CSCI-GA.2945-001: Assignment #1

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Contents

Everything is available at `https://github.com/wojzaremba/opt`.

Exercise 1.1

a) Function $f(x) = x^3 - 8$ evaluates to values having a different sign at $\{-2, 4\}$, i.e. $f(-2) = -16$, and $f(4) = 56$. This means that there is a zero of f in interval $[-2, 4]$.
b)

Listing 1: Bisection algorithm.

```
function bisection(f, x0, x1, maxit)
    assert(f(x0) * f(x1) < 0)
    i = 0;
    save(sprintf('Initial points a0=%f, f(a0)=%f, b0=%f, f(b0)=%f\n', ...
             x0, f(x0), x1, f(x1)));
    while (i < maxit) && abs(x0 - x1) > 1e-8
        x2 = (x0 + x1) / 2;
        if f(x2) * f(x0) > 0
            x0 = x2;
        elseif f(x2) * f(x1) > 0
            x1 = x2;
        elseif f(x2) == 0.0
            return
        end
        i = i + 1;
        save(sprintf('a%d=%.16f, f(a%d)=%.16f, b%d=%.16f, f(b%d)=%.16f\n', ...
                 i, x0, i, f(x0), i, x1, i, f(x1)));
    end
end
```

Listing 2: Program to call bisection algorithm.

```
global file
file = fopen('results/res_ex1_1b.txt', 'w');
bisection(@(x) x.^3 - 8, 0.5, 3.1, 12);
```

Listing 3: Execution results.

```
Initial points a0=0.500000, f(a0)=-7.875000, b0=3.100000, f(b0)=21.791000
a1=1.8000000000000000, f(a1)=-2.1679999999999993, b1=3.1000000000000001, f(b1)=21.7910000000000039
a2=1.8000000000000000, f(a2)=-2.1679999999999993, b2=2.4500000000000002, f(b2)=6.7061250000000037
a3=1.8000000000000000, f(a3)=-2.1679999999999993, b3=2.1250000000000000, f(b3)=1.5957031250000000
a4=1.9624999999999999, f(a4)=-0.4416152343750008, b4=2.1250000000000000, f(b4)=1.5957031250000000
a5=1.9624999999999999, f(a5)=-0.4416152343750008, b5=2.0437500000000002, f(b5)=0.5365681152343775
a6=1.9624999999999999, f(a6)=-0.4416152343750008, b6=2.0031249999999998, f(b6)=0.0375586242675752
a7=1.9828124999999999, f(a7)=-0.2044826164245617, b7=2.0031249999999998, f(b7)=0.0375586242675752
a8=1.9929687499999997, f(a8)=-0.0840787167549166, b8=2.0031249999999998, f(b8)=0.0375586242675752
a9=1.9980468749999998, f(a9)=-0.0234146192669895, b9=2.0031249999999998, f(b9)=0.0375586242675752
a10=1.9980468749999998, f(a10)=-0.0234146192669895, b10=2.0005859374999999, f(b10)=0.0070333101376878
a11=1.9993164062499997, f(a11)=-0.0082003215169566, b11=2.0005859374999999, f(b11)=0.0070333101376878
a12=1.9999511718749998, f(a12)=-0.0005859231950041, b12=2.0005859374999999, f(b12)=0.0070333101376878
```

Results are what I have expected. Exact value is not achieved.
c) Any polynomial can be uniquely factorized to the multiplication of monomials (uniquely up to the order, and constant multiplicative factor) $f(x) = (x - a_1) \dots (x - a_n)$. $\{a_i\}_{i=1,\dots,n}$ are all zeros of f, and f has no more zero values. This means that $f(x) = (x - 1)^7$ has only zeros at 1.
Another proof could be based on monotonicity of $(x - 1)^7$. We have that $\partial x(x - 1)^7 = 7(x - 1)^6 \geq 0$. This means that $(x-1)^7$ is non decreasing function. Moreover, $\lim_{x\to-\infty}(x-1)^7 = -\infty$, and $\lim_{x\to\infty}(x-1)^7 = \infty$. It can cross $x = 0$ only once, as it is non decreasing, and as 1 is its zero, than it is the only zero point.
d)

Listing 4: Program to call bisection algorithm.

```
global file
file = fopen('results/res_ex1_1d.txt', 'w');
f = @(x) x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35 * x^3 - 21 * x^2 + 7*x - 1;
bisection(f, 0.95, 1.01, 12);
```


