

Differential Equations

with Boundary Value Problems

EIGHTH EDITION



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Eighth Edition

DIFFERENTIAL EQUATIONS

with Boundary-Value Problems

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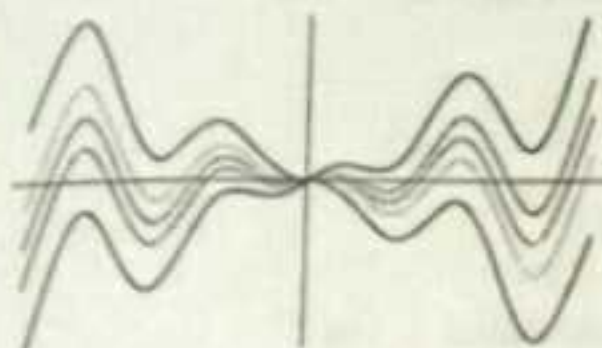
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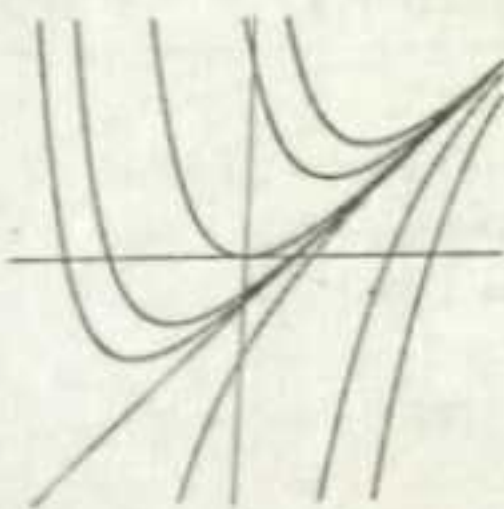
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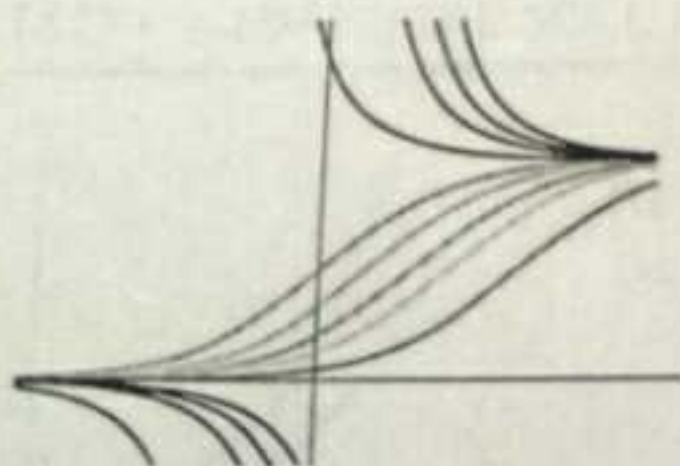
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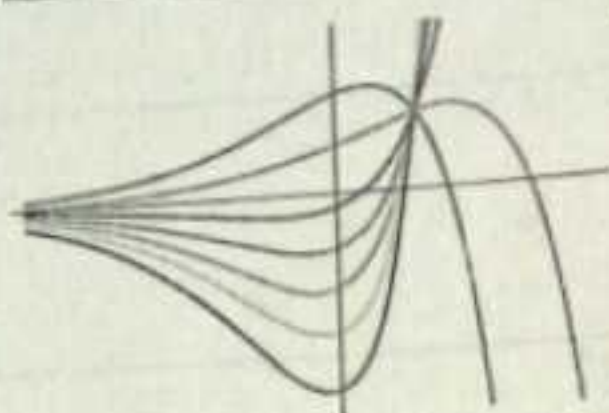
