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## Sorting

### Insertion Sort:

Best case  $O(n)$

Worst case  $O(n^2)$

Average case  $O(n^2)$

Small array  
Sorting  
Keep

### Simulation:

12 11 13 5 6

- \* first make a sub-short array with first two element and sort them

[11 12] 13 5 6

- \* then take <sup>next</sup> another element and make another sub array and sort them

[11 12 13] 5 6

keep the process repeating

[11 12 5 13] 6 → [11 5 12 13] 6

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5 11 12 13 6

5 11 12 6 13

5 11 6 12 13

5 6 11 12 13

Pseudo Code A: list of sortable items.

$n = \text{length}(A)$

for  $i = 1$  to  $n - 1$  do

$j = i$

while  $j > 0$  and  $A[j-1] > A[j]$  do

swap ( $A[j]$ ,  $A[j-1]$ )

$j = j - 1$

end swap

end for

end procedure

Algorithm :

1. Take a single list of sortable items A
2. The variable 'n' is assigned the length of A
3. The outer for loop starts at index '1' and runs for ' $n-1$ ' iterations, where 'n' is the length of array.
4. The inner while loop continues to move an element to its left. If an element is smaller than its left element, the elements are swapped.
5. The inner while loop continues as long as an element is smaller than its left element.
6. Once the inner loop is finished, the element at the current index is in its correct position in the sorted position.
7. The outer loop continues until the array is fully sorted.

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## Code

```
for (int i=1; i<=n-1; i++) {
```

```
    key = arr[i];
```

```
    int j = i-1;
```

```
    while (j >= 0 && arr[j] > key) {
```

~~```
        arr[j+1] = arr[j];
```~~~~```
        j = j-1;
```~~~~```
        arr[j+1] = key;
```~~~~```
}
```~~~~```
X
```~~

Time complexity of bubble sort is O(n^2).

It compares adjacent elements and swaps them if they are in wrong order.

Worst case time complexity of bubble sort is O(n^2).

Best case time complexity of bubble sort is O(n).

Space complexity of bubble sort is O(1).

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## Selection Sort:

### Time Complexity:

Best case:  $O(n^2)$

worst case:  $O(n^2)$

Average case:  $O(n^2)$

Swap with minimum,

### Simulation:

20 12 10 15 2

Set first element as minimum. Compare 2nd element with minimum. If 2nd element smaller than minimum, assign the 2nd element as minimum. Keep comparing until last element.

20 12 10 15  $\cancel{P2}$

20 12  $\cancel{\frac{10}{m}}$  15  $\cancel{P2}$

20 12  $\cancel{\frac{10}{m}}$  15  $\cancel{P2}$

20 12 10 15  $\frac{2}{m}$

$\cancel{(2)}$  12 10 15  $\cancel{(20)}$

Process

Swap minimum and first element

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minimum value at index 2

Step - 1 2  $\frac{12}{m}$  10 15 20

2 12  $\frac{10}{m}$  15 20

2 12  $\frac{10}{m}$   $\frac{15}{1}$  20

2 12  $\frac{10}{m}$  15 20

3 10 12 15 20

Step - 2 2 10  $\frac{12}{m}$  15 20

2 10  $\frac{12}{m}$  15 20

2 10 12 15 20

Step - 3 2 10 12 15 20

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### Pseudocode:

```
SelectionSort (Array, size)
repeat size-1 times
    set the first each of the unsorted element as the minimum
    for each of the first unsorted elements
        if element < minimum
            set element as new minimum
        swap minimum with first unsorted position
    end SelectionSort.
```

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### C++ Code:

```
for (int i=0; i<n-1; i++) {  
    int min_idx = i;  
    for (int j=i+1; j<n; j++) {  
        if (array[j] < array[min_idx])  
            min_idx = j;  
    }  
    swap (array[min_idx], array[i]);  
}
```

### Swap Swap