Listing 5: Execution results.

```
Initial points a0=0.950000, f(a0)=-0.000000, b0=1.010000, f(b0)=0.000000
a1=0.9800000000000000, f(a1)=-0.0000000000012861, b1=1.0100000000000000, f(b1)=0.0000000000000142
a2=0.9950000000000000, f(a2)=-0.0000000000000036, b2=1.0100000000000000, f(b2)=0.0000000000000142
a3=0.9950000000000000, f(a3)=-0.0000000000000036, b3=1.0024999999999999, f(b3)=0.0000000000000089
a4=0.9987500000000000, f(a4)=-0.0000000000000009, b4=1.0024999999999999, f(b4)=0.0000000000000089
a5=0.9987500000000000, f(a5)=-0.0000000000000009, b5=1.0006249999999999, f(b5)=0.0000000000000036
a6=0.9987500000000000, f(a6)=-0.0000000000000009, b6=0.9996875000000000, f(b6)=0.0000000000000044
a7=0.9992187500000000, f(a7)=-0.0000000000000009, b7=0.9996875000000000, f(b7)=0.0000000000000044
a8=0.9992187500000000, f(a8)=-0.0000000000000009, b8=0.9994531250000001, f(b8)=0.0000000000000018
a9=0.9992187500000000, f(a9)=-0.0000000000000009, b9=0.9993359375000000, f(b9)=0.0000000000000027
a10=0.9992187500000000, f(a10)=-0.0000000000000009, b10=0.9992773437500000, f(b10)=0.0000000000000036
a11=0.9992480468750000, f(a11)=-0.0000000000000036, b11=0.9992773437500000, f(b11)=0.0000000000000036
a12=0.9992480468750000, f(a12)=-0.0000000000000036, b12=0.9992626953125000, f(b12)=0.0000000000000071
```

Results are what I have expected. Exact value is not achieved.

e)

Listing 6: Program to call bisection algorithm.

```
global file
file = fopen('results/res_ex1_1e.txt', 'w');
f = @(x) (x - 1).^7;
bisection(f, 0.95, 1.01, 12);
```

Listing 7: Execution results.

```
Initial points a0=0.950000, f(a0)=-0.000000, b0=1.010000, f(b0)=0.000000
a1=0.9800000000000000, f(a1)=-0.0000000000012800, b1=1.0100000000000000, f(b1)=0.0000000000000100
a2=0.9950000000000000, f(a2)=-0.0000000000000001, b2=1.0100000000000000, f(b2)=0.0000000000000100
a3=0.9950000000000000, f(a3)=-0.0000000000000001, b3=1.0024999999999999, f(b3)=0.0000000000000000
a4=0.9987500000000000, f(a4)=-0.0000000000000000, b4=1.0024999999999999, f(b4)=0.0000000000000000
a5=0.9987500000000000, f(a5)=-0.0000000000000000, b5=1.0006249999999999, f(b5)=0.0000000000000000
a6=0.9996875000000000, f(a6)=-0.0000000000000000, b6=1.0006249999999999, f(b6)=0.0000000000000000
a7=0.9996875000000000, f(a7)=-0.0000000000000000, b7=1.0001562499999999, f(b7)=0.0000000000000000
a8=0.9999218749999998, f(a8)=-0.0000000000000000, b8=1.0001562499999999, f(b8)=0.0000000000000000
a9=0.9999218749999998, f(a9)=-0.0000000000000000, b9=1.0000390625000000, f(b9)=0.0000000000000000
a10=0.9999804687499999, f(a10)=-0.0000000000000000, b10=1.0000390625000000, f(b10)=0.0000000000000000
a11=0.9999804687499999, f(a11)=-0.0000000000000000, b11=1.0000097656250000, f(b11)=0.0000000000000000
a12=0.9999951171874999, f(a12)=-0.0000000000000000, b12=1.0000097656250000, f(b12)=0.0000000000000000
```

Error is much smaller in compare to d). Verbose formulation might make more numerical errors. Results are what I have expected. Exact value is not achieved.

Exercise 1.2

$f(x)'' > 0$ means that f is a strictly convex function, i.e. $f(ax_0 + (1-a)x_1) < af(x_0) + (1-a)f(x_1)$ for $a \in (0, 1)$. Or in other words, that line passing through $(x_0, f(x_0))$, and $(x_1, f(x_1))$ is above graph of f. Without lost of generality, we can assume that $x_0 < x_1$ and that $f(x_0) < 0$ and $f(x_1) > 0$. x_2 is defined as point lying on intersection of line passing through $(x_0, f(x_0))$, and $(x_1, f(x_1))$. It means that $f(x_2) < 0$. Regula falsi algorithm will keep points x_2, x_1. We can notice that interval $[x_2, x_1]$ has the same properties as $[x_0, x_1]$, i.e. $f(x)'' > 0$, $x_0 < x_1$, $x_2 < x_1$, and $x_0 < 0$, $x_2 < 0$. It means that point x_1 will be chosen in all future iterations of regula falsi.

