Qubits

QCP 2020-2

FABIO A. GONZÁLEZ

UNIVERSIDAD NACIONAL de COLOMBIA

1. Quantum Mechanics Postulates

1. The superposition principle:
   what are the possible states of quantum system
2. The measurement principle:
   How much information from a quantum state we can access
3. Unitary Evolution:
   How a quantum system evolves through time

2. The superposition Principle

If a quantum system can be in one of two states it can also be in any linear combination of these states with complex coefficients.

$|\alpha_i| = \sqrt{\alpha_i^* \alpha_i}$

Possible states $\{|0\rangle, |1\rangle, \dots, |k-1\rangle\}$

Quantum state $\alpha_0|0\rangle + \alpha_1|1\rangle \dots \alpha_{k-1}|k-1\rangle$ $\alpha_i \in \mathbb{C}$ $\sum_{i=0}^{k-1} |\alpha_i|^2 = 1$

for $k=3$

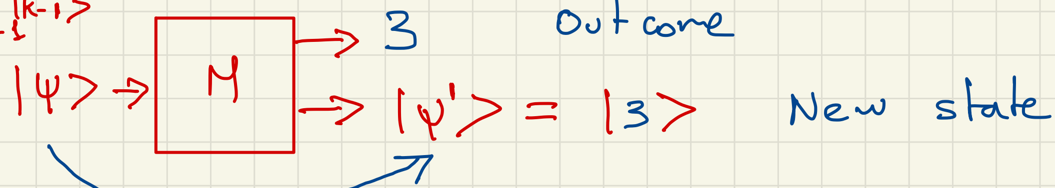
$$|\psi\rangle = |0\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{2}|1\rangle + \frac{i}{2}|2\rangle$$

3. The measurement Principle

- We can not measure the complex amplitudes α_i
- A measurement in a quantum system with k states produces k possible outcomes
- If we measure the system in the standard basis we get $|i\rangle$ with probability $|\alpha_i|^2$
- Measurement alters the state of the system, the new state is exactly the measurement outcome.

$$|\psi\rangle = \alpha_0|0\rangle \ldots \alpha_{k-1}|k-1\rangle$$



Collapse of the wave function

- In a general measurement you select an orthonormal basis $\{|e_0\rangle, \dots, |e_{k-1}\rangle\}$.
- The outcome of the measurement is $|e_i\rangle$ with probability $|\beta_i|^2$ where β_i is the amplitude of $|e_i\rangle$ in the representation of ψ in the basis.

$$|\psi\rangle = \beta_0|e_0\rangle + \dots + \beta_{k-1}|e_{k-1}\rangle$$

4. Qubits

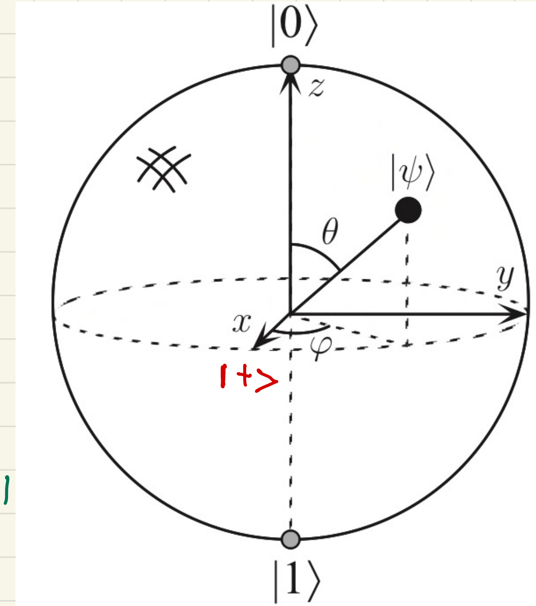
A qubit is a quantum system with two states.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$|\alpha|^2$ probability of getting $|0\rangle$

A qubit can be represented in an arbitrary basis $\{|v\rangle, |w\rangle\}$ $\langle v|v\rangle = \langle w|w\rangle = 1$, $\langle v|w\rangle = 0$

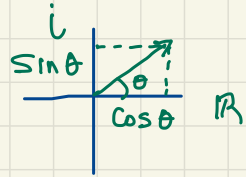
$$|\psi\rangle = \alpha'|v\rangle + \beta'|w\rangle$$



$\alpha = a e^{i\theta}$

magnitude (a) direction ($e^{i\theta}$)

$e^{i\theta} = \cos\theta + i\sin\theta$



$$|\psi\rangle = a e^{i\gamma}|0\rangle + b e^{i\gamma'}|1\rangle$$

$$= e^{i\gamma}\left(a|0\rangle + b e^{i(\gamma'-\gamma)}|1\rangle\right)$$

Global phase

$$= \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\varphi}|1\rangle$$

Local phase

$\theta = 2\arccos a$

5. Phase Estimation

$$|\psi\rangle = \underbrace{\frac{1}{\sqrt{2}}}_{\alpha}|0\rangle + \underbrace{\frac{e^{i\theta}}{\sqrt{2}}}_{\beta}|1\rangle$$

$|\psi\rangle \rightarrow \boxed{\begin{matrix}|0\rangle \\ |1\rangle\end{matrix}}$ $\rightarrow$ 0 # n ; $\rightarrow$ 1 # 100−n

$$|\alpha|^2 = P(|0\rangle) \approx \frac{n}{100}$$

$$|\beta|^2 = P(|1\rangle) \approx \frac{100-n}{100}$$

Only gives information about magnitudes.

Is there any measurement that yields information about θ?

$$|+\rangle \equiv \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \qquad |-\rangle \equiv \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$|\psi\rangle = \frac{1}{2}\left(|+\rangle + |-\rangle\right) + \frac{e^{i\theta}}{2}\left(|+\rangle - |-\rangle\right)$$

$$= \underbrace{\frac{1+e^{i\theta}}{2}}_{\alpha'}|+\rangle + \underbrace{\frac{1-e^{i\theta}}{2}}_{\beta'}|-\rangle$$

$|\psi\rangle \rightarrow \boxed{\begin{matrix}|+\rangle \\ |-\rangle\end{matrix}}$ $\rightarrow$ + $|\alpha'|$; $\rightarrow$ − $|\beta'|$

6. General Qubit Bases

7. Unitary Operators