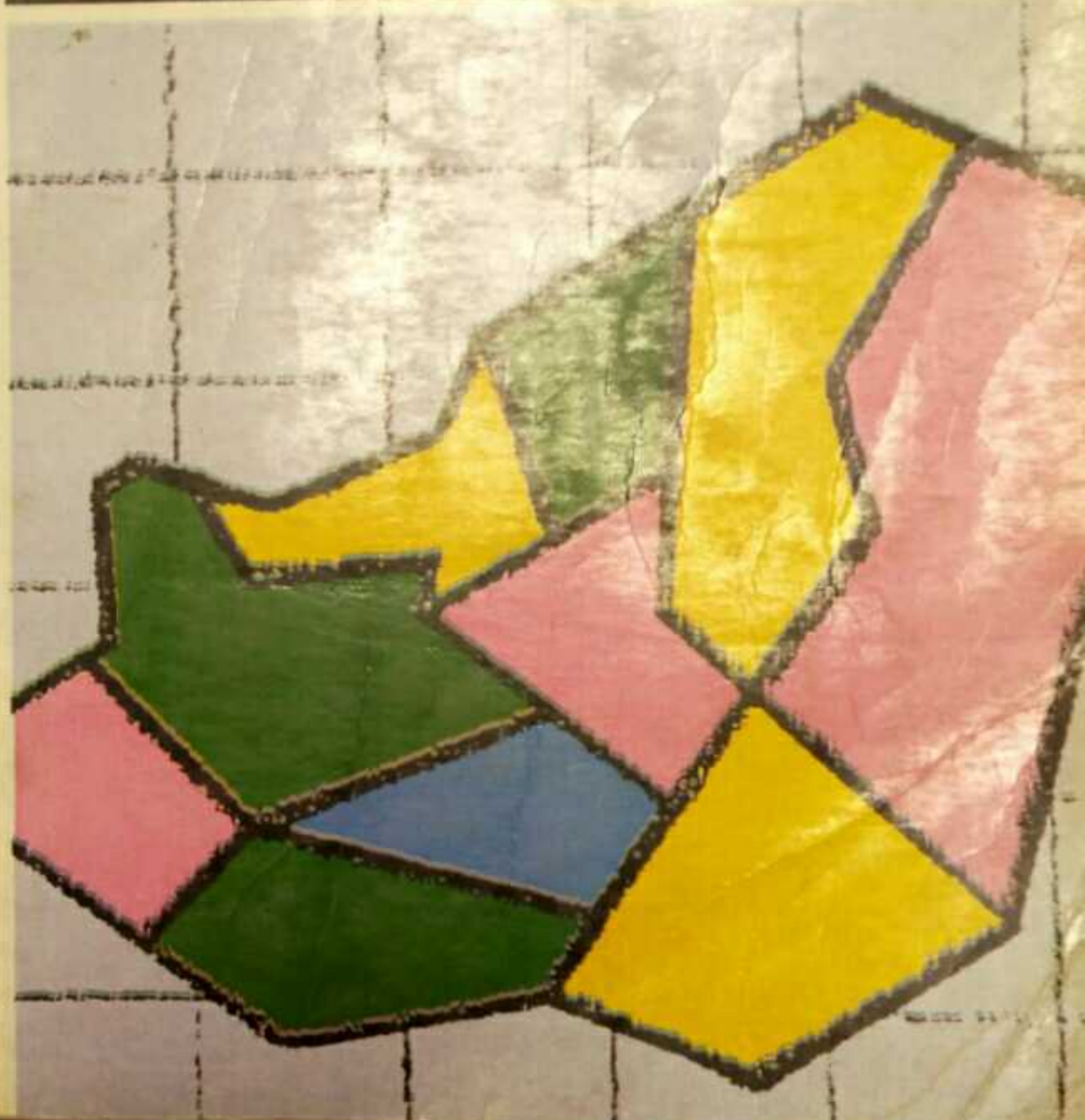
INTERNATIONAL COMPUTER SCIENCE SERIES

DISCRETE MATHEMATICS FOR COMPUTER SCIENTISTS

J. K. Truss



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