A note on risk measures for Credit Default Swap positions

Mascia Bedendo

November 2008

1 Pricing model for CDS

A CDS is a contract where one party (the protection seller) provides protection against credit risk of a single reference entity to the counterparty (protection buyer) in exchange for periodic premium payments.

The premium payments are normally paid on a regular basis until the contract expires or default occurs (before expiry). The premium payments are calculated according to the CDS spread (expressed in basis points on an annual basis), applied to the notional amount. The stream of premium payments identifies the **premium leg**.

In case of default of the reference entity, the protection seller pays to the protection buyer $N(1-R)$, i.e. the notional times the percentage loss (i.e. 1 - recovery rate). This payment identifies the **protection (or default) leg**.

At origination of a contract, the fair spread $S(0)$ is the spread that makes present value of premium and default leg equal. To calculate premium and default leg, we need to make some assumptions on the probabilities of survival and default. In particular, here we assume for simplicity to use the constant hazard rate approach. According to this approach, the probability of defaulting between t and $t+dt$, for small dt is approximately equal to λdt where λ is the default intensity rate. Under this assumption, the probability of surviving up to time t is equal to $\exp(-\lambda t)$. For simplicity, we assume that the premium leg is paid continuously and that default can happen at any instant in time.

The value of the protection leg for a CDS contract of new origination expiring at time T is given by:

$$Prot.leg = (1-R)\int_0^T \lambda e^{-(r+\lambda(0))s} ds = \frac{\lambda(0)(1-R)(1-e^{-(r+\lambda(0))T})}{r+\lambda(0)} \quad (1)$$

where r is the risk-free rate. The value of the premium leg is equal to:

$$Prem.leg = S(0)\int_0^T e^{-(r+\lambda(0))s}ds = \frac{S(0)(1-e^{-(r+\lambda(0))T})}{r+\lambda(0)} \tag{2}$$

Solving for $S(0)$, we obtain: $S(0) = \lambda(0)(1-R)$.

2 Valuation at Marked-to-Market for CDS

The value of a CDS position (for a protection buyer) at time t following initiation at time 0 is the difference between the market value of the protection at time t and the cost of the premium payments, which have been set contractually at $S(0)$. The evaluation of premium and protection legs must be carried out considering the new default intensity $\lambda(t)$, which reflects the changes in the market conditions that have led to changes in the CDS spreads.

Therefore we have:

$$MTM = \pm(S(t) - S(0)) \times RPV01(t) \tag{3}$$

and:

$$RPV01(t) = \frac{1-e^{-(r+\lambda(t))T}}{r+\lambda(t)} \tag{4}$$

where $\lambda(t) = S(t)/(1-R)$. The plus sign refers to a protection buyer, the minus sign to a protection seller. Changes in the MTM value of a CDS position reflect market risk arising from changes in the CDS spreads and appropriate VaR and ES measures must be estimated to account for this risk. For this purpose, our loss distribution is the distribution of the negative changes in the MTM values of the position. This is true if the position is held in the trading book. Clearly, a CDS position which represent a perfect hedge of an underlying credit is held in the banking book and is not the object of market risk measurement.