```
int temp;  
temp = array[min_idx];  
array[min_idx] = array[j];  
array[j] = temp;
```

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## Bubble Sort:

Best case:  $O(n)$ ,worst case:  $O(n^2)$ .Average case:  $O(n^2)$ .

keep swapping

to in  
the biggest to in  
last position

## Simulation:

-2 45 0 11 -9

keep starting from first zero index compare 1st and second element. if 1st element is smaller the swap the elements. keep swapping until the last element. (2nd with 3rd)  
(3rd with 4th) - - - - -

-2 45 0 11 -9-2 ~~45~~ 0 11 -9-2 0 45 11 -9-2 0 11 45 -9

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$$\begin{array}{r} -3 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} 11 \\ -9 \\ \underline{45} \end{array}$$

$$\begin{array}{r} -3 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} 11 \\ -9 \\ \underline{45} \end{array}$$

$$\begin{array}{r} -2 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} 11 \\ -9 \\ \underline{45} \end{array}$$

$$\begin{array}{r} -2 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} -9 \\ \underline{11} \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} -9 \\ \underline{11} \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} -9 \\ 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{-9} \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ -9 \\ \underline{\quad} \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ -9 \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{-9} \\ -2 \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{-9} \\ -2 \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{-9} \\ -2 \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

$$\begin{array}{r} -2 \\ \underline{-9} \\ -2 \\ 0 \end{array} \begin{array}{r} 11 \\ 45 \end{array}$$

Process will  
continue 0 to  
 $n-1$  time

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## Pseudocode

bubblesort (array)

for i=0 to n-1 times

    if leftelement > right element

        swap leftelement and rightelement

end bubblesort.

## Code:

