Supplementary Materials

1 Simulation study for Gaussian data

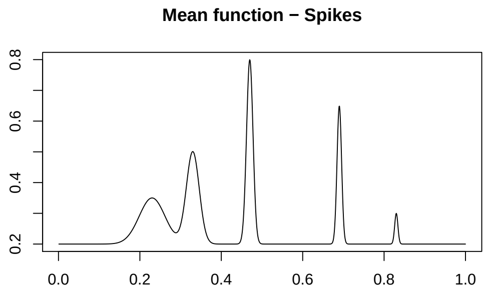
As noted in the main text, we performed an extensive simulation study for Gaussian errors using a variety of test functions (7 mean functions and 5 variance functions, including constant variance), signal-to-noise ratios (SNR = 1 and 3) and sample sizes ($T = 256, 512, 1024$). For details of where the results are saved and what information they contain, refer to README.md from the ashwave repo. In particular, we include an RShiny app as an interactive way to present the results for $T = 1024$, the code for which can be found in the dscr-smash repo. For more details on the repo refer to the README.md file from the repo. In particular, users can view the boxplots of MISEs for the methods and test functions they are interested in by checking the appropriate boxes. For more details on the RShiny display refer to graphs.Rmd in the dscr-smash repo. For reference, the mean and variance functions used in the simulation studes are shown below in Figures **??** and **??**.

In addition to the performance of the various methods, we also provide code in the ashwave repo generating a table that provide additional details for each method used, including the associated software, variance assumption, wavelet basis, shrinkage procedure used, and other relevant information.

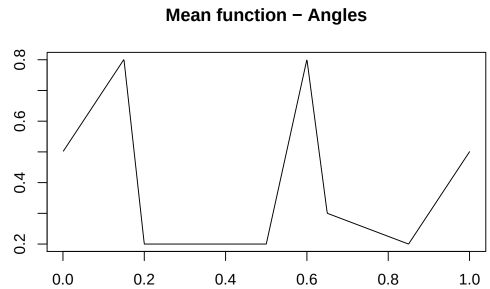
2 Simulation study for Poisson data

The results from the simulation study for Poisson noise are shown in the following tables. For reference, we first plot the intensity functions used in the simulation studies in Figure **??**. Each of the six tables (S1-S6) presents results for the corresponding mean function, and includes results for all three (min,max) intensity levels ((0.01,3), (1/8,8), (1/128,128)). As mentioned in the main text, we also explored several options for the Gaussian de-noising stage of the HF algorithm, and include the results in the following tables. Specifically, we considered four different options (all with 50 external cyclespins):

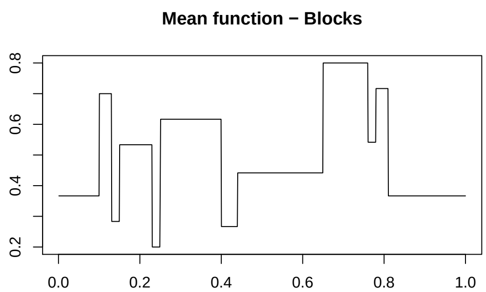
1. Hybrid of (1) Greedy tree de-noising ([**?**]) and (2) wavelet thresholding using "leave-half-out" crossvalidation (eg [**?**]). $j_0 = 3$ (default), and the noise level was estimated from the data. This corresponds to **H:CV+BT CS** in [**?**]. Note



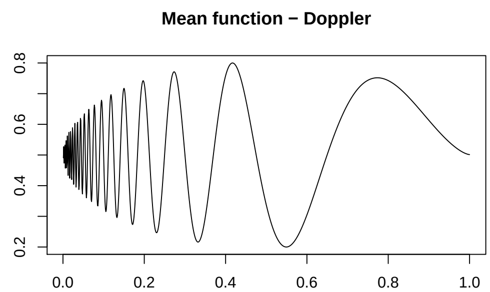
(a)



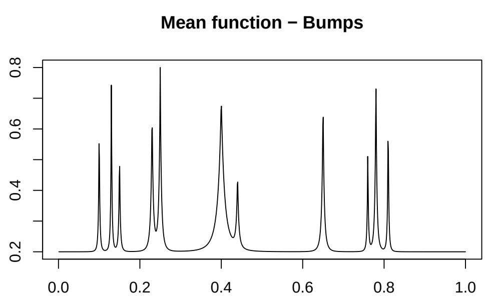
(b)



