DAI-Labor
TU Berlin

On the Spectral Evolution of Large Networks

Jérôme Kunegis
DAI Lab, Technische Universität Berlin, Germany jerome.kunegis@dai-labor.de

Summary

- Network evolution is **spectral**: The graph spectrum changes over time; eigenvectors stay constant
- Matrix decompositions separate **global** from **local** statistics
- Confirmed for over **one hundred** unipartite, bipartite, unweighted, weighted, signed networks
- Applied to link prediction, rating prediction, link sign prediction by learning **spectral transformations**

Kernels and Pseudokernels

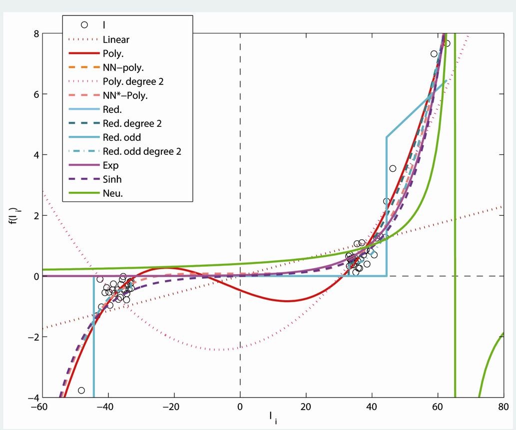
Adjacency matrix **A** and Laplacian **L** (ICML 2009):

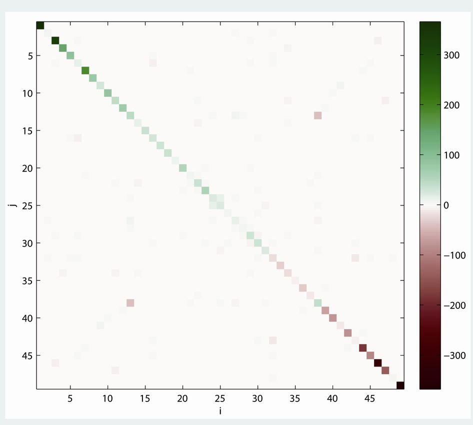
- $\exp(\mathbf{A})$ – Exponential kernel
- $(\mathbf{I} - \alpha\mathbf{A})^{-1}$ – Von Neumann kernel
- $(\alpha\mathbf{I} + \mathbf{L})^{+}$ – Laplacian kernel
- $\exp(-\alpha\mathbf{L})$ – Heat diffusion kernel
- $(\mathbf{A})_k$ – Rank reduction
- Poly($\mathbf{A}$) – Path counting

Are **spectral transformations**, e.g.:
$\exp(\mathbf{A}) = \mathbf{U} \exp(\mathbf{D}) \mathbf{U}^T$ with $\mathbf{A} = \mathbf{UDU}^T$

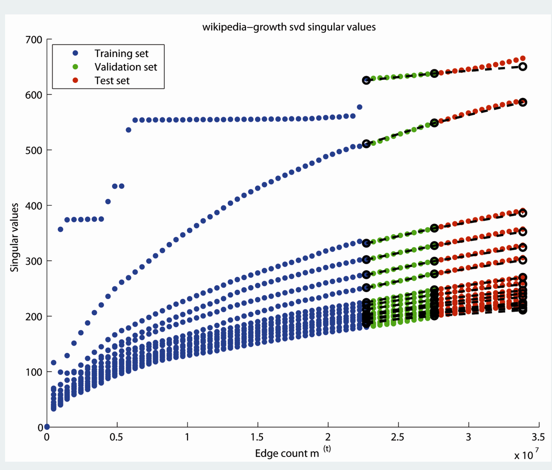
Learning Spectral Transformations

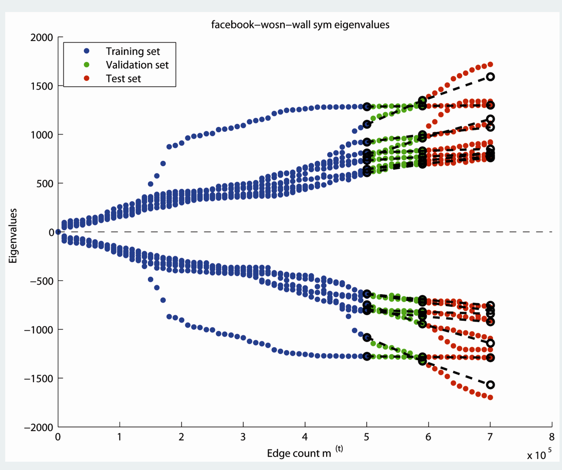
- By reduction to curve fitting (ICML 2009)