```
for (int i=0 ; i<n ; i++) {
```

```
    for (int j=0 ; j<n-i ; j++) {
```

```
        if (array [j] > array [j+1]) {
```

```
            int temp = array [j];
```

```
            array [j] = array [j+1];
```

```
            array [j+1] = temp; }
```

```
}
```

```
}
```

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## Merge Sort:

Stability Yes

Best case:  $O(n \log n)$

\* Divide and conqueror

Worst case:  $O(n \log n)$

$A[P, \dots, n]$   
subindex

Average case:  $O(n \log n)$

~~AFP..~~

$A[P_1, \dots, q]$   
 $A[q+1, \dots, n]$

### Algo

mergesort ( $A, P, n$ )

if  $P > n$

return

$q = (P+n)/2$

mergesort ( $A, P, q$ )

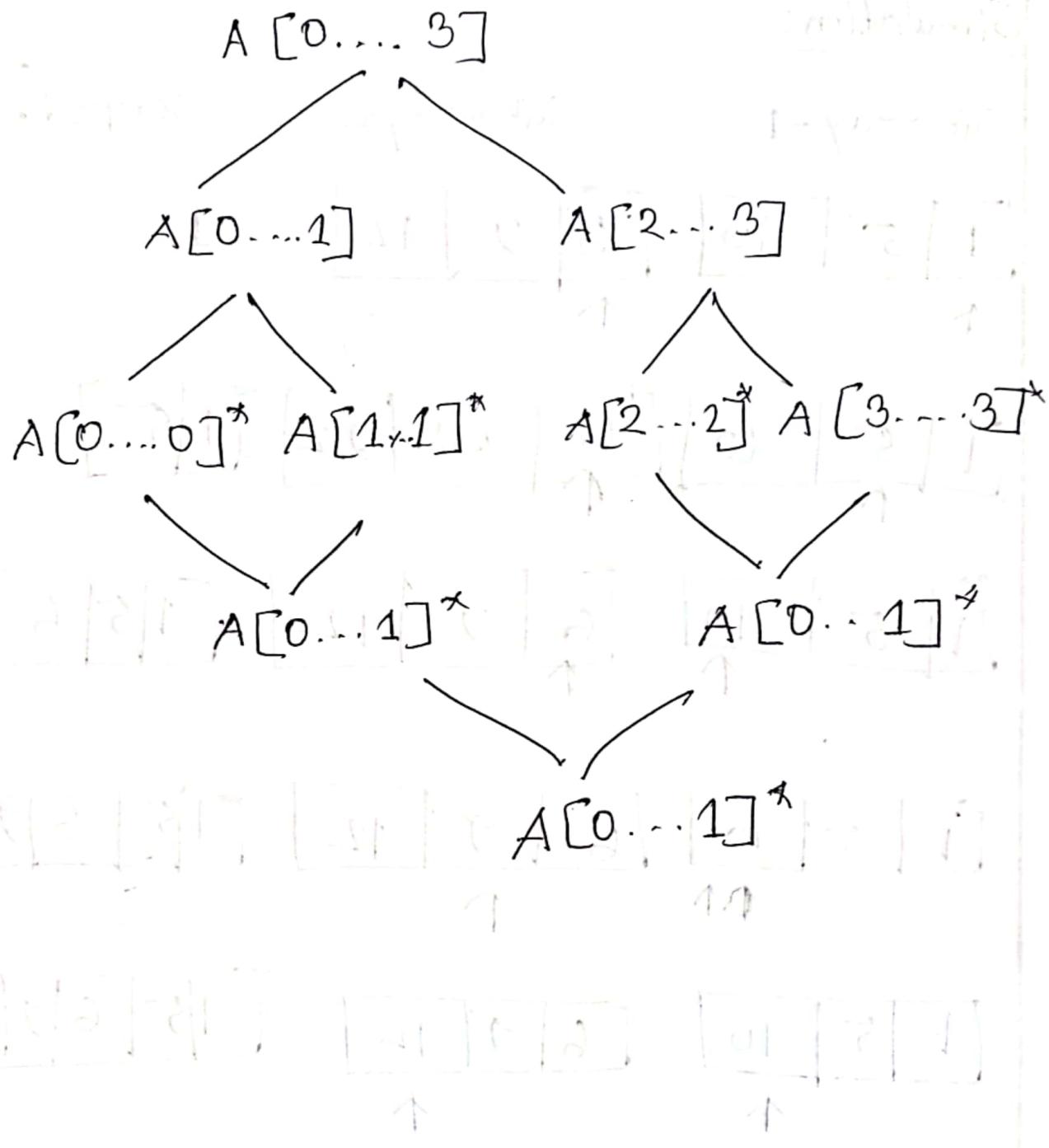
mergesort ( $A, q+1, n$ )

merge ( $A, P, q, n$ )

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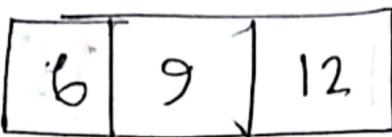
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## Simulation:

Subarray -1

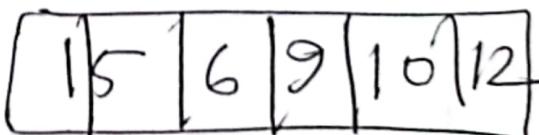
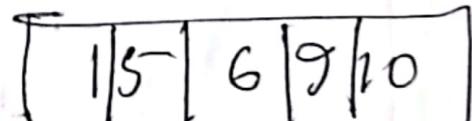
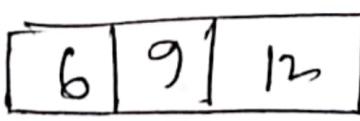
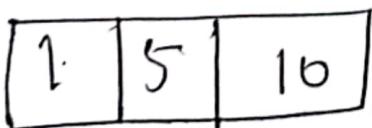
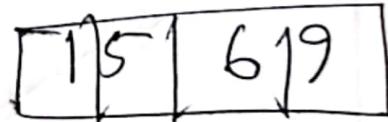
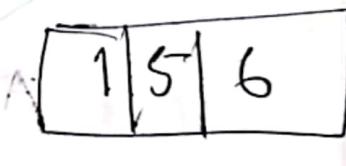
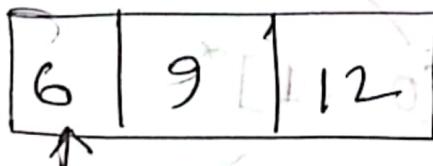
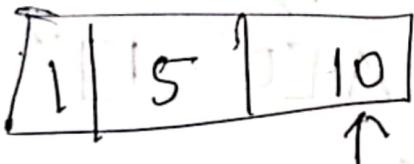
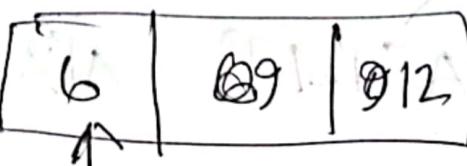
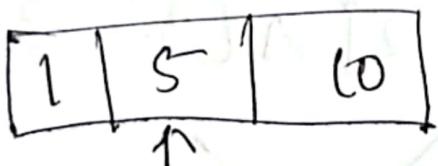


Subarray 2



sorted array <sup>comb'.</sup>

1



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Bstnode\* insertvalue (Bstnode\* root, int x)

{ if (root == NULL) {  
    root = new Bstnode(x);  
    return root; }

else if (x < root->data) {

    root->left = insertvalue (root->left, x); }

else if (x > root->data) {

    root->right = insertvalue (root->right, x); }

} // (if fail) addition fail

else if (root->data == x) {

    cout << "Value already present" << endl;

    return root; }

else if (x < root->data) {

    root->left = insertvalue (root->left, x); }

else if (x > root->data) {

    root->right = insertvalue (root->right, x); }

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void recursion (int n) {

visualgo.net.

printf()  
{ cout << n << endl; }  
recursion (int n-1);  
}

int recursion (int n) {

cout << n;  
if (n==0 (n==0) { return 0; }  
return recursion (int n-1);  
}

## Quick Sort:

\* Input → An Array, First element of the array, <sup>→ low</sup> last element of the array, <sup>→ high</sup>.