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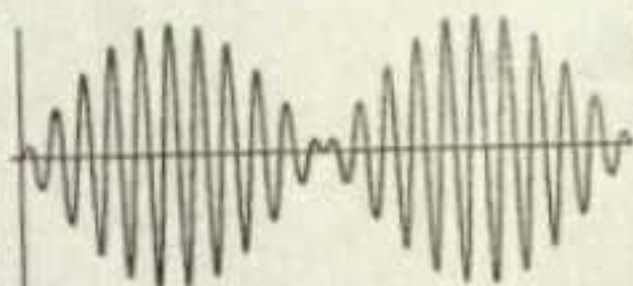
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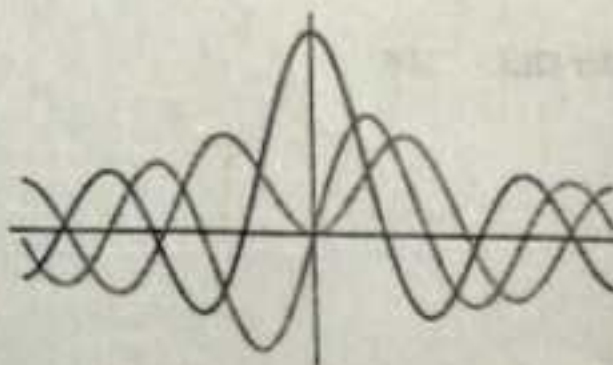
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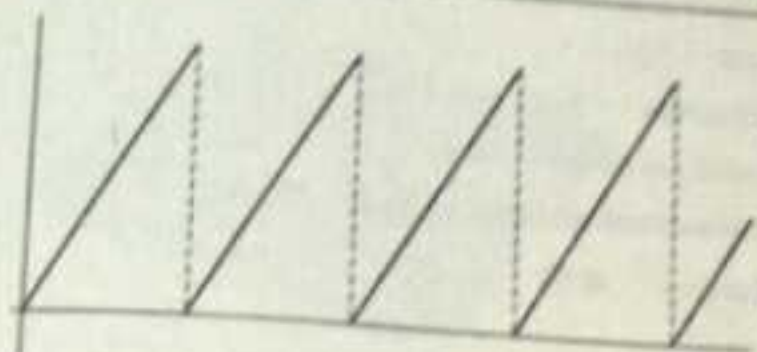
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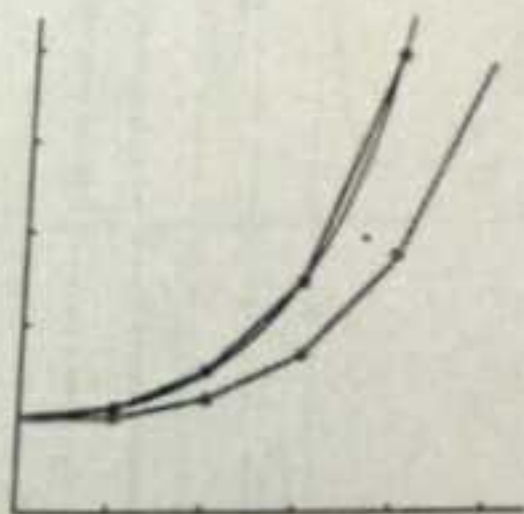
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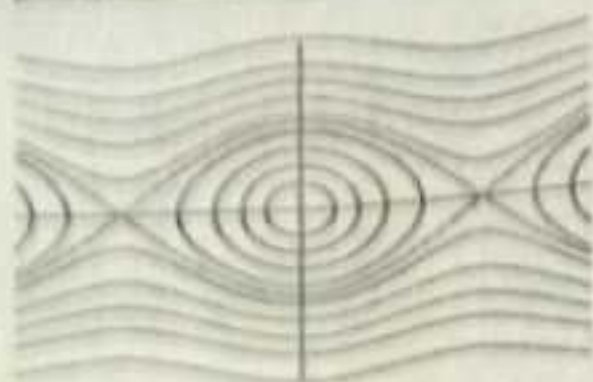
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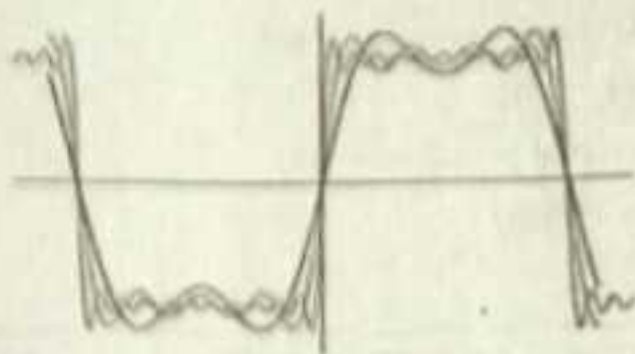
