Tarea

Demostrar $\frac{d\sigma(x)}{dx} = \sigma(x)(1-\sigma(x))$

donde $\sigma(x) = \frac{1}{1+\exp(-x)}$

Demostración

$$\frac{d\sigma(x)}{dx} = \frac{d}{dx}\left(\frac{1}{1+\exp(-x)}\right) \quad \left(\text{por } \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}\right)$$

$$= -\frac{\frac{d}{dx}(1+\exp(-x))}{(1+\exp(-x))^2} = -\frac{\frac{d}{dx}1 - \frac{d}{dx}\exp(-x)}{(1+\exp(-x))^2} \rightarrow \text{(Por regla de la cadena)}$$

$$= \frac{+\exp(-x)}{(1+\exp(-x))^2} = \frac{1}{1+\exp(-x)} \cdot \frac{\exp(-x)}{(1+\exp(-x))} \rightarrow \text{(se suma un 0)}$$

$$= \frac{1}{(1+\exp(-x))} \cdot \frac{(\exp(-x)+1)-1}{(1+\exp(-x))}$$

$$= \frac{1}{1+\exp(-x)} \cdot \left(1 - \frac{1}{(1+\exp(-x))}\right)$$

$$= \sigma(x)(1-\sigma(x))$$

$$\therefore \frac{d\sigma(x)}{dx} = \sigma(x)(1-\sigma(x))$$