy for credit spreads), the description of the bond and a pdf document containing the main elements of the amortizing swap.

In your report you have to answer to the questions raised in Section 5. The report should be no longer than 20 pages including relevant Tables and Figures. Be prudential rather than aggressive in your estimates. Your results can be questioned in a court. However, state when you have been prudential and why.

An important part of the pricing of the swap is related to the credit risk adjustment (see Section 5.3). This part is much more sophisticated so it's up to you to decide if try to take it into account or not. However, no penalisation will be given for skipping the credit adjustment part. A premium (higher marks) will be given to those that are able to do it correctly.

In the report you have to state the contributions of each component of the group. If possible, each component will be evaluated on the basis of the correctness and clarity (not necessary the difficulty) of its contribution.

Your report should make clear the procedure you are following, detailing the problems that you encountered and justifying why the solution you have adopted is preferable to others (if any).

3 The issue

The City of Milan (CofM) on June 24, 2005, has issued a bond, notional 1,685,347,000 Euros, maturity June 29, 2035, fixed annual coupon 4.019%. According to the Italian law art. 41 law n. 448/2001 subsection 2, "public agencies can issue bonds with bullet reimbursement, provided they open a sinking fund or if they stipulate a swap contract for the amortization of the debt." According to this rule, CofM has stipulated four amortizing swaps respectively with JP Morgan Chase Bank, UBS Limited, Deutsche Bank AG and Depfa Bank Plc[1],

[1] In the following, these will be indicated with JPM, UBS, DB and Dep.

each contract having a notional equal to 421,336,750 Euro. The amortizing swap allows to synthetically create an amortizing schedule: CofM pays periodically to the Banks exchange amounts in order to amortize the bullet notional. The above mentioned Banks will use these amounts to constitute a final amount of 421,336,750 Euro (per bank) that will be paid to CofM on June 2035. On its side, CofM will use this amount to pay back the notional to the subscribers of the bond. The details of the bond issue are given in Figure 1

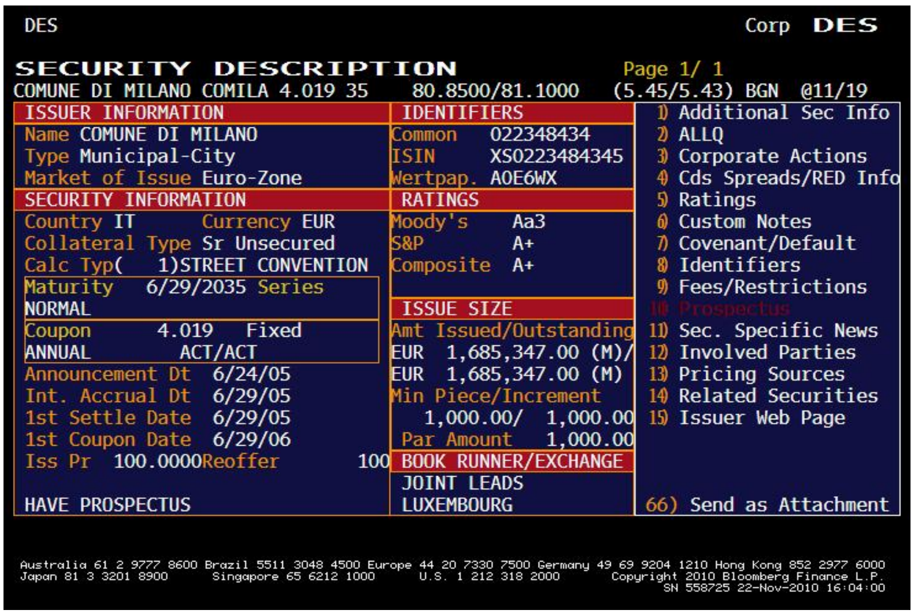


Figure 1: Bloomberg page detailing the CofM bond issue

4 Aim of the project

You have to answer to several questions. The most important is to find out if the Banks or CofM suffered any implicit costs entering the amortizing interest rate swap. The implicit cost is due to the fact that the swap at inception should have zero value, i.e. it should be fair to the parties that are entering the contract. Market data for your evaluation are provide in Section 6.

5 Main Questions

5.1 The bond

1. Price the CofM bond, assuming that CofM is a risk-free issuer and so you do not have to add any credit spread to the bootstrapped spot curve. In this point, you can use mid rates.

