CSCI-GA.2945-001: Assignment #2

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Contents

Listing 1: Script to inspect any function.

```
1   function fun = inspect(f)
2       function val = nested(x)
3           global xs
4           global fs
5           xs(length(xs) + 1) = x;
6           val = f(x);
7           fs(length(fs) + 1) = val;
8           method = '';
9           target = xs(length(xs));
10          if length(xs) > 3
11              a = xs(length(xs) - 3);
12              b = xs(length(xs) - 2);
13              c = xs(length(xs) - 1);
14              fa = fs(length(xs) - 3);
15              fb = fs(length(xs) - 2);
16              fc = fs(length(fs) - 1);
17              m = 0.5*(a - b);
18              s = fb/fc;
19              q = fc/fa;
20              r = fb/fa;
21              p = s*(2.0*m*q*(q - r) - (b - c)*(r - 1.0));
22              q = (q - 1.0)*(r - 1.0)*(s - 1.0);
23              if p > 0, q = -q; else p = -p; end;
24              quadratic = b + p/q;
25              if abs((quadratic - target) / target) < 1e-12
26                  method = 'Inverse quadratic interpolation between last 3 points.';
27              end
28          end
29          if length(xs) > 2
30              for a_idx = 1:length(xs) - 1
31                  for b_idx = (a_idx + 1):length(xs) - 1
32                      a = xs(a_idx);
33                      b = xs(b_idx);
34                      fa = fs(a_idx);
35                      fb = fs(b_idx);
36                      interesction = b - fb * (b - a) / (fb - fa);
37                      if abs((interesction - target) / target) < 1e-12
38                          method = sprintf('linear interpolation (secant) between x_%d, x_%d', a_idx, b_idx);
39                      end
40                      if abs(((a + b) / 2 - target) / target) < 1e-12
41                          method = sprintf('bisection between x_%d, x_%d', a_idx, b_idx);
42                      end
43                  end
44              end
45          end
46          if length(xs) <= 2
47              method = 'initial point';
48          end
49          save(sprintf('iter = %d,   x = %d,   f(x) = %d,   method = %s\n', length(xs), x, val, method));
50      end
51      fun = @(x) nested(x);
52   end
```

Exercise 1.1

a)

Listing 2: Program to call fzerotx $2x^3 - 4x^2 + 3x + 1$.

```
1   global file
2   init()
3   file = fopen('results/res_ex2_1a.txt', 'w');
4   fzerotx(inspect(@(x) 2 * x ^ 3 - 4 * x ^ 2 + 3 * x + 1), [-2, 2]);
```

Listing 3: Execution results for fzero.

```
1    iter = 1,            x = -2,            f(x) = -37,          method = initial point
2    iter = 2,            x = 2,             f(x) = 7,            method = initial point
3    iter = 3,            x = 1.363636e+00,  f(x) = 2.724267e+00, method = linear interpolation (secant) between x_1, x_2
4    iter = 4,            x = 9.859856e-01,  f(x) = 1.986373e+00, method = Inverse quadratic interpolation between last 3 points.
5    iter = 5,            x = -5.070072e-01, f(x) = -1.809906e+00, method = bisection between x_1, x_4
6    iter = 6,            x = 2.047889e-01,  f(x) = 1.463790e+00, method = linear interpolation (secant) between x_4, x_5
7    iter = 7,            x = -1.134814e-01, f(x) = 6.051210e-01, method = linear interpolation (secant) between x_5, x_6
8    iter = 8,            x = -2.815732e-01, f(x) = -2.065016e-01, method = Inverse quadratic interpolation between last 3 points.
9    iter = 9,            x = -2.388055e-01, f(x) = 2.823405e-02, method = linear interpolation (secant) between x_7, x_8
10   iter = 10,           x = -2.439496e-01, f(x) = 1.070015e-03, method = linear interpolation (secant) between x_8, x_9
11   iter = 11,           x = -2.441512e-01, f(x) = -3.612888e-07, method = Inverse quadratic interpolation between last 3 points.
12   iter = 12,           x = -2.441511e-01, f(x) = 7.492806e-11, method = linear interpolation (secant) between x_10, x_11
13   iter = 13,           x = -2.441511e-01, f(x) = 0,            method = linear interpolation (secant) between x_11, x_12
```

b)

