NESTED LOOPS

(Supplementary Notes)

Question 4 -- From Lab 8 (Least Common Multiple)

Write a function **smallest_multiple** that takes as input a *positive integer* \mathbf{n} and returns the smallest number \mathbf{x} , such that \mathbf{x} % \mathbf{j} == 0, for all $\mathbf{1}$ <= \mathbf{j} <= \mathbf{n} .

For example, for input n=5, the function should return 60

as it is the smallest number that can be divided by each of the numbers from 1 to 5, without any remainder.

Question 4 -- From Lab 8 (Least Common Multiple)

Write a function **smallest_multiple** that takes as input a *positive integer* \mathbf{n} and returns the smallest number \mathbf{x} , such that \mathbf{x} % \mathbf{j} == 0, for all $\mathbf{1}$ <= \mathbf{j} <= \mathbf{n} .

For example,

for input n=5, the function should return 60

as it is the smallest number that can be divided by each of the numbers from 1 to 5

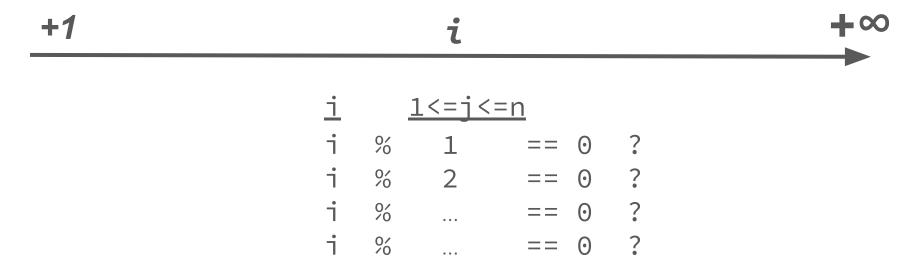
without any remainder.

X	<u>1</u>	<=j<=	<u>-n</u>		
60	0/	1	==	0	/
60	0/	2	==	0	
60	0/	3	==	0	
60	0/	4	==	0	/
60	0/	5	==	0	/

How do you find this magical number x?

You don't know how big or how small this magical number is...

In search of this magical number, we'll just have to traverse the entire number line, checking our condition(s) on each number



i	OUTER LOOP			
	1			
	2			
	3			
	4			
	x			
	+ infinity			

<u>i</u>	2	1<=j<	<= <u>n</u>		
i	0/	1	==	0	?
i	0/	2	==	0	?
i	0/	3	==	0	?
i	0/	• • •	==	0	?
i	0/	n	==	0	?

į	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
\	+ infinity				

<u>i</u>	-	1<= j <	<= <u>n</u>		
i	0/	1	==	0	?
i	0/	2	==	0	?
i	0/	3	==	0	?
i	0/		==	0	?
i	0/	n	==	0	?

i	OUTER LOOP	INNER LOOP	j_		
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
             j <= n:
           if i % j == 0:
               j = j + 1
           else:
                break
       if j == n+1:
           result = i
       i = i + 1
    return result
```

į	OUTER LOOP	INNER LOOP	j_		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
↓	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
                     initialization
    result = None
    while result == None:
                               The stopping
                               condition is
        j = 1
                               NOT
                              dependent on
        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
                      update
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
                  initialization
       j = 1
       while j <= n:
                        stopping condition
            if i % j == 0:
                j = j + 1
                              update
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —			-
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3	•••	n
	x	1	2	3		n
	+ infinity					

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:</pre>
           if i % j == 0:
               j = j + 1
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —			-
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3		n
	x	1	2	3	•••	n
	+ infinity					

```
def smallest_multiple(n=5):
    i = 1
    result = None
   while result == None:
       j = 1
       while j <= n:</pre>
           if i % j == 0:
               j = j + 1
            else:
                break
       if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
   result = None
    while result == None:
        j = 1
        while j <= n:</pre>
                                   Check
            if i % j == 0:
                                   condition
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —			-
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3		n
	X	1	2	3	•••	n
	+ infinity					