2. Compute the credit spread of the CofM bond knowing that it has been issued at par value.

5.2 The amortizing interest rate swap

Let us consider now the amortizing swap. The structure of the amortizing swap is as follows:

- Banks receive from the CofM every year an interim exchange amount necessary to amortize the notional. These amounts are detailed in the swap documentation.

- Banks pay back to CofM the notional in a unique solution on June 2035.

- In addition Banks pay each year to CofM a fixed interest (4.019%) computed on the full notional.

- CofM pays a floating amount (with a lower floor at 3,48% and an upper cap set at 6.19%) to the Banks. The interest payments are computed on an amortizing notional.

1. In June 2005, a possibility for the CofM was to enter into an amortizing swap fixed vs fixed, i.e. CofM is paying a fixed rate on an amortizing notional. What should be the fixed rate the CofM should pay to the Banks in this case? How this fixed rate compares to the floor in the amortizing interest rate swap?

2. Price now the amortizing interest rate swap (for the moment assume that Banks and CofM are risk-free) considering the two legs. For pricing the swap use bid and ask quotations as appropriate. In particular:

   - Identify and price the components of the floating leg. Explain which model you are using in order to price the different components (no derivation of models is required).
   - Price separately the floating component, the floor component and of the cap component. Has the collar a zero value?
   - Price the fixed leg.
   - What is the estimated implicit cost (related to the fact that the swap does not have a zero value at the issue date) at the inception of the contract? Does the implicit cost depends on the CofM counterparty?
   - What should be the floor that makes fair the amortizing swap?

- Alternatively, what should be the cap that makes fair the amortizing swap?
- Alternatively, what should be the cap the fixed rate paid by the Banks to CofM?

3. Have a look to the forward curve and compare it to the floor and to the cap. Explain in words (taking into account also the amortization) how this implicit cost is generated.

4. Compute the sensitivity (DV01) of your implicit cost to parallel shifts in the term structure and to changes in the slope of the term structure. If you are using a parametric curve, the sensitivity to changes in the level are measured by numerically measuring the partial derivative to the β_0 parameter in the Nelson-Siegel curve. If you are using a non parametric interpolation, the sensitivity can be computed numerically, by shifting the term structure up and down by 1 basis point. Does the DVO1 size can justify the implicit cost?

5. How your estimate is sensitive to changes in the implied volatilities you are using for pricing the collar.

6. Given the term structure of cap volatilities extract the forward-forward volatilities using at least one of the methods among those presented during the second part of the course. Reprice the caplets and the floorlets in the collar using the bootstrapped forward-forward volatilities. How this procedure affects your estimate of the implicit cost?

7. There is any credit exposure of CofM versus the Banks or viceversa? No computations are required, just a motivated answer.

5.3 Credit Adjustment

In this Section, if you want, you can try to give a value to the credit components of the swap and try to adjust the fair value you have computed in previous section (where you have assumed that the counterparties are risk-free). Otherwise, you can stop here. No penalty will be given to those that stop here. But you can proceed.

We can start with some preliminary analysis

- Have a look to the differential of interest rate payments. How this differential compares to those you have in a standard plain vanilla swap without notional exchange?

- What about the credit exposure related to the payments of the exchange amounts and the obligation of the Banks to pay back the notional at maturity?

In this Section, we outline the credit value adjustment on the currency swap that you have to estimate and that is associated with the potential default of the counterparties, both CofM and the relevant Bank.[2]

The (risk-free) fair value of the swap has to be adjusted for two credit components. The exposure on the amortizing currency swap under examination has similarities and differences with a plain vanilla swap with the same coupon rate, face value, and maturity. Indeed,

- As in a plain-vanilla swap contract, the credit risk due to the loss of interest payments is limited by the fact that, at each coupon date, there is an exchange only of the difference between the fixed-rate payment and the floating-rate payment. However, this difference is increasing over time due to the amortization of the notional on the floating side.

- Concerning the repayment of the notional at the expiry, CofM has an exposure to the default of the Banks. The impact of the default risk of the depositary banks will be accounted for in the next subsection.

- The Banks are not exposed to the risk that CofM will not pay the exchange amounts at the payment dates, because if CofM fails to perform at any time, there is no further obligation on the Banks'side and the swap is closed. In practice, the Bank bears default risk if interest rates move enough to make the currency swap move in their favor.