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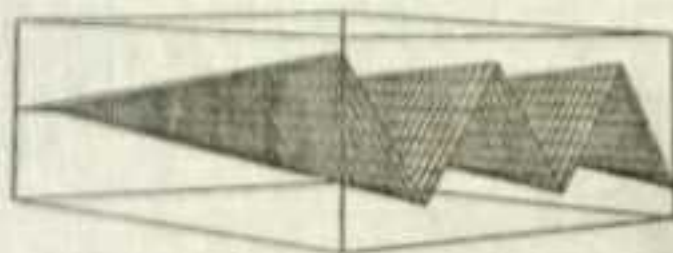
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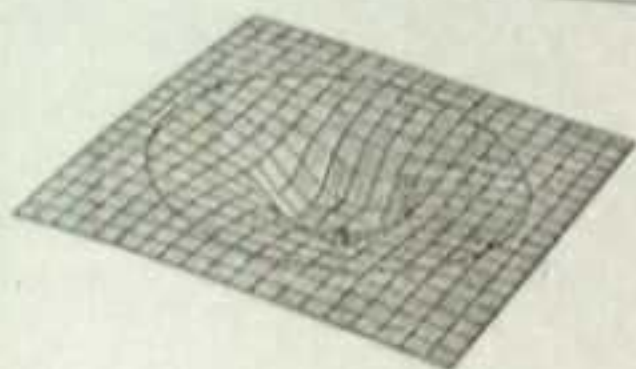
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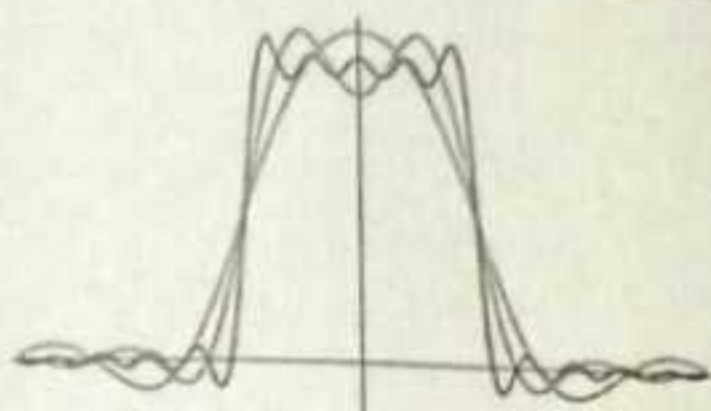
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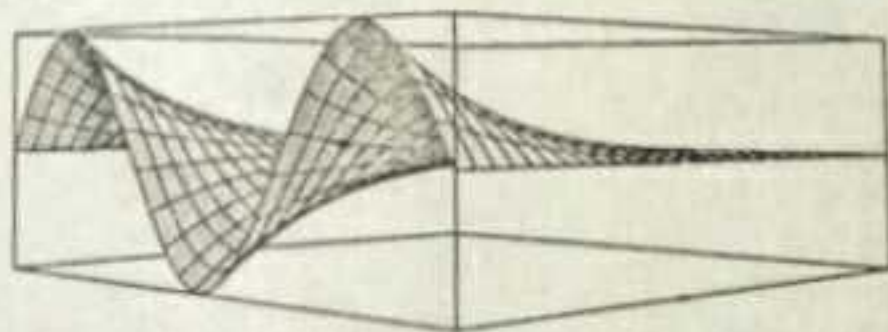
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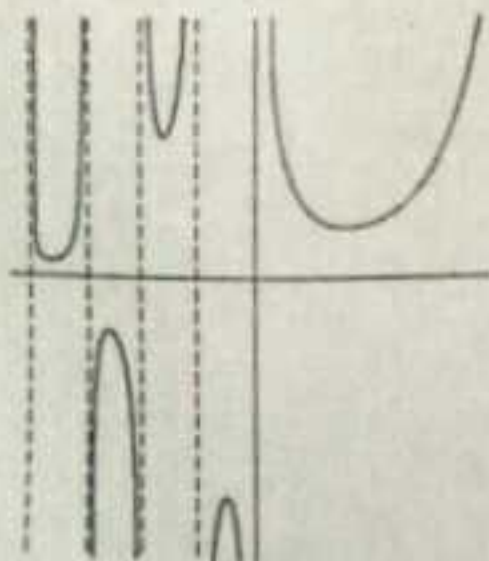
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TO THE STUDENT

