

# Discrete Mathematics with Applications

---

H. F. Mattson, Jr.

LIQUAT ALI  
PHOTO STATE  
DEPT. OF MATH  
UNIVERSITY OF KAN



John Wiley & Sons, Inc.

New York • Chichester • Brisbane • Toronto • Singapore

# Contents

$$(A \cup B) \cap C$$



## CHAPTER 1. Sets

1. Beginnings	1
First Notation for Sets	2
2. Membership	3
3. Inclusion	4
✓ 4. Equality of Sets	5
✓ 5. Terminology	6
6. Membership vs. Inclusion	7
7. The Empty Set	8
8. Singleton, Proper Subset, Universal Set	9
9. Shoeboxes	9
✓ 10. Union and Intersection	10
11. The Distributive Laws	13
Epsilon-Arguments	14
✓ 12. The Complement	16
✓ 13. Venn Diagrams	17
✓ 14. The Cartesian Product	19
✓ 15. The Power Set	21
16. Postscript	22
✓ 17. More on Ordered Pairs	23
18. Summary	24
19. More About Sets	25
Ordered $n$ -tuples	26
Strings	26
20. Further Reading	27
21. Problems	27
22. Answers to Practice Problems for Chapter 1	31

## CHAPTER 2. Logic


I. PROPOSITIONAL LOGIC	33
1. Propositions	33
✓ 2. Logical Connectives	34
✓ 3. Truth-tables	36
4. Terminology	36
5. The Common Meaning of the "Logical Words"	37
6. Propositional Formulas; Truth-tables	38
7. Logical Implication and Equivalence	40
✓ 8. Tautologies and Contradictions	44
9. A Shortcut	45



10. More Tautologies; Rules of Inference	47
The Substitution Theorem	50
11. Some Wrong Inferences	51
12. Theorems and Proofs	52
Types of Proof	53
✓▶ 13. Conjunctive and Disjunctive Normal Forms	55
Karnaugh Maps	59
Basic Facts on Propositional Formulas and Sets	61
14. Logical Puzzles	61
15. Summary of Propositional Logic	62
16. Problems on Propositional Logic	63
17. Answers to Practice Problems	68
II. PREDICATE LOGIC	70
✓ 18. Predicates	70
✓ 19. Set-Builder: The Second Notation for Sets	72
✓ 20. Logical Implication Between Predicates	76
✓ 21. Quantification	78
✓ 22. Negation of Quantified Predicates	80
✓ 23. Quantifying Predicates in More Than One Variable	81
Negation	86
24. Epsilon-Arguments with Predicates	87
25. Quantification over the Empty Set	90
26. Analysis of Syllogisms: Truth-sets and Venn Diagrams	90
27. The Unwritten $\forall$	93
28. Proofs	94
▶ 29. A Correspondence Between Sets and Propositional Formulas	95
30. Postscript	98
Paradoxes	99
31. Further Reading	100
32. Problems on Predicate Logic	101
33. Answers to Practice Problems	105

### ✱ CHAPTER 3. *Mathematical Induction*

1. Introduction	109
2. Sequences of Propositions	110
3. Mathematical Induction	111
4. Standard Induction Problems	113
5. The Second Form of Induction	114
6. An Extension	116
7. Wrong Proofs by Induction	116
Proofs in General	117
8. The Domino Analogy	118
9. How to Solve Problems by Induction	119
10. In Logical Terms	120

47	11. Subsets and Strings	120
50	12. Examples	122
51	Maximizing a Function	124
52	✓ 13. Recursion	126
53	14. Applications	128
55	▶ 15. Fermat's Method of Infinite Descent	128
59	16. Postscript	130
61	Regions in the Plane	132
61	17. Further Reading	132
62	18. Problems	132
63	19. Answers to Practice Problems	136
68		
70	<b>CHAPTER 4. Equivalence Relations and Partitions</b>	
70		
72	✓ 1. Relations, Graphs, and Matrices	137
76	✓ 2. Equivalence Relations	139
78	Examples	139
80	The Complete Relation	144
81	3. The Main Theorem	144
86	4. The Infix Notation	149
87	5. Partitions	149
90	The Vertical Bar Notation	150
90	6. The Connection between Partitions and Equivalence Relations	151
93	7. Refinements	152
94	8. A Relation on Partitions	154
95	9. Postscript	154
98	10. Further Reading	155
99	11. Problems	155
100	12. Answers to Practice Problems	159
101		
105	 <b>CHAPTER 5. Functions</b>	
109	✓ 1. Functions	161
110	✓ 2. Equality, Partial Functions	162
111	3. Some Terminology, $f$	163
113	4. What Is a "Rule"?	165
114	5. Alternate Definition of Function	166
116	✓ 6. Functions as Relations	167
116	7. Injectivity, Surjectivity, and Bijectivity	168
117	✓ 8. The Pigeonhole Principle	172
118	✓ 9. The Inverse "Function"	176
119	✓ 10. Composition of Functions	179
120	11. Identity Functions	181
	12. The Inverse of a Bijective Function	181
	✓ 13. The Restriction of a Function	184
	✓ 14. A Property of Injective Functions	184