\* Output → No return value but when finish the algorithm array will be sorted.

\* Step 1 → If the array has less than one element, no return the array.

\* Step 2 → We partitioned the array and last element of the array should suit for this position.

→ From the first element or low element of the array to  $(P-1)$  part then we have to quick sort.

Step-3 → Initialize the PIVOT value.

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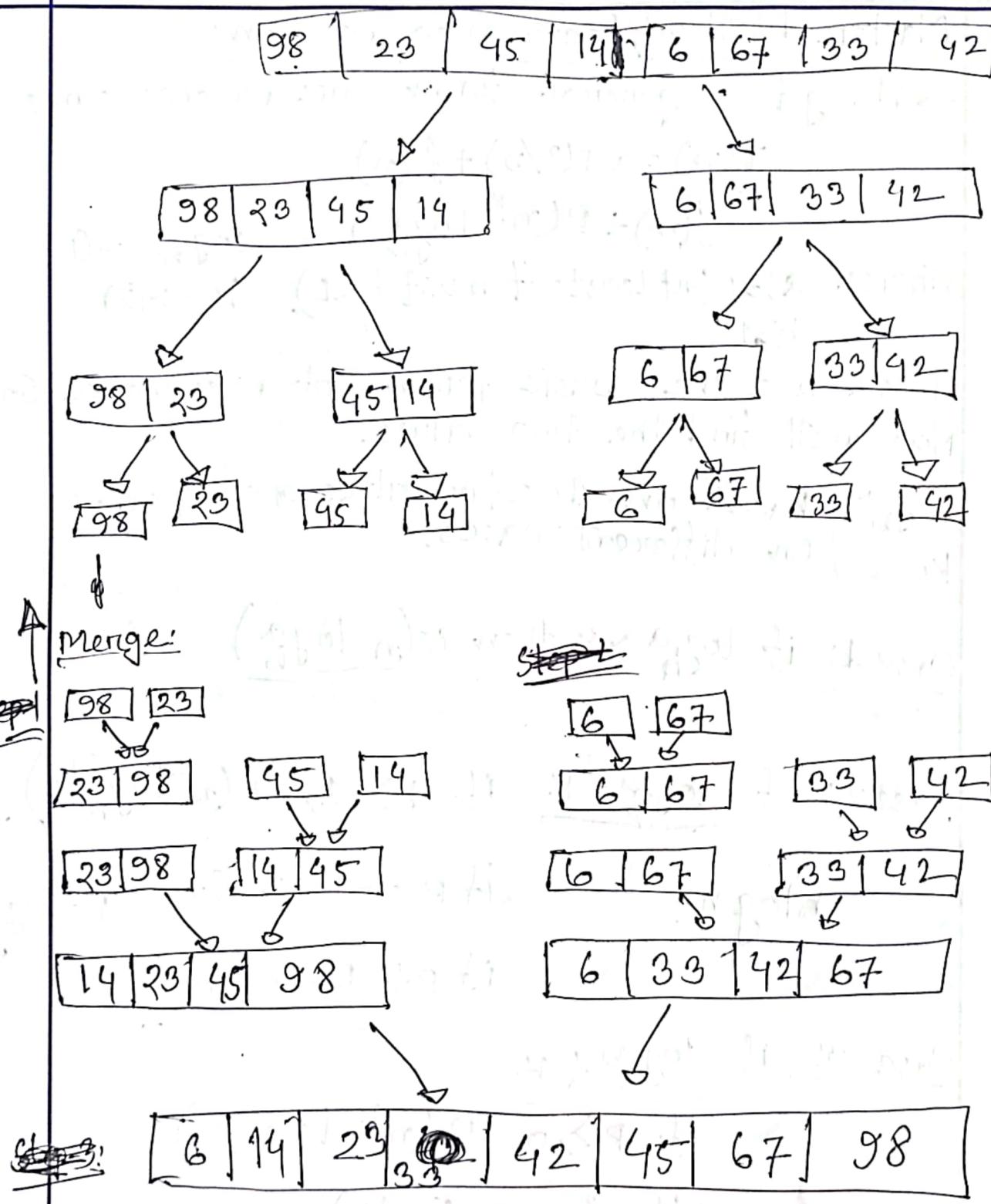
```
int Partition(int A[], int low, int high) {  
    int pivot; i, j, temp;  
    pivot = A[high];  
    for (i = low; i <= high; i++) {  
        if (A[i] < pivot) {  
            for (j = low + 1; j >= low; j--) {  
                if (A[i] < pivot) {  
                    i = i + 1;  
                    temp = A[j];  
                    A[j] = A[i];  
                    A[i] = temp;  
                }  
            }  
        }  
    }  
}
```

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# Merge Sort

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## Master Method/Recurrence Solution:

→ We get a general form for recurrence Sol<sup>n</sup>.

$$T(n) = aT(n/b) + f(n).$$

$$f(n) = O(n^k \log_n^p), \quad \log_b \rightarrow ①$$

Where  $a \geq 1$  (at least it must be),  $k \rightarrow ②$   
 $b > 1$

These are the basic form of recurrence Sol<sup>n</sup>.  
Now we'll find the two values.

$\log_b$  based on these two values are depends  
on different cases.  
K

Case 1: if  $\log_b^a > k$  then  $\Theta(n \log_b^a)$

Case 2: if  $\log_b^a = k$  if  $p > -1$ ,  $\Theta(n^k \log_n^{p+1})$ .

$n \log_n$  if  $p = -1$ ,  $\Theta(n^k \log \log n)$   
if  $p < -1$ ,  $\Theta(n^k)$

Case 3: if  $\log_b^a < k$

if  $p > 0$ ,  $\Theta(n^k \cdot \log_n^p)$

if  $p \leq 0$ ,  $\Theta(n^k)$ ,

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defug, take some recurrence that are  
satisfy case 1)

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$$a=2, b=2,$$

$$T(n) = 2T(n/2)$$

$$\boxed{P=K}$$

$$f(n) = \Theta(n^k \log_n P)$$

$$= \Theta(n^0 \log_1^0) = (\text{both are zero}) \quad \underline{\underline{P=K=0}}$$

$$= \Theta(1)$$

Complexity should consider the case 1.

Solve this comp  $f(n)$  based on the case 1.

$$T(n) = 8T(n/2) + n$$

$$\log_b a = \log_2 8 = 3$$

$$f(n) = \Theta(n^0, n^1, n^2) \log_n^{P=0,1,2}$$