Authors of books live with the hope that someone actually reads them. Contrary to what you might believe, almost everything in a typical college-level mathematics text is written for you, and not the instructor. True, the topics covered in the text are chosen to appeal to instructors because they make the decision on whether to use it in their classes, but everything written in it is aimed directly at you, the student. So we want to encourage you—no, actually we want to tell you—to read this textbook! But do not read this text like you would a novel; you should not read it fast and you should not skip anything. Think of it as a workbook. By this we mean that mathematics should always be read with pencil and paper at the ready because, most likely, you will have to work your way through the examples and the discussion. Before attempting any of the exercises, work *all* the examples in a section; the examples are constructed to illustrate what we consider the most important aspects of the section, and therefore, reflect the procedures necessary to work most of the problems in the exercise sets. We tell our students when reading an example, copy it down on a piece of paper, and do not look at the solution in the book. Try working it, then compare your results against the solution given, and, if necessary resolve, any differences. We have tried to include most of the important steps in each example, but if something is not clear you should always try—and here is where the pencil and paper come in again—to fill in the details or missing steps. This may not be easy, but that is part of the learning process. The accumulation of facts followed by the slow assimilation of understanding simply cannot be achieved without a struggle.

Specifically for you, a *Student Resource Manual (SRM)* is available as an optional supplement. In addition to containing solutions of selected problems from the exercises sets, the *SRM* contains hints for solving problems, extra examples, and a review of those areas of algebra and calculus that we feel are particularly important to the successful study of differential equations. Bear in mind you do not have to purchase the *SRM*; by following my pointers given at the beginning of most sections, you can review the appropriate mathematics from your old precalculus or calculus texts.

In conclusion, we wish you good luck and success. We hope you enjoy the text and the course you are about to embark on—as undergraduate math majors it was one of our favorites because we liked mathematics that connected with the physical world. If you have any comments, or if you find any errors as you read/work your way through the text, or if you come up with a good idea for improving either it or the *SRM*, please feel free to contact us through our editor at Cengage Learning.

molly.taylor@cengage.com

TO THE INSTRUCTOR

In case you are examining this book for the first time, *Differential Equations with Boundary-Value Problems, Eighth Edition* can be used for either a one-semester course, or a two-semester course that covers ordinary and partial differential equations. The shorter version of the text, *A First Course in Differential Equations with Modeling Applications, Tenth Edition*, is intended for either a one-semester or a two-quarter course in ordinary differential equations. This book ends with Chapter 9. For a one-semester course, we assume that the students have successfully completed at least two semesters of calculus. Since you are reading this, undoubtedly you have already examined the