Background Chapter

Group 23

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1 Introduction and Aims

As of 2014, there were more than 100 000 patients on the waiting list in the United States for a kidney transplant from a deceased donor (Anderson et al., 2015b). Despite the possibility to pair up with a living donor, issues of donor compatibility place restrictions on kidney transplants. Frequently, a willing donor for a patient proves biologically incompatible. Within pools of such incompatible donor-recipient pairs, it is possible to find 'exchanges', which transplant kidneys across more than one pair, thus overcoming individual incompatibilities and massively increasing the number of transplant operations conducted. Such Kidney Exchange Programs (KEPs) have saved many lives to date, but designing such a program is a challenging task and many operational researchers have tried to tackle this problem. The goal of this project is to investigate the methods for optimising kidney exchange as proposed by Anderson et al. (2015a). The paper discusses the application of operational research to improve kidney transplantation in collaboration with the Alliance for Paired Donation (APD). The APD developed and implemented Nonsimultaneous Extended Altruistic Donor (NEAD) chains, starting with an altruistic donor and extending through a series of transplants. This method offered an alternative to cyclic exchanges, where donors are swapped between incompatible pairs. In k-way cyclic exchanges, k pairs are assigned to one another based on compatibility, with the k-th donor being paired with the 1st recipient, forming a cycle. Usually these were two-way exchanges, because performing a cycle exchange of more than three pairs poses logistic difficulties. A simultaneous cycle involving k donor-recipient pairs required $2k$ operating rooms with $2k$ operating teams, which was logistically very demanding and often infeasible. Importantly, the transplants in NEAD chains do not need to occur at the same time, thus relaxing the requirement for simultaneity. This allows for longer chains and better optimization of donor-recipient matching, as well as overcoming operational challenges. The APD tackled the problem of optimising the matching process as a classical OR combinatorial optimization problem, using integer programming techniques and the theory of random graphs. The objective of the problem was to find a maximal set of compatible matches using chains and small cycles. Allowing long chains significantly increased the size of the optimization models and resulted in computational challenges.

Moreover, the implementation of NEAD chains faced skepticism and resistance within the medical community. An influential team of medical professionals argued against non-simultaneous chains based on computer simulations they were using, which suggested that even a modest risk of withdrawal of one pair in the long chain suggests that small chains of up to three pairs are more effective. A collaborative effort between physicians and operations researchers was essential to address concerns and demonstrate the effectiveness of the approach. A crucial motivation for the success of this work was the fact that long chains increase the number of transplants, and kidney exchange pools include a high percentage of very highly sensitized patients who are struggling to find a compatible donor.

There are two models proposed in this paper, and our aim is to reproduce both methods. The Recursive Algorithm aims to maximize the weighted number of selected exchanges, with a binary variable for each selected exchange and the constraint that each pair in the pool is used at most once. This approach allows to consider only cycles, by adding a zero weight node from the end on a chain to an altruistic donor. It is essentially an integer programming problem that finds the largest disjoint cycle cover of a graph, where each node represents a donor-recipient pair. To incorporate a bound on the length of the cycles, the algorithm is run recursively by eliminating long cycles one at a time. The algorithm works well in many instances, but its computational efficiency could be vastly improved.

The Prize-Collecting Travelling-Salesman-Problem-Based Algorithm (PC-TSP) utilizes a similar idea to a common solution to the Travelling Salesman Problem, where the goal is to find an optimal route where each node of the graph is visited only once. Similarly to PC-TSP, where not visiting a node accrues some penalty, the formulation for the kidney exchange seeks to find the longest paths possible without the requirement to include every single node in the dataset.

The key differences between the two models are as follows. The Recursive Algorithm solves the problem iteratively by first finding a solution, and then checking for cycles exceeding the allowed length. If such a cycle exists, it is removed by adding appropriate constraints and re-solving the model. The PC-TSP model, in contrast, explicitly includes binary variables for allowed cycles. Then, given a solution it tries to detect if any other cycles were formed that are not part of the set of allowable cycles, using a technique called cutting planes.

Both algorithms described in the paper performed well in finding optimal solutions for large-scale practical instances of the KEP. The PC-TSP–based algorithm has often outperformed the recursion-based algorithm. In this project, we will apply the described methods and offer new ideas on possible extensions to the approaches.

2 Literature review

In order to contextualize Anderson et al. (2015a) efforts and understand the state-of-the art in kidney exchange optimization, we looked into other methods that have been proposed either as and alternative, or building on the findings of Anderson et al. (2015a). This Section outlines the key results of our research.

Delorme et al. (2024) offers a computationally effective approach to optimising KEPs using integer linear programming (ILP), building on the argument that using the simple objective function of maximising the number of transplants is too vague and needs redefining. In the paper, they investigate a hierarchical objective approach using case studies of KEPs in UK, Spain and the Netherlands. For example, objective functions that are optimized in turn, according to the ordering can be: to maximize the number of effective two-way exchanges, to maximize the number of transplants, to minimize the number of three-way exchanges, to maximize the number of cross arcs, and to maximize the sum of the scores. Once the solution to highest order objective function is found, it is added to the set of constraints when optimising with respect to the next objective, and so on. The paper offers valuable insight into the data from UK Living Kidney Sharing Scheme (UKLKSS), and the authors suggests the constraints on cycle and chain sizes should be 3 and 4, respectively, while the average compatibility rate of patients in the pool is about 10%.

