## Entanglement

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1. Two Oubits - 9, 92 each one has two possible states / 10>, 11>3 - 4 possible combinations 10>00 (0>= 100> (10) = (11) - General state 14> = 400 (00> + 40,101> + 4,0 (10> + 4,111) - More than z qubits, n ez" terms [Ψ>= x<sub>000.0</sub>|00...ο> + x<sub>0...0</sub>|[0...οι> -... + x<sub>0...ο</sub>|

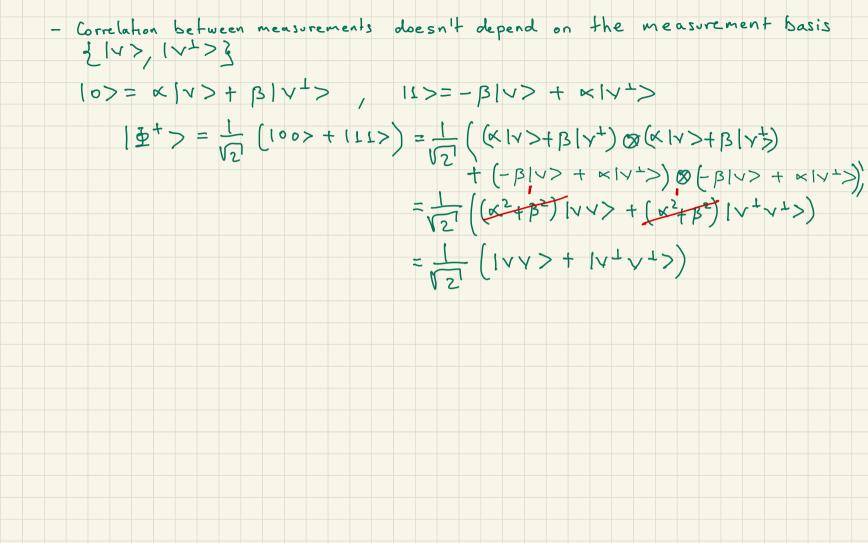
[Ψ>= x<sub>000.0</sub>|00...ο> + x<sub>0...ο</sub>|0...οι> -... + x<sub>0...ο</sub>|

[Ψ] = x<sub>000.0</sub>|00...ο> + x<sub>0...ο</sub>|0...οι> -... + x<sub>0..</sub>

2. Measurement on a two Oubits System - Measurement only gives information about the basis states
- 2 qubits system -> 2 bits
- Probability of getting  $x \in \{0,13^2 \mid 13 \mid \alpha_x \mid x = 00,01,10,11$ - If we measure 12>=1ij> the new state of qi is 1i)
and the new state of qz is 1j>
what does happen if you only measure I qubit? - ( Ynew > = x10/10> + x11/11>

- what does happen if you only measure I qubit (
$$|\psi\rangle = \chi_{00}|_{00} + \chi_{01}|_{00} + \chi_{01}|_{$$

3. Entanglement 14>=14,>8142> - Suppose  $q_1 \rightarrow | \psi_1 \rangle = \frac{3}{5} | 0 \rangle + \frac{1}{5} | 1 \rangle$ 9192 > 14>=3 100>+3101>+410>+410> - Consider [47 = 1 (100>+111>), Can you decompose in 14, > and 142>  $|\Psi_1\rangle = \alpha|0\rangle + b|1\rangle$   $|\Psi_1\rangle = c|0\rangle + d|1\rangle$   $|\Psi_2\rangle = c|0\rangle + d|1\rangle$   $|\Psi_1\rangle \otimes |\Psi_2\rangle = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} \sqrt{\sqrt{2}} \\ 0 \\ 0 \\ 0 \end{pmatrix} = |\Psi\rangle$ - This is an entangled state => non-separable state -  $P_r(q_1=0)=|\frac{1}{\sqrt{2}}|^2=\frac{1}{2}=P_r(q_r=1)$ - If you measure 9, = 0 what's the new state of the system? [4new>= 100> => 91, 92 = 10> - In entangled states we cannot determine the state of a quisit separately



2. Two qubit operators - Unitary transformations on C4, 4x4 matrix U, uut=utu=] CNOT = CX = (000 ) CNOT | 10> = | LL> CNOT = CX = 000 CNOT | 00> = | 00> CNOT | 00 | 0 | CNOT | 01> = | 120> - Example - Any unitary transform on two qubits Control Controlled Can be closely approximated by Sequences bit bit bit of croop and single qubit operations - How to apply a one-qubit operator to one qubit of a two qubits system?