

Supplementary Materials

1 Simulation study for Gaussian data

As noted in the main text, we performed an extensive simulation study for Gaussian errors using a variety of test functions (7 mean functions and 5 variance functions, including constant variance), signal-to-noise ratios ($\text{SNR} = 1$ and 3) and sample sizes ($T = 256, 512, 1024$). For details of where the results are saved and what information they contain, refer to README.md from the [ashwave repo](#). In particular, we include an RShiny app as an interactive way to present the results for $T = 1024$, the code for which can be found in the [dscr-smash repo](#). For more details on the repo refer to the README.md file from the repo. In particular, users can view the boxplots of MISEs for the methods and test functions they are interested in by checking the appropriate boxes. For more details on the RShiny display refer to graphs.Rmd in the dscr-smash repo. For reference, the mean and variance functions used in the simulation studies are shown below in Figures ?? and ??

In addition to the performance of the various methods, we also provide code in the [ashwave repo](#) generating a table that provide additional details for each method used, including the associated software, variance assumption, wavelet basis, shrinkage procedure used, and other relevant information.

2 Simulation study for Poisson data

The results from the simulation study for Poisson noise are shown in the following tables. For reference, we first plot the intensity functions used in the simulation studies in Figure ??. Each of the six tables (??-??) presents results for the corresponding mean function, and includes results for all three (min,max) intensity levels $((0.01,3), (1/8,8), (1/128,128))$.

In addition we explore the results from trying out different Gaussian de-noising procedures for the second step of the Haar-Fisz (HF) algorithm.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	690.01	329.26	48.87
BMSM	1007.34	397.79	41.88
BMMIM	930.08	436.54	368.36
Haar-Fisz	722.19	287.44	18.06
Anscombe	1329.87	388.59	18.17
Haar thresholds	803.89	326.11	120.02
L1_penalty	3085.65	2209.67	1516.78

Table S1: Comparison of methods for denoising Poisson data for the “Spikes” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	145.26	68.47	10.25
BMSM	147.40	73.87	10.49
BMMIM	245.89	100.17	71.26
Haar-Fisz	314.41	122.79	9.08
Anscombe	419.04	146.64	9.50
Haar thresholds	264.38	162.16	89.37
L1_penalty	655.82	284.75	2979.23

Table S2: Comparison of methods for denoising Poisson data for the “Angles” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	81.41	43.21	7.21
BMSM	85.29	44.22	7.35
BMMIM	205.09	81.49	11.22
Haar-Fisz	274.26	105.47	9.23
Anscombe	372.06	128.43	9.10
Haar thresholds	226.27	143.01	85.74
L1_penalty	385.47	71.17	8.68

Table S3: Comparison of methods for denoising Poisson data for the “Heavisine” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	487.34	234.35	33.11
BMSM	706.04	301.86	34.42
BMMIM	654.11	290.91	531.79
Haar-Fisz	618.39	299.39	25.20
Anscombe	869.16	343.15	26.03
Haar thresholds	540.58	271.16	107.44
L1_penalty	1836.73	1280.16	719.59

Table S4: Comparison of methods for denoising Poisson data for the “Bursts” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	307.80	137.28	6.82
BMSM	355.15	143.09	6.91
BMMIM	472.24	205.20	150.03
Haar-Fisz	632.21	338.55	29.72
Anscombe	804.70	384.46	28.94
Haar thresholds	467.64	228.14	82.99
L1_penalty	1021.58	627.67	1961.42

Table S5: Comparison of methods for denoising Poisson data for the “Clipped Blocks” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	(0.01,3)	(1/8,8)	(1/128,128)
SMASH	2597.46	1194.62	141.21
BMSM	4036.77	1889.94	171.07
BMMIM	3440.75	2226.20	291.22
Haar-Fisz	3113.37	1658.74	184.66
Anscombe	4038.42	2241.74	195.75
Haar thresholds	3517.89	1652.55	173.72
L1_penalty	4705.77	3676.42	1774.51