Risk Neutral default probabilities. The evaluation of the credit exposure requires a preliminary estimate of the underlying risk-neutral probabilities of default. These numbers are computed by using price data for credit derivatives (such as Credit Default Swaps - "CDS") and bond prices depending on the availability and quality of the data. The calibration procedure is described here.

To calibrate risk-neutral default probabilities using CDS data, we adopt the following standard approximation. Let CDS^i denote the CDS quoted spread for entity i and let LGD^i be the "loss given default" of bank i, both referring to a maturity as close as possible to the one under consideration. If we assume that default occurs on coupon payment dates[3] and that there are m coupon dates per year, then the per-coupon-period risk-neutral default probability q_m^i of entity i reads as[4]:

$$q_m^i = \frac{CDS^i}{m \times LGD^i}. \tag{1}$$

A CDS spread typically provides the protection seller with quarterly payments, that is $m = 4$. The (annual) risk-neutral default probability, q^i, is computed according to the formula:

$$q^i = 1 - (1 - q_4^i)^4. \tag{2}$$

[2] For details see Chapter 12.2.3 in Duffie and Singleton (2003).

[3] For default between coupon dates, a more complicated formula can be used. See, for details, D. Duffie and K. Singleton, *Credit Risk*, Princeton University Press, 2003.

[4] More accurate estimations of the default probability are possible, similar to the bootstrapping procedure to extract spot rates from swap rates.

The annual risk-neutral survival probability of bank entity i is $1-q^i$. A standard assumption is to take as recovery rate $R = 0.4$, so that $LGD = 1 - R = 0.6$.

Measuring the expected losses on the principal balance of exchange amounts. By using the probabilities computed above, for each bank i we calculate the expected payments made by CofM conditional to its survival and provided that bank i defaults **between year** $n-1$ **and year** n. This results into the following expression:

$$(1-q^i)^{n-1} \times q^i \times w^i \times SF(t_{n-1}) \times LGD^i, \tag{3}$$

where:

- w^i is the weight of bank i at the inception date of the amortizing swap ($w^i = 1/4$);
- $SF(t)$ is the aggregate principal balance of exchange amounts at time t;
- n is the number of years and
- LGD^i is the percentage recovery amount (assumed to be equal to 0.6).

The present value of losses due to the bank default and assuming that CofM survives due to default of bank entity i during year n reads as:

$$Q^n \times P(t, t_n) \times (1-q^i)^{n-1} \times q^i \times w^i \times SF(t_{n-1}) \times LGD^i, \tag{4}$$

where:

- Q is the annual risk-neutral survival probability for CofM and
- $P(t, t_n)$ is the discount factor quoted at inception t for date t_n.

The present value of expected losses due to default of the banks during period n is thus:

$$\sum_{i=1}^{I} Q^n \times P(t, t_n) \times (1-q^i)^{n-1} \times q^i \times w^i \times SF(t_{n-1}) \times LGD^i, \tag{5}$$

where:

- I is the number of banks.

The market value of protection sold by CofM can then be computed summing over the N (=35) years and over the I (=4) banks as:

$$\sum_{n=1}^{N} \sum_{i=1}^{I} Q_2^n \times P(t, t_n) \times (1-q_2^i)^{n-1} \times q_2^i \times w^i \times SF(t_{n-1}) \times LGD^i, \tag{6}$$

where:

- N is the number of years.

This expression provides the adjustment to the risk-free value of the amortizing swap due to the potential losses of the aggregate value of the exchange amounts paid by CofM.

- Quantify the credit adjustment. Does it depend on the CofM counterparty? Quantify the exposure for each Bank.

Measuring the Credit Exposure of the Amortizing Currency Swap. We can also consider a second component of the credit risk exposure.

The exposure to a given counterparty on a swap position is the amount that would be lost by that counterparty in the event of default by the other counterparty. We follow the general method for computing credit risk adjustments when both counterparties are defaultable, as follows. One first calculates the market value at time T, X_T say, of default losses to counterparty A that are caused by any default of counterparty B that occurs before the default of A. (If counterparty A defaults first, then, under standard master agreements, there is no default loss to A caused by the default of B because all contracts are settled at the time of the default by A.) The same algorithm can be used to calculate the market value Y_T of losses to counterparty B through any default by counterparty A that occurs before default by B. After accounting for the collateral, the difference, if positive, $X_T - Y_T$, multiplied by the percentage loss given default $L = LGD$, is the net market value of the default losses to party A, as of time T. The risk-neutral expectation of this loss is then discounted to present value, and this amount is summed over all potential default dates, T.

