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CALCULUS WITH ANALYTIC GEOMETRY

$$\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$$

$$\int_a^b f = \lim S(P, f)$$

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