$$K=0,1,2.$$

$$= \Theta(n^3 \log_1^0) \quad \left| \begin{array}{l} \Theta(n^3 \log_1^1) \\ \vdots \\ \Theta(n^3) \end{array} \right.$$

Best cases

$$\left. \begin{array}{l} \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{array} \right\} \text{at } T(n/2) \approx \Theta(n^3)$$

$$\log_a b = 3$$

$a/b$

Q12 → Insertion, Selection, Merge.  
Subject Simulations.

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Int main() {

int a = 5;

int b = 10;

int c = a + b; } complexity depends on operation}.

cout << c;

}

for loop  $O(n)$ .

double for loop  $O(n) \times O(n) = O(n^2)$ .

Mid

\* dynamic programming ✓ ✗

\* longest +~~sub~~ common subsequence,

\* Matrix chain (L.C.S) ✓ ✗

Multiplication ✓

\* ~~Graph~~

\* ~~Tree~~ → Knapsack algorithm

\* Sorting (Merge / insertion / selection /  
Quick / ~~linear~~ search / Counting )

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## Dynamic Programming

- Dynamic Programming are used in optimization Problems
  - Dynamic Programming solves the problem by combining the solutions of subproblems.
  - Dynamic programming algorithm solves each subproblem once and then saves its answer. In that case avoiding the work of recomputing.
  - Two types of properties can be given to solve that using the dynamic programming.
    - \* Overlapping sub-problems. approach similar to divide and conqueror each. Dynamic programming also combine the solutions of the subproblems. It is also used to solve the problem using repeated approach.
- Example: Fibonacci numbers have many overlapping subproblems.

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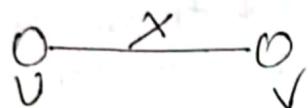
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## Optimal Sub-Structure:

If the optimal solution of the given problem can be obtained using optimal solutions of its sub-problems.

Eg: The shortest path problem has the following optimal sub-structure properties.



$$U \rightarrow X$$

$$X \rightarrow V$$

Dynamic programming algorithm is used to follow 4 steps.

1. It's optimal Solution.
2. Recursive, the values of optimal solution.
3. Compute the value of optimal solution.
4. Compute an optimal solution from the computed information.

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Applications:

Longest common Subsequence.

Matrix Chain Multiplication.