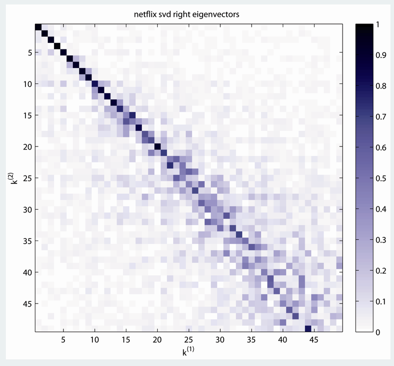


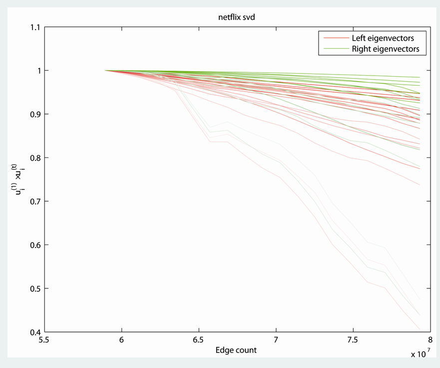
- By spectral extrapolation (Submitted 2010)



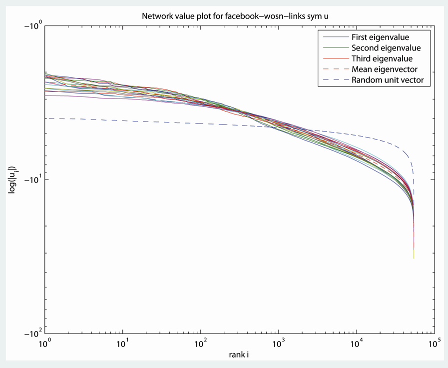


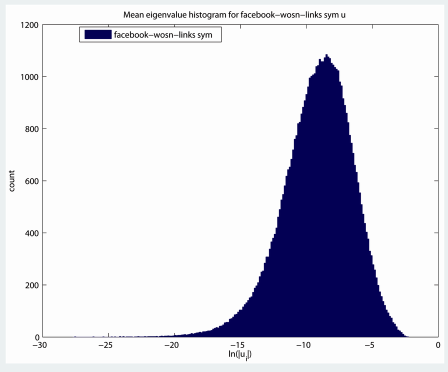
Eigenvector Evolution





Cosine similarity between new and old eigenvectors





Distribution of eigenvector components

Applications and Special Cases

- **Link prediction** in unweighted networks (ICML 2009, Submitted 2010)
- **Rating prediction** / collaborative filtering in weighted networks (ICML 2009)
- **Bipartite networks**: spectral transformations are odd functions (IPMU 2010)
- **Signed networks**: special treatment of the Laplacian (SDM 2010)

References

- J. Kunegis, A. Lommatzsch, *Learning Spectral Graph Transformations for Link Prediction*, ICML 2009.
- J. Kunegis, A. Lommatzsch, C. Bauckhage, *The Slashdot Zoo: Mining a Social Network with Negative Edges*, WWW 2009.
- J. Kunegis, S. Schmidt, A. Lommatzsch, J. Lerner, E. W. De Luca, S. Albayrak, Spectral Analysis of Signed Graphs for Clustering, Prediction and Visualization, SDM 2010.
- J. Kunegis, E. W. De Luca, S. Albayrak, *The Link Prediction Problem in Bipartite Networks*, IPMU 2010.
- J. Kunegis, C. Bauckhage, D. Fay, E. W. De Luca, S. Albayrak, *Network Growth and the Spectral Evolution Hypothesis*, Submitted 2010.
- J. Kunegis, N. Neubauer, C. Bauckhage, E. W. De Luca, S. Albayrak, *A Survey of Seventy-Seven Large Network Datasets*, 2010.