The study introduces new preprocessing techniques to remove unnecessary variables in KEP optimization models, significantly improving computational efficiency. Furthermore, a diving algorithm is proposed to streamline hierarchical optimization by reducing the need for solving multiple complex ILPs while maintaining solution optimality. A particularly interesting part of this study, and one that aligns with our approach to focus on realistic assumptions and modelling the logistics of kidney exchange as accurately as possible, is the introduction of cross arcs – additional, hypothetical arcs between compatible donor-recipient pairs that offer an alternative, should one of the pairs involved in the chain or cycle withdraw from the exchange. This adds robustness to the proposed exchanges and accounts for the risk of reneging.

The main result of this study is a hybrid model – the authors propose a method to transition between two existing ILP models: Cycle Formulation (CF), good for smaller instances but computationally heavy; and Position-Indexed Chain-Edge Formulation (PICEF), more efficient but unable to handle some objective functions. The paper introduces a technique that allows the dynamic switch between PICEF and Cycle Formulation. The hybrid approach balances efficiency with flexibility, reducing the overall computational cost. The model is tested on data from UK, Spanish and Dutch KEP. The authors conclude that this improvement in computational efficiency could be particularly beneficial in facilitating international exchange. Merging pools across country can potentially increase the number of successful transplants in the future.

Ding et al. (2018) presents a nonasymptotic approach to analyzing kidney exchange programs (KEPs) using a random graph model. The key innovation is a two-phase random walk method to model and optimize kidney exchanges, particularly the impact of long chains versus short cycles in medium-sized exchange pools. The authors introduce a two-phase random walk procedure, where random walks are used to allocate chains, followed by allocation in cycles. This method maintains a probabilistic structure throughout the process, facilitating straightforward analysis. The authors emphasize here the relative importance of long chains versus cycles in kidney exchanges and offer an argument for prioritizing chains in KEPs. Moreover, the paper compares the two-phase method with previously developed ILP-based models, and proposes a hybrid ILP method that involves preselecting a subset of chains using random walks. Their two-phase random-walk method provides analytical insight and practical benefits, offering an alternative to computationally expensive ILP-based methods.

Another innovative approach to the problem is presented by Riascos-Álvarez et al. (2024). They developed a new technique for modeling KEPs, using the branch-and-price algorithm that incorporates cycles and long chains and solves the pricing problems via multivalued decision diagrams (MDDs). This method not only includes both cycles and chains, which has not been done before in B&P, but also provides an exact optimal solution and can scale to larger datasets than was possible before. The approach allows for prioritising chains over cycles (or vice-versa) to imitate real-life considerations. The proposed two-phase solution method shifts between MDDS and linear programs, as well as presents the first use of Lagrangian relaxation in this context. This framework can potentially be used in different applications and help solve OR problems such as multicommodity pickup-and-delivery TSP.

In order to understand the real-life challenges of KEPs, we turn to the findings of Ashlagi and Roth (2021). Their work provides a comprehensive review of the operational challenges and innovations in Kidney Exchange Programs (KEPs). The authors analyze how KEPs have evolved from small, hospital-based programs into large-scale, multi-hospital networks. The paper discusses key operational issues. One critical aspect is the development of matching algorithms, which may employ various methods such as integer linear programs and graph-based matching to optimize pairings within both static and dynamic pools. Deciding when to match pairs, particularly in the presence of new arrivals and departures, remains a significant challenge. The paper also examines the trade-offs between batch and greedy matching in dynamic pools, where waiting for a larger pool can enhance compatibility but frequent matching may facilitate transplants for patients already in the system. Frictions in the matching process further complicate operations, as last-minute refusals, incompatibilities, and hospital rejections based on non-medical preferences can lead to failure. Additionally, the study explores the incentives influencing hospital participation, noting that some hospitals prioritize internal transplants over KEPs, which contributes to a higher concentration of highly sensitized patients in exchange pools. Finally, while increasing pool size and fostering international collaboration can improve transplant rates by broadening the pool of compatible matches, cross-border exchanges face regulatory and logistical hurdles. Global Kidney Exchange initiatives aim to integrate developing countries into these networks, addressing financial barriers that often hinder successful transplantation.

The papers outlined have improved our understanding about what has been proposed in scientific literature so far, and what are the advantages and disadvantages of various methods. In particular, we gained a new perspective and inspiration for proposing extensions to the models explained in Anderson et al. (2015a).

References

Anderson, R., Ashlagi, I., Gamarnik, D., Rees, M., Roth, A. E., Sönmez, T., and Ünver, M. U. (2015a). Kidney exchange and the alliance for paired donation: Operations research changes the way kidneys are transplanted. *Interfaces (Providence)*, 45(1):26–42.

Anderson, R., Ashlagi, I., Gamarnik, D., and Roth, A. E. (2015b). Finding long chains in kidney exchange using the traveling salesman problem. *Proceedings of the National Academy of Sciences - PNAS*, 112(3):663–668.

Ashlagi, I. and Roth, A. E. (2021). Kidney exchange: An operations perspective. *Management science*, 67(9):5455–5478.

Delorme, M., Garcia, S., Gondzio, J., Kalcsics, J., Manlove, D., and Pettersson, W. (2024). New algorithms for hierarchical optimization in kidney exchange programs. *Operations research*, 72(4):1654–1673.

Ding, Y., Ge, D., He, S., and Ryan, C. T. (2018). A nonasymptotic approach to analyzing kidney exchange graphs. *Operations research*, 66(4):918–935.

Riascos-Álvarez, L. C., Bodur, M., and Aleman, D. M. (2024). A branch-and-price algorithm enhanced by decision diagrams for the kidney exchange problem. *Manufacturing service operations management*, 26(2):485–499.