PROJECT: APOLLO MISSION SIMULATION

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1. INTRODUCTION

1.1. OBJECTIVES

The main objective of the project is to demonstrate the translunar trajectory followed by the Apollo XI mission, using the data provided in the mission's final report. Similarly, the path shown here, as well as Delta-V values obtained by the software, will be compared.

The theoretical fundamentals behind the maneuver carried out, the calculations made using the software developed, and the final result will be explained.

Likewise, and due to library limitations, some of the parameters that were not considered during the orbit calculation process will be shown, as well as the difficulties to simulate other mission parameters, such as takeoff and landing.

Since the translunar trajectory is based on the same principle for all dates, it is possible to determine for different dates, the amount of Delta-V that must be added to the vehicle to achieve the trajectory to the moon.

1.2. OVERVIEW OR APOLLO MISSIONS

The Apollo mission program was created by NASA, resulting in a total of 11 space flights to the Moon, some of them with extravehicular activities on the lunar surface.

In the first four flights the equipment used during all Apollo programs was tested. Six of the last seven flights landed on the moon.

The first flight of the Apollo missions took place in 1968, while the first landing was in 1969. The last landing was made in 1972.

A total of 12 astronauts walked on the moon. During the walks, scientific experiments were carried out, such as collecting rocks for later analysis on the ground.

1.3. VEHICLES USED

NASA designed the Apollo Command Module for this program. It was a three-man capsule. The astronauts traveled in the command module en-route to the Moon and back to Earth. It was a larger vessel than the one used during the Mercury and Gemini programs. The spacecraft had enough space to allow astronauts to move, its area can be compared to that of a car.

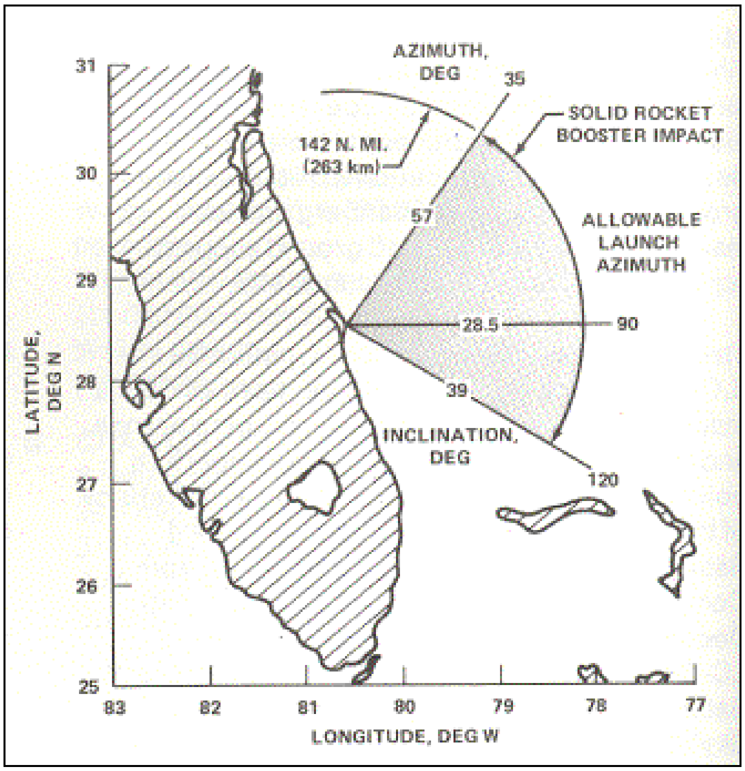
Another vehicle, the lunar module, was used for the moon landing. This vehicle carried astronauts from orbit around the Moon to its surface, and back. It had room for two astronauts.

Two types of rockets were used by the Apollo program. The first flights were made with the Saturn IB rocket, and was composed of two stages. It was mainly used for testing the Apollo capsule in Earth orbit.

The other flights were carried out with the Saturn V rocket. This three-stage rocket had the capability of sending the Apollo ships to the moon.

1.4. LAUNCH PAD RESTRICTIONS (UNITED STATES)

Apollo missions were launched from Cape Canaveral, which has restrictions on the type of orbit and inclination of the orbital plane during the launch. For the platform mentioned above, it has an allowable path of no less than 35° northeast and no more than 120° southeast. The two angles described above represent the launch limits from Cape Canaveral. Any azimuth angle greater than those mentioned above may cause the spacecraft to fly over inhabited land areas, adversely affecting the safety factors required for separation of the stages or for aborting the mission, and if some parts require recovery, they may fall into foreign territories.