Listing 4: A program to call fzerotx for f.

```
1  global file
2  init();
3  file = fopen('results/res_ex2_1b_fzero.txt', 'w');
4  fzerotx(inspect(@(x) 1.1 * x^3 - 2.6*x - 2.6049), [-1.95, 2.4]);
5
6  init();
7  file = fopen('results/res_ex2_1b_wheeler.txt', 'w');
8  wheeler(inspect(@(x) 1.1 * x^3 - 2.6*x - 2.6049), -1.95, 2.4, 11);
```

Listing 5: Execution results for fzero for $1.1x^3 - 2.6x - 2.6049$.

```
1   iter = 1,         x = -1.950000e+00,    f(x) = -5.691263e+00,   method = initial point
2   iter = 2,         x = 2.400000e+00,     f(x) = 6.361500e+00,    method = initial point
3   iter = 3,         x = 1.040512e-01,     f(x) = -2.874194e+00,   method = linear interpolation (secant) between x_1, x_2
4   iter = 4,         x = 1.547562e+00,     f(x) = -2.551598e+00,   method = Inverse quadratic interpolation between last 3 points.
5   iter = 5,         x = 1.973781e+00,     f(x) = 7.216948e-01,    method = bisection between x_2, x_4
6   iter = 6,         x = 1.879808e+00,     f(x) = -1.854987e-01,   method = linear interpolation (secant) between x_4, x_5
7   iter = 7,         x = 1.899023e+00,     f(x) = -9.089825e-03,   method = linear interpolation (secant) between x_5, x_6
8   iter = 8,         x = 1.900001e+00,     f(x) = 1.027322e-05,    method = Inverse quadratic interpolation between last 3 points.
9   iter = 9,         x = 1.900000e+00,     f(x) = -6.758447e-09,   method = linear interpolation (secant) between x_7, x_8
10  iter = 10,        x = 1.900000e+00,     f(x) = -3.996803e-15,   method = linear interpolation (secant) between x_8, x_9
11  iter = 11,        x = 1.900000e+00,     f(x) = 3.108624e-15,    method = linear interpolation (secant) between x_9, x_10
```

