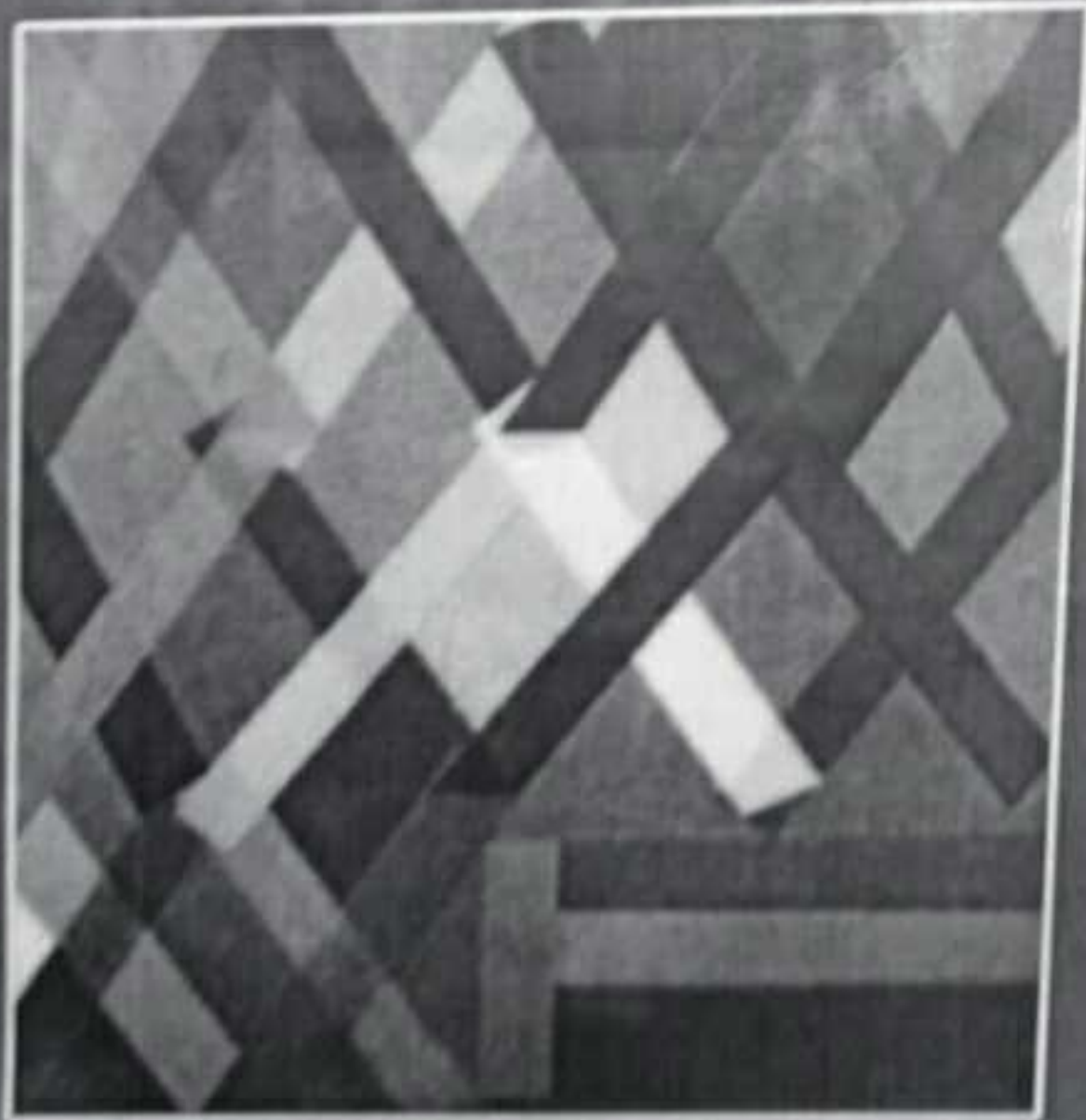


CALCULUS

Fifth Edition



HOWARD ANTON

CONTENTS

CHAPTER 1

INTRODUCTION TO DIFFERENTIAL EQUATIONS 1

- 1.1 Basic Definitions and Terminology 3
- [O] 1.2 Some Mathematical Models 12
- Chapter 1 Review 29 Chapter 1 Review Exercises 30