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
      j = 1
      while j <= n:</pre>
          j = j + 1
          else:
              break
      if j == n+1:
          result = i
       i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
•	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
           if i % j == 0:
              j = j + 1 Update j
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —			
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3	•••	n
	4	1	2	3	•••	n
	X	1	2	3	•••	n
	+ infinity					

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
        j = 1
       while j <= n:
                                  Check
            if i % j == 0:
                                  condition
                                  for new
                j = j + 1
                                  value of j
            else:
                break
       if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
	+ infinity				

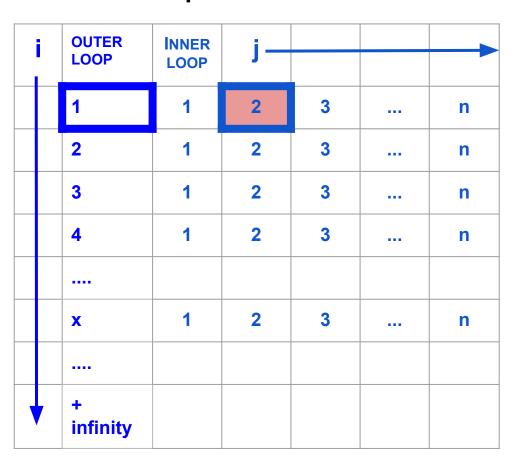
```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:</pre>
           if i % j == 0: FALSE
               j = j + 1
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
•	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
            if i % j == 0:
                j = j + 1
            else:
                                   For this i,
                                  no need
                break
                                  to check
                                   for
       if j == n+1:
                                  further j
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
▼	+ infinity				

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def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```



```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
           if i % j == 0:
               j = j + 1
           else:
               break
       if j == n+1:
           result = i
                                  Move
       i = i + 1
                                   onto next
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
\	+ infinity				

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def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
\	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
                                  Reinitialize
        j = 1
                                 j with 1.
                                 Starting
        while j <= n:</pre>
                                 over with
            if i % j == 0:
                                 new i
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
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\	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
                                  Check
           if i % j == 0:
                                  condition
               j = j + 1
           else:
                break
       if j == n+1:
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i	OUTER LOOP	INNER LOOP	j —		
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
\	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
                               TRUE,
           if i % j == 0:
                               check for
              j = j + 1 next j
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	X	1	2	3	 n
▼	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
    result = None
   while result == None:
        j = 1
        while j <= n:</pre>
                                   TRUE for
            if i % j == 0:
                                   next j
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
	+ infinity				

```
def smallest_multiple(n=5):
   i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:</pre>
           if i % j == 0: _______FALSE
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j-		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	x	1	2	3	 n
▼	+ infinity				

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   result = None
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       j = 1
       while j <= n:</pre>
           if i % j == 0:
               j = j + 1
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j—			-
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3	•••	n
	X	1	2	3	•••	n
\	+ infinity					

```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
                               Eventually...
                               You'll get to
       j = 1
                               the number
                               you are
       while j <= n:
                               looking for
            if i % j == 0:
               j = j + 1
            else:
                        x % 1 == 0 🗸
                break
       if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —			-
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3		n
	х	1	2	3	•••	n
V	+ infinity					

