

HORST A. EISELT
HELMUT VON FRAJER

7

OPERATIONS RESEARCH HANDBOOK

STANDARD ALGORITHMS AND METHODS

Horst A. Eiselt · Helmut von Frajer

Operations Research Handbook

Standard Algorithms and Methods with Examples

145

SEMINAR LIBRARY
Department of Computer Science
UNIVERSITY OF KARACHI
22-6-94

M

Contents

	Definitions and Symbols	15
0.	Summary of Matrix Algebra and Allied Topics	17
0.1	Definitions	18
0.2	Elementary Operations	
1.	Linear Programming	
1.1	General Methods	22
1.1.1	The Primal Simplex-Algorithm	25
1.1.2	The Two-Phase Method	
1.1.3	The Primal Simplex-Algorithm without Explicit Identity Matrix	30
1.1.4	The Dual Simplex-Algorithm	34
1.1.5	Sensitivity Analysis and Parametric Programming (S.A. and P.P.)	38
1.1.5.1	S.A. and P.P. with Expected Alterations	39
1.1.5.2	S.A. and P.P. with Unexpected Alterations	
1.1.5.2.1	Subsequent Alterations of the Restriction Vector	43
1.1.5.2.2	Subsequent Alterations of Coefficients of the Objective Function	45
1.2	Shortened Methods	48
1.2.1 ✓	The Transportation Problem	49
1.2.1.1	The Northwest-Corner Rule	51
1.2.1.2	The Row Minimum Method	54
1.2.1.3	The Column Minimum Method	57
1.2.1.4	The Matrix Minimum Method	60
1.2.1.5	The Double Preference Method	66
1.2.1.6	VOGEL's Approximation Method (VAM)	71
1.2.1.7	The Frequency Method	72
1.2.1.8	The Stepping-Stone Method	75
1.2.2	The Hungarian Method (Kuhn)	

1.2.3	The Decomposition Principle (Dantzig Wolfe).....	81
1.2.4	FLOOD's Technique	89
1.3	Theorems and Rules	
1.3.1	The Dual Problem	91
1.3.2	Theorems of Duality	93
1.3.3	The Lexicographic Selection Rule.....	94
2.	Integer Programming	
2.1	Cutting Plane Methods	
2.1.1	The GOMORY-I-All Integer Method	96
2.1.2	The GOMORY-II-All Integer Method	100
2.1.3	The GOMORY-III-Mixed Integer Method	103
2.1.4	The GOMORY-III-Mixed Integer Method with Intensified Cuts	106
2.1.5	The Primal Cutting Plane Method (Young; Glover; Ben-Israel; Charnes).....	108
2.2	Branch and Bound Methods	
2.2.1	The Method of LAND and DOIG	111
2.2.2	The Method of DAKIN	118
2.2.3	The Method of DRIEBEEK	122
2.2.4	The Additive Algorithm (Balas).....	129
2.3	Primal-Dual Methods	
2.3.1	A Partitioning Procedure for Mixed Integer Problems (Benders).....	134
3.	Theory of Graphs	
3.0.1	Definitions	143
3.0.2	The Determination of Rank in Graphs.....	146
3.0.3	The Number of Paths in a Graph.....	148
3.0.4	The Determination of the Strongly Connected Components of a Graph.....	149
3.1	Shortest Paths in Graphs	
3.1.1	The Algorithm of DIJKSTRA	151
3.1.2	The Algorithm of DANTZIG	154

3.1.3	The FORD Algorithm I (shortest path(s))	159
3.1.4	The FORD Algorithm II (longest path(s))	160
3.1.5	The Tripel Algorithm	162
3.1.6	The HASSE Algorithm	166
3.1.7	The Cascade Algorithm	168
3.1.8	The Algorithm of LITTLE	169
3.1.9	The Method of EASTMAN	174
3.2	Flows in Networks	
3.2.1	The Algorithm of FORD and FULKERSON	178
3.2.2	The Algorithm of BUSACKER and GOWEN	183
3.2.3	The Method of KLEIN	187
3.2.4	The Out-of-Kilter Algorithm (Ford; Fulkerson)	191
3.3	Shortest Spanning Subtrees of a Graph	
3.3.1	The Method of KRUSKAL	200
3.3.2	The Method of SOLLIN	203
3.3.3	The Method of WOOLSEY	205
3.3.4	The Method of BERGE	207
3.4	Gozinto Graphs	210
3.4.1	The Method of VAZSONYI	210
3.4.2	The Method of TISCHER	212
3.4.3	The Method of FLOYD	213
3.4.4	The Gozinto List Method	214
4.	Planning Networks	
4.0.1	The Critical Path Method (CPM)	217
4.0.2	The CPM Project Acceleration	220
4.0.3	The Program Evaluation and Review Technique (PERT)	224
4.0.4	The Metra Potential Method (MPM)	227
4.0.5	The Graphical Evaluation and Review Technique (GERT)	230