Exercise 1.3

a) In general $x_{n+1} = x_n - \frac{f(x_n)}{f(x_n)'}$. We have that $f(x) = x^3 - c$, and $f(x)' = 3x^2$. This gives us an update rule $x_{n+1} = x_n - \frac{x_n^3 - c}{3x_n^2} = \frac{2}{3}x_n - \frac{c}{3x_n^2}$

b)

Listing 8: Newton algorithm.

```
function newton(f, df, x0, maxit)
    i = 0;
    save(sprintf('Initial points a0=%f, f(a0)=%f\n', ...
```

```
            x0, f(x0)));

    x1 = x0;
    x0 = Inf;
    while (i < maxit) && abs(x0 - x1) > 1e-4
        x0 = x1;
        x1 = x1 - f(x1) / df(x1);
        i = i + 1;
        save(sprintf('a%d=%.16f, f(a%d)=%.16f\n', ...
                 i, x1, i, f(x1)));
    end
end
```

Listing 9: Secant method.

```
function secant(f, x0, x1, maxit)
    assert(f(x0) * f(x1) < 0)
    i = 0;
    save(sprintf('Initial points a0=%f, f(a0)=%f, b0=%f, f(b0)=%f\n', ...
            x0, f(x0), x1, f(x1)));
    while (i < maxit) && abs(x0 - x1) > 1e-8
        x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0));
        x0 = x1;
        x1 = x2;
        i = i + 1;
        save(sprintf('a%d=%.16f, f(a%d)=%.16f, b%d=%.16f, f(b%d)=%.16f\n', ...
                 i, x0, i, f(x0), i, x1, i, f(x1)));
    end
end
```

Listing 10: Regula falsi.

```
function regula_falsi(f, x0, x1, maxit)
    assert(f(x0) * f(x1) < 0)
    i = 0;
    save(sprintf('Initial points a0=%f, f(a0)=%f, b0=%f, f(b0)=%f\n', ...
            x0, f(x0), x1, f(x1)));
    while (i < maxit) && abs(x0 - x1) > 1e-8
        x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0));
        if f(x2) * f(x1) < 0
            x0 = x1;
            x1 = x2;
        else
            x1 = x2;
        end
        i = i + 1;
        save(sprintf('a%d=%.16f, f(a%d)=%.16f, b%d=%.16f, f(b%d)=%.16f\n', ...
                 i, x0, i, f(x0), i, x1, i, f(x1)));
    end
end
```

Listing 11: Wheeler method.

```
function wheeler(f, x0, x1, maxit)
    assert(f(x0) * f(x1) < 0)
    i = 0;
    save(sprintf('Initial points a0=%f, f(a0)=%f, b0=%f, f(b0)=%f\n', ...
            x0, f(x0), x1, f(x1)));
    u = 1;
    while (i < maxit) && abs(x0 - x1) > 1e-8
        x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - u * f(x0));
        if f(x2) * f(x1) < 0
            u = 1;
            x0 = x1;
            x1 = x2;
        else
            u = u / 2;
        end
        i = i + 1;
        save(sprintf('a%d=%.16f, f(a%d)=%.16f, b%d=%.16f, f(b%d)=%.16f, u=%f\n', ...
                 i, x0, i, f(x0), i, x1, i, f(x1), u));
    end
end
```

c)

Listing 12: Implementation

```
global file
file = fopen('results/res_ex1_3c_i.txt', 'w');
newton(@(x) x.^3 - 8, @(x) 3 * x.^2, 0.1, 12);
file = fopen('results/res_ex1_3c_ii.txt', 'w');
newton(@(x) x.^3 - 8, @(x) 3 * x.^2, 4, 12);
file = fopen('results/res_ex1_3c_iii.txt', 'w');
newton(@(x) x.^3 - 8, @(x) 3 * x.^2, -0.2, 12);
file = fopen('results/res_ex1_3c_iv.txt', 'w');
newton(@(x) x.^3 - 8, @(x) 3 * x.^2, 1000, 12);
```

Listing 13: Results for i)

```
Initial points a0=0.100000, f(a0)=-7.999000
a1=266.7333333333332916, f(a1)=18977180.7410370260477066
a2=177.8222597034792329, f(a2)=5622866.2936413511633873
a3=118.5482574684455983, f(a3)=1666032.3833036711439490
a4=79.0323613941441465, f(a4)=493637.1506170315551572
a5=52.6886678615353219, f(a5)=146260.7853968200215604
a6=35.1267391560279236, f(a6)=43334.4550296254892601
a7=23.4199872945730512, f(a7)=12837.7647813685161964
a8=15.6181866443840391, f(a8)=3801.7091906359942186
a9=10.4230566282518531, f(a9)=1124.3620131651696283
a10=6.9732502949344273, f(a10)=331.0828006958400920
a11=4.7036736282498737, f(a11)=96.0666416808744543
a12=3.2563122012182433, f(a12)=26.5285316054768430
```

Listing 14: Results for ii)

```
Initial points a0=4.000000, f(a0)=56.000000
a1=2.8333333333333330, f(a1)=14.7453703703703631
a2=2.2210688196847368, f(a2)=2.9568583222447167
a3=2.0212735368091126, f(a3)=0.2580074495430384
a4=2.0002231146078984, f(a4)=0.0026776739866570
a5=2.0000000248863623, f(a5)=0.0000002986363512
a6=2.0000000000000004, f(a6)=0.0000000000000053
```

