University of Dhaka
Department of Mathematics
2nd Year B.S. Honors, Session 2023-24
Subject: Mathematics
Course Code: MTH 250 Course Title: Math Lab – II
Assignment 4: Linear Algebra, Deadline: 2 lab classes

Name: Roll:

Write a Script file (if necessary) to solve each of the following problems.

1. Let $u = (-1,2,0,1), v = (-2,1,2,5), w = (-1,-3,1,6)$.

   (a) Verify $\langle u+v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$.

   (b) Compute the inner product $\langle 2u - v, u + 2v\rangle$.

   (c) Test whether the set of vectors $\{u_1, u_2, u_3\}$ is orthonormal basis for $\mathbb{R}^3$ with Euclidean inner product.

2. Let $u_1 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), u_2 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right), u_3 = \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right), u_4 = \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ be the orthonormal basis for the vector space $\mathbf{W}$ and $v = (1,1,1,1)$ be any vector. Then by using loop, compute $proj_W v = \sum_{i=1}^{3}\langle v, u_i\rangle\, u_i$ and also find the component of v orthogonal on $\mathbf{W}$ by the formula $v - \mathrm{proj}_W v$.

3. Let $A = \begin{pmatrix} -2 & 3 & 1 \\ 0 & -4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$. By using "For" and "while" loop find the inner product $\langle A, B\rangle$, norm $\|A\|$, $\|B\|$, where $\langle A, B\rangle = \sum_{i=1}^{2}\sum_{j=1}^{3} a_{ij}b_{ij}$.

4. Apply the Gram-Schmidt process to transform the basis vectors $u_1 = (0,2,1,0), u_2 = (1,-1,0,0), u_3 = (1,2,0,-1)$, $u_4 = (1,0,0,1)$ into an orthonormal basis $\{u_1, u_2, u_3, u_4\}$. The Gram-Schmidt orthogonalizing process is as follows:

$$u_1 = v_1$$

$$u_k = v_k - \sum_{j=1}^{k-1} \mathrm{proj}_{u_j}(v_k)\,, k = 2, 3, \ldots$$

Let T be the transformation from the uv-plane to xy-plane defined by

$$T(u, v) = (x(u, v), y(u, v))$$

where, $x = 4u - 3v$ and $y = 2u + 5v$. By using two-dimensional array

(i) Find $T(4, -1)$.

(ii) Find the integer coordinate point of the image under T of the rectangle $-3 \le u \le 3, -2 \le v \le 2$.

6. Let $u_1 = (1,1,1), u_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right), u_3 = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$ be the orthogonal basis for the vector space $\mathbf{W}$ and $v = (1,2,3)$ be any vector. Then by using loop, compute

$$\mathrm{proj}_W \mathrm{v} = \sum_{i=1}^{3} \frac{\langle v, u_i\rangle}{\|u_i\|^2} u_i$$

7. Let $S = \{u_1 = (1, -2, 3, -4), u_2 = (2, 1, -4, -3), u_3 = (-3, 4, 1, -2), u_4 = (4, 3, 2, 1)\}$ be the orthonormal basis for $\mathbb{R}^4$. Express $v = (-1, 2, 3, 7)$ as a linear combination of the given vectors. Also find the coordinate vector $(v)_S$. Show the output as
$v = a_1 u_1 + a_2 u_2 + a_3 u_3 + a_4 u_4$
and $(v)_S = (a_1, a_2, a_3, a_4)$, where $a_i = \langle v, u_i \rangle,\ i = 1, 2, 3, 4.$

8. Verify that the quadratic function $f(x, y, z) = -5x^2 + 2y^2 + 4xy - 5yz - 4xz + z^2$ can be written as $Q_A(\boldsymbol{x}) = \boldsymbol{x}^T A \boldsymbol{x}$, where A is a symmetric matrix and $\boldsymbol{x}$ is a column vector. Hence evaluate Q_A for $x = 0, y = -2$ and $z = 2$.