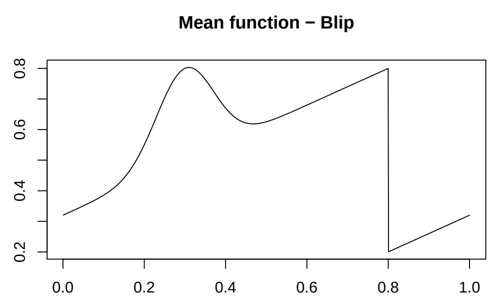
(c)



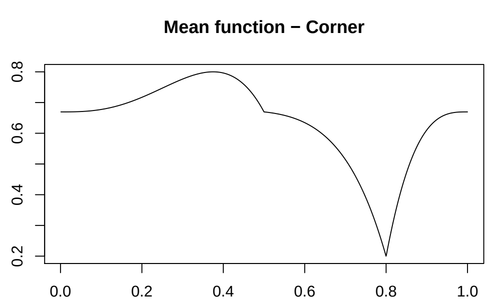
(d)



(e)



(f)



(g)

Figure S1: The seven mean functions used in the Gaussian simulations, all scaled to be between 0.2 and 0.8.

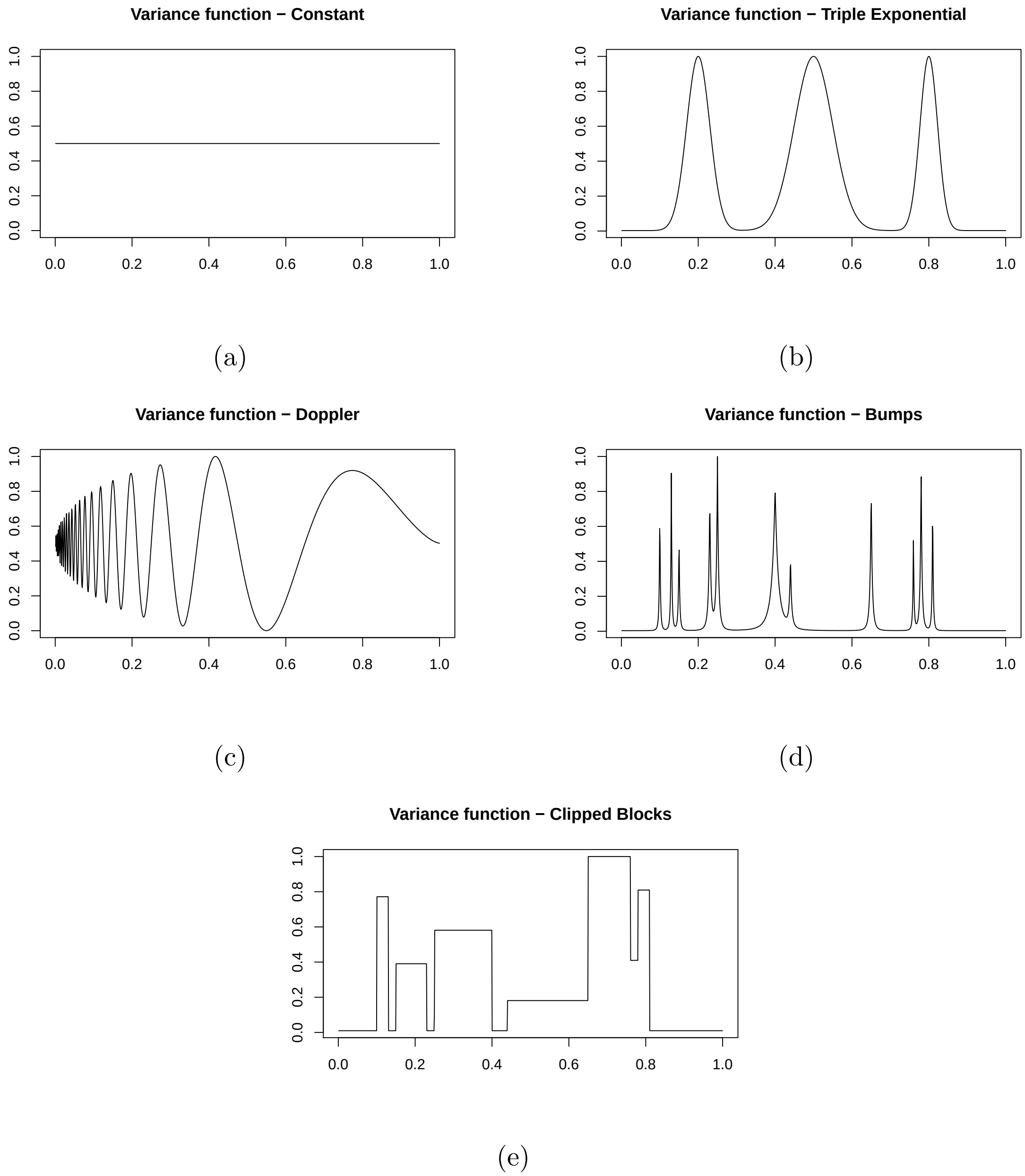


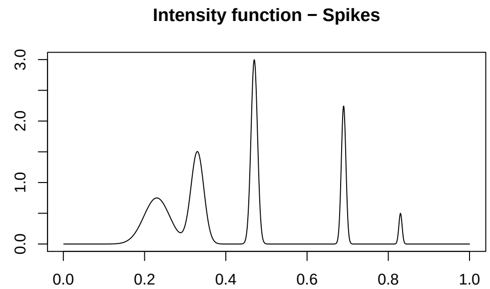
Figure S2: The five variance functions used in the Gaussian simulations, which are rescaled in the simulations to achieve the desired signal to noise ratios.

that the algorithm does not always converge for this option, resulting in NA values in the tables below.

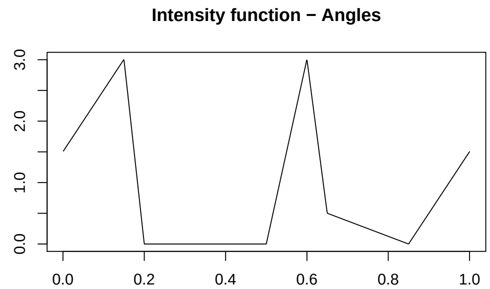
2. Wavelet thresholding using the universal threshold ([**?**]). $j_0 = 3$ (default), and the noise level was estimated from the data. This corresponds to **F⋈U CS** in [**?**].
3. Wavelet thresholding using the universal threshold for the non-decimated wavelet transform. Results averaged over $j_0 = 4, 5, 6, 7$, and the noise level was estimated from the data.
