

# Elementary Linear Algebra

Applications Version

NINTH EDITION

$$\frac{a}{1-k} = a + ak + ak^2 + ak^3 + ak^4 + ak^5 + ak^6 + ak^7 + ak^8 + ak^9 + ak^{10} + ak^{11} + ak^{12} + ak^{13} + ak^{14} + ak^{15} + \dots$$



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Applications of Linear Algebra

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