Listing 15: Results for iii)

```
Initial points a0=-0.200000, f(a0)=-8.008000
a1=66.5333333333333172, f(a1)=294514.0717037034919485
a2=44.3561579627747875, f(a2)=87261.3546271644590888
a3=29.5721273546105508, f(a3)=25853.1422747241740581
a4=19.7178008939133385, f(a4)=7658.1167795218898391
a5=13.1520594534180493, f(a5)=2266.9994198309764215
a6=8.7834559729687491, f(a6)=669.6357129194425397
a7=5.8902024198586265, f(a7)=196.3575368339476768
a8=4.0036630135882154, f(a8)=56.1759857134060070
a9=2.8354705100505502, f(a9)=14.7968795433616762
a10=2.2219930454971513, f(a10)=2.9705420392554736
a11=2.0214400941186876, f(a11)=0.2600490507709363
a12=2.0002265971029196, f(a12)=0.0027194733241522
```

Listing 16: Results for iv)

```
Initial points a0=1000.000000, f(a0)=999999992.000000
a1=666.6666693333334024, f(a1)=296296291.8518519401550293
a2=444.4444522222222531, f(a2)=87791491.8079562038183212
a3=296.2963149814810322, f(a3)=26012291.7949501499533653
a4=197.5309070293168361, f(a4)=7707343.6429487364366651
a5=131.6873396965982579, f(a5)=2283655.3016162859275937
a6=87.7917135709433722, f(a6)=676636.5338184607680887
a7=58.5281550358087372, f(a7)=200482.8248561182990670
a8=39.0195484888232329, f(a8)=59400.2444727503243485
a9=26.0147838021330884, f(a9)=17597.9986016998918785
a10=17.3471294923635213, f(a10)=5212.1485417036656145
a11=11.5736146224297780, f(a11)=1542.2689597608691656
a12=7.7356512221603424, f(a12)=454.9036885214264316
```

Newton method converges very fast (quadratic speed of convergence). Close to convergence point precision doubles.

d)

Listing 17: Implementation

```
global file
file = fopen('results/res_ex1_3d_i.txt', 'w');
secant(@(x) x.^3 - 8, 0.9, 10, 12);
file = fopen('results/res_ex1_3d_ii.txt', 'w');
secant(@(x) x.^3 - 8, -0.2, 3, 12);
file = fopen('results/res_ex1_3d_iii.txt', 'w');
secant(@(x) x.^3 - 8, 0.1, 6, 12);
file = fopen('results/res_ex1_3d_iv.txt', 'w');
secant(@(x) x.^3 - 8, -100, 100, 12);
```

Listing 18: Results for i)

```
Initial points a0=0.900000, f(a0)=-7.271000, b0=10.000000, f(b0)=992.000000
a1=10.0000000000000000, f(a1)=992.0000000000000000, b1=0.9662143702759316, f(b1)=-7.0979710480860518
a2=0.9662143702759316, f(a2)=-7.0979710480860518, b2=1.0303938108438389, f(b2)=-6.9060191389474692
```

```
a3=1.0303938108438389, f(a3)=-6.9060191389474692, b3=3.3394328137792897, f(b3)=29.2407253154423614
a4=3.3394328137792897, f(a4)=29.2407253154423614, b4=1.4715474413449063, f(b4)=-4.8134348382134142
a5=1.4715474413449063, f(a5)=-4.8134348382134142, b5=1.7355664183185524, f(b5)=-2.7721428151507341
a6=1.7355664183185524, f(a6)=-2.7721428151507341, b6=2.0941130161653545, f(b6)=1.1833333362905378
a7=2.0941130161653545, f(a7)=1.1833333362905378, b7=1.9868490293274914, f(b7)=-0.1567762343267933
a8=1.9868490293274914, f(a8)=-0.1567762343267933, b8=1.9993975882692594, f(b8)=-0.0072267635881422
a9=1.9993975882692594, f(a9)=-0.0072267635881422, b9=2.0000039793744642, f(b9)=0.0000477525885820
a10=2.0000039793744642, f(a10)=0.0000477525885820, b10=1.9999999988011501, f(b10)=-0.0000000143861989
a11=1.9999999988011501, f(a11)=-0.0000000143861989, b11=1.9999999999999976, f(b11)=-0.0000000000000293
```

Listing 19: Results for ii)

```
Initial points a0=-0.200000, f(a0)=-8.008000, b0=3.000000, f(b0)=19.000000
a1=3.0000000000000000, f(a1)=19.0000000000000000, b1=0.7488151658767772, f(b1)=-7.5801212506244724
a2=0.7488151658767772, f(a2)=-7.5801212506244724, b2=1.3908082493289478, f(b2)=-5.3096934197633203
a3=1.3908082493289478, f(a3)=-5.3096934197633203, b3=2.8921933206380119, f(b3)=16.1925672188360288
a4=2.8921933206380119, f(a4)=16.1925672188360288, b4=1.7615550544044121, f(b4)=-2.5337604186048761
a5=1.7615550544044121, f(a5)=-2.5337604186048761, b5=1.9145357424606759, f(b5)=-0.9823704974150713
a6=1.9145357424606759, f(a6)=-0.9823704974150713, b6=2.0114061117779363, f(b6)=0.1376554215787174
a7=2.0114061117779363, f(a7)=0.1376554215787174, b7=1.9995003770081257, f(b7)=-0.0059939782884051
a8=1.9995003770081257, f(a8)=-0.0059939782884051, b8=1.9999971609542038, f(b8)=-0.0000340685011935
a9=1.9999971609542038, f(a9)=-0.0000340685011935, b9=2.0000000007093450, f(b9)=0.0000000085121403
a10=2.0000000007093450, f(a10)=0.0000000085121403, b10=1.9999999999999989, f(b10)=-0.0000000000000133
```