4. Wavelet thresholding using the universal threshold for the non-decimated wavelet transform. Results average over $j_0 = 4, 5, 6, 7$, and the noise level was set to be 1 (which is the asymptotic variance under the Fisz transform). This is the option for which we included the results in the main text, as it had the best overall performance.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	690.01	329.26	48.87
BMSM	1007.34	397.79	41.88
BMMIM	930.08	436.54	368.36
Haar-Fisz (1)	8194.60	NA	184.81
Haar-Fisz (2)	8074.05	408.32	158.68
Haar-Fisz (3)	5904.98	317.12	73.00
Haar-Fisz (4)	722.19	287.44	18.06
Anscombe	1329.87	388.59	18.17
Haar thresholds	803.89	326.11	120.02
L1_penalty	3085.65	2209.67	1516.78

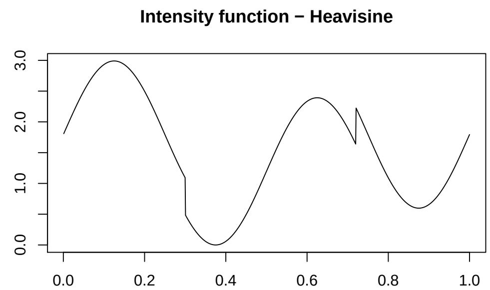
Table S1: Comparison of methods for denoising Poisson data for the “Spikes” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.