Listing 6: Execution results for Wheeler for $1.1x^3 - 2.6x - 2.6049$.

```
1   iter = 1,         x = -1.950000e+00,    f(x) = -5.691263e+00,   method = initial point
2   iter = 2,         x = 2.400000e+00,     f(x) = 6.361500e+00,    method = initial point
3   iter = 3,         x = -1.950000e+00,    f(x) = -5.691263e+00,   method =
4   iter = 4,         x = 2.400000e+00,     f(x) = 6.361500e+00,    method =
5   iter = 5,         x = 1.040512e-01,     f(x) = -2.874194e+00,   method = linear interpolation (secant) between x_3, x_4
6   iter = 6,         x = 2.400000e+00,     f(x) = 6.361500e+00,    method = bisection between x_2, x_4
7   iter = 7,         x = 1.040512e-01,     f(x) = -2.874194e+00,   method = linear interpolation (secant) between x_3, x_6
8   iter = 8,         x = 8.185620e-01,     f(x) = -4.129842e+00,   method = linear interpolation (secant) between x_6, x_7
9   iter = 9,         x = 1.040512e-01,     f(x) = -2.874194e+00,   method = bisection between x_5, x_7
10  iter = 10,        x = 1.193905e+00,     f(x) = -3.837070e+00,   method =
11  iter = 11,        x = 1.040512e-01,     f(x) = -2.874194e+00,   method = bisection between x_7, x_9
12  iter = 12,        x = 1.582134e+00,     f(x) = -2.362102e+00,   method =
13  iter = 13,        x = 1.040512e-01,     f(x) = -2.874194e+00,   method = bisection between x_9, x_11
14  iter = 14,        x = 1.902448e+00,     f(x) = 2.283172e-02,    method =
15  iter = 15,        x = 1.040512e-01,     f(x) = -2.874194e+00,   method = bisection between x_11, x_13
16  iter = 16,        x = 1.902448e+00,     f(x) = 2.283172e-02,    method =
17  iter = 17,        x = 1.888274e+00,     f(x) = -1.083417e-01,   method = linear interpolation (secant) between x_15, x_16
18  iter = 18,        x = 1.902448e+00,     f(x) = 2.283172e-02,    method = bisection between x_14, x_16
19  iter = 19,        x = 1.888274e+00,     f(x) = -1.083417e-01,   method = linear interpolation (secant) between x_15, x_18
20  iter = 20,        x = 1.899981e+00,     f(x) = -1.807775e-04,   method = linear interpolation (secant) between x_18, x_19
21  iter = 21,        x = 1.888274e+00,     f(x) = -1.083417e-01,   method = bisection between x_17, x_19
22  iter = 22,        x = 1.901096e+00,     f(x) = 1.021918e-02,    method =
23  iter = 23,        x = 1.888274e+00,     f(x) = -1.083417e-01,   method = bisection between x_19, x_21
24  iter = 24,        x = 1.901096e+00,     f(x) = 1.021918e-02,    method =
25  iter = 25,        x = 1.899991e+00,     f(x) = -8.104269e-05,   method = linear interpolation (secant) between x_23, x_24
26  iter = 26,        x = 1.901096e+00,     f(x) = 1.021918e-02,    method = bisection between x_22, x_24
27  iter = 27,        x = 1.899991e+00,     f(x) = -8.104269e-05,   method = linear interpolation (secant) between x_23, x_26
28  iter = 28,        x = 1.900000e+00,     f(x) = -5.979504e-08,   method = linear interpolation (secant) between x_26, x_27
29  iter = 29,        x = 1.899991e+00,     f(x) = -8.104269e-05,   method = bisection between x_25, x_27
30  iter = 30,        x = 1.900009e+00,     f(x) = 7.965962e-05,    method =
31  iter = 31,        x = 1.899991e+00,     f(x) = -8.104269e-05,   method = bisection between x_27, x_29
32  iter = 32,        x = 1.900009e+00,     f(x) = 7.965962e-05,    method =
33  iter = 33,        x = 1.900000e+00,     f(x) = -4.667036e-10,   method = linear interpolation (secant) between x_31, x_32
34  iter = 34,        x = 1.900009e+00,     f(x) = 7.965962e-05,    method = bisection between x_30, x_32
35  iter = 35,        x = 1.900000e+00,     f(x) = -4.667036e-10,   method = linear interpolation (secant) between x_31, x_34
36  iter = 36,        x = 1.900000e+00,     f(x) = -3.108624e-15,   method = linear interpolation (secant) between x_34, x_35
37  iter = 37,        x = 1.900000e+00,     f(x) = -4.667036e-10,   method = bisection between x_33, x_35
38  iter = 38,        x = 1.900000e+00,     f(x) = 4.666938e-10,    method = linear interpolation (secant) between x_23, x_28
39  iter = 39,        x = 1.900000e+00,     f(x) = -4.667036e-10,   method = bisection between x_35, x_37
40  iter = 40,        x = 1.900000e+00,     f(x) = 4.666938e-10,    method = linear interpolation (secant) between x_23, x_28
```

Exercise 1.2

a)

Listing 7: Program to call fzero for $2x^3 - 4x^2 + 3x + 1$.

```
1  global file
2  init();
3  file = fopen('results/res_ex2_2a_-2.txt', 'w');
4  fzero(inspect(@(x) 2 * x ^ 3 - 4 * x ^ 2 + 3 * x + 1), -2);
5
6  init();
7  file = fopen('results/res_ex2_2a_2.txt', 'w');
8  fzero(inspect(@(x) 2 * x ^ 3 - 4 * x ^ 2 + 3 * x + 1), 2);
```