Table S6: Comparison of methods for denoising Poisson data for the “Bumps” test function for 3 different (min,max) intensities ((0.01,3), (1/8,8), (1/128,128)). Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance. Values colored in red indicates the smallest MISE amongst all methods for a given (min, max) intensity.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	NA	280.29	312.35	408.32
j0 = 5				
(where applicable)	NA	246.95	234.90	408.32
j0 = 6				
(where applicable)	NA	254.54	261.18	408.32
j0 = 7				
(where applicable)	NA	367.98	460.04	408.32
Average				
(where applicable)	NA	287.44	317.12	408.32

Table S7: Comparison of different options for HF for the “Spikes” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default oness used in Fryzlewicz2004HaarFisz, with primary resolution $j_0=3$. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with $j_0 = 4, 5, 6$ and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Note here that BT + CV do not converge due to numerical issues. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	91.86	70.11	77.37	78.84
j0 = 5				
(where applicable)	91.86	65.09	65.43	78.84
j0 = 6				
(where applicable)	91.86	121.87	123.01	78.84
j0 = 7				
(where applicable)	91.86	234.08	237.28	78.84
Average				
(where applicable)	91.86	122.79	125.77	78.84

Table S8: Comparison of different options for HF for the “Angles” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default oness used in Fryzlewicz2004HaarFisz, with primary resolution $j_0=3$. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with $j_0 = 4, 5, 6$ and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	62.01	38.51	37.96	61.66
j0 = 5				
(where applicable)	62.01	59.24	58.61	61.66
j0 = 6				
(where applicable)	62.01	110.77	110.22	61.66
j0 = 7				
(where applicable)	62.01	213.36	213.13	61.66
Average				
(where applicable)	62.01	105.47	104.98	61.66

Table S9: Comparison of different options for HF for the “Heavisine” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default oness used in Fryzlewicz2004HaarFisz, with primary resolution j0=3. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with j0 = 4, 5, 6 and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	NA	331.11	340.45	384.56
j0 = 5				
(where applicable)	NA	323.51	294.31	384.56
j0 = 6				
(where applicable)	NA	211.35	235.24	384.56
j0 = 7				
(where applicable)	NA	331.58	404.25	384.56
Average				
(where applicable)	NA	299.39	318.56	384.56

Table S10: Comparison of different options for HF for the “Bursts” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default oness used in Fryzlewicz2004HaarFisz, with primary resolution j0=3. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with j0 = 4, 5, 6 and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Note here that BT + CV do not converge due to numerical issues. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	141.71	337.24	486.16	316.44
j0 = 5				
(where applicable)	141.71	333.16	384.27	316.44
j0 = 6				
(where applicable)	141.71	341.66	346.94	316.44
j0 = 7				
(where applicable)	141.71	342.12	341.76	316.44
Average				
(where applicable)	141.71	338.54	389.78	316.44

Table S11: Comparison of different options for HF for the “Clipped Blocks” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default ones used in Fryzlewicz2004HaarFisz, with primary resolution $j_0 = 3$. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with $j_0 = 4, 5, 6$ and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.

	BT + CV	TI-universal sd = 1	TI-universal sd estimated	Universal thresholding
j0 = 4				
(where applicable)	NA	1918.04	2553.46	1288.30
j0 = 5				
(where applicable)	NA	1815.30	1853.65	1288.30
j0 = 6				
(where applicable)	NA	1621.46	1440.67	1288.30
j0 = 7				
(where applicable)	NA	1280.15	1176.50	1288.30
Average				
(where applicable)	NA	1658.74	1756.07	1288.30

Table S12: Comparison of different options for HF for the “Bumps” test function for (min,max) intensity = (1/8,8). Options BT + CV and Universal thresholding correspond to the two default ones used in Fryzlewicz2004HaarFisz, with primary resolution $j_0 = 3$. TI-universal corresponds to universal thresholding with translational invariance, with sd either estimated from the data or set to be 1 (which is the asymptotic variance). For TI-universal, we ran the HF with $j_0 = 4, 5, 6$ and 7, corresponding to the first 4 rows, as well as the average of all 4 values, corresponding to the 5th row. Note here that BT + CV do not converge due to numerical issues. Performance is measured using MISE over 100 independent datasets, with smaller values indicating better performance.