PPGSC0098 - TÓPICOS AVANÇADOS EM FUNDAMENTOS DA COMPUTAÇÃO IV
Label Transition System - LTS

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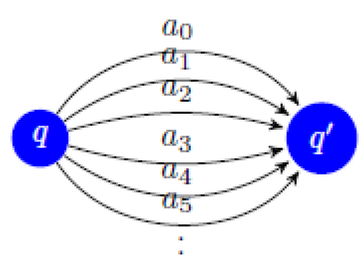
1 Prove that a finite-state LTS is boundedly-branching. Does the converse hold ?

boundely-branching: $\exists k \cdot \forall q \in Q \cdot \{(q, a, q')|q \xrightarrow{a} q'\} \leq k$

Let us suppose, by contrast, that $(i)\neg(boundely{-}branching) \rightarrow (ii)\neg(finite{-}states)$. Following from (i), we have that it is true that $\neg(\exists k \cdot \forall q \in Q \cdot (q, a, q')|q \xrightarrow{a} q' \leq k)$, therefore (ii) $\forall k \cdot (\neg(\forall q \in Q \cdot \{(q, a, q')|q \xrightarrow{a} q'\} \leq k))$, so we have that $\forall k \cdot (\exists q \in Q \cdot \{(q, a, q')|q \xrightarrow{a} q'\} > k)$, so we do not have an upper bound, k, therefore $\{(q, a, q')|q \xrightarrow{a} q'\}$ is infinite.
The opposite is not true. For example:

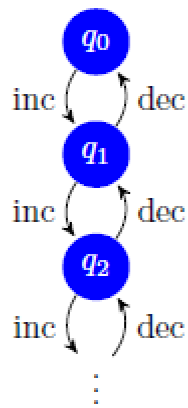


2 Prove that a boundedly-branching LTS is finitely branching. Does the converse hold ?

finitely branching: $\forall q \in Q \cdot \{(q, a, q')|q \xrightarrow{a} q'\}$ is finite

Let us suppose, by contrast, that $(i)\neg(finitelybranching) \rightarrow (ii)\neg(boundedly{-}branchingLTS)$. Following from (i), we have that it is true that $\forall q \in Q \cdot$

$\{(q,a,q')|q \xrightarrow{a} q'\}$ is finite, therefore $\exists q \in Q \cdot \{(q,a,q')|q \xrightarrow{a} q'\}$ is not finite, therefore $\neg\exists k \cdot (\neg\exists q \in Q \cdot \{(q,a,q')|q \xrightarrow{a} q'\} > k)$, therefore $\forall k \cdot \exists q \in Q \cdot \{(q,a,q')|q \xrightarrow{a} q'\} > k$
The opposite is not true. For example:



3 Prove that if L is a deterministic LTS, then it is image-finite

deterministic: if $q \xrightarrow{a} q'$ and $q \xrightarrow{a} q''$ then $q' = q''$
image-finite: $\forall q \in Q$ and $a \in A \cdot post(q,a)$ is finite.

Let us suppose, by contrast, that $(i)\neg(\forall q \in Q, \forall a \in A \cdot post(q,a)$ is finite), therefore, $\exists q \in Q, \exists a \in A \cdot post(q,a)$ is infinite, therefore $q \xrightarrow{a} q'$ and $q \xrightarrow{a} q''$ therefore $q' \neq q''$