Source: http://ccar.colorado.edu/asen5050/projects/projects_2003/vellone/index_files/image002.gif

1.5. PLANNED RETURN TO MOON

The different space agencies and some private companies have their eyes set on missions back to the Moon, considering the potential for surface and lunar orbit exploitation.

Among the possible applications for the return to the Moon is found:

- **Space Mining:** There are different chemical elements that can be exploited in large quantities from the surface of the moon, including Helium-3, a helium isotope that can be used for more efficient energy production.
- **Space stations and launch/refuel platforms:** Due to the Moon's low gravity characteristics, as well as the possibility of exploiting chemical components for manufacturing rocket propellants, the Moon could be used as a launch pad for interplanetary spacecrafts. Similarly, Lunar orbit represents an opportunity to place an intermediate space station between Earth and manned long-distance trips, such as to Mars.

2. SOFTWARE TOOLS

2.1. PYTHON

Python is an interpreted programming language whose philosophy emphasizes a syntax that favors a readable code.

It is a multiparadigm programming language, as it supports object orientation, imperative programming and, to a minor degree, functional programming. It is an interpreted language, uses dynamic typing and is multiplatform.

2.2. POLIASTRO

Poliastro is a library of functions for Python applied in astrodynamics and orbital mechanics, focused on interplanetary applications. It provides simple and intuitive functions and handling of physical quantities with units. Some of the functions are:

- Analytical and numerical orbit propagation analysis.
- Conversion between position and velocity vectors and classical orbital mechanical element.
- Transformations of coordinate systems.
- Calculation of Hohmann and bieliptical transfer maneuvers.
- Trajectory plotting.
- Initial determination of orbits (Solutions to the Lambert problem)
- Planetary ephemeris and other bodies.
- Near-Earth Objects calculation.

2.3. SPYDER

Spyder is an interactive programming environment for Python that provides MATLAB-like tools in lightweight, easy-to-use software. It also provides ready-to-use widgets written in pure Python for PyQt5 and PyQt4 applications: Source code editor with syntax highlighting and code analysis tools.

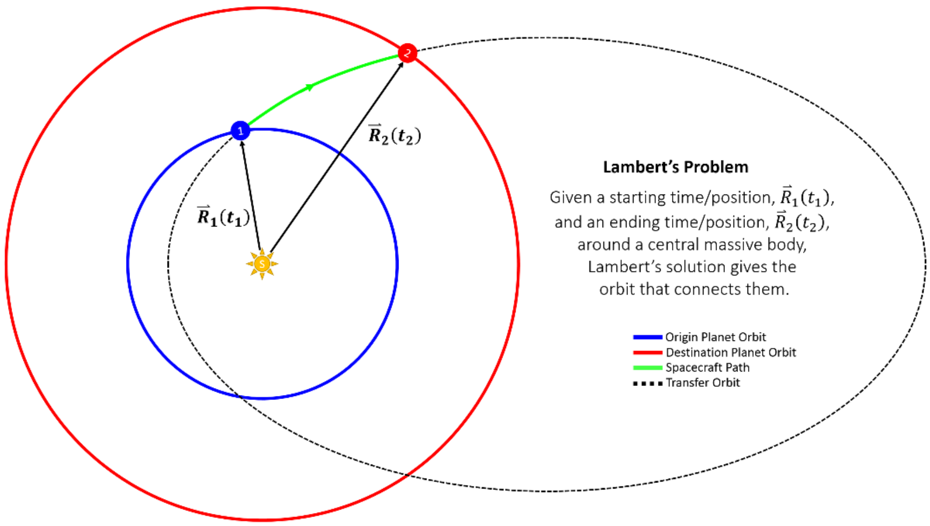
2.4. JUPYTER NOTEBOOK

Jupyter Notebook is an open source web application that allows to create and share documents containing code, equations, visualization and narrative text. Uses include: Data cleaning and transformation, numerical simulations, statistical modeling, data visualization, among others.

3. THEORETICAL APPROACH

3.1. LAMBERT'S PROBLEM

The Lambert problem aims to find the transfer orbit that connects two given position vectors at a specific transfer time. Lambert's theorem indicates that the transfer time depends only on the semi-major axis of the transfer orbit, the sum of the radius to points 1 and 2, and the length of the chord between the points to be connected.



Source: http://ccar.colorado.edu/ASEN5050/projects/projects_2016/Marino_John/img/lambert.png

3.2. SPHERE OF INFLUENCE (SOI)

The Sphere of Influence is defined as the locus of points measured with respect to the gravitational centers of attraction, where the ratios of the perturbative to primary gravitational accelerations of the centers are equal.