To grasp the idea, it can be convenient for you to construct a plot where at each future point in time you provide the present value (computed using the forward discount curve implied from the one quoted in June 2005) of the difference of the remaining payments in the amortizing swap (without including the exchange amounts and the notionals: they are accounted for in the next point). The remaining payments at each future point in time need to be estimated. Make as a rough approximation that they are equal to the amount you have estimated at the inception of the swap[5]

In your report, consider the following questions:

1. Due to the different timings in the prepayment of the notionals, relative to those of an at-market plain vanilla interest-rate swap, is mean exposure positive or negative at the inception (that is, initially, the Banks are exposed to default by CofM or viceversa)?

2. What's happening after the the first prepayments of the notional by CofM?

3. What's happening after as maturity approaches?

[5] For example, if at time 0 you estimate that the future net payments will be 1, -2, 0.23, 0.35, the remaining net payments after one period will be -2, 0.23, 0.35. You can compute their present value in one period time by using the forward discount curve.

The net credit adjustment for the market value of the swap to the receiver is the present value of cash flows lost due to default by the payer, provided that the payer defaults before the receiver, minus the present value of the payer's cash flows lost owing to default by the receiver, provided that the receiver defaults before the payer. Thus, a key intermediate calculation is the risk-neutral probability $Q_{T,i,j}$ that the payer j defaults before the receiver i, and that this happens during a particular coupon period T. Summing up, the net loss in market value to Firm B due to default in period T is given by the following expected value

$$Z_T = E_0\left(\frac{Q_{T,B,A} L \max\left(X_T - Y_T, 0\right) - Q_{T,A,B} L \max\left(Y_T - X_T, 0\right)}{MMA_T}\right), \quad (7)$$

where:

- L is the percentage loss given default (set equal to 0.6 in our calculations);
- X_T is the present value at period T of the remaining risk-free leg paid by A. This only includes the future fixed interest rate payments due by the Banks.
- Y_T is the present value at period T of the remaining risk-free leg paid by B. This only includes the future floating interest rate payments due by CofM.
- MMA_T is the money market account (cumulated value of the money invested in a risk-free account) up to time T, and
- $Q_{T,i,j}$ is the probability that j defaults exactly in period T and i survives at least to period T, that is,

  $$Q_{T,i,j} = q_i^{T-t} q_j^{T-t-1}(1 - q_j), \quad (8)$$

  where q_i is the risk-neutral probability of survival of counterparty i during any period, conditional on survival to the beginning of that period.

So, the total value of the swap to B is the value of the swap to B ignoring the effect of counterparty credit risk, less the total credit value adjustment $\sum_{T=1}^{n} Z_T$, where n is the number of coupon payments. The computation of $\sum_{T=1}^{n} Z_T$ requires the annual risk-neutral survival probabilities q_i and q_j. Those relative to the Banks have been assumed to be constant and can be computed using the procedure previously describeve. The risk-neutral per-period survival probability for CofM can be estimated assuming that the credit spread on the bond is rougly equal to the CDS spread (which are the assumptions that guarantee that this no-arbitrage relationship holds?).

In general Z_T can be computed by doing a Monte Carlo simulation of the term structure of interest rates. However, for the purpose of computing the credit adjustment we make the rough simplification that the future payments are equal to their expected value computed at the inception and that interest rates are deterministic, so that $P(t,T) = 1/MMA(T)$ and the above expression simplifies into

$$Z_T = (Q_{T,B,A} L \max(X_T - Y_T, 0) - Q_{T,A,B} L \max(Y_T - X_T, 0)) \times P(t,T). \quad (9)$$

Another strategy is to obtain a possible range of values of the credit risk adjustment assuming that the interest rate will be always below the floor (lower estimate) or will be always above the cap.