15. A Property of Surjective Functions	185
16. Permutations	188
17. The Disjoint-Cycle Notation for Permutations	191
18. Composition of Permutations	191
19. Characteristic Functions	192
20. Boolean Functions	194
21. Propositions and Functions	195
22. The Cartesian Product Again	196
23. The Axiom of Choice	197
24. Factoring Functions	201
25. Numbers as Sets	202
26. A Confusion	202
27. Orders of Growth	206
28. Postscript	207
29. Further Reading	207
30. Problems	212
31. Answers to Practice Problems	
<b>CHAPTER 6. Divisibility in the Integers</b>	
1. Basic Ideas	216
Divisibility	216
Floor and Ceiling Functions	217
2. Division	219
3. The Division Algorithm on a Computer	222
4. Integers in Base $b$	223
5. Proof of Correctness	225
6. Analysis of the Algorithm in Base $b$	227
7. Greatest Common Divisor	227
8. The Euclidean Algorithm	231
The Formal Version of the Euclidean Algorithm	233
9. Finding the Coefficients	233
Algorithm EE	234
Proof that EE is Correct	235
10. Application: Pouring Problems	237
11. Primes	239
Unique Factorization	241
GCD and LCM	242
The Square-Root Bound	243
12. The Sieve of Eratosthenes	243
13. Mersenne and Fermat Primes	244
14. Fermat's Method of Descent: An Example	245
15. Summary	246
16. Further Reading	247
17. Problems	247
18. Answers to Practice Problems	251



# CHAPTER 7. Congruences

1. Introduction	200
Complete System of Residues	200
2. An Application to Computers	204
Two's Complement	204
One's Complement	208
3. Basic Properties of Congruence Mod $m$	210
4. Some Applications	243
5. Solving Linear Congruences	267
6. An Algorithm To Solve $ax \equiv b \pmod{m}$	269
7. The Chinese Remainder Theorem	273
8. An Algorithm for the CRT	274
9. Applications	276
10. A Generalization of the CRT	277
11. Fermat's Theorem	281
12. The Euler Phi Function	282
Euler's Theorem	283
13. Application: A Cryptographic Scheme	286
14. The Euler Triangle	288
15. A Short Descent	289
16. Further Reading	289
17. Problems	289
18. Answers to Practice Problems	290

# CHAPTER 8. The Binomial Theorem

Introduction	298
1. Polynomials	298
Degree	300
Multiplication	302
2. Binomial Coefficients	304
3. The Basic Recursion	306
4. Pascal's Triangle	307
5. Symmetry	308
6. The Counting Property	309
7. The Binomial Theorem	311
8. Identities Involving Binomial Coefficients	314
The Knight's Move Identity	314
The Vandermonde Convolution	315
Some Suggestions on References	317
9. Applications to Computer Science	318
Zipper Merge	319
Analysis of Zipper Merge	320
The $k$ -Subsets of an $n$ -set in Lexicographical Order	322

Construction of the Algorithm 1st	324
Analysis of the Algorithm 1st	324
Construction of the Algorithm 2nd	325
Analysis of the Algorithm 2nd	326
Answers to Practice Problems	330

## CHAPTER 9. Counting

1. Introduction	332
Bijections	333
2. Basic Results	334
Sums	334
Products	335
The Number of Subsets of an $n$ -Set	335
The Number of $k$ -Subsets of an $n$ -Set	336
Example: Committees with Cochairs	339
3. Counting Functions	339
Examples	341
Zero to the Power Zero	342
4. The Method of Inclusion-Exclusion	343
5. Bijections, Injections, and Surjections	346
The Number of Permutations of an $n$ -Set	347
The Birthday Problem	349
6. Partitions	351
7. Sets with Repetition	352
First Proof	353
Second Proof	354
Third Proof	356
Summary	357
8. Flag Counts	357
Examples	359
Kaplansky's Lemmas	360
The Problème des Ménages	364
9. On Terminology	364
10. Further Reading	365
11. Problems	371
12. Answers to Practice Problems	371

