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The Importance of Being Nonplanar: Street Network Representation in Urban Form Studies

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Abstract

Keywords
street network, GIS, urban form, transportation, urban design

Introduction

Background

Why it's appealing. Planarity allows for easy polygonal spatial analysis (Fohl et al. 1996). In mathematics, there is an exact bijection between planar graphs and trees, and classifying planar graphs presents a trivial computational problem (Louf and Barthélemy 2014). Planar graphs offer computational simplicity and tractability. But Masucci et al. (2009) and Masucci et al. (2013) argue that planar graphs have also presented a compelling research domain for urban scholars and geographers as they were understudied until recently because they appear topologically and geometrically trivial and the planarity constraint did not lend itself to certain popular graph-theoretic analyses. By contrast, Barthélemy (2011, p. 3) argue that "planar spatial networks are the most important and most studies have focused on these examples". As Viana et al. (2013, p. 1) put it, "there is still a lack of global, high-level metrics allowing to characterize their structure and geometrical patterns."

Street networks, however, are frequently nonplanar in reality: they tend to have some overpasses or underpasses that result in the failure of formal proofs of their planarity, such as the Kuratowski (1930) theorem or the Hopcroft and Tarjan (1974) algorithm (Gastner and Newman 2006). As Levinson (2012, p. 7) points out, "Real networks are neither perfect, nor planar, nor grids, though they may approximate them." "Quite often the transportation network has overpasses and underpasses that require a non-planar network representation" Mandloi and Thill (2010, p. 199). "A planar graph is one which can be drawn in two dimensions with no edges intersecting except at vertices on which they are both incident. For many infrastructure networks, this is approximately true, although bridges and tunnels in ground-transport networks are an obvious (but generally minor) exception." O'Sullivan (2014, p. 1258).

Twenty years ago, (Fohl et al. 1996, p. 18) claimed, "The most commonly used data model for transportation networks is the fully intersected, planar data model" and called for a nonplanar model to better represent truly nonplanar spatial networks.

"The planar network data model has received widespread acceptance and use. Despite its popularity, the model has limitations for some areas of transportation analysis, especially where complex network structures are involved. One major problem is caused by the planar embedding requirement... intersections at grade cannot be distinguished from intersections with an overpass or underpass that do not cross at grade." (Fischer et al. 2004, p. 395)

Kwan et al. (1996, p. 6) "the difficulty in accurately representing overpasses or underpasses may lead to problems when running various routing algorithms (e.g. recommending that a traveler make a left-turn at an intersection that proves to be an overpass!)".

Methods

Cardillo et al use one square mile of different world cities.

We formally test for planarity using the method described by Boyer (2012).

Results

Discussion

The point: street networks are regularly nonplanar in the formal sense. They are embedded in three dimensions and have a z-value along with their x and y. Because they are mostly planar, typically with only a few overpasses or underpasses, they could often be described accurately as *quasi-planar*. However, claiming that urban street networks broadly are planar misrepresents them in several ways.

1. Forces false nodes where grade-separated edges cross.
2. Accordingly, underestimates average edge length (a proxy for street segment lengths and block sizes)
3. Misrepresents connectivity for routing, accessibility analysis, and other connectivity studies

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Table 1. Recent statements in the urban studies and urban physics literatures regarding the representation of street networks as planar graphs.