Listing 8: Execution results for fzero for initial point −2

```
1   iter = 1,             x = -2,               f(x) = -37,           method = initial point
2   iter = 2,             x = -1.943431e+00,    f(x) = -3.461839e+01, method = initial point
3   iter = 3,             x = -2.056569e+00,    f(x) = -3.948401e+01, method =
4   iter = 4,             x = -1.920000e+00,    f(x) = -3.366138e+01, method =
5   iter = 5,             x = -2.080000e+00,    f(x) = -4.054342e+01, method =
6   iter = 6,             x = -1.886863e+00,    f(x) = -3.233701e+01, method =
7   iter = 7,             x = -2.113137e+00,    f(x) = -4.207259e+01, method =
8   iter = 8,             x = -1.840000e+00,    f(x) = -3.052141e+01, method =
9   iter = 9,             x = -2.160000e+00,    f(x) = -4.429779e+01, method =
10  iter = 10,            x = -1.773726e+00,    f(x) = -2.806624e+01, method =
11  iter = 11,            x = -2.226274e+00,    f(x) = -4.757216e+01, method =
12  iter = 12,            x = -1.680000e+00,    f(x) = -2.481286e+01, method =
13  iter = 13,            x = -2.320000e+00,    f(x) = -5.246394e+01, method =
14  iter = 14,            x = -1.547452e+00,    f(x) = -2.063186e+01, method =
15  iter = 15,            x = -2.452548e+00,    f(x) = -5.992174e+01, method =
16  iter = 16,            x = -1.360000e+00,    f(x) = -1.550931e+01, method =
17  iter = 17,            x = -2.640000e+00,    f(x) = -7.159789e+01, method =
18  iter = 18,            x = -1.094903e+00,    f(x) = -9.705132e+00, method =
19  iter = 19,            x = -2.905097e+00,    f(x) = -9.050927e+01, method =
20  iter = 20,            x = -7.200000e-01,    f(x) = -3.980096e+00, method =
21  iter = 21,            x = -3.280000e+00,    f(x) = -1.224487e+02, method =
22  iter = 22,            x = -1.898066e-01,    f(x) = 2.727977e-01,  method =
23  iter = 23,            x = -1.966758e-01,    f(x) = 2.400316e-01,  method = linear interpolation (secant) between x_21, x_22
24  iter = 24,            x = -2.468984e-01,    f(x) = -1.463175e-02, method = Inverse quadratic interpolation between last 3 points.
25  iter = 25,            x = -2.440128e-01,    f(x) = 7.342134e-04,  method = linear interpolation (secant) between x_23, x_24
26  iter = 26,            x = -2.441507e-01,    f(x) = 2.072306e-06,  method = linear interpolation (secant) between x_24, x_25
27  iter = 27,            x = -2.441511e-01,    f(x) = -1.366907e-12, method = Inverse quadratic interpolation between last 3 points.
28  iter = 28,            x = -2.441511e-01,    f(x) = -2.220446e-16, method = linear interpolation (secant) between x_26, x_27
29  iter = 29,            x = -2.441511e-01,    f(x) = 2.220446e-15,  method = bisection between x_27, x_28
```

Listing 9: Execution results for fzero for initial point 2

```
1   iter = 1,             x = 2,                f(x) = 7,             method = initial point
2   iter = 2,             x = 1.943431e+00,     f(x) = 6.402984e+00,  method = initial point
3   iter = 3,             x = 2.056569e+00,     f(x) = 7.648216e+00,  method =
4   iter = 4,             x = 1.920000e+00,     f(x) = 6.170176e+00,  method =
5   iter = 5,             x = 2.080000e+00,     f(x) = 7.932224e+00,  method =
6   iter = 6,             x = 1.886863e+00,     f(x) = 5.854996e+00,  method =
7   iter = 7,             x = 2.113137e+00,     f(x) = 8.349804e+00,  method =
8   iter = 8,             x = 1.840000e+00,     f(x) = 5.436608e+00,  method =
9   iter = 9,             x = 2.160000e+00,     f(x) = 8.972992e+00,  method =
10  iter = 10,            x = 1.773726e+00,     f(x) = 4.897414e+00,  method =
11  iter = 11,            x = 2.226274e+00,     f(x) = 9.921786e+00,  method =
12  iter = 12,            x = 1.680000e+00,     f(x) = 4.233664e+00,  method =
13  iter = 13,            x = 2.320000e+00,     f(x) = 1.140474e+01,  method =
14  iter = 14,            x = 1.547452e+00,     f(x) = 3.475004e+00,  method =
15  iter = 15,            x = 2.452548e+00,     f(x) = 1.380180e+01,  method =
16  iter = 16,            x = 1.360000e+00,     f(x) = 2.712512e+00,  method =
17  iter = 17,            x = 2.640000e+00,     f(x) = 1.784109e+01,  method =
18  iter = 18,            x = 1.094903e+00,     f(x) = 2.114626e+00,  method =
19  iter = 19,            x = 2.905097e+00,     f(x) = 2.499257e+01,  method =
20  iter = 20,            x = 7.200000e-01,     f(x) = 1.832896e+00,  method =
21  iter = 21,            x = 3.280000e+00,     f(x) = 3.838150e+01,  method =
22  iter = 22,            x = 1.898066e-01,     f(x) = 1.438990e+00,  method =
23  iter = 23,            x = 3.810193e+00,     f(x) = 6.498981e+01,  method =
24  iter = 24,            x = -5.600000e-01,    f(x) = -2.285632e+00, method =
25  iter = 25,            x = -4.115260e-01,    f(x) = -1.051379e+00, method = linear interpolation (secant) between x_23, x_24
26  iter = 26,            x = -2.870642e-01,    f(x) = -2.381273e-01, method = Inverse quadratic interpolation between last 3 points.
27  iter = 27,            x = -2.509627e-01,    f(x) = -3.642960e-02, method =
28  iter = 28,            x = -2.444577e-01,    f(x) = -1.628936e-03, method =
29  iter = 29,            x = -2.441534e-01,    f(x) = -1.196376e-05, method =
30  iter = 30,            x = -2.441511e-01,    f(x) = -3.974370e-09, method =
31  iter = 31,            x = -2.441511e-01,    f(x) = -9.769963e-15, method = linear interpolation (secant) between x_29, x_30
32  iter = 32,            x = -2.441511e-01,    f(x) = 0,             method = linear interpolation (secant) between x_30, x_31
```

