University of Dhaka
Department of Mathematics
2nd Year B.S. Honors, Session 2023-24
Subject: Mathematics
Course Code: MTH 250 Course Title: Math Lab - II
Assignment 1: Calculus I, Deadline: 1 lab class

Name: Roll:

1) Consider the following functions

i. $f(x) = \frac{\sin x}{x}$

ii. $g(x) = \frac{\ln(x+1)}{(x+1)}$

Compute the following limits: (a) $\lim_{x \to 0} f(x)$ (b) Plot all three functions $f(x)$ & $g(x)$ on the same graph for $x \in [-10,10]$.

I. Use distinct styles or colors for each function. Include a grid on the plot. Highlight key points where the limits are evaluated (e.g., $x = 0, x = \pi$).

II. Adjust the appearance of the plot: Make the lines for each function visually distinct. Use markers to indicate specific points. Focus the y-axis to show $y \in [-1,5]$.

III. Add appropriate labels to the axes, include a title, and provide a legend to differentiate the functions.

Commands you will need: *syms, ezplot, plot, figure, fplot, limit, grid, hold, LineWidth, MarkerFaceColor, MaekerSize, axis, xlabel, ylabel, title.*

2(a) Consider the function representing the growth of a certain population or process:

$$f(x) = 6x^4 - 12x^3 + 8x^2 - 4x$$

You are tasked with modeling the population growth of a species over time. Sketch the graphs of $f(x), f'(x)$ & $f''(x)$ on the same axes. Analyze the following:

i. Identify the intervals of population growth (increasing) and decline (decreasing) based on the first derivative.

ii. Determine the population's local maxima and minima, indicating periods of rapid growth or stability.

iii. Identify intervals of concavity and locate inflection points to understand where the population experiences changes in growth behavior.

2(b) A company tracks the temperature $T(t)$ in degrees Celsius inside a storage facility over a 6-hour period, where t is time in hours. The temperature is modeled by the function:

$$T(t) = -2t^3 + 9t^2 - 12t + 25, \qquad 0 \le t \le 6$$

i. Verify that $T(t)$ satisfies the conditions of the Mean Value Theorem on the interval $[0,6]$

ii. Use the Mean Value Theorem to find a time c in $(0,6)$ where the rate of change of temperature is equal to the average rate of change over the interval.

iii. Plot the temperature function $T(t)$ over the interval $[0,6]$ along with the secant line connecting the endpoints $(0, T(0))$ and $(6, T(6))$ and the tangent line at c.

iv. Interpret the result: What does the time c represent in terms of the temperature variation inside the facility? How can this information help in maintaining stable storage conditions?

2(c) A biotech company is studying the growth rate of a bacterial colony in a controlled environment. The size of the colony at time t (in hours) is modeled by the function:

$$P(t) = 50e^{0.2t} - 10t^2, 0 \le t \le 10$$

i. Graph P(t), P′(t) (growth rate), and P″(t) (acceleration of growth or decay) on the same set of axes over the interval [0,10].

ii. Analyze the graphs to answer the following:

iii. When is the bacterial population increasing, and when is it decreasing?

iv. At what time does the population reach its maximum size?

v. Over what intervals is the growth rate increasing or decreasing?

vi. Based on the behavior of $P'(t)$ & $P''(t)$, identify the points of inflection and explain their biological significance in terms of changes in growth trends.

vii. Discuss how this information can be used to optimize the conditions for bacterial growth in the lab.

Commands you will need: *diff, solve, legend*

3 Sketch the region enclosed by the given curves and find its area.

i. $y = e^x$ & $y = x^2$

ii. $y = \cos x$ & $y = 1 - \cos x, 0 \le x \le \pi$

Commands you will need: *solve, int*