FOCUS	MATHEMATICS
KNOWLEDGE AREA	EXPONENTS
TOPIC	2
SUBTOPIC	
NAME OF ASSET	Simplify expressions involving exponents

TYPE OF ASSET SUBTYPE	INTERACTIVE: Type your answer
OBJECTIVE	Take the user step-by-step through a method to solve the given problem along with having access to a step-by-step interactive solution

SUBHEADING: Type the missing information in the blanks and check your answer

TEXT FOR BUTTONS

Instructions

1. Read each step carefully. Type the missing information in the blank spaces.
2. After each step, click on the CHECK button to see whether your answers are correct.
3. Click on the REVEAL ANSWER button to see the correct answers.
4. Go to the next step and repeat the process.
5. Continue until you have the final answer.
6. Click on GO TO INTERACTIVE SOLUTION button to see the step-by-step solution.
7. Click on the coloured dots to reveal the different steps of the solution.
8. Click on GO TO EXERCISE button to go back to solving the given exercise.
9. Click on the numbered buttons at the bottom to solve different exercises.

Check

- There must be a check button after each step, as described below. The button only functions once all the blanks of a certain step are completed. If the user attempts before that: Complete all the blanks.
- When clicked, a green tick appears next to each correct answer and an orange cross appears next to each incorrect answer.
- As soon as answers are marked:
    - If all answers are correct (with green tick), the next step loads.
    - If some answers are incorrect (with red cross), a pop-up comes which asks user to either Try again or Reveal answer.
- When Try again button is clicked, only the incorrect answers disappear and are replaced by ? symbol. The correct answers will still have green ticks and cannot be edited. Only allow for one Try again.
- When Reveal answer button is clicked, the correct answers appear in the blanks, and the next step is loaded after a pause of 2 seconds.

USER INTERACTION: EXERCISE

- Note the idea is to force the user to work through the method in a step-by-step manner, so the user must not be able to continue to the next step until the previous one is complete.
- The user will click each box to enter values in it through a custom numeric keypad generated through code (not using image). The numeric keypad must have a toggle +/- button (only – will be displayed). Three new buttons, need to be available for inputting variables: x, y and n. In case a screen has some other variables (e.g. p, q and r), it should be possible to easily change the code of the numeric keypad and get these variables on the keypad.
- A textbox that is clicked for entering the answer must be highlighted in red border and a cursor shown. But direct entry through the native keyboard must NOT be possible. All entry should be through the numeric keypad.
- All the steps in the exercise must be coded using Mathjax (LaTeX) text and not as images.
- Once all boxes are filled, the user will click the Check answer button.
- If all answers are correct, this step will move forward (with ticks and boxes removed) and the next step with empty boxes will load. If all answers are not correct, please refer to the flow mentioned for the Check button.
- The dots at the bottom of the central pane denote the number of steps (or screens) for one exercise. The user can go back to a previous step by clicking on these buttons and also come to the current step (the state of the exercise must be preserved in this case). The buttons for the later steps are disabled.
- When the user is on any dot (e.g. step #3), the text of the previous 2 steps will be visible on the same screen.
- The numbered buttons in the bottom bar shall load different exercises.

USER INTERACTION: SOLUTION

- Note the idea is to force the user to go through the method in a step-by-step manner, so that each step is gradually unlocked.
- The user can move to the interactive solution whenever he clicks on GO TO INTERACTIVE SOLUTION button.
- Here, each step has some coloured dots. When user clicks on any one of the dots, that particular step gets expanded in that particular colour. On clicking on that step again, the step collapses and the coloured dot appears again.
- All the steps in the interactive solution must be coded using Mathjax (LaTeX) text and not as images.
- Once all the coloured dots have been expanded for a particular step, the yellow arrow button appears which when clicked, shall take the user to the next step.
- For certain steps, the clicking of the coloured dot, might additionally result in a sliding pop-up that gives which particular law has been applied to the particular step.
- The user can return to the exercise at any point of time by clicking on the GO TO EXERCISE button.

- This section will NOT have any dots at the bottom of the central pane denoting step numbers.