b)

Listing 10: Program to call fzero.

```
1   global file
2   init();
3   file = fopen('results/res_ex2_2b_minus.txt', 'w');
4   fzero(inspect(@(x) 1.1 * x^3 - 2.6*x - 2.6049), -1.95);
5
6   init();
7   file = fopen('results/res_ex2_2b_plus.txt', 'w');
8   fzero(inspect(@(x) 1.1 * x^3 - 2.6*x - 2.6049), 2.4);
```

Listing 11: Execution results for fzero for initial point -1.95 for $1.1x^3 - 2.6x - 2.6049$.

```
1   iter = 1,             x = -1.950000e+00,    f(x) = -5.691263e+00, method = initial point
2   iter = 2,             x = -1.894846e+00,    f(x) = -5.161964e+00, method = initial point
3   iter = 3,             x = -2.005154e+00,    f(x) = -6.259711e+00, method =
4   iter = 4,             x = -1.872000e+00,    f(x) = -4.953928e+00, method =
5   iter = 5,             x = -2.028000e+00,    f(x) = -6.506899e+00, method =
6   iter = 6,             x = -1.839691e+00,    f(x) = -4.670709e+00, method =
```

```
 7  iter = 7,           x = -2.060309e+00,    f(x) = -6.868418e+00,    method =
 8  iter = 8,           x = -1.794000e+00,    f(x) = -4.291762e+00,    method =
 9  iter = 9,           x = -2.106000e+00,    f(x) = -7.403968e+00,    method =
10  iter = 10,          x = -1.729383e+00,    f(x) = -3.797899e+00,    method =
11  iter = 11,          x = -2.170617e+00,    f(x) = -8.211035e+00,    method =
12  iter = 12,          x = -1.638000e+00,    f(x) = -3.180409e+00,    method =
13  iter = 13,          x = -2.262000e+00,    f(x) = -9.454934e+00,    method =
14  iter = 14,          x = -1.508765e+00,    f(x) = -2.460074e+00,    method =
15  iter = 15,          x = -2.391235e+00,    f(x) = -1.142809e+01,    method =
16  iter = 16,          x = -1.326000e+00,    f(x) = -1.721921e+00,    method =
17  iter = 17,          x = -2.574000e+00,    f(x) = -1.467187e+01,    method =
18  iter = 18,          x = -1.067531e+00,    f(x) = -1.167560e+00,    method =
19  iter = 19,          x = -2.832469e+00,    f(x) = -2.023750e+01,    method =
20  iter = 20,          x = -7.020000e-01,    f(x) = -1.160243e+00,    method =
21  iter = 21,          x = -3.198000e+00,    f(x) = -3.026736e+01,    method =
22  iter = 22,          x = -1.850615e-01,    f(x) = -2.130712e+00,    method =
23  iter = 23,          x = -3.714939e+00,    f(x) = -4.934197e+01,    method =
24  iter = 24,          x = 5.460000e-01,     f(x) = -3.845452e+00,    method =
25  iter = 25,          x = -4.446000e+00,    f(x) = -8.771738e+01,    method =
26  iter = 26,          x = 1.579877e+00,     f(x) = -2.374850e+00,    method =
27  iter = 27,          x = -5.479877e+00,    f(x) = -1.693683e+02,    method =
28  iter = 28,          x = 3.042000e+00,     f(x) = 2.045085e+01,     method =
29  iter = 29,          x = 2.123865e+00,     f(x) = 2.411420e+00,     method = linear interpolation (secant) between x_27, x_28
30  iter = 30,          x = 2.002856e+00,     f(x) = 1.025430e+00,     method = Inverse quadratic interpolation between last 3 points.
31  iter = 31,          x = 1.913952e+00,     f(x) = 1.311595e-01,     method =
32  iter = 32,          x = 1.900957e+00,     f(x) = 8.917273e-03,     method =
33  iter = 33,          x = 1.900009e+00,     f(x) = 8.712318e-05,     method =
34  iter = 34,          x = 1.900000e+00,     f(x) = 5.882251e-08,     method =
35  iter = 35,          x = 1.900000e+00,     f(x) = 3.876899e-13,     method = linear interpolation (secant) between x_33, x_34
36  iter = 36,          x = 1.900000e+00,     f(x) = 2.220446e-15,     method = linear interpolation (secant) between x_34, x_35
37  iter = 37,          x = 1.900000e+00,     f(x) = -6.661338e-15,    method = bisection between x_35, x_36
```