CHAPTER 2

FIRST-ORDER DIFFERENTIAL EQUATIONS 33

- 2.1 Preliminary Theory 34
- ✓ 2.2 Separable Variables 38
- ✓ 2.3 Homogeneous Equations 44
- ✓ 2.4 Exact Equations 52
- ✓ 2.5 Linear Equations 62
- [O] 2.6 Equations of Bernoulli, Riccati, and Clairaut 71
- [O] 2.7 Substitutions 74
- [O] 2.8 Picard's Method 80
- Chapter 2 Review 83 Chapter 2 Review Exercises 84

CHAPTER 3

APPLICATIONS OF FIRST-ORDER DIFFERENTIAL EQUATIONS 87

- 3.1 Orthogonal Trajectories 88
- ✓ 3.2 Applications of Linear Equations 94
- 3.3 Applications of Nonlinear Equations 108
- Chapter 3 Review 120 Chapter 3 Review Exercises 121
- Essay: Population Dynamics by Michael Glinick 123

CHAPTER 4

LINEAR DIFFERENTIAL EQUATIONS OF HIGHER-ORDER 129

- 4.1 Preliminary Theory 130
 - 4.1.1 Initial-Value and Boundary-Value Problems 130
 - 4.1.2 Linear Dependence and Linear Independence 134
 - 4.1.3 Solutions of Linear Equations 139

[O] = optional

4.2	Constructing a Second Solution from a Known Solution	154
4.3	Homogeneous Linear Equations with Constant Coefficients	159
4.4	Undetermined Coefficients—Superposition Approach	169
4.5	Differential Operators	181
4.6	Undetermined Coefficient—Annihilator Approach	187
4.7	Variation of Parameters	194
	Chapter 4 Review	203
	Chapter 4 Review Exercises	204
	Essay: Chaos by John H. Hubbard	207

CHAPTER 5

APPLICATIONS OF SECOND-ORDER DIFFERENTIAL EQUATIONS: VIBRATIONAL MODELS 211

5.1	Simple Harmonic Motion	212
5.2	Damped Motion	222
5.3	Forced Motion	232
5.4	Electric Circuits and Other Analogous Systems	242
	Chapter 5 Review	248
	Chapter 5 Review Exercises	249
	Essay: Tacoma Narrows Suspension Bridge Collapse by Gilbert N. Lewis	253

CHAPTER 6

DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS 257

6.1	Cauchy-Euler Equation	258
6.2	Review of Power Series; Power Series Solutions	267
6.3	Solutions About Ordinary Points	277
6.4	Solutions About Singular Points	286
6.4.1	Regular Singular Points; Method of Frobenius—Case I	286
6.4.2	Method of Frobenius—Cases II and III	296
6.5	Two Special Equations	306
6.5.1	Solution of Bessel's Equation	307
6.5.2	Solution of Legendre's Equation	314
	Chapter 6 Review	321
	Chapter 6 Review Exercises	322

CHAPTER 7

LAPLACE TRANSFORM 323

7.1	Laplace Transform	324
7.2	Inverse Transform	335

7.3	Translation Theorems and Derivatives of a Transform	343
7.4	Transforms of Derivatives, Integrals, and Periodic Functions	345
7.5	Applications	348
(O) 7.6	Dirac Delta Function	379
	Chapter 7 Review	384
	Chapter 7 Review Exercises	385

