TCCS

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Introduction

A extension of Milner's CCS to make a interleaving model for "real time"systems.(YI, 1991)

- What is a real time system ?

We use the term "real time"to describe the class of system that have to respond to externally generated stimuli or inputs within specified time limits.

This type of system includes many safety-critical system such as robotics and flight control systems.

To deal with real time, CCS action prefix shall be extended to the form $\mu@t.P$ and a delay-construct $\epsilon(d).P$ in prefix form is introduced.

First, Some Change

In CCS we process like: $A \stackrel{\text{def}}{=} \alpha x.P$ who depends on the contents v of the received message, but not the time when v is available.
For a behavior on a real time system depends on time when an external stimulus such as a message arises.
We need a extra variable to recording the time delay before a message on α is available.

$$A \stackrel{\text{def}}{=} \alpha x@t.P \qquad\qquad B \stackrel{\text{def}}{=} \bar{\alpha} v@t.Q$$

$$P[v/x, d/t] \qquad\qquad Q[e/t]$$

where

- v is the message
- d is the delay before the message is available
- e is the delay before message v is delivered

First, Some Changes

$$A \stackrel{\text{def}}{=} \epsilon(d).P$$

means that the process A will idles for a d units of time and then behaves like P

Instantaneousness of Actions

If the environment is ready to offer $\bar{\alpha}$, then $\alpha.P$ will synchronize immediately and become P.
But, in TCCS, we can put a delay before an action using the delay-construct, with the goal to separate the time delay for enabling an atomic action and the moment at which the action is committed

$$A \stackrel{\text{def}}{=} \epsilon(3).\alpha.P$$

Exist two types of atomic actions:

- Controllable Actions
- Non-Controllable Actions

Controllable Actions

In CCS they are called visible actions. A controllable actions α needs the external stimulus $\bar{\alpha}$ to occur.

$$P \xrightarrow{\alpha} Q$$

means that, if the environment is ready to offer $\bar{\alpha}$, P may will do α immediately and become Q. This transition takes no time, because α is instantaneous.

Non-Controllable Actions

They also called invisible actions in CCS.
A non-controllable action is autonomous and does not require external stimulus to occur.

$$P \xrightarrow{\tau} Q$$

This transition takes no times either, because τ is instantaneous.

Idling

When, during the time no action is performed, the behavior of a system is considered to be idling.

$$P \xrightarrow{\epsilon(d)} Q$$

Means P will process to Q in d units of time

- $\epsilon(4).P \xrightarrow{\epsilon(3.5)} \epsilon(0.5).P$
  means that $\epsilon(4).P$ process to $\epsilon(0.5).P$ in 3.5 units of time

Maximal Progress

Until now, we have three types of transitions

1. $P \xrightarrow{\alpha} Q$
2. $P \xrightarrow{\tau} Q$
3. $P \xrightarrow{\epsilon(d)} Q$

is $\tau @ t.P$ allowed to wait?

NO.

If an agent can performed a $\tau - action$, then it should never wait unnecessarily. This is called maximal progress assumption.

$$\tau @ t.P \xrightarrow{\tau} P[0/t]$$

for controllable actions:

$$\alpha @ t.P \xrightarrow{\alpha} P[0/t]$$

$$\alpha @ t.P \xrightarrow{\epsilon(d)} \alpha @ t.P[d + t/t]$$

Time Determinacy

$$P \xrightarrow{\epsilon(d)} P1 \qquad\qquad P \xrightarrow{\epsilon(d)} P2$$

P1 and P2 are identical.
Let $X \stackrel{\text{def}}{=} \bar{\alpha}x + \bar{\beta}z$

$$\bar{\alpha}x + \bar{\beta}z \xrightarrow{\epsilon(d)} \bar{\alpha}x + \bar{\beta}z \qquad\qquad \forall d > 0$$

This motivates the following rule for summation:

$$\frac{P \xrightarrow{\epsilon(d)} P' \quad Q \xrightarrow{\epsilon(d)} Q'}{P + Q \xrightarrow{\epsilon(d)} P' + Q'}$$

Syntax

Let a time domain $\mathscr{T}$ with the last element 0, let δt be the set $\{\epsilon(d) | d \in \mathscr{T} - \{0\}\}$

- $t + d$
- $c \dot{-} b$

where $\dot{-}$ is defined by $c \dot{-} b = 0$ if $b \geq c$, otherwise $c - b$
Let Λ denote the set of controllable actions and non-controllable actions, let $\Lambda = \Delta \cup \bar{\Delta}$, where Δ is the set of names and $\bar{\Delta} = \{\bar{\alpha} | \alpha \in \Delta\}$ is the set of co-names.

