lec 1

$C = \{ Z : Z = (x,y) \ , x,y \in R \}$

$R \times R$

$Z_1 , Z_2 \in C$

$Z_1 = (x_1 , y_1) \quad \& \quad Z_2 = (x_2 , y_2)$

$Z_1 \oplus Z_2 = (x_1 + x_2 , y_1 + y_2)$

→ $(C, \oplus)$ Commutative Group

→ $(C, \oplus, \otimes)$ is a Field of Complex numbers

• Commutative Property :-

$\forall Z_1 , Z_2 \in C \Rightarrow Z_1 \oplus Z_2 = Z_2 \oplus Z_1$

• Associative Property :-

$\forall Z_1 , Z_2 , Z_3 \in C \Rightarrow Z_1 \oplus (Z_2 \oplus Z_3) = (Z_1 \oplus Z_2) \oplus Z_3$

• Identity For addition :-

$\forall Z \in C \quad \exists \ (0,0) \in C : Z + (0,0) = (0,0) + Z = Z$

• Inverse :-

$\forall Z \in C \quad \exists \ -Z \in C : Z \oplus (-Z) = (-Z) \oplus Z = (0,0)$

$\therefore$ $(C, \oplus)$ is Commutative Group #

$\otimes : C \times C \to C$

$z_1 = (x_1, y_1)$ s $z_2 = (x_2, y_2)$

$z_1 \otimes z_2 = (x_1 x_2 - y_1 y_2 , x_1 y_2 + x_2 y_1)$

- Commutative Property :-

$\forall z_1, z_2 \in C : z_1 \otimes z_2 = z_2 \otimes z_1$

- associative Property :-

$\forall z_1, z_2, z_3 \in C : z_1 \otimes (z_2 \otimes z_3) = (z_1 \otimes z_2) \otimes z_3$

- Identity For multiplication :-

$\forall z \in C \; \exists \; (1,0) \in C : z \otimes (1,0) = (1,0) \otimes z = z$

- Inverse :-

$\forall z \in C - \{(0,0)\} \; \exists \; \frac{1}{z} = z^* = (x^*, y^*) \in C - \{(0,0)\}$

$: z \otimes \frac{1}{z} = (1,0)$

$z = (x,y)$ s $z^* = (x^*, y^*)$

$z \otimes z^* = (xx^* - yy^*, xy^* + x^* y) = (1,0)$

$xx^* - yy^* = 1$
$xy^* - x^* y = 0$

$$\begin{pmatrix} x & -y \\ y & x \end{pmatrix} \begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} x & -y \\ y & x \end{vmatrix} = x^2 + y^2$$

$$\Delta_1 = \begin{vmatrix} 1 & -y \\ 0 & x \end{vmatrix} = x \quad , \quad \Delta_2 = \begin{vmatrix} x & 1 \\ y & 0 \end{vmatrix} = -y$$

$$x^* = \frac{\Delta_1}{\Delta} = \frac{x}{x^2+y^2} \quad , \quad y^* = \frac{-y}{x^2+y^2}$$

$$\therefore z^* = \frac{1}{z} = \left(\frac{x}{x^2+y^2} , \frac{-y}{x^2+y^2} \right) \in C - \{(0,0)\}$$

$\therefore$ ($C - \{(0,0)\}$) is Commutative Group #

• Distribution :

$$\forall z_1, z_2, z_3 \in C \quad z_1 \otimes (z_2 \oplus z_3) = (z_1 \otimes z_2) \oplus (z_1 \otimes z_3)$$

$$(z_1 \oplus z_2) \otimes z_3 = (z_1 \otimes z_3) \oplus (z_2 \otimes z_3)$$

→ Algebric Form of a ComPlex numbers :-

$$z = (x,y) = (x,0) + (0,y)$$
$$= x(1,0) + (0,1)\,y$$
$$= x + iy$$

$(0,1) = i$

$iy = (0,1)(y,0)$

$= (0,y)$

$z_1 = (x_1, y_1) = x_1 + iy_1$, $z_2 = (x_2, y_2) = x_2 + iy_2$

$x_2 \to Re(z) = x_2$, $y_2 \to Img(z) = y_2$

$\therefore z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$= (x_1 x_2 + i x_1 y_2 + i y_1 x_2 + i^2 y_1 y_2)$

$i^2 = i\, i = (0,1)(0,1) = (-1,0) = -1$

$z_1 z_2 = x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)$ #

$z_1 - z_2 = x_1 + iy_1 - (x_2 + iy_2) = x_1 - x_2 + i(y_1 - y_2)$

Conjugate: المرافق

$\bar{z} = x - iy$, $z = x + iy$

- $\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \otimes \frac{x_2 - iy_2}{x_2 - iy_2} = \frac{x_1 x_2 + y_1 y_2 + i(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}$
- $z + \bar{z} = 2x = 2\,Re(z)$
- $z - \bar{z} = 2iy = 2\,img(z)$
- $z\bar{z} = (x+iy)(x-iy) = x^2 + y^2$
- $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq (0,0)$

Scanned by CamScanner

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$4n,\ 4n+1,\ 4n+2,\ 4n+3 \qquad n \in Z^+$

$/ n \in Z$

$i^{4n} = 1$

$i^{4n+1} = i^{4n}\ i = i$

$i^{4n+2} = i^{4n}\ i^2 = -1$

Example:-

$$L = \frac{2+i}{1+i} * \frac{1-i}{1-i} \quad M = \frac{1+2i}{1+i} * \frac{1-i}{1-i} \text{ hence}$$

$$Z = \frac{15(L^2+M^2)}{8LM(L+M)}$$

$$L = \frac{3}{2} - \frac{1}{2}i \qquad M = \frac{3}{2} + \frac{1}{2}i$$

$\therefore Z = 1$