"In a planar graph, no links intersect, except by nodes. This feature represents a transportation network well." (Dill 2004, p. 6)
"Street networks are planar graphs composed of junctions and street segments..." (Batty 2005, p. 18)
"The number of long-range connections and the number of edges that can be connected to a single node are limited by the spatial embedding. This is particularly evident in planar networks e.g., those networks forming vertices whenever two edges cross, as urban streets or ant networks of galleries..." (Crucitti et al. 2006, p. 1)
"Any of these street networks (SNS) can be described by an embedded planar graph... Street networks are planar graphs and such planarity strongly constrains their heterogeneity..." (Buhl et al. 2006, pp. 514 & 521)
"Planar graphs are those graphs forming vertices whenever two edges cross, whereas nonplanar graphs can have edge crossings that do not form vertices. The graphs representing urban street patterns are, by construction, planar..." (Cardillo et al. 2006, p. 3)
"The connection and arrangement of a road network is usually abstracted in network analysis as a directed planar graph..." (Xie and Levinson 2007, p. 340)
"Urban street patterns form planar networks... The simplest description of the street network consists of a graph whose links represent roads and whose vertices represent road intersections and end points. For these graphs, links intersect essentially only at vertices and are thus planar." (Barthélemy and Flammini 2008, p. 1)
"Urban street networks as spatial networks are embedded in planar space, which give many constraints." (Hu et al. 2008, p. 1)
"...a street network is a strange network when compared to other social or biological networks in the sense that it is embedded in the Euclidian space and the edges do not cross each other. In graph theory, such a network is called a planar graph." (Masucci et al. 2009, p. 259)
"...street networks are embedded in space and are planar in nature..." (Porta et al. 2010, p. 114)
"Roads, rail, and other transportation networks are spatial and to a good accuracy planar networks. For many applications, planar spatial networks are the most important..." (Barthélemy 2011, p. 3)
"...urban road systems can be (in good approximation) considered as planar networks, i.e., links cannot 'cross' each other without forming a physical intersection (node) as long as there are no tunnels or bridges... The meaningful definition of link angles requires the presence of a planar network, which is assumed to be the case in urban road systems." (Chan et al. 2011, pp. 563 & 567)
"Road networks are planar graphs consisting of a series of land cells surrounded by street segments." (Strano et al. 2012, p. 3)
"Planar graphs are basic tools for understanding transportation systems embedded in two-dimensional space, in particular urban street networks... As these graphs are embedded in a two-dimensional surface, the planarity criteria requires that the links do not cross each other." (Masucci et al. 2013, p. 1)
"...street networks are essentially planar; in the absence of tunnels and bridges, the streets (the links) cannot cross without generating an intersection or a junction, that is, a node." (Gudmundsson and Mohajeri 2013, p. 1).
"Networks of street patterns belong to a particular class of graphs called planar graphs, that is, graphs whose links cross only at nodes." (Strano et al. 2013, p. 1074)
"In city science, planar networks are extensively used to represent, to a good approximation, various infrastructure networks... in particular, transportation networks and more recently streets patterns..." (Viana et al. 2013, p. 1)
"...finding a typology of street patterns essentially amounts to classifying planar graphs..." (Louf and Barthélemy 2014, p. 2)
"...we are dealing with spatial graphs, which tend to be planar..." (Zhong et al. 2014, p. 2191)
"Urban transport systems as networks can be represented as planar graphs..." (Wang 2015, p. 2)
"In graph theory, a spatial street network is a type of planar graph embedded in Euclidean space." (Law 2017, p. 168)

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Figure 1. Map of world cities in Table 2 grouped by OPN score tertiles.

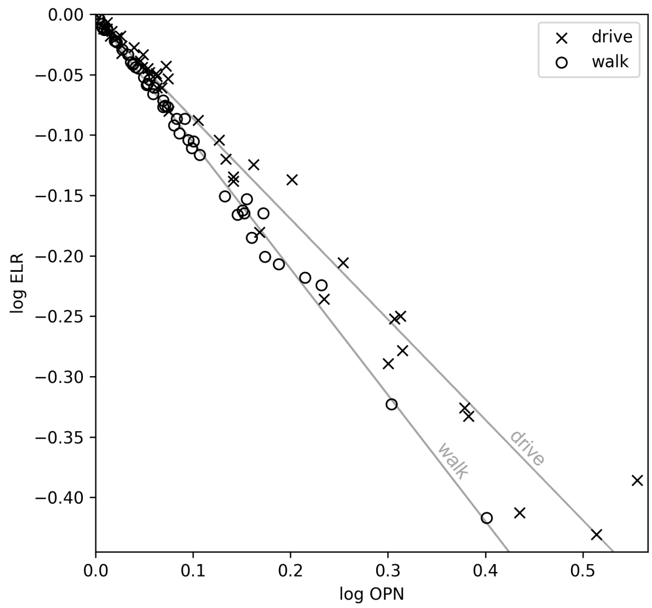

Figure 2. Log-log plot of ELR vs OPN with simple regression lines: drive $r^2 = 0.98$ and walk $r^2 = 0.99$.

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Table 2. Across many cities.