- $\bar{\bar{\alpha}} = \alpha$
- $\bar{\tau} = \tau$
- $\overline{\epsilon(d)} = \epsilon(d)$

Syntax

Grammar:

$E ::= NIL|X|\epsilon(d).E|\mu@t.E|E+F|E|F|E \setminus L|E[S]|recX : E$

When E does not depend on time variable t, we shall write $\mu.E$ instead of $\mu@t.E$

Given a time interval d, $Sort_d(P)$ shall include all controllable actions which P may perform within d units of time.

Transition Rules

$$Sort_d(X)\rho = \rho(X)$$
$$Sort_d(\alpha@t.E)\rho = \{\alpha\}$$
$$Sort_d(\epsilon(e).E) = Sort_{d \dot{-} e}(E)\rho$$
$$Sort_d(E+F)\rho = Sort_d(E)\rho \cup Sort_d(F)\rho$$
$$Sort_d(E|F)\rho = Sort_d(E)\rho \cup Sort_d(F)\rho$$
$$Sort_d(E \setminus F)\rho = Sort_d(E)\rho - \{L \cup \bar{L}\}$$
$$Sort_d(E[S])\rho = \{S(\alpha)|\alpha \in Sort_d(E)\rho\}$$
$$Sort_d(recX : E)\rho = \mu S : H(S)$$

$$Sort_d(NIL)\rho = \emptyset$$
$$Sort_d(\tau@t.E)\rho = \emptyset$$

where $H(S) \equiv Sort_d(E)\rho_{[x \mapsto s]}$ and $\mu S : H(S)$ is the least fixed point of function H

Properties of Agents

1. Maximal progress: If $P \xrightarrow{\tau} P_1$, for some P_1, then $P \xrightarrow{\tau} P_2$ for no d and P_2
2. Time determinacy: Whenever $P \xrightarrow{\epsilon(d)} P_1$ and $P \xrightarrow{\epsilon(d)} P_2$ the $P_1 \equiv P_2$, where " $\equiv$ " is the syntactical identity
3. Time continuity: $P \xrightarrow{\epsilon(e+d)} P_2$ iff there exists P_1 such that $P \xrightarrow{\epsilon(e)} P_1$ and $P_1 \xrightarrow{\epsilon(d)} P_2$
4. Persistency: If $P \xrightarrow{\epsilon(d)} P_1$ and $P \xrightarrow{\alpha} Q$, for some Q, then $P_1 \xrightarrow{\alpha} Q_1$, for some Q_1

Rules

$$\frac{}{NIL \xrightarrow{\epsilon(d)} NIL}\ Inaction$$

$$\frac{}{\mu@t.P \xrightarrow{\mu} P[0/t]}\ Prefix_0 \qquad \frac{}{\alpha@t.P \xrightarrow{\epsilon(d)} \alpha@t.P[t+d/t]}\ Prefix_{\epsilon(d)}$$

$$\frac{}{\epsilon(d+e).P \xrightarrow{\epsilon(d)} \epsilon(e).P}\ Prefix \qquad \frac{P \xrightarrow{\epsilon(d)} P_1}{\epsilon(e).P \xrightarrow{\epsilon(d+e)} P_1}\ Prefix$$

$$\frac{P \xrightarrow{\sigma} P_1}{\epsilon(0).P \xrightarrow{\sigma} P_1}\ Prefix$$

Rules part 2

$$\frac{P \xrightarrow{\mu} P_1}{P + Q \xrightarrow{\mu} P_1} \; Sum_L \qquad \frac{Q \xrightarrow{\mu} Q_1}{P + Q \xrightarrow{\mu} Q_1} \; Sum_R$$

$$\frac{P \xrightarrow{\epsilon(d)} P_1 \quad Q \xrightarrow{\epsilon(d)} Q_1}{P + Q \xrightarrow{\epsilon(d)} P_1 + Q_1} \; Sum_{\epsilon(d)}$$

$$\frac{P \xrightarrow{\mu} P_1}{P|Q \xrightarrow{\mu} P_1|Q} \; Comp_L \qquad \frac{Q \xrightarrow{\mu} Q_1}{P|Q \xrightarrow{\mu} P|Q_1} \; Comp_R$$

$$\frac{P \xrightarrow{\alpha} P_1 \quad Q \xrightarrow{\bar{\alpha}} Q_1}{P|Q \xrightarrow{\tau} P_1|Q_1} \; Comp_{p|q}$$

Rules part 3

$$\frac{P \xrightarrow{\epsilon(d)} P_1 \quad Q \xrightarrow{\epsilon(d)} Q_1}{P|Q \xrightarrow{\epsilon(d)} P_1|Q_1}\; Comp_{p|q} \qquad [Sort_d(P) \cap \overline{Sort_d(Q)} = \emptyset]$$

$$\frac{P \xrightarrow{\sigma} P_1}{P \setminus L \xrightarrow{\sigma} P_1 \setminus L}\; Res \qquad [\alpha, \bar{\alpha} \notin L]$$

$$\frac{P \xrightarrow{\alpha} P_1}{P[S] \xrightarrow{S(\alpha)} P_1[S]}\; Relabelling$$

$$\frac{E[recX : E/X] \xrightarrow{\alpha} P}{recX : E \xrightarrow{\alpha} P}\; Rec$$