Listing 20: Results for iii)

```
Initial points a0=0.100000, f(a0)=-7.999000, b0=6.000000, f(b0)=208.000000
a1=6.0000000000000000, f(a1)=208.0000000000000000, b1=0.3184922152417373, f(b1)=-7.9676930124272767
a2=0.3184922152417373, f(a2)=-7.9676930124272767, b2=0.5281000007639206, f(b2)=-7.8527183963198510
a3=0.5281000007639206, f(a3)=-7.8527183963198510, b3=14.8442244559566685, f(b3)=3262.9396986237547935
a4=14.8442244559566685, f(a4)=3262.9396986237547935, b4=0.5624710275794094, f(b4)=-7.8220489841233922
a5=0.5624710275794094, f(a5)=-7.8220489841233922, b5=0.5966259381220809, f(b5)=-7.7876235335110628
a6=0.5966259381220809, f(a6)=-7.7876235335110628, b6=8.3230486979104938, f(b6)=568.5637145792458114
a7=8.3230486979104938, f(a7)=568.5637145792458114, b7=0.7010248832360269, f(b7)=-7.6554912147566583
a8=0.7010248832360269, f(a8)=-7.6554912147566583, b8=0.8022890128679131, f(b8)=-7.4835925083083588
a9=0.8022890128679131, f(a9)=-7.4835925083083588, b9=5.2108124830632701, f(b9)=133.4869336829014799
a10=5.2108124830632701, f(a10)=133.4869336829014799, b10=1.0363208641040105, f(b10)=-6.8870318775085044
a11=1.0363208641040105, f(a11)=-6.8870318775085044, b11=1.2411299021431332, f(b11)=-6.0881592367063000
a12=1.2411299021431332, f(a12)=-6.0881592367063000, b12=2.8019669780588972, f(b12)=13.9982958311780514
```

Listing 21: Results for iv)

```
Initial points a0=-100.000000, f(a0)=-1000008.000000, b0=100.000000, f(b0)=999992.000000
a1=100.0000000000000000, f(a1)=999992.0000000000000000, b1=0.0007999999999981, f(b1)=-7.9999999994879998
a2=0.0007999999999981, f(a2)=-7.9999999994879998, b2=0.0015999935999469, f(b2)=-7.9999999959040489
a3=0.0015999935999469, f(a3)=-7.9999999959040489, b3=1785724.4577114155981690, f(b3)=5694339291707603968.0000000000000000
a4=1785724.4577114155981690, f(a4)=5694339291707603968.0000000000000000, b4=0.0015999937895685, f(b4)=-7.9999999959040480
a5=0.0015999937895685, f(a5)=-7.9999999959040480, b5=0.0015999937920773, f(b5)=-7.9999999959040480
```

Secant method converges very fast.

e)

Listing 22: Implementation

```
global file
file = fopen('results/res_ex1_3e_i.txt', 'w');
regula_falsi(@(x) x.^3 - 8, 0.9, 10, 12);
file = fopen('results/res_ex1_3e_ii.txt', 'w');
regula_falsi(@(x) x.^3 - 8, -0.2, 3, 12);
file = fopen('results/res_ex1_3e_iii.txt', 'w');
regula_falsi(@(x) x.^3 - 8, 0.1, 6, 12);
file = fopen('results/res_ex1_3e_iv.txt', 'w');
regula_falsi(@(x) x.^3 - 8, -100, 100, 12);
```

Listing 23: Results for i)

```
Initial points a0=0.900000, f(a0)=-7.271000, b0=10.000000, f(b0)=992.000000
a1=10.0000000000000000, f(a1)=992.0000000000000000, b1=0.9662143702759316, f(b1)=-7.0979710480860518
a2=10.0000000000000000, f(a2)=992.0000000000000000, b2=1.0303938108438389, f(b2)=-6.9060191389474692
a3=10.0000000000000000, f(a3)=992.0000000000000000, b3=1.0924059229187364, f(b3)=-6.6963766267332998
a4=10.0000000000000000, f(a4)=992.0000000000000000, b4=1.1521323885134833, f(b4)=-6.4706490514479471
a5=10.0000000000000000, f(a5)=992.0000000000000000, b5=1.2094715263459188, f(b5)=-6.2307592011760322
a6=10.0000000000000000, f(a6)=992.0000000000000000, b6=1.2643402685335974, f(b6)=-5.9788808798875364
a7=10.0000000000000000, f(a7)=992.0000000000000000, b7=1.3166755132389951, f(b7)=-5.7173660271796507
a8=10.0000000000000000, f(a8)=992.0000000000000000, b8=1.3664348399922914, f(b8)=-5.4486691521661239
a9=10.0000000000000000, f(a9)=992.0000000000000000, b9=1.4135966054198152, f(b9)=-5.1752730023070441
a10=10.0000000000000000, f(a10)=992.0000000000000000, b10=1.4581594650072747, f(b10)=-4.8996190218539386
a11=10.0000000000000000, f(a11)=992.0000000000000000, b11=1.5001413893337758, f(b11)=-4.6240455320349394
a12=10.0000000000000000, f(a12)=992.0000000000000000, b12=1.5395782596438816, f(b12)=-4.3507357766247567
```

Exercise 1.3 continued on next page...