The sphere of influence can be described as the area in which the gravitational force on a spacecraft is predominant due to the mass of a nearby massive body. It is calculated using:

$$r_{SOI} = a\left(\frac{m}{M}\right)^{2/5}$$

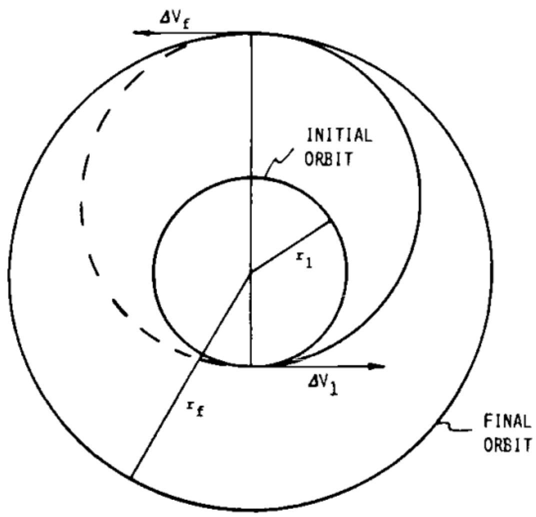
Being A the semi-major axis of the orbit of the smallest object, m and M being the masses of the smallest and largest objects, respectively.

For the case of the Moon, the radius of the Influence Sphere is 66,280 km. This value will be considered during the demonstration of the path obtained by the software.

3.3. HOHMANN TRANSFER MANEUVER

To perform Apollo missions to the moon, half of Hohmann's transfer maneuver was used, which is the most basic transfer maneuver possible with two firing moments, for transfer between circular coplanar orbits.

The Hohmann transfer operation is a relatively simple operation. A Delta-V is applied tangentially to the circular orbit. The magnitude of the Delta-V is determined by the requirement that the radius of the transfer ellipse apogee must be equal to the radius of the final circular orbit. When the satellite reaches the apogee of the transfer orbit, another Delta-V must be applied, otherwise the satellite will remain in elliptical orbit. This Delta-V is the difference between the velocity at the height of the transfer orbit and the velocity in the final circular orbit. After the last Delta-V has been applied, the satellite is in final orbit, and the transfer has been completed.



Source: Orbital Mechanics-Chobotov Pag 94

To calculate the Delta-V required to reach the transfer orbit, in the case of two coplanar orbits, it is used:

$$\Delta V_1 = \sqrt{\frac{\mu}{R_1}}\left(\sqrt{\frac{2R_2}{R_2 + R_1}} - 1\right)$$

To circularize the orbit, the ignition is performed in the apoapsis. Such ignition shall provide the next Delta-V to the spacecraft:

$$\Delta V_2 = \sqrt{\frac{\mu}{R_2}}\left(1 - \sqrt{\frac{2R_2}{R_2 + R_1}}\right)$$

In case the orbits are not coplanar, it is necessary to analyze the force triangle to determine the Delta-V required in each of the components to reach the target orbit.

3.4. 3-4 BODY PROBLEM

The effects of solar gravity on the spacecraft are neglected, as well as the influence that the mass of the spacecraft can have on the bodies of the other planets, because it is insignificant. The only bodies taken as a reference for the calculations are the Earth and the Moon, the force of each of them being applicable within their spheres of influence.

3.5. PARKING ORBIT DATA AROUND EARTH AND MOON

According to the Apollo mission report, the parking orbits around the Earth and Moon were, with an average height above ground of 114 miles x 116 miles.

Earth Orbit:

- Launch at 9:32 EDT, July 16th -1969
- Insertion into Earth orbit of 100nm (average) at 11 minutes, 43 seconds after take-off.
- Orbit Inclination: 32,521°

Impulse for Translunar Trajectory

- At 2:44:26 GET on the Pacific Ocean, during the second revolution.
- Duration of flight to the moon, 73:10, from 2:44 to 75:54 GET (Ground Elapsed Time)

Lunar Orbit Insertion

- Duration 25:00, from 69:00 to 94:00
- 69x190 miles orbit

Lunar Orbit

- Revolution starts at 180° longitude
- Duration of revolution 1hr, 58.2 min.
- Orbit inclination 1.25°.
- 62x70.5 miles orbit