The credit value adjustement (due to the expected losses on the interest payments) to the risk-free value of the swap is

$$\sum_{T=1}^{n} Z_T. \quad (10)$$

6 Market Data

- Euribor and Swap rates (bid-mid-ask quotations) are given in Table 1. Bootstrap cash and swap rates to extract the term structure of discount factors up to the bond maturity. If you need to interpolate missing rates, explain the procedure you are following. If necessary, interpolate at the swap payment dates the term structure of continuously compounded spot rates using parametric or non parametric approach. Discuss, if any, the problems that you encounter using this interpolation on your data. At the end, try to verify how your results are sensitive to the procedure you have adopted. If not, explain why.
- Volatility surface. Figure 2 provides the volatility surface for caps up to 20 years. The volatilities refer to flat volatilities of the 6m Euribor rate.
- CDS Spread. You have been provided in Table 2 of a CDS spread curve for each Bank, except for Depfa for which no CDS or similar data are available. No CDS quotations are available for Depfa and for CofM. For CofM you have two possibilities: to try to recover CDS spreads related to Republic of Italy or to consider the credit spread implied in the bond price issued by CdM (issue price is 100).

7 Financial Libraries

7.1 Excel

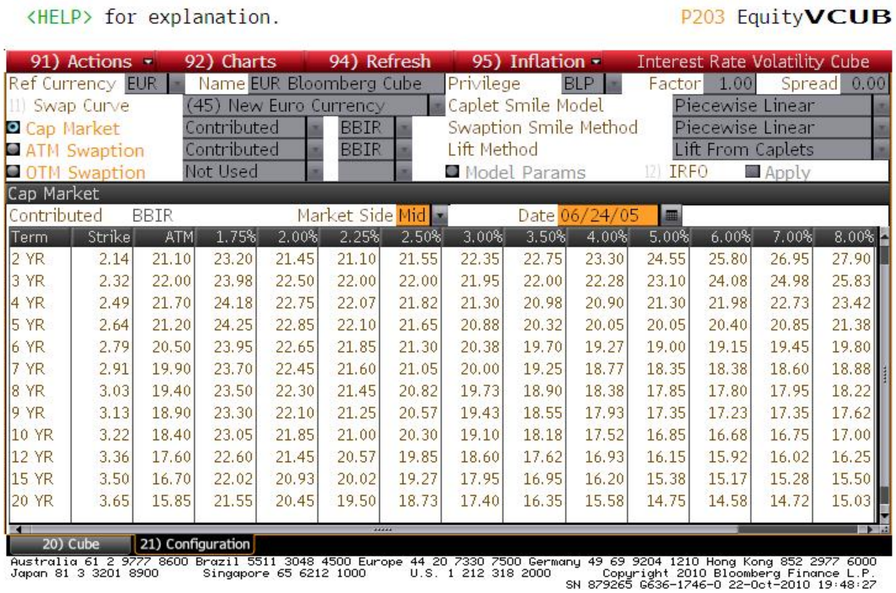
You are provided with a file LibraryFixedIncome.xla. This file is an Excel add-in containing the main functions you need to use (Nelson-Siegel functions and Black formula). In order to make available these functions in Excel, you need to follow the following steps:

Term	Mty/Term	Bid	Mid	Ask	Source
o/n	06/27/2005	2.08	2.08	2.08	Cash Rate
1w	07/05/2005	2.108	2.108	2.108	Cash Rate
1m	07/28/2005	2.104	2.104	2.104	Cash Rate
2m	08/29/2005	2.103	2.103	2.103	Cash Rate
3m	09/28/2005	2.104	2.104	2.104	Cash Rate
4m	10/28/2005	2.096	2.096	2.096	Cash Rate
5m	11/28/2005	2.091	2.091	2.091	Cash Rate
6m	12/28/2005	2.086	2.086	2.086	Cash Rate
7m	01/30/2006	2.08	2.08	2.08	Cash Rate
8m	02/28/2006	2.074	2.074	2.074	Cash Rate
9m	03/28/2006	2.068	2.068	2.068	Cash Rate
10m	04/28/2006	2.064	2.064	2.064	Cash Rate
11m	05/29/2006	2.059	2.059	2.059	Cash Rate
1y	06/28/2006	2.063	2.073	2.083	Swap Rate
2y	06/28/2007	2.163	2.166	2.169	Swap Rate
3y	06/30/2008	2.315	2.325	2.335	Swap Rate
4y	06/29/2009	2.47	2.49	2.51	Swap Rate
5y	06/28/2010	2.62	2.64	2.66	Swap Rate
6y	06/28/2011	2.783	2.78505	2.7871	Swap Rate
7y	06/28/2012	2.885	2.905	2.925	Swap Rate
8y	06/28/2013	3.03	3.03245	3.0349	Swap Rate
9y	06/30/2014	3.11	3.13	3.15	Swap Rate
10y	06/29/2015	3.205	3.225	3.245	Swap Rate
11y	06/28/2016	3.2975	3.3025	3.3075	Swap Rate
12y	06/28/2017	3.35	3.37	3.39	Swap Rate
15y	06/29/2020	3.517	3.522	3.527	Swap Rate
20y	06/30/2025	3.66	3.68	3.7	Swap Rate
25y	06/28/2030	3.73	3.75	3.77	Swap Rate
30y	06/28/2035	3.76	3.78	3.8	Swap Rate
35y	06/28/2040	3.782	3.786	3.79	Swap Rate
40y	06/28/2045	3.785	3.805	3.825	Swap Rate
45y	06/28/2050	3.787	3.797	3.807	Swap Rate
50y	06/28/2055	3.79	3.81	3.83	Swap Rate