Listing 24: Results for ii)

```
Initial points a0=-0.200000, f(a0)=-8.008000, b0=3.000000, f(b0)=19.000000
a1=3.0000000000000000, f(a1)=19.0000000000000000, b1=0.7488151658767772, f(b1)=-7.5801212506244724
a2=3.0000000000000000, f(a2)=19.0000000000000000, b2=1.3908082493289478, f(b2)=-5.3096934197633203
a3=3.0000000000000000, f(a3)=19.0000000000000000, b3=1.7422859377612148, f(b3)=-2.7111859953874848
a4=3.0000000000000000, f(a4)=19.0000000000000000, b4=1.8993430765314379, f(b4)=-1.1481120216218939
a5=3.0000000000000000, f(a5)=19.0000000000000000, b5=1.9620624739697443, f(b5)=-0.4466693788827509
a6=3.0000000000000000, f(a6)=19.0000000000000000, b6=1.9859027985537820, f(b6)=-0.1679768323751141
a7=3.0000000000000000, f(a7)=19.0000000000000000, b7=1.9947897477143053, f(b7)=-0.0623602884963592
a8=3.0000000000000000, f(a8)=19.0000000000000000, b8=1.9980781758199193, f(b8)=-0.0230397368099746
a9=3.0000000000000000, f(a9)=19.0000000000000000, b9=1.9992916525014934, f(b9)=-0.0084971598004246
a10=3.0000000000000000, f(a10)=19.0000000000000000, b10=1.9997389881677921, f(b10)=-0.0031317332412177
a11=3.0000000000000000, f(a11)=19.0000000000000000, b11=1.9999038320842908, f(b11)=-0.0011539594997920
a12=3.0000000000000000, f(a12)=19.0000000000000000, b12=1.9999645689467009, f(b12)=-0.0004251651074760
```

Listing 25: Results for iii)

```
Initial points a0=0.100000, f(a0)=-7.999000, b0=6.000000, f(b0)=208.000000
a1=6.0000000000000000, f(a1)=208.0000000000000000, b1=0.3184922152417373, f(b1)=-7.9676930124272767
a2=6.0000000000000000, f(a2)=208.0000000000000000, b2=0.5281000007639206, f(b2)=-7.8527183963198510
a3=6.0000000000000000, f(a3)=208.0000000000000000, b3=0.7271676340374997, f(b3)=-7.6154935573560190
a4=6.0000000000000000, f(a4)=208.0000000000000000, b4=0.9134029562284072, f(b4)=-7.2379433827318742
a5=6.0000000000000000, f(a5)=208.0000000000000000, b5=1.0844531940952675, f(b5)=-6.7246410426728396
a6=6.0000000000000000, f(a6)=208.0000000000000000, b6=1.2383958791902547, f(b6)=-6.1007659202816331
a7=6.0000000000000000, f(a7)=208.0000000000000000, b7=1.3740770012135406, f(b7)=-5.4056222453303926
a8=6.0000000000000000, f(a8)=208.0000000000000000, b8=1.4912528844181487, f(b8)=-4.6836993952975643
a9=6.0000000000000000, f(a9)=208.0000000000000000, b9=1.5905440675170039, f(b9)=-3.9761932166045995
a10=6.0000000000000000, f(a10)=208.0000000000000000, b10=1.6732554725177538, f(b10)=-3.3152463070962996
a11=6.0000000000000000, f(a11)=208.0000000000000000, b11=1.7411361582096831, f(b11)=-2.7216497624808813
a12=6.0000000000000000, f(a12)=208.0000000000000000, b12=1.7961430157229581, f(b12)=-2.2054096123809579
```