Travelling Salesman Problem.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times B \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$$

$$\begin{aligned} C &= A \times B \\ &= (a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31}) + (a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) \\ &\quad + (a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31}) + (a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) \end{aligned}$$

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## Parenthesized

|                       |                       |                       |                       |                       |                       |                       |
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## Matrix Multiplication (A, B):

If columns[A] != rows[B] then

error "Incompatible dimensions"

else for i=1 to rows[A] do

for j=1 to column[B], do {

$$C[i, j] = 0$$

for k=1 to column[A] do

$$C[i, j] = C[i, j] + A[i, k] \times B[k, j]$$

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## Merge Sort

### Pseudocode

#### bubble sort

begin BubbleSort(list)

for all elements of the list

if list[i] > list[i+1]

swap (list[i], list[i+1])

endif

end for

end BubbleSort.

void bubblesort (int arr[], int n) {

int i, j;

for (i=0; i<n-1; i++) {

for (j=0; j<n-1-i; j++) {

if (arr[j] > arr[j+1]) {

swap (arr[j], arr[j+1]);

}

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## Selection Sort

Start procedure selectionsort (list[], n) {

    max = length (list - 1)

    for i from 0 to max

        key = list [i]

        key [j] = i

        for j from i+1 to max

            if list [j] < key

                key = list [j]

                key [j] = j

                list [key] = list [j]

                list [i] = key

    return list

end for

end for

end procedure selectionsort

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## Insertion sort:

start procedure insertionsort (array [ $n$ ],  $n$ )

for i from 1 to  $n$

key = array [i]

while  $j \geq 0$  and  $a[j] > \text{key}$

$a[j+1] = a[j]$

$j = j - 1$

array [ $j+1$ ] = key

end for

end procedure insertionsort;

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Recursion :  $f(n) = 2n + 1$  for natural numbers

Basic clause:  $f(0) = 1$

Inductive clause  $f(n+1) = 2f(n) + 2$

Master method