CONTENTS OF SCREENS: EXERCISE

SCREEN 5 **Variant 1**
Simplify:

$$\frac{45^{x-3} \cdot 3 \cdot 75^{4-x}}{25^{-x} \cdot 15^{x+2}}$$

Step-by-step exercise: (the ones marked in **bold** have to be shown as blank boxes in the Exercise)

$$\frac{45^{x-3} \cdot 3 \cdot 75^{4-x}}{25^{-x} \cdot 15^{x+2}}$$

$$= \frac{(5 \cdot \mathbf{3^2})^{x-3} \cdot 3 \cdot (3 \cdot \mathbf{5^2})^{4-x}}{(5^{\mathbf{2}})^{-x} \cdot (3 \cdot \mathbf{5})^{x+2}}$$

$$= \frac{5^{x-\mathbf{3}} \cdot 3^{2x-6} \cdot 3^1 \cdot 3^{\mathbf{4}-x} \cdot \mathbf{5}^{8-2x}}{5^{\mathbf{-2}x} \cdot 3^{x+2} \cdot \mathbf{5}^{x+2}}$$

$$= 5^{x-\mathbf{3}+8-2x+\mathbf{2}x-x-2} \cdot \mathbf{3}^{2x-\mathbf{6}+1+4-\mathbf{x}-x-2}$$

$$= 5^{\mathbf{3}} \cdot 3^{-\mathbf{3}}$$

$$= \frac{5^{\mathbf{3}}}{3^{\mathbf{3}}}$$

$$= \frac{\mathbf{125}}{27}$$

Interactive solution

$$\frac{45^{x-3} \cdot 3 \cdot 75^{4-x}}{25^{-x} \cdot 15^{x+2}}$$

$$= \frac{(5 \cdot 3^2)^{x-3} \cdot 3 \cdot (3 \cdot 5^2)^{4-x}}{(5^2)^{-x} \cdot (3 \cdot 5)^{x+2}}$$

$$= \frac{(5 \cdot 3^2)^{x-3} \cdot 3 \cdot (3 \cdot 5^2)^{4-x}}{(5^2)^{-x} \cdot (3.5)^{x+2}}$$

$$= \frac{5^{x-3} \cdot 3^{2x-6} \cdot 3^1 \cdot 3^{4-x} \cdot 5^{8-2x}}{5^{-2x} \cdot 3^{x+2} \cdot 5^{x+2}}$$

$$= \frac{5^{x-3} \cdot 3^{2x-6} \cdot 3^1 \cdot 3^{4-x} \cdot 5^{8-2x}}{5^{-2x} \cdot 3^{x+2} \cdot 5^{x+2}}$$

$$= 5^{x-3+8-2x+2x-x-2} \cdot 3^{2x-6+1+4-x-x-2}$$

(Applying multiplication law and division law of exponents)

$$= 5^3 \cdot 3^{-3}$$

$$= \frac{5^3}{3^3}$$

(Applying division law of exponents)

$$= \frac{125}{27}$$

(please refer to given layout images to see how the coloured dots would expand, along with the colour coding shown above)

SCREEN 5 VARIANT 2
Exercise

Simplify:

$$\frac{\sqrt[3]{a} \times \sqrt{m^3}}{a^{-\frac{2}{3}} m^{\frac{1}{2}}}$$

Step-by-step exercise:

$$\frac{\sqrt[3]{a} \times \sqrt{m^3}}{a^{-\frac{2}{3}}m^{\frac{1}{2}}}$$

$$=\frac{a^{\frac{1}{3}}m^{\frac{3}{2}}}{a^{-\frac{2}{3}}m^{\frac{1}{2}}}$$

$$= a^{\frac{1}{3}+\frac{2}{3}}m^{\frac{3}{2}-\frac{1}{2}}$$

$$= am$$

Interactive exercise:

$$\frac{\sqrt[3]{a} \times \sqrt{m^3}}{a^{-\frac{2}{3}}m^{\frac{1}{2}}}$$

$$=\frac{a^{\frac{1}{3}}m^{\frac{3}{2}}}{a^{-\frac{2}{3}}m^{\frac{1}{2}}}$$

$$=\frac{a^{\frac{1}{3}}m^{\frac{3}{2}}}{a^{-\frac{2}{3}}m^{\frac{1}{2}}}$$

$$= a^{\frac{1}{3}+\frac{2}{3}}m^{\frac{3}{2}-\frac{1}{2}}$$

$$= a^{\frac{1}{3}+\frac{2}{3}}m^{\frac{3}{2}-\frac{1}{2}}$$

$$= am$$

SCREEN 5 VARIANT 3
Exercise

Simplify:

$$\frac{6^{x+1} \times 9^{x-1}}{2^x \times 27^x}$$

Step-by-step exercise:

$$\frac{6^{x+1} \times 9^{x-1}}{2^x \times 27^x}$$

$$=\frac{2^{x+1}3^{x+1}3^{2x-2}}{2^x 3^{3x}}$$

$$= 2^{x+1-x}3^{x+1+2x-2-3x}$$

$$= 2.3^{-1}$$

$$= \frac{2}{3}$$

Interactive exercise:

$$\frac{6^{x+1} \times 9^{x-1}}{2^x \times 27^x}$$

$$= \frac{2^{x+1}3^{x+1}3^{2x-2}}{2^x 3^{3x}}$$

$$= \frac{2^{x+1}3^{x+1}3^{2x-2}}{2^x 3^{3x}}$$

$$= 2^{x+1-x}3^{x+1+2x-2-3x}$$

$$= 2^{x+1-x}3^{x+1+2x-2-3x}$$

$$= 2.3^{-1}$$

$$= 2.3^{-1}$$

$$= \frac{2}{3}$$

SCREEN 1 VARIANT 1
Exercise

Simplify:

$$\frac{2^{n+2} - 2^{n+1}}{2^x}$$

Step-by-step exercise:
$$\frac{2^{n+2} - 2^{n+1}}{2^x}$$

$$= \frac{2^n(2^2 - 2^1)}{2^x}$$

$$= 2^{n-x}(4 - 2)$$

$$= 2^{n-x} \cdot 2^1$$

$$= 2^{n-x+\textbf{1}}$$

Interactive solution

$$\frac{2^{n+2} - 2^{n+1}}{2^x}$$

$$= \frac{2^n(2^2 - 2^1)}{2^x}$$

$$= \frac{2^n(2^2 - 2^1)}{2^x}$$

$$= 2^{n-x}(4 - 2)$$

(Applying division law of exponents)

$$= 2^{n-x}(4 - 2)$$

$$= 2^{n-x} \cdot 2^1$$

$$= 2^{n-x} \cdot 2^1$$

$$= 2^{n-x+1}$$

(Applying multiplication law of exponents)

Exercise

Simplify:

$$\frac{3^x - 3^{x-1}}{3^{x+2}}$$

Step-by-step exercise:

$$\frac{3^x - 3^{x-1}}{3^{x+2}}$$

$$= \frac{3^x(1 - 3^{-1})}{3^x 3^2}$$

$$= \frac{\frac{2}{3}}{9}$$

$$= \frac{2}{27}$$

Interactive solution

$$\frac{3^x - 3^{x-1}}{3^{x+2}}$$

$$= \frac{3^x(1 - 3^{-1})}{3^x 3^2}$$

$$= \frac{3^x(1 - 3^{-1})}{3^x 3^2}$$

$$= \frac{\frac{2}{3}}{9}$$

$$= \frac{\frac{2}{3}}{9}$$

$$= \frac{2}{27}$$

SCREEN 1 VARIANT 3

Exercise

Simplify: $\dfrac{3^x - 3^{x-2}}{3^{x-2}}$

Step-by-step exercise

$$\frac{3^x - 3^{x-2}}{3^{x-2}}$$

$$= \frac{3^x(1 - 3^{-2})}{3^x 3^{-2}}$$

$$= \frac{\frac{8}{9}}{\frac{1}{9}}$$

$$= 8$$

Interactive solution

$$\frac{3^x - 3^{x-2}}{3^{x-2}}$$

$$= \frac{3^x(1 - 3^{-2})}{3^x 3^{-2}}$$

$$= \frac{3^x(1 - 3^{-2})}{3^x 3^{-2}}$$

$$= \frac{\frac{8}{9}}{\frac{1}{9}}$$

$$= \frac{\frac{8}{9}}{\frac{1}{9}}$$

$$= 8$$

Simplify:

$$\frac{3 \cdot 2^n - 2^{n-2}}{2^{n+1}}$$

Step-by-step exercise:

$$\frac{3 \cdot 2^n - 2^{n-2}}{2^{n+1}}$$

$$= \frac{2^n(3 - 2^{-2})}{2^n 2^{1}}$$

$$= \frac{2\frac{3}{4}}{2}$$

$$= \frac{11}{8}$$

Interactive solution

$$\frac{3 \cdot 2^n - 2^{n-2}}{2^{n+1}}$$

$$= \frac{2^n(3 - 2^{-2})}{2^n 2^1}$$

(Applying multiplication law of exponents)

$$= \frac{2^n(3 - 2^{-2})}{2^n 2^1}$$

$$= \frac{2\frac{3}{4}}{2}$$

$$= \frac{2\frac{3}{4}}{2}$$

$$= \frac{11}{8}$$

SCREEN 2 VARIANT 2

Simplify:

$$\frac{3^n - 3^{n+2}}{3^n}$$

Step-by-step exercise:

$$\frac{3^n - 3^{n+2}}{3^n}$$

$$= \frac{3^n - 3^n 3^2}{3^n}$$

$$= \frac{3^n(1 - 3^2)}{3^n}$$

$$= 1 - \mathbf{9}$$

$$= \mathbf{-8}$$

Interactive solution:

$$\frac{3^n - 3^{n+2}}{3^n}$$

$$= \frac{3^n - 3^n 3^2}{3^n}$$

$$= \frac{3^n - 3^n 3^2}{3^n}$$

$$= \frac{3^n(1 - 3^2)}{3^n}$$

$$= \frac{3^n(1 - 3^2)}{3^n}$$

$$= 1 - 9$$

$$= 1 - 9$$

$$= -8$$

SCREEN 2 VARIANT 3

Simplify:

$$\frac{x^{n+1} - x^n}{x - 1}$$

Step-by-step exercise:

$$\frac{x^{n+1} - x^n}{x - 1}$$

$$= \frac{x^n x^{\mathbf{1}} - \mathbf{x^n}}{x - 1}$$

$$= \frac{x^n(x^{\mathbf{1}} - \mathbf{1})}{x - 1}$$

$$= \boxed{x^n}$$

Interactive solution:

$$\frac{x^{n+1} - x^n}{x - 1}$$

$$= \frac{x^n x^1 - x^n}{x - 1}$$

$$= \frac{x^n x^1 - x^n}{x - 1}$$

$$= \frac{x^n(x^1 - 1)}{x - 1}$$

$$= \frac{x^n(x^1 - 1)}{x - 1}$$

$$= x^n$$

Simplify:

$$\frac{3^{n+4} - 6.3^{n+1}}{3^{n+3} + 2 \cdot 9^{\frac{1}{2}n+1}}$$

Step-by-step exercise:

$$\frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot 9^{\frac{1}{2}n+1}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot (3^2)^{\frac{1}{2}n+1}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot 3^{1n+2}}$$

$$= \frac{3^{n+1}(3^3 - 6)}{3^{n+1}(3^2 + 2 \cdot 3^1)}$$

$$= \frac{21}{15}$$

$$= \frac{7}{5}$$

Interactive solution

$$\frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot 9^{\frac{1}{2}n+1}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot (3^2)^{\frac{1}{2}n+1}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot (3^2)^{\frac{1}{2}n+1}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot 3^{1n+2}}$$

$$= \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+3} + 2 \cdot 3^{1n+2}}$$

$$= \frac{3^{n+1}(3^3 - 6)}{3^{n+1}(3^2 + 2 \cdot 3^1)}$$

(Applying multiplication law of exponents)

$$= \frac{3^{n+1}(3^3 - 6)}{3^{n+1}(3^2 + 2 \cdot 3^1)}$$

$$= \frac{21}{15}$$

$$= \frac{21}{15}$$

$$= \frac{7}{5}$$

Simplify:

$$\frac{2^{2n+1} - 2^{2n}}{4^n + 2^{2n-1}}$$

Step-by-step exercise:

$$\frac{2^{2n+1} - 2^{2n}}{4^n + 2^{2n-1}}$$

$$= \frac{2^{2n}2^1 - 2^{2n}}{2^{2n} + 2^{2n}2^{-1}}$$

$$= \frac{2^{2n}(2^1 - 1)}{2^{2n}(1 + 2^{-1})}$$

$$= \frac{1}{\frac{3}{2}}$$

$$= \frac{2}{3}$$

Interactive exercise:

$$\frac{2^{2n+1} - 2^{2n}}{4^n + 2^{2n-1}}$$

$$= \frac{2^{2n}2^1 - 2^{2n}}{2^{2n} + 2^{2n}2^{-1}}$$

$$= \frac{2^{2n}2^1 - 2^{2n}}{2^{2n} + 2^{2n}2^{-1}}$$

$$= \frac{2^{2n}(2^1 - 1)}{2^{2n}(1 + 2^{-1})}$$

$$= \frac{2^{2n}(2^1 - 1)}{2^{2n}(1 + 2^{-1})}$$

$$= \frac{1}{\frac{3}{2}}$$

$$= \frac{1}{\frac{3}{2}}$$

$$= \frac{2}{3}$$

SCREEN 3 VARIANT 3

Simplify:

$$\frac{5 \cdot 2^x - 4 \cdot 2^{x-2}}{2^x - 2^{x-1}}$$

Step-by-step exercise:

$$\frac{5 \cdot 2^x - 4 \cdot 2^{x-2}}{2^x - 2^{x-1}}$$

$$= \frac{5 \cdot 2^x - 2^2 \cdot 2^x 2^{-2}}{2^x - 2^x 2^{-1}}$$

$$= \frac{2^x (5 - 2^2 \cdot 2^{-2})}{2^x (1 - 2^{-1})}$$

$$= \frac{5 - 1}{1 - \frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= 8$$