		Drive				Walk			
Country	City	Planar	OPN	ONC	ELR	Planar	OPN	ONC	ELR
Argentina	Buenos Aires	Yes	1.000	1.052	1.000	No	1.057	2.241	0.947
Australia	Sydney	No	1.350	1.172	0.749	No	1.100	1.619	0.901
Brazil	Sao Paulo	No	1.264	1.146	0.790	No	1.174	1.404	0.831
Canada	Toronto	Yes	1.075	1.054	0.958	No	1.165	4.763	0.848
	Vancouver	No	1.077	1.064	0.948	No	1.077	1.538	0.926
Chile	Santiago	No	1.143	1.074	0.887	No	1.028	1.261	0.971
China	Beijing	No	1.223	1.754	0.872	No	1.188	1.743	0.848
	Hong Kong	No	1.183	1.317	0.835	No	1.190	1.837	0.818
	Shanghai	No	1.466	1.615	0.717	No	1.494	1.751	0.659
Denmark	Copenhagen	Yes	1.008	1.175	0.988	No	1.009	1.703	0.987
Egypt	Cairo	No	1.111	1.110	0.916	No	1.090	1.233	0.906
France	Lyon	No	1.009	1.225	0.989	No	1.042	1.647	0.957
	Paris	No	1.012	1.163	0.993	No	1.087	1.632	0.917
Germany	Berlin	No	1.065	1.267	0.950	No	1.061	2.183	0.936
India	Delhi	Yes	1.000	1.462	1.000	Yes	1.007	1.465	0.992
Indonesia	Jakarta	Yes	1.017	1.218	0.986	No	1.040	1.374	0.960
Iran	Tehran	No	1.040	1.350	0.973	No	1.045	1.478	0.956
Italy	Bologna	Yes	1.000	1.218	1.000	Yes	1.004	2.267	0.996
	Florence	Yes	1.000	1.212	1.000	No	1.021	1.464	0.978
	Milan	Yes	1.000	1.259	1.000	No	1.142	2.399	0.860
Japan	Osaka	No	1.152	1.215	0.871	No	1.051	2.475	0.949
	Tokyo	No	1.078	1.333	0.923	No	1.084	1.723	0.912
Kenya	Nairobi	No	1.027	1.372	0.974	No	1.054	1.382	0.943
Mexico	Mexico City	No	1.064	1.249	0.952	No	1.096	1.386	0.917
Nigeria	Lagos	No	1.050	1.085	0.967	No	1.012	1.076	0.987
Peru	Lima	No	1.065	1.344	0.941	No	1.072	1.747	0.931
Philippines	Manila	No	1.057	1.192	0.953	No	1.104	1.392	0.895
Russia	Moscow	No	1.743	1.186	0.680	No	1.168	1.875	0.858
Singapore	Singapore	No	1.152	1.375	0.874	No	1.113	1.907	0.890
Somalia	Mogadishu	Yes	1.000	1.044	1.000	Yes	1.000	1.047	1.000
South Africa	Johannesburg	No	1.176	1.057	0.883	No	1.003	1.056	0.997
Spain	Barcelona	Yes	1.000	1.265	1.000	No	1.106	1.923	0.900
Switzerland	Geneva	No	1.015	1.249	0.982	No	1.207	2.613	0.813
Thailand	Bangkok	No	1.012	1.258	0.988	No	1.007	1.349	0.989
Turkey	Istanbul	No	1.026	1.222	0.982	No	1.020	1.490	0.978
UAE	Dubai	No	1.460	1.303	0.722	No	1.163	1.457	0.850
UK	Edinburgh	No	1.027	1.333	0.968	No	1.012	2.231	0.988
	London	No	1.022	1.276	0.981	No	1.157	2.734	0.847
USA	Atlanta	No	1.359	1.042	0.777	No	1.355	2.030	0.724
	Chicago	No	1.289	1.414	0.814	No	1.240	2.671	0.804
	Cincinnati	No	1.370	1.141	0.757	No	1.074	1.482	0.927
	Dallas	No	1.672	1.311	0.650	Yes	1.039	1.819	0.959
	Los Angeles	No	1.717	1.111	0.635	No	1.261	2.089	0.799
	Miami	No	1.545	1.189	0.662	No	1.037	1.617	0.961
	New York	No	1.135	1.144	0.901	No	1.062	2.026	0.941
	Phoenix	No	1.047	1.206	0.962	No	1.022	2.710	0.977
	San Francisco	No	1.070	1.093	0.941	No	1.055	1.706	0.944
	Seattle	No	1.367	1.047	0.779	No	1.072	3.122	0.926
	Washington DC	No	1.055	1.351	0.956	No	1.034	3.169	0.967
Venezuela	Caracas	No	1.049	1.230	0.957	Yes	1.000	1.254	1.000

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Table 3. 100 random square mile sample runs across one city, Oakland CA.

	OPN	ONC	OPC	ELR
mean	1.091	1.149	1.256	0.947
σ	0.154	0.120	0.226	0.082
min	1.000	1.000	1.000	0.637
max	1.759	1.833	2.011	1.000

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