Strong Bisimulation

As in CCS this equality depends on the capability of the observer.
In TCCS we first require that the observer is able to observe all kinds of actions and moreover, it can record the time delay between actions.
So the set of observations is $\Gamma = \Lambda \cup \{\tau\} \cup \delta_{R^{+0}}$
Example:

$$\alpha|\beta \sim \alpha.\beta + \beta.\alpha$$
$$\epsilon(1).\alpha|\epsilon(1).\beta \sim \epsilon(1).\alpha.\beta + \epsilon(1).\beta.\alpha$$

Theorem 1: $\sim$ is preserved by all operators

Weak Bisimulation

To define weak bisimulation we shall define a weak equivalence over agents in terms of controllable actions and time delays between them.

Let $\Psi = \Lambda \cup \Delta_\tau$ be the set of observations. where Λ is the set of controllable actions and $\Delta_\tau = \{\epsilon(d) | d \geq 0\}$

Definition:

1. $P \overset{\alpha}{\Rightarrow} Q$ if $P(\overset{\tau}{\rightarrow})^* \overset{\alpha}{\rightarrow} (\overset{\tau}{\rightarrow})^* Q$
2. $P \overset{\epsilon(d)}{\Longrightarrow} Q$ if $P(\overset{\tau}{\rightarrow})^* \xrightarrow{\epsilon(d_1)} (\overset{\tau}{\rightarrow})^* \ldots (\overset{\tau}{\rightarrow})^* \xrightarrow{\epsilon(d_n)} (\overset{\tau}{\rightarrow})^* Q$ where d $= \sum_{i \leq n} d_i$

Example:

$$\epsilon(2).\tau.P \approx \epsilon(2).P$$
$$\epsilon(2).\epsilon(3).\tau.P \approx \epsilon(5).P$$

Weak Bisimulation

Proposition:
$P \sim Q$ implies $P \approx Q$
Like in CCS, $\approx$ is not a congruence.But $\approx$ is preserved by all operators except summation and recursion
summation:

$$\tau.\alpha + \beta \not\approx \alpha + \beta$$

Recursion:

$$recX : \tau.\alpha(X + \beta.P) \not\approx recX : \alpha(X + \beta.P)$$

Basic Laws

$$E + F \sim F + E \qquad E|F \sim F|E$$
$$(E + F) + G \sim E + (F + G) \qquad (E|F)|G \sim E|(F|G)$$
$$E + E \sim E \qquad E|NIL \sim E$$
$$E + NIL \sim E$$
$$(E + F) \setminus L \sim F \setminus L + E \setminus L \qquad (E + F)[S] \sim E[S] + F[S]$$
$$(\mu@t.E) \setminus L \sim NIL, (\mu \in L \cup \overline{L}) \qquad (E|F)[S] \sim E[S]|F[S]$$
$$(\mu@t.E) \setminus L \sim \mu@t.(E \setminus L), (\mu \notin L \cup \overline{L}) \qquad (\mu@t.E)[S] \sim S(\mu)@t.(E[S])$$
$$(\epsilon(d).E) \setminus L \sim \epsilon(d).E \setminus L \qquad (\epsilon(d).E[S]) \sim \epsilon(d).(E[S])$$
$$NIL \setminus L \sim NIL \qquad NIL[S] \sim NIL$$
$$recX : E \sim E[recX : E/X]$$

Laws part 2

$$\alpha - \textit{conversion} \qquad \mu@t.E \sim \mu@u.E[u/t]$$

Real Time Laws

1. maximal progress

$$\tau@t.E \sim \tau.E[0/t] \qquad \tau.E + \epsilon(d).E \sim \tau.E$$

2. time determinacy

$$\epsilon(d).(E + F) \sim \epsilon(d).E + \epsilon(d).F \quad \epsilon(d).(E|F) \sim \epsilon(d).E|\epsilon(d).F$$

Laws part 3

1. maximal progress
2. time determinacy
3. time continuity

$$\epsilon(c+d).E \sim \epsilon(c).\epsilon(d).E \qquad \epsilon(0).E \sim E$$
$$\epsilon(d).NIL \sim NIL$$

4. persistency

$$\alpha@t.E + \epsilon(d).\alpha@t.E[t+d/t] \sim \alpha@t.E$$

References

YI, W. Ccs+ time= an interleaving model for real time systems. In: SPRINGER. *International Colloquium on Automata, Languages, and Programming*. [S.l.], 1991. p. 217–228.