Interactive exercise:

$$\frac{5 \cdot 2^x - 4 \cdot 2^{x-2}}{2^x - 2^{x-1}}$$

$$= \frac{5 \cdot 2^x - 2^2 \cdot 2^x 2^{-2}}{2^x - 2^x 2^{-1}}$$

$$= \frac{5.2^x - 2^2.2^x 2^{-2}}{2^x - 2^x 2^{-1}}$$

$$= \frac{2^x(5 - 2^2.2^{-2})}{2^x(1 - 2^{-1})}$$

$$= \frac{2^x(5 - 2^2.2^{-2})}{2^x(1 - 2^{-1})}$$

$$= \frac{5 - 1}{1 - \frac{1}{2}}$$

$$= \frac{5 - 1}{1 - \frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= 8$$

SCREEN 4 variant 1

Simplify:

$$\frac{5^{2x-1} \cdot 9^{x-2}}{15^{2x-3}}$$

Step-by-step exercise:

$$\frac{5^{2x-1} \cdot 9^{x-2}}{15^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot (3^{\mathbf{2}})^{x-2}}{(5 \cdot \mathbf{3})^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot 3^{\mathbf{2x-4}}}{5^{\mathbf{2x-3}} \cdot 3^{2x-3}}$$

$$= 5^{2x-1-2x+3} \cdot 3^{2x-4-2x+3}$$

$$= 5^{2} \cdot 3^{-1}$$

$$= \frac{25}{3}$$

Interactive solution

$$\frac{5^{2x-1} \cdot 9^{x-2}}{15^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot \left(3^2\right)^{x-2}}{(5 \cdot 3)^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot \left(3^2\right)^{x-2}}{(5 \cdot 3)^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot 3^{2x-4}}{5^{2x-3} \cdot 3^{2x-3}}$$

$$= \frac{5^{2x-1} \cdot 3^{2x-4}}{5^{2x-3} \cdot 3^{2x-3}}$$

$$= 5^{2x-1-2x+3} \cdot 3^{2x-4-2x+3}$$

(Applying division law of exponents)

$$= 5^{2x-1-2x+3} \cdot 3^{2x-4-2x+3}$$

$$= 5^{2} \cdot 3^{-1}$$

$$= 5^2 \cdot 3^{-1}$$

$$= \frac{25}{3}$$

SCREEN 4 variant 2

Simplify:

$$\frac{2^{n+1}.3^{n-1}}{6^n}$$

Step-by-step exercise:

$$\frac{2^{n+1}.3^{n-1}}{6^n}$$

$$= \frac{2^{n+1}3^{n-1}}{2^n3^n}$$

$$= 2^{n+1-n}3^{n-1-n}$$

$$= 2.3^{-1}$$

$$= \frac{2}{3}$$

Interactive exercise:

$$\frac{2^{n+1}.3^{n-1}}{6^n}$$

$$= \frac{2^{n+1}3^{n-1}}{2^n3^n}$$

$$= \frac{2^{n+1}3^{n-1}}{2^n3^n}$$

$$= 2^{n+1-n}3^{n-1-n}$$

$$= 2^{n+1-n}3^{n-1-n}$$

$$= 2.3^{-1}$$

$$= 2.3^{-1}$$

$$= \frac{2}{3}$$

SCREEN 4 variant 3

Simplify:

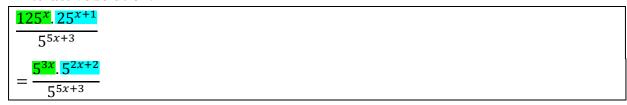$$\frac{125^x.25^{x+1}}{5^{5x+3}}$$

Step-by-step exercise

$$\frac{125^x.25^{x+1}}{5^{5x+3}}$$

$$= \frac{5^{3x} \cdot 5^{2x+2}}{5^{5x+3}}$$

$$= 5^{3x+2x+2-5x-3}$$

$$= 5^{-1}$$

$$= \frac{1}{5}$$

Interactive solution:

$$\frac{125^{x} \cdot 25^{x+1}}{5^{5x+3}}$$

$$= \frac{5^{3x} \cdot 5^{2x+2}}{5^{5x+3}}$$

$$= \frac{5^{3x} \cdot 5^{2x+2}}{5^{5x+3}}$$

$$= 5^{3x+2x+2-5x-3}$$

$$= 5^{3x+2x+2-5x-3}$$

$$= 5^{-1}$$

$$= 5^{-1}$$

$$= \frac{1}{5}$$