University of Dhaka
Department of Mathematics
2nd Year B.S. Honors, Session 2023-24
Subject: Mathematics
Course Code: MTH 250 Course Title: Math Lab - II
Assignment 3: Ordinary Differential Equations, Deadline: 2 lab classes

Name: Roll:

***Write a Script file (if necessary) to solve each of the following problems.***

Q1. (a) Sketch (by hand, without using MATLAB) the direction field of the differential equation $\frac{dy}{dx} = x + y$ for $x$ and $y$ values between -6 and 6.

(b) Add a curve (by hand) on your direction field that approximates the solution passing through the point $x = -1, y = 0$.

(c) Now solve the differential equation given in part (a), either working it out by hand or using the `dsolve` command. Compare your answers to parts (a) and (b).

Q2. Consider the differential equation $\frac{dy}{dx} = (e^{-x} - y)(e^{-x} + 2 + y)$

(a) Plot a direction field for Eq. (2.1) for and between -10 and 10. Plot at least two solution curves on this direction field, one passing through the point (2, 3).

(b) Considering how complicated differential equation (2.1) appears, why do you think we might want to plot a direction field?

(c) Solve the equation using `ode45` choosing an appropriate initial condition.

(d) Also, solve it using a function m file named `odefun`(say).

Q3. A radioactive substance decays at a rate proportional to the amount present, and half the original quantity $Q_0$ is left after 1500 years. In how many years would the original amount be reduced to $\frac{3Q_0}{4}$? How much will be left after 2000 years?

Q4. Consider solving the second order ODE: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 9x^2 + 4$ with $y(0) = 6$ and $y'(0) = 8$ using `ode45`.

Hint: First express the given ODE as a system of first-order ODEs. Hence, solve the system using `ode45`. Finally draw the solution curve. Try solving it using `dsolve` as well.

Q5. The logistic model has been applied to the natural growth of the halibut population in certain areas of the Pacific Ocean. Let $P(t)$, measured in kilograms, be the total mass, or biomass, of the halibut population at time $t$. The parameters in the logistic equation are estimated to have the values $r = 0.71/year$ and $K = 106\ kg$. If the initial biomass is $P_0 = 0.25K$, find the biomass 2 years later. Also find the time $\tau$ for which $P(\tau) = 0.75K$ and find the time $\tau_1$ for which population grows fastest. The logistic differential equation and solution of the equation are, $P'(t) = r\,P(t)\left(1 - \frac{P(t)}{K}\right)$ and $P(t) = \frac{KP_0}{P_0 + (K - P_0)e^{-rt}}$, respectively where $r$ is the intrinsic growth rate, and $K$ is the carrying capacity.