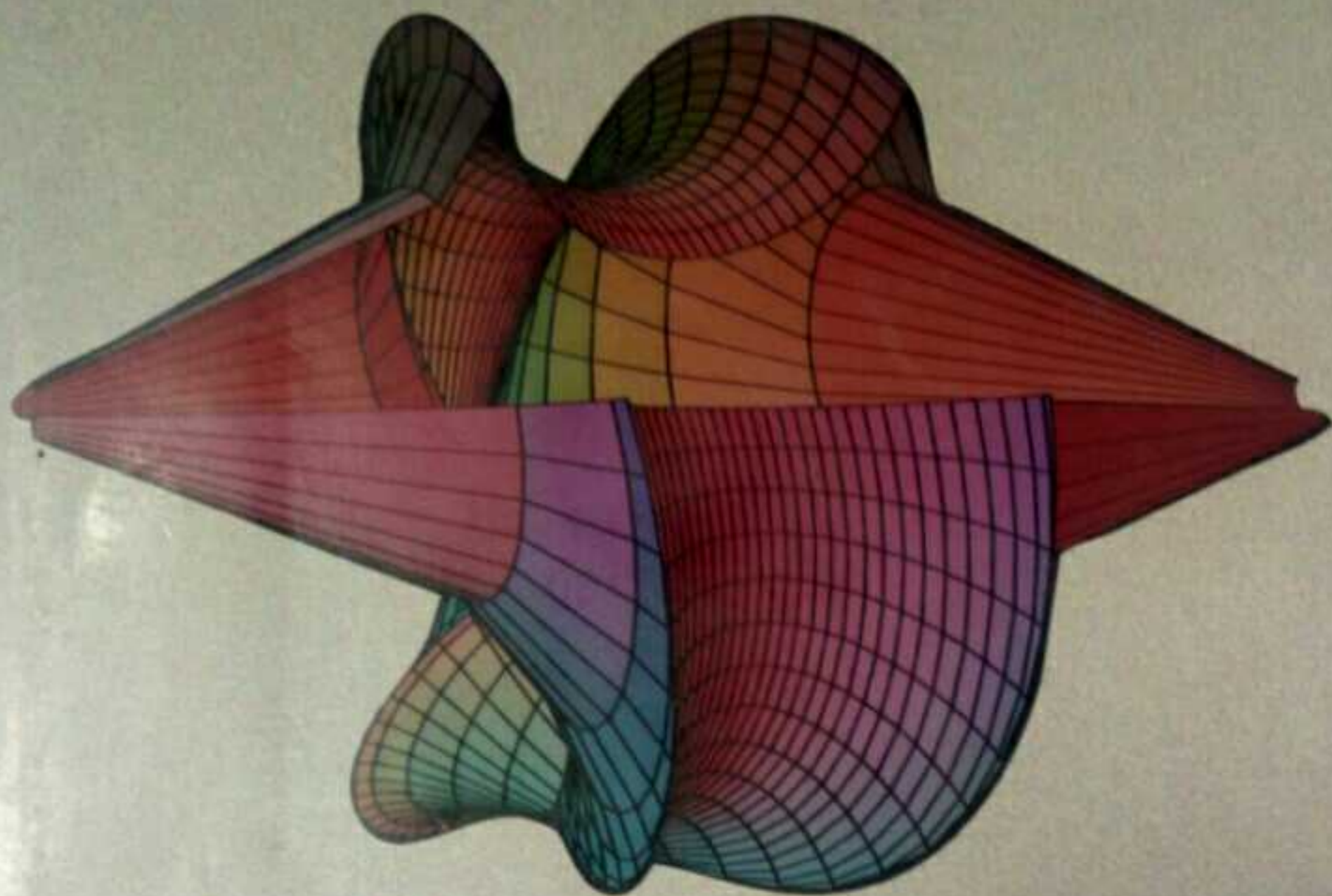


DIFFERENTIAL EQUATIONS

WITH **MAPLE V**[®]



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Preface

Maple V's diversity makes it particularly well suited to performing many calculations encountered when solving ordinary and partial differential equations. In some cases, Maple's built-in functions can immediately solve a differential equation by providing an explicit, implicit, or numerical solution; in other cases, Maple can be used to perform the calculations encountered when solving a differential equation. Since one goal of differential equations courses is to introduce the student to basic methods and algorithms and for the student to gain proficiency in them, nearly every topic covered in *Differential Equations with Maple V* includes typical examples solved by traditional methods and examples solved using Maple. Consequently, we feel that we have addressed one issue frequently encountered when implementing computer-assisted instruction. In addition, *Differential Equations with Maple V* uses Maple to establish well-known algorithms for solving elementary differential equations.

Taking advantage of the capabilities of Release 2 of Maple V, *Differential Equations with Maple V* introduces the fundamental concepts of differential equations as encountered in typical introductory differential equations courses and uses Maple V to solve typical problems of interest to students, instructors, and scientists. Other features to help make *Differential Equations with Maple V* as easy to use as possible include the following:

1. **Getting Started.** The Appendix provides a brief introduction to Maple V, including discussions about entering and evaluating commands, loading miscellaneous library functions and packages, and taking advantage of Maple's extensive help facilities.
2. **Release 2 Compatibility.** All examples illustrated in *Differential Equations with Maple V* were completed using Release 2 of Maple V. Although most computations can continue to be carried out with Release 1 of Maple V, we have taken advantage of the new features in Release 2 as much as possible.
3. **Detailed Table of Contents.** The table of contents includes all chapter, section, and subsection headings. Along with the comprehensive index, we hope that users will be able to locate information quickly and easily.
4. **Comprehensive Index.** In the index, mathematical examples are listed by topic, or name, as well as commands along with frequently used options: particular mathematical examples as