Table 1: Euribor and Swap rates at the trade date. Terms are in format mm/dd/year

Date	UBS	JPM	DB
10Y	18	54.5	26.5
7Y	14.655	41.25	22.96
5Y	11.5	35.67	19.375
4Y	11.5	29.235	19.375
3Y	11.5	21.5	19.375
2Y	11.5	21.5	19.375
1Y	11.5	21.5	19.375
6M	11.5	21.5	19.375

Table 2: CDS quotations (basis points)



Term	Strike	ATM	1.75%	2.00%	2.25%	2.50%	3.00%	3.50%	4.00%	5.00%	6.00%	7.00%	8.00%
2 YR	2.14	21.10	23.20	21.45	21.10	21.55	22.35	22.75	23.30	24.55	25.80	26.95	27.90
3 YR	2.32	22.00	23.98	22.50	22.00	22.00	21.95	22.00	22.28	23.10	24.08	24.98	25.83
4 YR	2.49	21.70	24.18	22.75	22.07	21.82	21.30	20.98	20.90	21.30	21.98	22.73	23.42
5 YR	2.64	21.20	24.25	22.85	22.10	21.65	20.88	20.32	20.05	20.05	20.40	20.85	21.38
6 YR	2.79	20.50	23.95	22.65	21.85	21.30	20.38	19.70	19.27	19.00	19.15	19.45	19.80
7 YR	2.91	19.90	23.70	22.45	21.60	21.05	20.00	19.25	18.77	18.35	18.38	18.60	18.88
8 YR	3.03	19.40	23.50	22.30	21.45	20.82	19.73	18.90	18.38	17.85	17.80	17.95	18.22
9 YR	3.13	18.90	23.30	22.10	21.25	20.57	19.43	18.55	17.93	17.35	17.23	17.35	17.62
10 YR	3.22	18.40	23.05	21.85	21.00	20.30	19.10	18.18	17.52	16.85	16.68	16.75	17.00
12 YR	3.36	17.60	22.60	21.45	20.57	19.85	18.60	17.62	16.93	16.15	15.92	16.02	16.25
15 YR	3.50	16.70	22.02	20.93	20.02	19.27	17.95	16.95	16.20	15.38	15.17	15.28	15.50
20 YR	3.65	15.85	21.55	20.45	19.50	18.73	17.40	16.35	15.58	14.75	14.58	14.72	15.03

Figure 2: Volatility surface of Black implied volatilities (flat volatilities) in the cap market

- Copy the file LibraryFixedIncome.xll into a directory, e.g. c:\FixedIncome
- Open the Excel file where you want to build your project.
- Now, the above VBA functions are available in your project. Check it by inserting a new formula and selecting the function among the ones called user defined

For pricing using the Black model, use the Undiscounted version. If you have to use the pricing formula for a cash or nothing, the strike is already included in the payoff ($K1_{(L>K)}$).

7.2 Matlab

You can also develop the project using Matlab rather than Excel. In this case you can find useful routines (but not guarantee of being error free is given) at the following address:

http://www.unibocconi.it/wps/wcm/connect/SitoPubblico_IT/Albero+di+navigazione/Home/Dipartimen

then select Risorse (bottom, left)

then select FixedIncome

Remember that if you want to do the final thesis with me, you have to be Matlab or VBA literate! Remember that if you want to get a quantitative job you have to be at least Matlab or VBA literate!