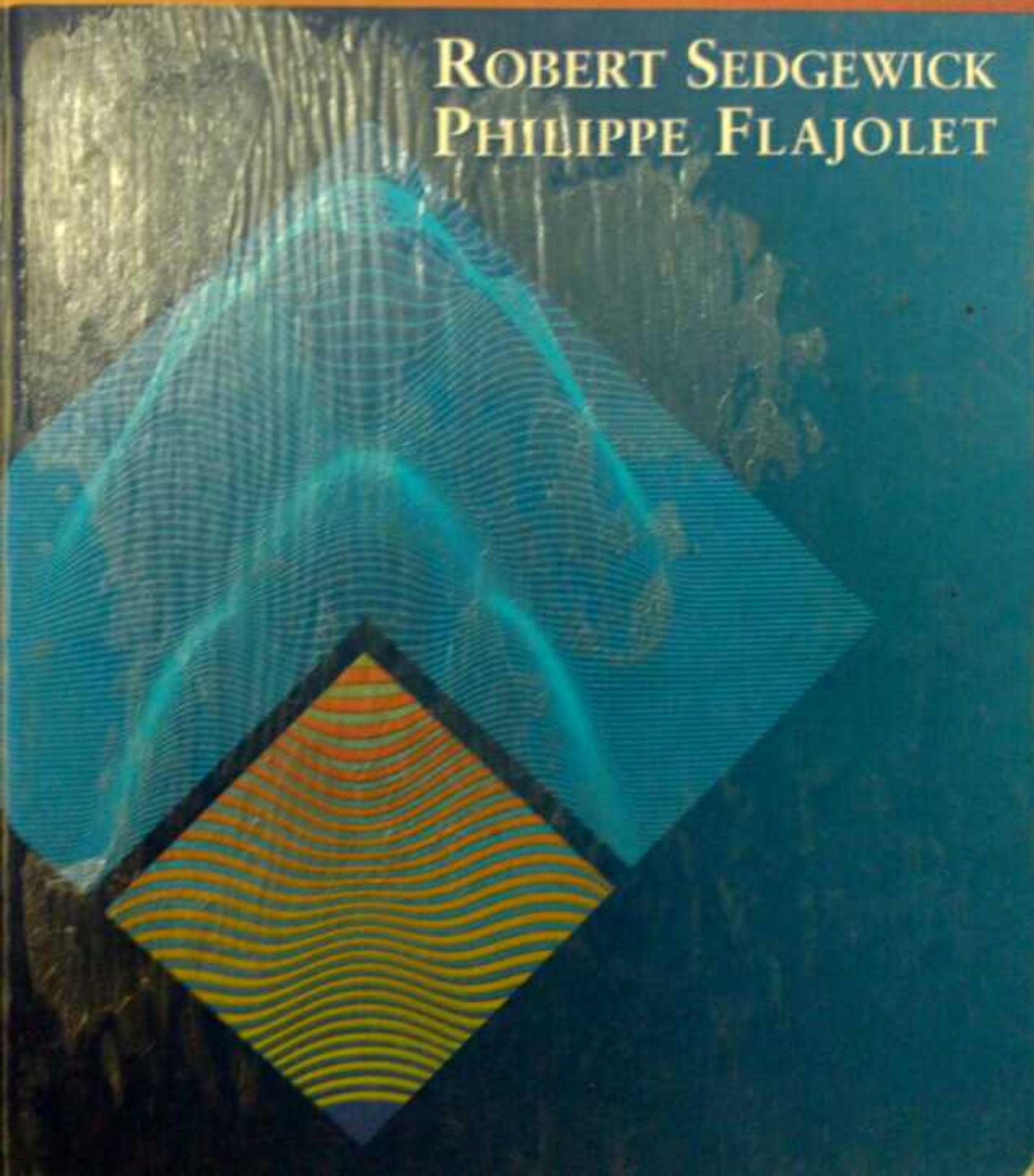


AN INTRODUCTION TO THE

# ANALYSIS OF ALGORITHMS

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Sydney • Singapore • Tokyo • Madrid • San Juan • Milan • Paris

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