4. CALCULATIONS: DELTA-V REQUIRED FOR MANEUVER

4.1. LIBRARIES

The libraries are loaded with the modules necessary to perform the calculations.

```
In [1]: from astropy import units as u
        from astropy import time
        from astropy.time import Time

        from poliastro.bodies import Earth, Moon
        from poliastro.twobody import Orbit
        from poliastro.maneuver import Maneuver
        from poliastro import iod

        from poliastro.util import time_range
        from astropy.coordinates import solar_system_ephemeris, get_body_barycentric_posvel
        solar_system_ephemeris.set("jpl")

        from poliastro.frames import HeliocentricEclipticJ2000
        from astropy.coordinates import (
            ICRS, GCRS,
            CartesianRepresentation, CartesianDifferential
        )

        from poliastro.patched_conics import compute_soi

        from poliastro.plotting import OrbitPlotter3D
        from poliastro.plotting import OrbitPlotter, plot
        import matplotlib.pyplot as plt
        from plotly.offline import init_notebook_mode
        init_notebook_mode(connected=True)
```

4.2. TIMES

Subsequently, the launch and completion times of the translunar injection maneuver are determined, and from these, the total duration of the maneuver.

```
In [2]: #Times
        date_liftoff=time.Time('1969-07-16 14:32', scale='tdb')
        date_launch=date_liftoff + (2 * u.h + 44 * u.min)
        #date_launch is the time at which Translunar Insertion Maneuver is performed
        date_arrival=date_launch + (75 * u.h + 54 * u.min)
        #date_arrival is the time at which Cislunar Insertion Maneuver is performed
        tof=date_arrival - date_launch
```

4.3. APOLLO'S ORBIT AROUND EARTH

The parking orbit of the Apollo spacecraft around the Earth is specified, according to the next parameters:

- Attracting Body
- Height
- Orbit Inclination
- Time

```
In [3]: #Apollo Parking Orbit Around Earth
        apollo=Orbit.circular(Earth,alt=185.21 * u.km, inc=32.521 * u.deg, epoch=date_launch)
        apollo

Out[3]: 6563 x 6563 km x 32.5 deg orbit around Earth (♁)
```

4.4. LUNAR ORBIT

The orbit of the moon is acquired and transformed.

Because orbit is by default in the International Celestial Reference System (ICRS), it is not possible to plot or obtain appropriate parameters for calculating the transfer orbit from the Apollo spacecraft's parking orbit to the moon.

The position and velocity information transformation function (values that determine the orbit) is used to convert from ICRS to GCRS (Geocentric Celestial Reference System).

With the transformation already carried out, it is possible to express the moon's trajectory around the Earth in a satisfactory way. With the characteristics of the orbit, it is now possible to determine the transfer orbit required for the Apollo mission.

```
In [4]: #Moon Orbit aqcuisition and conversion to GCRS
        EPOCH=date_arrival
        moon = Orbit.from_body_ephem(Moon, EPOCH)
        moon_icrs = ICRS(
        x=moon.r[0], y=moon.r[1], z=moon.r[2],
        v_x=moon.v[0], v_y=moon.v[1], v_z=moon.v[2],
        representation=CartesianRepresentation,
        differential_cls=CartesianDifferential
        )

        moon_gcrs = moon_icrs.transform_to(GCRS(obstime=EPOCH))
        moon_gcrs.representation = CartesianRepresentation
        moon_gcrs

        moon = Orbit.from_vectors(
        Earth,
        [moon_gcrs.x, moon_gcrs.y, moon_gcrs.z] * u.km,
        [moon_gcrs.v_x, moon_gcrs.v_y, moon_gcrs.v_z] * (u.km / u.s),
        epoch=EPOCH
        )
        moon

Out[4]: 366158 x 406669 km x 28.5 deg orbit around Earth (♁)
```

4.5. LAMBERT'S PROBLEM SOLUTION

The solution of Lambert's problem is realized for the parking orbits of the Apollo mission and the lunar orbit, both around the Earth. The following parameters are indicated for the solution:

- Characteristics of the main attractor body
- Vector R for the Apollo spacecraft orbit
- Vector R of the lunar orbit
- Duration of the transfer operation.

```
In [5]: ss0=apollo
        ssf=moon
```

```
In [6]: #Solve for Lambert's Problem (Determining Translunar Trajectory)
        (v0, v), = iod.lambert(Earth.k, ss0.r, ssf.r, tof)
        ss0_trans=Orbit.from_vectors(Earth,ss0.r,v0,date_launch)
        ss0_trans

Out[6]: 6558 x 512347 km x 19.1 deg orbit around Earth (♁)
```

4.6. FINAL ORBIT

With the transfer orbit already calculated with the previous function, the Delta-V values can be obtained on all three axes required to perform the maneuver. The result obtained for the Delta-V required for each axis is:

- X=-312.87077937 m/s
- Y=3775.34881645 m/s
- Z=-614.599664056 m/s

From the above it can be deduced that changes were made in the three axes. Anti-Normal, Prograde and Anti-Radial impulses were performed, respectively for each axis.