Listing 26: Results for iv)

```
Initial points a0=-100.000000, f(a0)=-1000008.000000, b0=100.000000, f(b0)=999992.000000
a1=100.0000000000000000, f(a1)=999992.0000000000000000, b1=0.0007999999999981, f(b1)=-7.9999999994879998
a2=100.0000000000000000, f(a2)=999992.0000000000000000, b2=0.0015999935999469, f(b2)=-7.9999999959040489
a3=100.0000000000000000, f(a3)=999992.0000000000000000, b3=0.0023999807995886, f(b3)=-7.9999999861763316
a4=100.0000000000000000, f(a4)=999992.0000000000000000, b4=0.0031999615983598, f(b4)=-7.9999999672331796
a5=100.0000000000000000, f(a5)=999992.0000000000000000, b5=0.0039999359953905, f(b5)=-7.9999999360030722
a6=100.0000000000000000, f(a6)=999992.0000000000000000, b6=0.0047999039895031, f(b6)=-7.9999998894146360
a7=100.0000000000000000, f(a7)=999992.0000000000000000, b7=0.0055998655792133, f(b7)=-7.9999998243966459
a8=100.0000000000000000, f(a8)=999992.0000000000000000, b8=0.0063998207627295, f(b8)=-7.9999997378780243
a9=100.0000000000000000, f(a9)=999992.0000000000000000, b9=0.0071997695379531, f(b9)=-7.9999996267878402
a10=100.0000000000000000, f(a10)=999992.0000000000000000, b10=0.0079997119024785, f(b10)=-7.9999994880553125
a11=100.0000000000000000, f(a11)=999992.0000000000000000, b11=0.0087996478535933, f(b11)=-7.9999993186098077
a12=100.0000000000000000, f(a12)=999992.0000000000000000, b12=0.0095995773882780, f(b12)=-7.9999991153808390
```

Sometimes it takes a very long time for regula falsi to converge.

f)

Listing 27: Implementation

```
global file
file = fopen('results/res_ex1_3f_i.txt', 'w');
wheeler(@(x) x.^3 - 8, 0.9, 10, 12);
file = fopen('results/res_ex1_3f_ii.txt', 'w');
wheeler(@(x) x.^3 - 8, -0.2, 3, 12);
file = fopen('results/res_ex1_3f_iii.txt', 'w');
wheeler(@(x) x.^3 - 8, 0.1, 6, 12);
file = fopen('results/res_ex1_3f_iv.txt', 'w');
wheeler(@(x) x.^3 - 8, -100, 100, 12);
```

Listing 28: Results for i)

```
Initial points a0=0.900000, f(a0)=-7.271000, b0=10.000000, f(b0)=992.000000
a1=10.0000000000000000, f(a1)=992.0000000000000000, b1=0.9662143702759316, f(b1)=-7.0979710480860518, u=1.000000
a2=10.0000000000000000, f(a2)=992.0000000000000000, b2=0.9662143702759316, f(b2)=-7.0979710480860518, u=0.500000
a3=10.0000000000000000, f(a3)=992.0000000000000000, b3=0.9662143702759316, f(b3)=-7.0979710480860518, u=0.250000
a4=10.0000000000000000, f(a4)=992.0000000000000000, b4=0.9662143702759316, f(b4)=-7.0979710480860518, u=0.125000
a5=10.0000000000000000, f(a5)=992.0000000000000000, b5=0.9662143702759316, f(b5)=-7.0979710480860518, u=0.062500
a6=10.0000000000000000, f(a6)=992.0000000000000000, b6=0.9662143702759316, f(b6)=-7.0979710480860518, u=0.031250
a7=0.9662143702759316, f(a7)=-7.0979710480860518, b7=2.6492842842475985, f(b7)=10.5945507303995115, u=1.000000
a8=2.6492842842475985, f(a8)=10.5945507303995115, b8=1.6414364596336282, f(b8)=-3.5774553395360744, u=1.000000
a9=2.6492842842475985, f(a9)=10.5945507303995115, b9=1.6414364596336282, f(b9)=-3.5774553395360744, u=0.500000
a10=1.6414364596336282, f(a10)=-3.5774553395360744, b10=2.0477057540563921, f(b10)=0.5862326531114181, u=1.000000
a11=2.0477057540563921, f(a11)=0.5862326531114181, b11=1.9905044635735956, f(b11)=-0.1134063020117573, u=1.000000
a12=2.0477057540563921, f(a12)=0.5862326531114181, b12=1.9905044635735956, f(b12)=-0.1134063020117573, u=0.500000
```

Listing 29: Results for ii)

```
Initial points a0=-0.200000, f(a0)=-8.008000, b0=3.000000, f(b0)=19.000000
a1=3.0000000000000000, f(a1)=19.0000000000000000, b1=0.7488151658767772, f(b1)=-7.5801212506244724, u=1.000000
a2=3.0000000000000000, f(a2)=19.0000000000000000, b2=0.7488151658767772, f(b2)=-7.5801212506244724, u=0.500000
a3=3.0000000000000000, f(a3)=19.0000000000000000, b3=0.7488151658767772, f(b3)=-7.5801212506244724, u=0.250000
a4=0.7488151658767772, f(a4)=-7.5801212506244724, b4=2.1327637624371505, f(b4)=1.7012625717801484, u=1.000000
a5=2.1327637624371505, f(a5)=1.7012625717801484, b5=1.8790882336962398, f(b5)=-1.3649909525976476, u=1.000000
a6=2.1327637624371505, f(a6)=1.7012625717801484, b6=1.8790882336962398, f(b6)=-1.3649909525976476, u=0.500000
a7=1.8790882336962398, f(a7)=-1.3649909525976476, b7=2.0353715547478970, f(b7)=0.4320097932970146, u=1.000000
a8=2.0353715547478970, f(a8)=0.4320097932970146, b8=1.9978001038859310, f(b8)=-0.0263697267578440, u=1.000000
a9=2.0353715547478970, f(a9)=0.4320097932970146, b9=1.9978001038859310, f(b9)=-0.0263697267578440, u=0.500000
a10=1.9978001038859310, f(a10)=-0.0263697267578440, b10=2.0018877797292256, f(b10)=0.0226747457520471, u=1.000000
a11=2.0018877797292256, f(a11)=0.0226747457520471, b11=1.9999979233250242, f(b11)=-0.0000249200738338, u=1.000000
a12=2.0018877797292256, f(a12)=0.0226747457520471, b12=1.9999979233250242, f(b12)=-0.0000249200738338, u=0.500000
```