Listing 12: Execution results for fzero for initial point 2.4 for $1.1x^3 - 2.6x - 2.6049$.

```
 1  iter = 1,           x = 2.400000e+00,     f(x) = 6.361500e+00,     method = initial point
 2  iter = 2,           x = 2.332118e+00,     f(x) = 5.283839e+00,     method = initial point
 3  iter = 3,           x = 2.467882e+00,     f(x) = 7.512151e+00,     method =
 4  iter = 4,           x = 2.304000e+00,     f(x) = 4.858350e+00,     method =
 5  iter = 5,           x = 2.496000e+00,     f(x) = 8.010632e+00,     method =
 6  iter = 6,           x = 2.264235e+00,     f(x) = 4.277105e+00,     method =
 7  iter = 7,           x = 2.535765e+00,     f(x) = 8.737858e+00,     method =
 8  iter = 8,           x = 2.208000e+00,     f(x) = 3.495341e+00,     method =
 9  iter = 9,           x = 2.592000e+00,     f(x) = 9.811585e+00,     method =
10  iter = 10,          x = 2.128471e+00,     f(x) = 2.468157e+00,     method =
11  iter = 11,          x = 2.671529e+00,     f(x) = 1.142269e+01,     method =
12  iter = 12,          x = 2.016000e+00,     f(x) = 1.166394e+00,     method =
13  iter = 13,          x = 2.784000e+00,     f(x) = 1.389231e+01,     method =
14  iter = 14,          x = 1.856942e+00,     f(x) = -3.894625e-01,    method =
15  iter = 15,          x = 1.882223e+00,     f(x) = -1.635840e-01,    method = linear interpolation (secant) between x_13, x_14
16  iter = 16,          x = 1.900318e+00,     f(x) = 2.965389e-03,     method = Inverse quadratic interpolation between last 3 points.
17  iter = 17,          x = 1.899996e+00,     f(x) = -3.579448e-05,    method = linear interpolation (secant) between x_15, x_16
18  iter = 18,          x = 1.900000e+00,     f(x) = -7.670544e-09,    method = linear interpolation (secant) between x_16, x_17
19  iter = 19,          x = 1.900000e+00,     f(x) = -4.440892e-16,    method = linear interpolation (secant) between x_17, x_18
20  iter = 20,          x = 1.900000e+00,     f(x) = 7.549517e-15,     method = linear interpolation (secant) between x_18, x_19
```