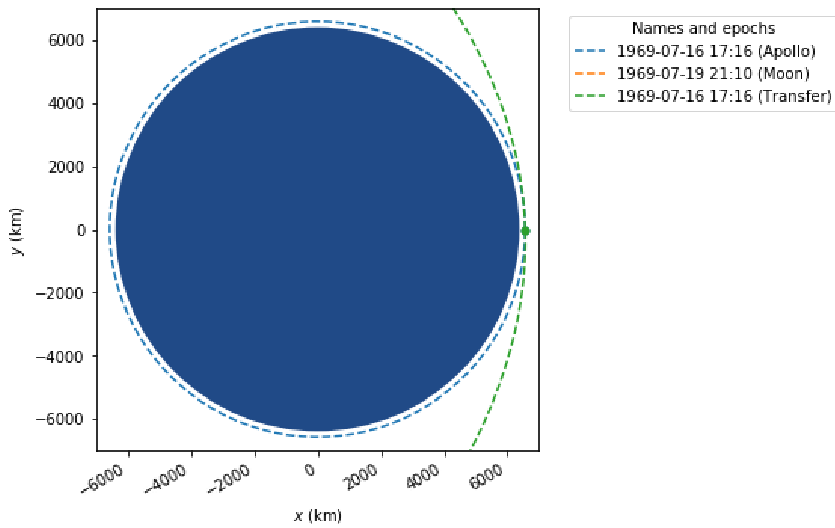
```
In [7]: dv=(ss0_trans.v-ss0.v)
        dv=dv.to(u.m/u.s)
        man = Maneuver.impulse(dv)
        ss_f = ss0.apply_maneuver(man)
        print("The required Delta-V to perform the Translunar Injection maneuver is: " + str(dv))

        The required Delta-V to perform the Translunar Injection maneuver is: [ -312.87077937  3775.34881645  -614.59664056] m / s
```

4.7. ORBIT PLOTS

4.7.1. 2D

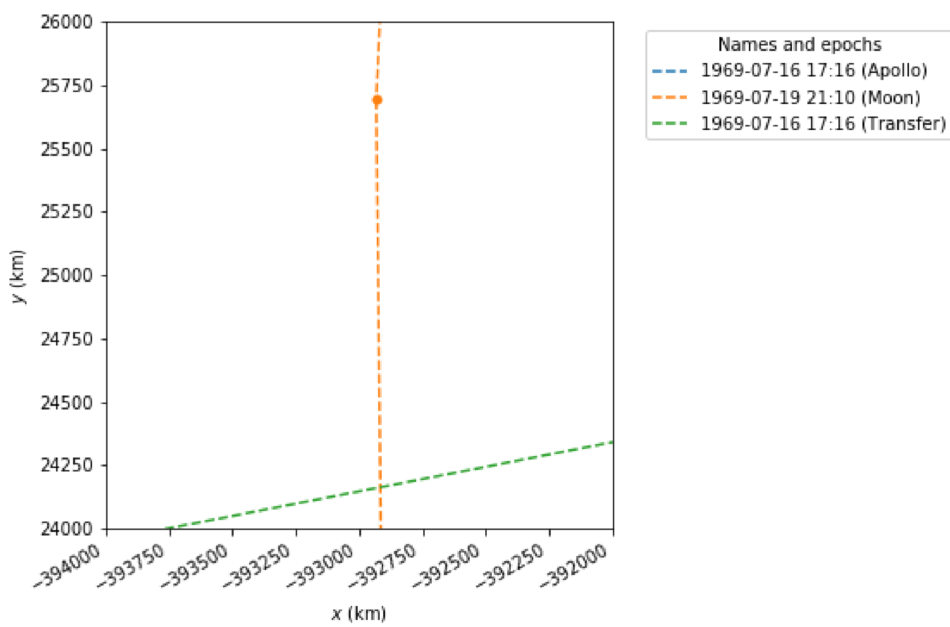
The graphing of the resulting orbit is then carried out, making close-ups to the starting point of the maneuver in the parking orbit around the Earth, as well as to the intersection of the transfer orbit with that of the Moon.

```
In [8]: #Plotting Solution in 2D - Earth close-up
        plt.ion()
        op=OrbitPlotter()
        op.plot(ss0, label="Apollo")
        op.plot(ssf, label="Moon")
        op.plot(ss0_trans, label="Transfer")
        plt.xlim(-7000, 7000)
        plt.ylim(-7000, 7000)
        plt.gcf().autofmt_xdate()
```