CHAPTER 8

SYSTEMS OF LINEAR DIFFERENTIAL EQUATIONS 389

8.1	Operator Method	390
8.2	Laplace Transform Method	398
8.3	Systems of Linear First-Order Equations	407
8.4	Introduction to Matrices	413
	8.4.1 Basic Definitions and Theory	413
	8.4.2 Gaussian and Gauss-Jordan Elimination Methods	423
	8.4.3 The Eigenvalue Problem	427
8.5	Matrices and Systems of Linear First-Order Equations	436
	8.5.1 Preliminary Theory	436
	8.5.2 A Fundamental Matrix	446
8.6	Homogeneous Linear Systems	453
	8.6.1 Distinct Real Eigenvalues	453
	8.6.2 Complex Eigenvalues	457
	8.6.3 Repeated Eigenvalues	461
(O) 8.7	Undetermined Coefficients	469
8.8	Variation of Parameters	473
(O) 8.9	Matrix Exponential	478
	Chapter 8 Review	481
	Chapter 8 Review Exercises	482

CHAPTER 9

NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS 487

9.1	Direction Fields	488
9.2	The Euler Methods	494
9.3	The Three-Term Taylor Method	503
9.4	The Runge-Kutta Method	507
9.5	Multistep Methods	513
9.6	Errors and Stability	517
9.7	Higher-Order Equations and Systems	523
9.8	Second-Order Boundary-Value Problems	528
	Chapter 9 Review	533
	Chapter 9 Review Exercises	534
	Essay: Nerve Impulse Models by C. J. Knickerbocker	537

CHAPTER 10

PLANE AUTONOMOUS SYSTEMS AND STABILITY 543

- 10.1 Autonomous Systems, Critical Points, and Periodic Solutions 544
- 10.2 Stability of Linear Systems 554
- 10.3 Linearization and Local Stability 565
- 10.4 Applications of Autonomous Systems 578
- Chapter 10 Review 593 Chapter 10 Review Exercises 594

CHAPTER 11

ORTHOGONAL FUNCTIONS AND FOURIER SERIES 597

- 11.1 Orthogonal Functions 598
- 11.2 Fourier Series 604
- 11.3 Fourier Cosine and Sine Series 610
- 11.4 Sturm-Liouville Problem 622
- 11.5 Bessel and Legendre Series 631
- Chapter 11 Review 639 Chapter 11 Review Exercises 640

CHAPTER 12

BOUNDARY-VALUE PROBLEMS IN RECTANGULAR COORDINATES 643

- 12.1 Separable Partial Differential Equations 644
- 12.2 Classical Equations and Boundary-Value Problems 649
- 12.3 Heat Equation 657
- 12.4 Wave Equation 660
- 12.5 Laplace's Equation 666
- 12.6 Nonhomogeneous Equations and Boundary Conditions 670
- 12.7 Use of Generalized Fourier Series 674
- [0] 12.8 Boundary-Value Problems Involving Fourier Series in Two Variables 680
- Chapter 12 Review 683 Chapter 12 Review Exercises 683
- Essay: Where Is Middle C? by Ruth G. Favro 687

CHAPTER 13

BOUNDARY-VALUE PROBLEMS IN OTHER
COORDINATE SYSTEMS 693

- 13.1 Problems Involving Laplace's Equation in
Polar Coordinates 694
- 13.2 Problems in Polar and Cylindrical Coordinates 699
- 13.3 Problems in Spherical Coordinates 707
- Chapter 13 Review 710 Chapter 13 Review Exercises 710

CHAPTER 14

INTEGRAL TRANSFORM METHOD 715

- 14.1 Error Function 716
- 14.2 Applications of the Laplace Transform 718
- 14.3 Fourier Integral 728
- 14.4 Fourier Transform 735
- Chapter 14 Review 743 Chapter 14 Review Exercises 744

CHAPTER 15

NUMERICAL METHODS FOR PARTIAL
DIFFERENTIAL EQUATIONS 747

- 15.1 Elliptic Equations 748
- 15.2 Parabolic Equations 757
- 15.3 Hyperbolic Equations 767
- Chapter 15 Review 773 Chapter 15 Review Exercises 774

APPENDICES APP-1

- I Gamma Function APP-2
- II Laplace Transforms APP-4
- III Review of Determinants APP-7
- IV Complex Numbers APP-12

ANSWERS TO ODD-NUMBERED PROBLEMS A-1

INDEX I-1