Listing 30: Results for iii)

```
Initial points a0=0.100000, f(a0)=-7.999000, b0=6.000000, f(b0)=208.000000
a1=6.0000000000000000, f(a1)=208.0000000000000000, b1=0.3184922152417373, f(b1)=-7.9676930124272767, u=1.000000
a2=6.0000000000000000, f(a2)=208.0000000000000000, b2=0.3184922152417373, f(b2)=-7.9676930124272767, u=0.500000
a3=6.0000000000000000, f(a3)=208.0000000000000000, b3=0.3184922152417373, f(b3)=-7.9676930124272767, u=0.250000
a4=6.0000000000000000, f(a4)=208.0000000000000000, b4=0.3184922152417373, f(b4)=-7.9676930124272767, u=0.125000
a5=6.0000000000000000, f(a5)=208.0000000000000000, b5=0.3184922152417373, f(b5)=-7.9676930124272767, u=0.062500
a6=0.3184922152417373, f(a6)=-7.9676930124272767, b6=2.4774569544640985, f(b6)=7.2061178568415407, u=1.000000
a7=2.4774569544640985, f(a7)=7.2061178568415407, b7=1.4521539179599152, f(b7)=-4.9377689712478139, u=1.000000
a8=2.4774569544640985, f(a8)=7.2061178568415407, b8=1.4521539179599152, f(b8)=-4.9377689712478139, u=0.500000
a9=1.4521539179599152, f(a9)=-4.9377689712478139, b9=2.0449195818100172, f(b9)=0.5512322320321594, u=1.000000
a10=2.0449195818100172, f(a10)=0.5512322320321594, b10=1.9853911671030091, f(b10)=-0.1740286045600961, u=1.000000
a11=2.0449195818100172, f(a11)=0.5512322320321594, b11=1.9853911671030091, f(b11)=-0.1740286045600961, u=0.500000
a12=1.9853911671030091, f(a12)=-0.1740286045600961, b12=2.0084307947669577, f(b12)=0.1015966062524711, u=1.000000
```

Listing 31: Results for iv)

```
Initial points a0=-100.000000, f(a0)=-1000008.000000, b0=100.000000, f(b0)=999992.000000
a1=100.0000000000000000, f(a1)=999992.0000000000000000, b1=0.0007999999999981, f(b1)=-7.9999999994879998, u=1.000000
a2=100.0000000000000000, f(a2)=999992.0000000000000000, b2=0.0007999999999981, f(b2)=-7.9999999994879998, u=0.500000
a3=100.0000000000000000, f(a3)=999992.0000000000000000, b3=0.0007999999999981, f(b3)=-7.9999999994879998, u=0.250000
a4=100.0000000000000000, f(a4)=999992.0000000000000000, b4=0.0007999999999981, f(b4)=-7.9999999994879998, u=0.125000
a5=100.0000000000000000, f(a5)=999992.0000000000000000, b5=0.0007999999999981, f(b5)=-7.9999999994879998, u=0.062500
a6=100.0000000000000000, f(a6)=999992.0000000000000000, b6=0.0007999999999981, f(b6)=-7.9999999994879998, u=0.031250
a7=100.0000000000000000, f(a7)=999992.0000000000000000, b7=0.0007999999999981, f(b7)=-7.9999999994879998, u=0.015625
a8=100.0000000000000000, f(a8)=999992.0000000000000000, b8=0.0007999999999981, f(b8)=-7.9999999994879998, u=0.007812
a9=100.0000000000000000, f(a9)=999992.0000000000000000, b9=0.0007999999999981, f(b9)=-7.9999999994879998, u=0.003906
a10=100.0000000000000000, f(a10)=999992.0000000000000000, b10=0.0007999999999981, f(b10)=-7.9999999994879998, u=0.001953
a11=100.0000000000000000, f(a11)=999992.0000000000000000, b11=0.0007999999999981, f(b11)=-7.9999999994879998, u=0.000977
a12=100.0000000000000000, f(a12)=999992.0000000000000000, b12=0.0007999999999981, f(b12)=-7.9999999994879998, u=0.000488
```

e) Last example where initial values are big is quite difficult. None of methods has guarantees far from convergence point, and it might take a long time to get over there. Scent method seems to perform the best in case of this example. However, it has no guarantees. Newton method has a nice behavious close to the convergence point (doubles precision in every step).