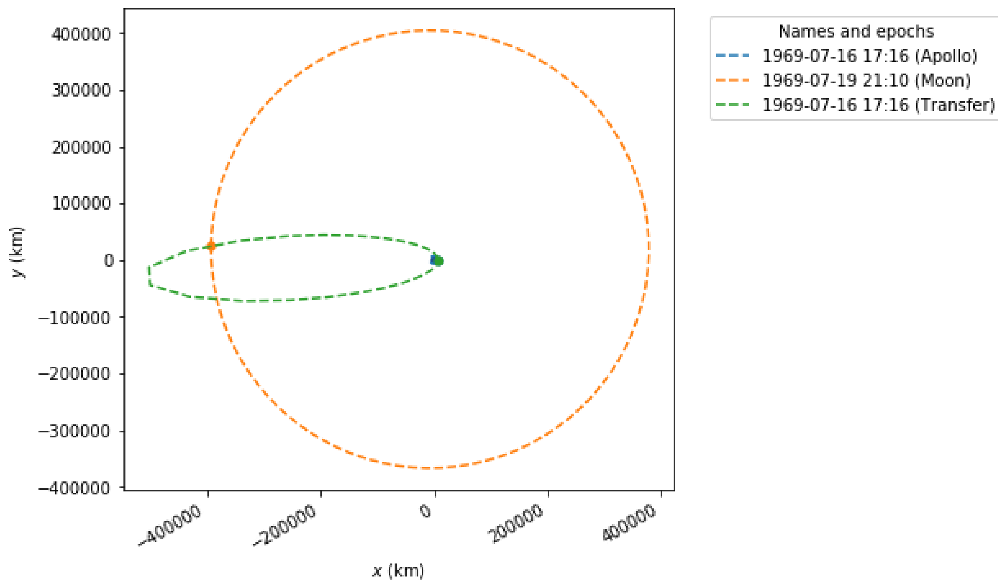
A second graph is also made showing that the transfer orbit passes through the Moon's sphere of influence and returns to Earth. This type of orbit for the Earth-Moon system is known as Free-Return Trajectory. This type of orbit coincides with that followed by the Apollo missions.

In [9]:
```
#Plotting Solution in 2D - Moon close-up
plt.ion()
op=OrbitPlotter()
op.plot(ss0, label="Apollo")
op.plot(ssf, label="Moon")
op.plot(ss0_trans, label="Transfer")
plt.xlim(-394000, -392000)
plt.ylim(24000, 26000)
plt.gcf().autofmt_xdate()
```



In the same way, a graph of the complete set is made:

```
In [10]: #Plotting Solution in 2D
         plt.ion()
         op=OrbitPlotter()
         op.plot(ss0, label="Apollo")
         op.plot(ssf, label="Moon")
         op.plot(ss0_trans, label="Transfer")
         plt.gcf().autofmt_xdate()
```