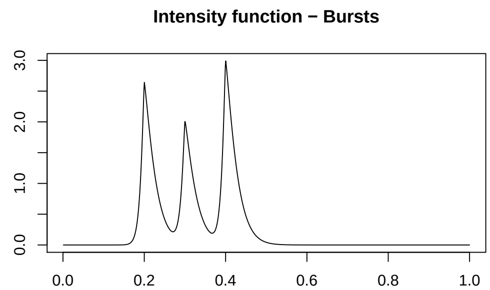
(a)



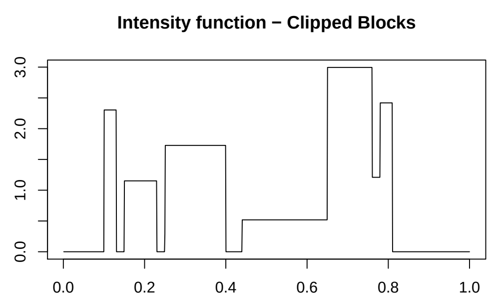
(b)



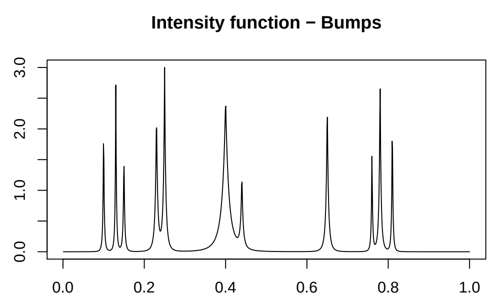
(c)



(d)



(e)



(f)

Figure S3: The six intensity functions used in the Poisson simulations, which are rescaled to achieve the desired (min,max) intensities in the simulations.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	145.26	68.47	10.25
BMSM	147.40	73.87	10.49
BMMIM	245.89	100.17	71.26
Haar-Fisz (1)	NA	91.86	25.23
Haar-Fisz (2)	1020.76	78.84	12.22
Haar-Fisz (3)	614.95	125.78	11.29
Haar-Fisz (4)	314.41	122.79	9.08
Anscombe	419.04	146.64	9.50
Haar thresholds	264.38	162.16	89.37
L1_penalty	655.82	284.75	2979.23

Table S2: Comparison of methods for denoising Poisson data for the "Angles" test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	81.41	43.21	7.21
BMSM	85.29	44.22	7.35
BMMIM	205.09	81.49	11.22
Haar-Fisz (1)	135.42	62.01	8.10
Haar-Fisz (2)	105.72	61.66	6.29
Haar-Fisz (3)	273.97	104.99	9.98
Haar-Fisz (4)	274.26	105.47	9.23
Anscombe	372.06	128.43	9.10
Haar thresholds	226.27	143.01	85.74
L1_penalty	385.47	71.17	8.68

Table S3: Comparison of methods for denoising Poisson data for the "Heavisine" test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	487.34	234.35	33.11
BMSM	706.04	301.86	34.42
BMMIM	654.11	290.91	531.79
Haar-Fisz (1)	6870.35	NA	157.15
Haar-Fisz (2)	6807.29	384.56	149.23
Haar-Fisz (3)	5619.13	318.57	94.20
Haar-Fisz (4)	618.39	299.39	25.20
Anscombe	869.16	343.15	26.03
Haar thresholds	540.58	271.16	107.44
L1_penalty	1836.73	1280.16	719.59

Table S4: Comparison of methods for denoising Poisson data for the "Bursts" test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	307.80	137.28	6.82
BMSM	355.15	143.09	6.91
BMMIM	472.24	205.20	150.03
Haar-Fisz (1)	NA	141.71	31.23
Haar-Fisz (2)	879.20	316.44	26.71
Haar-Fisz (3)	741.49	389.79	48.41
Haar-Fisz (4)	632.21	338.55	29.72
Anscombe	804.70	384.46	28.94
Haar thresholds	467.64	228.14	82.99
L1_penalty	1021.58	627.67	1961.42

Table S5: Comparison of methods for denoising Poisson data for the "Clipped Blocks" test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	2597.46	1194.62	141.21
BMSM	4036.77	1889.94	171.07
BMMIM	3440.75	2226.20	291.22
Haar-Fisz (1)	9515.68	NA	187.6
Haar-Fisz (2)	9249.05	1288.30	130.19
Haar-Fisz (3)	6286.05	1756.08	248.28
Haar-Fisz (4)	3113.37	1658.74	184.66
Anscombe	4038.42	2241.74	195.75
Haar thresholds	3517.89	1652.55	173.72
L1_penalty	4705.77	3676.42	1774.51

Table S6: Comparison of methods for denoising Poisson data for the "Bumps" test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods (excluding Haar-Fisz (1-3)) for a given (min, max) intensity.