## ★ CHAPTER 10. Probability

1. Introduction	373
2. Sample Spaces and Events	373
Probability	374
Odds	376
3. Random Variables	376
4. Expected Value	378

324	5. Variance	379
324	6. Independence	379
325	7. Mean and Variance of Independent Random Variables	380
326	8. Real Life	382
330	9. Repeated Trials	385
	10. The Binomial Distribution	388
	11. The Poisson Distribution	391
	12. Mean and Variance	393
332	X 13. Chebyshev's Inequality	394
333	X 14. The Law of Large Numbers	395
334	X 15. Little's Formula	396
334	16. More about Queues	400
334	17. Further Reading	403
335	18. Problems	403
335	19. Answers to Practice Problems	408

## CHAPTER 11. Recurrence Relations

339	1. Introduction	411
341	A Problem on Strings	412
342	2. The Simplest Recurrences	414
345	3. Repeated Roots	415
346	4. Proof for the Simplest Case	418
347	Solution of the Problem on Strings	426
349	5. Proof When Roots Are Repeated	428
351	6. The Nonhomogeneous Case	437
352	7. Summary, and a Look Outward	442
353	8. Chebyshev and Legendre Polynomials	443
354	9. Further Reading	444
356	10. Problems	444
357	11. Answers to Practice Problems	446

## CHAPTER 12. Matrices and Order Relations

359	1. Introduction	449
360	2. Comparison with Functions	450
364	3. The Converse of a Relation	451
364	4. The Composition of Two Relations	451
365	5. Graphs of Relations	453
371	6. Relation-Matrices $M(R)$	454
	7. General Matrices	455
373	8. Matrix Multiplication	458
373	9. Compositions, Products, and Transposes	463
374	10. Order Relations	467
376	11. Comparison with Equivalence Relations	468
376	Strongly Connected Components	469
378		



12. Examples of Partial Order Relations	471
13. The Hasse Diagram	473
14. The "Idea" of Order Relations	474
15. Transfinite Induction	477
Example	479
16. More on Composition	481
17. Transitivity	484
18. Closures	487
19. Algorithms for Finding Transitive Closures	488
Warshall's Algorithm	490
20. Complexity Analysis	493
21. The Equivalence Closure	494
22. Appendix. Proof that Hasse Diagrams Exist	495
23. Further Reading	496
24. Problems	497
25. Answers to Practice Problems	501

## ✱ CHAPTER 13. Trees 504

1. Binary Search	506
2. Trees	511
3. Forests	512
4. Binary Trees	515
5. Forests and Binary Trees	518
6. Traversal of Binary Trees	520
A Sorting Algorithm	523
Analysis of Treesort	526
The Complete Binary Tree	530
A Lower Bound	531
10. Extended Binary Trees	535
11. Counting Trees	539
12. The Equivalence Closure	540
13. Further Reading	540
14. Problems	544
15. Answers to Practice Problems	

## CHAPTER 14. Graphs

1. Introduction	546
2. Basic Concepts	547
The Edge-Degree Lemma	552
3. Euler Tours	554
4. Free Trees	560
5. Subgraphs	566
6. Spanning Trees	567
7. Minimum Spanning Trees	570

471	8. A Second Proof of Correctness	576
473	9. Interval Graphs	576
474	10. Directed Graphs	577
477	11. The Shortest-Path Problem	578
479	12. Complexity and Finding the Paths	581
481	13. Finite-State Machines	584
484	Formal Languages	585
487	Operations on Languages	586
488	The Pumping Lemma	587
490	Examples	588
493	How to Use the Pumping Lemma	592
494	14. Application to Computerized Tomography	593
495	The Surface Digraph	596
496	An $O(n^2)$ Algorithm	598
497	Euler's Theorem for Directed Graphs	600
501	Postscript. Strongly Connected Digraphs	601
	15. Mathematical Structures	601
	16. Further Reading	602
	17. Problems	603
	18. Answers to Practice Problems	606

## *Answers to Selected Problems*

609

## *Index*

627