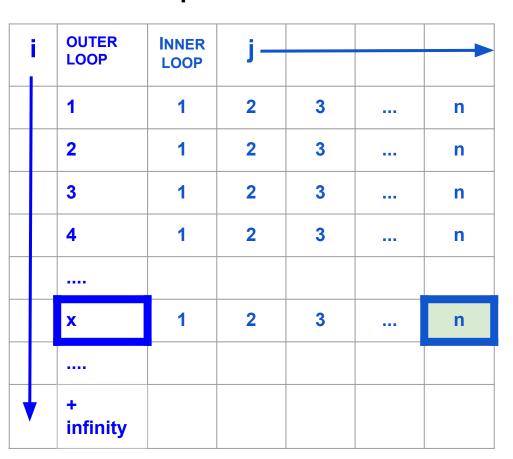
```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
           if i % j == 0:
               j = j + 1
           else:
                     x % 1 == 0 
x % 2 == 0
               break
       if j == n+1:
           result = i
       i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —		
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	х	1	2	3	 n
	••••				
↓	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:
            if i % j == 0:
                j = j + 1
            else:
                       x % 1 == 0 
x % 2 == 0 
x % 3 == 0 
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
	3	1	2	3	 n
	4	1	2	3	 n
	х	1	2	3	 n
\	+ infinity				

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def smallest_multiple(n=5):
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   while result == None:
       j = 1
       while j <= n:
           if i % j == 0:
               j = j + 1
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
    return result
```



```
def smallest_multiple(n=5):
   i = 1
   result = None
   while result == None:
       j = 1
       while j <= n:
           if i % j == 0:
               j = j + 1
                               updated
                               to n + 1
           else:
               break
       if j == n+1:
           result = i
       i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —			
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3	•••	n
	х	1	2	3		n
\	+ infinity					

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def smallest_multiple(n=5):
    i = 1
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   while result == None:
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        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
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i	OUTER LOOP	INNER LOOP	j —			
	1	1	2	3		n
	2	1	2	3		n
	3	1	2	3		n
	4	1	2	3	•••	n
	х	1	2	3		n
\	+ infinity					

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def smallest_multiple(n=5):
   i = 1
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      j = 1
      while j <= n:</pre>
          if i % j == 0:
             j = j + 1
          else:
              break
      result = i
      i = i + 1
    return result
```

i	OUTER LOOP	INNER LOOP	j —		-
	1	1	2	3	 n
	2	1	2	3	 n
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\	+ infinity				

```
def smallest_multiple(n=5):
    i = 1
    result = None
    while result == None:
        j = 1
        while j <= n:</pre>
            if i % j == 0:
                j = j + 1
            else:
                break
        if j == n+1:
            result = i
        i = i + 1
     return result
```

Largest Palindrome Problem

i	OUTER LOOP	INNER LOOP	j —		
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
V	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i	OUTER LOOP	INNER LOOP	j —		
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
\	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i	OUTER LOOP	INNER LOOP	j —		-
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
\	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i	OUTER LOOP	INNER LOOP	j —		-
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
V	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

	JTER DOP	INNER LOOP	j —		
10	00	100	101	102	 999
10	01	100	101	102	 999
10)2	100	101	102	 999
10)3	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
99	99	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

į	OUTER LOOP	INNER LOOP	j —		
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
↓	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i OUT				
100	100	101	102	 999
101	100	101	102	 999
102	100	101	102	 999
103	100	101	102	 999
	100	101	102	 999
	100	101	102	 999
	100	101	102	 999
999	100	101	102	 999

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- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

į	OUTER LOOP	INNER LOOP	j —		
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
V	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i	OUTER LOOP	INNER LOOP	j —		-
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
\	999	100	101	102	 999

Find the largest palindrome obtained from multiplying two n-digit numbers

- 1. Check if **i*****j** is a palindrome
- Check if i*j is the largest palindrome you've seen so far

i	OUTER LOOP	INNER LOOP	j —		
	100	100	101	102	 999
	101	100	101	102	 999
	102	100	101	102	 999
	103	100	101	102	 999
		100	101	102	 999
		100	101	102	 999
		100	101	102	 999
\	999	100	101	102	 999

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Assignment Questions

Question 1.

For `n=4`, the function should return `1634` since it is the smallest number that can be written as the sum of fourth power of its digits i.e.

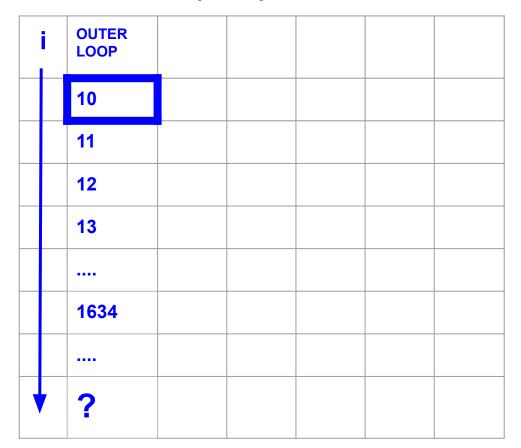
$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

As $1 = 1^4$ is not a sum, it is not included.