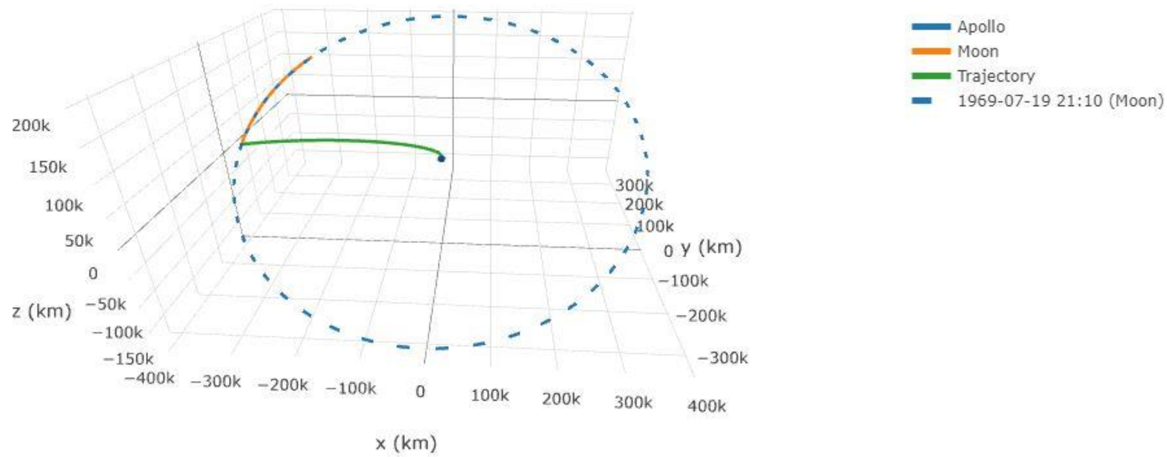


4.7.2. 3D

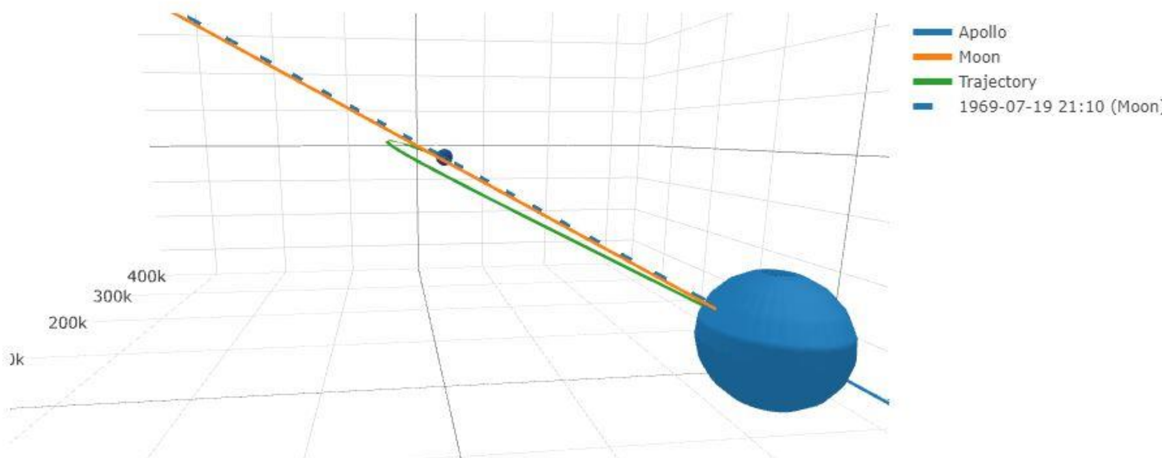
Finally, a 3D graph is made to observe the resulting transfer orbit. This type of graph makes it possible to better appreciate other parameters of the orbit that are not visible through the 2D graph, such as the inclination of the lunar orbit relative to the Earth and the inclination change of the transfer orbit.

```
In [11]: #Plotting Solution in 3D
         N=100
         times_vector = time_range(date_launch, end=date_arrival, periods=N)

         frame = OrbitPlotter3D()

         frame.set_attractor(Earth)

         frame.plot_trajectory(ss0.sample(times_vector), label="Apollo")
         frame.plot_trajectory(ssf.sample(times_vector), label="Moon")
         frame.plot_trajectory(ss0_trans.sample(times_vector), label="Trajectory")

         frame.plot(moon, label="Moon")

         frame.set_view(30 * u.deg, 260 * u.deg, distance=3 * u.km)
         frame.show(title="Apollo Mission: Trans-Lunar Trajectory")
```