5.	Game Theory	
5.1	Non Matrix Games	
5.1.1	The Normal Form	236
5.1.2	NASH's Solution of the Bargaining Problem	241
5.1.3	The Extensive Form	242
5.2	Matrix Games	
5.2.1	A Method for Determining Pure Strategy Pairs for Two-Person Zero-Sum Games	251
5.2.2	A Method for Solving Two-Person Zero-Sum Games with the Simplex-Algorithm	253
5.2.3	An Approximization Method for Two-Person Zero-Sum Games ("learning method"; Gale; Brown).....	256
5.2.4	The LEMKE-HOWSON Algorithm for the Solution of Bimatrix Games	260
5.3	Decisions under Uncertainty (games against nature)	264
5.3.1	The Solution of WALD	265
5.3.2	The Solution of HURWICZ	266
5.3.3	The Solution of SAVAGE and NIEHANS	266
5.3.4	The Solution of BAYES	267
5.3.5	The Solution of LAPLACE	268
5.3.6	The Solution of HODGES and LEHMANN	268
6.	Dynamic Programming	
6.0.1	The n-Period Model	270
6.0.2	The Infinite-Period Model (policy iteration routine)	276
7.	Queueing Models	280
7.0.1	The 1-Channel, 1-Stage Model	282
7.0.2	The 1-Channel, r-Phase Model	284
7.0.3	The k-Channel, 1-Stage Model	285
8.	Nonlinear Programming	
8.1	Theorems and Special Methods	
8.1.1	The	288

11.0.6	An Inventory Model with Respect to Transportation Capacity	340
12.	Sequencing Models	
12.0.1	JOHNSON's Algorithm for Two Machines	343
12.0.2	JOHNSON's Algorithm for Three Machines (special case)	345
12.0.3	A Heuristic Solution for a Sequencing Problem	347
13.	Plant Location Models	
13.1	Exact Methods	
13.1.1	The Optimal Plant Location in a Transportation Network I	350
13.1.2	The Optimal Plant Location in a Transportation Network II	351
13.1.3	The Optimal Plant Location on a Straight Line	353
13.1.4	The Optimal Plant Location with Respect to Rectangular Transportation Movements	354
13.2	Heuristic Methods	
13.2.1	The Center of Gravity-Method	356
13.2.2	A Solution by Vector Summation	358
13.2.3	An Iterative Method	363
	Appendix	367
	Table 1 : $q^k = (1 + i)^k$	367
	Table 2 : $q^{-k} = (1 + i)^{-k}$	367
	Table 3 : e^{-k}	368
	Table 4 : Random numbers with an equal distribution.....	370
	Table 5 : Area under the standardized normal distribution function.....	372
	Bibliography	373

Definitions and Symbols

$A : A_{[m \times n]} : [m \times n]$ - dimensional matrix A

I : identity matrix

θ : null matrix

A^T : transpose of matrix A

A^{-1} : inverse of matrix A

\mathbb{R}^n : n -dimensional real euclidian space

If not otherwise defined, a problem P is given as follows:

$$P : \begin{cases} \min \\ \max \end{cases} \pi = f(x); A \cdot x \begin{cases} \geq \\ \leq \end{cases} b; x \geq 0,$$

where $A: A_{[m \times n]}$; $x: x_{[n \times 1]}$; $b: b_{[m \times 1]}$.

$x \in [a; b] = a \leq x \leq b$, where $b \geq a$: closed interval

$x \in (a; b] = a < x \leq b$, where $b > a$ }
 $x \in [a; b) = a \leq x < b$, where $b > a$ } : half-closed intervals

$x \in (a; b) = a < x < b$, where $b > a$: open interval

$a : = a + b$: valuation

\exists : there is ...

\forall : for all ...

\Leftrightarrow : iff : equivalence relation

\Rightarrow : implication

$[a]$: largest integer smaller than a

$\langle a \rangle$: smallest integer larger than a

\emptyset : empty set

$|a|$: absolute value of a

$|M| = |\{m_i\}|$: the number of elements in M

$M_1 \cup M_2$: union of the sets M_1 and M_2

$M_1 \cap M_2$: intersection of the sets M_1 and M_2