Similarly, for input `n=5`, the function should output `4150` since:

$$4150 = 4^5 + 1^5 + 5^5 + 0^5$$

Question 1. (n=4)



$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

As $1 = 1^4$ is not a sum, it is not included.

Question 1. (n=4)

i	OUTER LOOP	INNER LOOP	j —			
	10	?	?			
	11	?	?			
	12	?	?			
	13	?	?			
	1634	?	?	?	?	
	+∞					

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

As $1 = 1^4$ is not a sum, it is not included.

Question 1. (n=4)

i	OUTER LOOP	INNER LOOP	j—			-
	10	1 ⁴	04			
	11	1 ⁴	14			
	12	14	2 ⁴			
	13	14	3 ⁴			
	1634	14	6 ⁴	34	44	
*	+∞					

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

As $1 = 1^4$ is not a sum, it is not included.

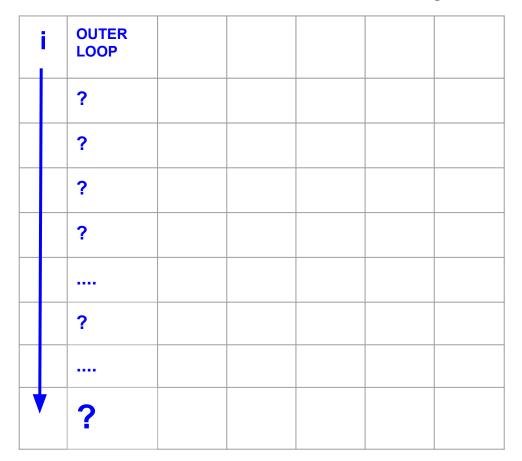
- Remember:i % 10 and i // 10are your friends
- Keep a running sum of digits raised to the power of n

Question 2. Goldbach's Conjecture:

Every even natural number greater than 2 is the sum of two prime numbers.

Given an even natural number greater than 2, find the smallest two primes that sum to the given natural number

Return the product of these two prime numbers



Find the smallest two primes that sum to the given natural even number greater than 2

i	OUTER LOOP	INNER LOOP	j —		
	1	?	?	?	 ?
	2	?	?	?	 ?
	3	?	?	?	 ?
	5	?	?	?	 ?
	7	?	?	?	 ?
	11	?	?	?	 ?
	••••				
	+∞				

Find the smallest two primes that sum to the given natural even number greater than 2

i	OUTER LOOP	INNER LOOP	j			
	1	1	2	3	5	 +∞
	2	1	2	3	5	 +∞
	3	1	2	3	5	 +∞
	5	1	2	3	5	 +∞
	7	1	2	3	5	 +∞
	11	1	2	3	5	 +∞
	••••					
	+∞					

Find the smallest two primes that sum to the given natural even number greater than 2

For every pair of prime numbers (i, j)

Check if they meet the requirements

i	OUTER LOOP	INNER LOOP	j				-
	1	1	2	3	5		+∞
	2	1	2	3	5		+∞
	3	1	2	3	5	7	
	5	1	2	3	5		+∞
	7	1	2	3	5		+∞
	11	1	2	3	5		+∞
	+∞						

Find the smallest two primes that sum to the given natural even number greater than 2

For every pair of prime numbers (i, j)

Check if they meet the requirements

Question 3. Goldbach's (disproven) Conjecture:

Every odd composite number can be written as sum of a prime number and twice a square.

$$c = p + 2x^2$$

c: composition number

p: prime number

x: an arbitrary number

i	OUTER LOOP	INNER LOOP	j			
	?	?	?	?	?	 ?
	?	?	?	?	?	 ?
	?	?	?	?	?	 ?
	?	?	?	?	?	 ?
	••••	?	?	?	?	 ?
	?	?	?	?	?	 ?
🔻	?					

Every odd composite number can be written as sum of a prime number and twice a square.

$$c = p + 2x^2$$

c: odd composite number

p: prime number

x: an arbitrary number

Example:

$$n=9$$

$$9 = 7 + 2 \times 1^2$$