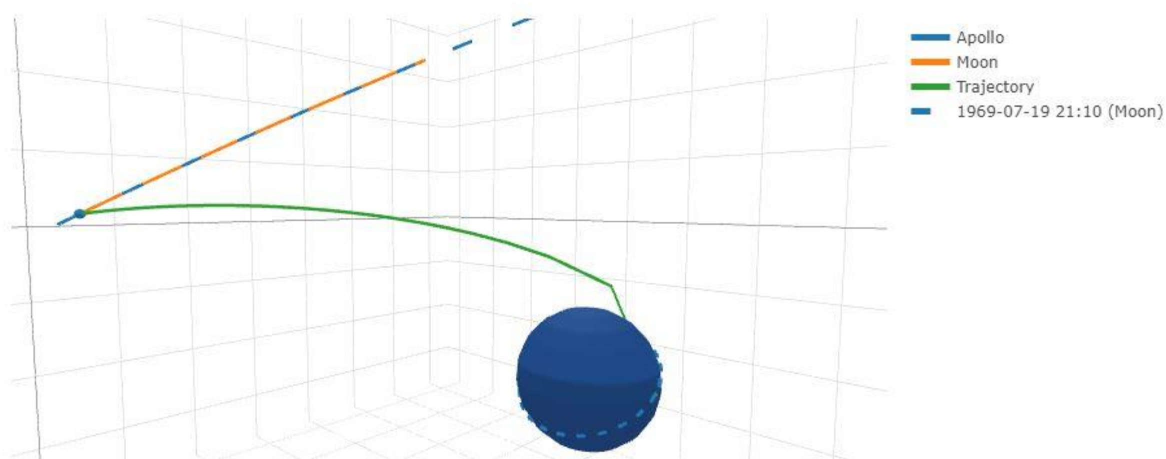
Apollo Mission: Trans-Lunar Trajectory

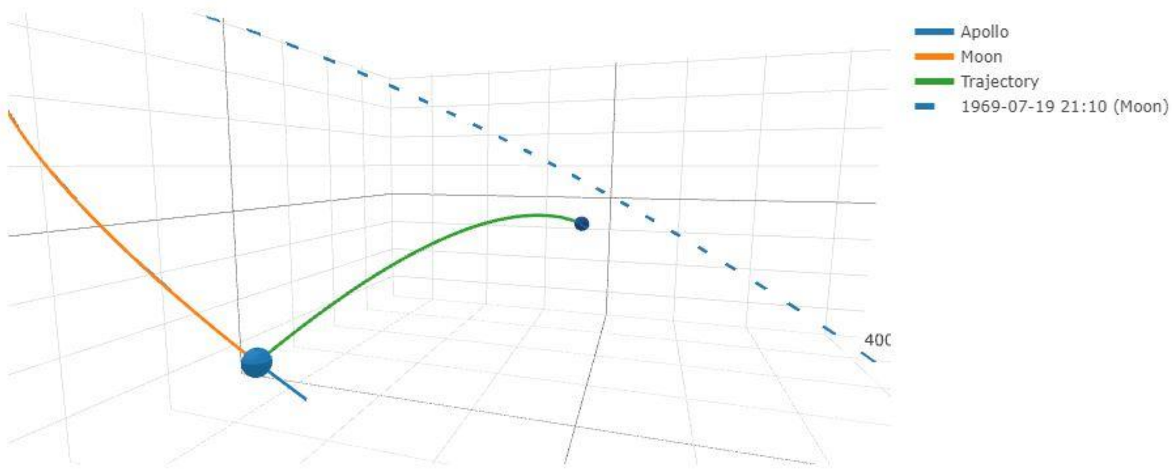


Apollo Mission: Trans-Lunar Trajectory



Apollo Mission: Trans-Lunar Trajectory





4.8. NEGLECTED PARAMETERS DURING CALCULATIONS

To carry out the transfer maneuver it is necessary to neglect certain variables and perform some assumptions, in order to facilitate the process of preliminary orbit calculation.

- Newtonian Gravity: It is assumed that the Earth and Moon can be represented as points of mass, indicating that there are no irregularities or special concentrations of mass at points other than the center.
- The speed changes made to complete the maneuver are instantaneous, it does not consider the real time required to apply the necessary Delta-V.

Other parameters not considered during simulation are:

- Take-off of the spacecraft from the Earth's surface. Only the parking orbit around the Earth, from which the translunar injection maneuver starts, is considered.
- Circularization of the orbit around the moon, due to lack of modules within the libraries to perform maneuvers that in addition change the attraction body. This is the case of the retrograde thrust necessary to exit the free return path to Earth.

5. CONCLUSIONS

The proximity of the moon facilitates mission planning, because the launch windows are extremely wide compared to interplanetary missions or bodies with the same Moon configuration (Asteroids, for example).

From the 2D graphs obtained, it can be seen that, according to what was said in the sphere of influence section, the spacecraft actually passes through the sphere of influence of the Moon, so that, after carrying out a retrograde thrust in the apogee of the transfer orbit, it is possible to obtain an orbit around the Moon.

According to the official report of the Apollo 11 mission, a total Delta-V of 10,451.2 ft./s (3,185.52576 m/s) was required to reach the Moon. According to the calculations made by the software, the total Delta-V is 3,837,822242 m/s. The difference between the Delta-V obtained and the real is 652,296482 m/s.

When comparing the values obtained by means of the simulation, and the real values applied during the Apollo mission, a difference is perceived which, although significant enough to affirm that there are errors in the calculation not attributable to the input parameters, is not large enough to say that the simulation is not successful. This statement refers not only to the Delta-V value, but also to the characteristics of the orbit type achieved.

Installed libraries allow a basic approach to the solution of astrodynamics problems. Unfortunately, it does not allow for more complex operations, so it was impossible to simulate the retrograde thrust required to circulate the orbit around the Moon after the initial maneuver.

Both the programming language and the libraries used to perform the calculations demonstrate the potential to be used in preliminary calculations for determining trajectories, satellite tracking, analysis of the propagation of bodies in orbits, among other preliminary applications for astrodynamics.

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