$$T(n) = \underset{= b}{\cancel{a}} T(\frac{n}{b}) + f(n) \quad f(n) = \log n$$

\*  $T(n)$  function solution:

funcA (Array A, leftIndex, rightIndex, increment)

if (leftIndex == RightIndex) return;

for (i = leftIndex; i < rightIndex; i++)

    Array A[i] = Array[i] + increment;

    mid = (l + r) / 2

    funcA (Array A, l, m, increment)

    funcA (l ~ u , m, r, increment),

    i (u ~ , l , m , increment + 1),

}

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## Quicksort $O(n \log n)$

### Algorithm:

1. The Quicksort function is called with input index A, starting index p, ending index n, if  $p \leq n$ , The PARTITION subroutine

$A[0:n] \leftarrow \text{PARTITION}(A, p, n)$

$A[0:p-1] \leftarrow \text{QUICKSORT}(A, 0, p-1)$

~~$A[p+1:n] \leftarrow \text{QUICKSORT}(A, p+1, n)$~~

$T(n) = (n)^{1.5}$

$T(n) = 78$

$T(n) = 78$

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Quick sort: (Pivot point) let First element

$n \log n$

Pivot left  $\rightarrow$  small

Pivot right  $\rightarrow$  big - pivot

left ++

right ++

left < right

$$f(n) \geq n^{(\log_b)^a}$$

$$n^{(\log_b)^a}$$

$$f(n) = n^{(\log_b)^a}$$

$$n \log n$$

$$f(n) \gg n \log_b a$$

~~Don't do~~

$$T(n) = O(f(n))$$

$2T\left(\frac{n}{2}\right) + w$

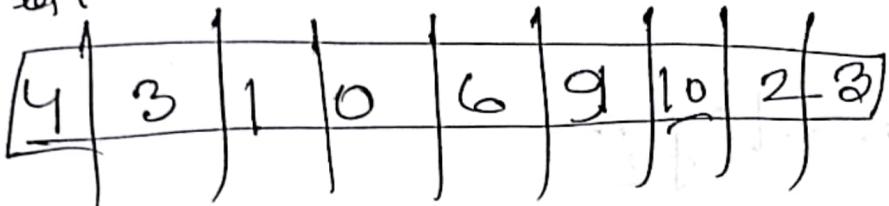
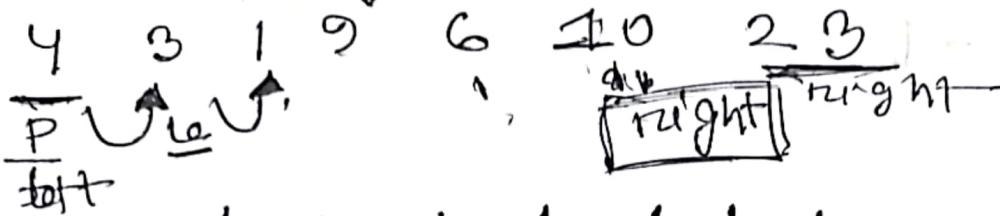
$$3^k T\left(\frac{n}{3^k}\right) +$$

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~~left~~ ~~right~~



Merge Sort: (Divide and Conqueror)

Repeated Substitution:

Quick Sort

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{q}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$$

$$= 2 \left[ 2T\left(\frac{n}{4}\right) + \frac{n}{2} \right] + n$$

first substitution

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$= 2^2 \left[ 2T\left(\frac{n}{4}\right) + \frac{n}{2} \right] + 2n$$

→ 2nd sub.

$$= 2^3 \cdot \left[ 2T\left(\frac{n}{2^3}\right) + \frac{3n}{2} \right] + 4n$$

$$2^k T\left(\frac{n}{2^k}\right) + nk$$

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$$T\left(\frac{n}{2^k}\right) = 1$$

$$\frac{n}{2^k} = 1$$

$$2^k = n; k = \log_2 n$$

~~$$2 \log_2 n + kn$$~~

double loop

$$T(n)$$

$$\underbrace{2 \log_2 n}_{\text{loop}} + \underbrace{\log_2 n n}_{\text{inner}}$$

$$= n \log n$$

$$T(n) =$$

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## Quick Sort:

```
int partition(int arr[], int low, int high) {  
    int left, right, pivot;  
    pivot = arr[low];  
    left = low;  
    right = high;  
    while (arr[left] < pivot) {  
        while (arr[right] > pivot) {  
            right--;  
        }  
        if (left < right) { swap(arr[left], arr[right]); }  
        left++;  
    }  
    arr[low] = arr[right];  
    arr[right] = pivot;  
    return right;  
}
```

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```
void Quick_Sort(int arr[], int low, int high){  
    int pivot;  
    if (low < high){  
        pivot = partition(arr[], low, high);  
        quickSort(arr[], low, pivot - 1);  
        quickSort(arr[], pivot + 1, high);  
    }  
}
```

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More voids Merge(int arr[ ], int low, int mid, int high)

int i = low;

int j = mid+1;

int k = 0;

int arr2[high - low + 1];

while (i <= mid && j <= high) {

if arr[i] < arr[j] {

arr2[k] = arr[i];

i++;

k++;

else { arr2[k] = arr[j];

j++;

k++; }

while (j <= high)

{ arr2[k] = arr[j];

j++;

k++; }

}

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for ( $i = low; i \leq high; i++$ ) {

    arr[i] = arr[2[i - low]];

}  
}

void mergeSort(int arr[], int low, int

int mid = (low + high) / 2

if ( $low < high$ ) {

    mergeSort(arr[], low, mid);

    mergeSort(arr[], mid + 1, high);

    merge(arr[], mid, low, high);

}

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## QuickSort Simulation

5 2 4 6 1 3 9 7  
5 2 4 6 1 3 9 7  
pivot = 5, left

partition the list into two small sub array  
low = 5, high = 7

{2, 4, 1, 3}, [5], {6, 9, 7}

one less than pivot      another higher than pivot

Recursively sort the two sub arrays.

{2, 4, 1, 3}

6, 9, 7

2 pivot

{1} {2} {4, 3}

6, 9, 7  
sub array

1, 2, 3, 4

6, 7, 9

now sorted

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Merge Sort,

5, 2, 4, 6, 1, 3, 9, 7

5, 2, 4, 6, 1, 3, 9, 7

5, 2      4, 6      1, 3      9, 7

1)      1)      1)      1)  
5, 2      4, 6      1, 3      9, 7

1, 2, 3, 4, 5, 6, 7, 9

2, 7, 8, 6      1, 3, 7, 9

2, 5      9, 6      1, 3      7, 9

5, 2      4, 6      1, 3      9, 7

## Merge Sort Time Complexity.

$\Theta(n \log n)$  with  $S$  of  $n$  integers and another integer  $x$ , two element  $S$ , whose sum is  $x$

- \* Sort the set using merge sort algorithm,
- \* Initialize two pointers, left and right , while  $left < right$   
check if the sum of the elements point by the left and right is equal to  $x$ .  
if yes, return true  
if sum less than  $x$ ,  $left = left + 1$   
if sum greater  $x$ ;  $right = right - 1$ ,

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## Analysing Binary search $O(\log n)$

1. Initialize the search interval by setting the lower bound 'lo' to the first index and upper bound 'hi' to the last.

Analysize → Algorithm, Implement → Code

Recursive (to count zero)

```
int CountZero (int arr[] ; int size) {  
    if (size == 0) return 0;  
    if (arr[0] == 0) return 1 + CountZero (arr+1, size-1);  
    return CountZero (arr+1, size-1);  
}  
for find mind  
if (size == 1) return arr[0];
```

$$T(n) = 2T(n/2) + O(1)$$

recursion

$$\alpha \log_b a = \log_2 2 = 1,$$

relation

$$\log_b a = f(n)$$

So, the time complexity is  $\log(n)$

$$T(n) = \underline{T(n/2)} + T(n/4) + \underbrace{O(n)}_{f(n)}$$

Using Master method.

$$T(n) = aT(n/b) + \underbrace{f(n)}_{\text{amount of work outside}}$$

the recursive calls.

$$a=1, b=2, d=1$$

$$\log_2 1 = 0$$

$$a < b^d \quad O(n^{d \log n}) = \Theta(n)$$

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## Recursive definition of Sum of Integers

$F(0) = 0$  (base case)

$F(n) = n + F(n-1)$  for  $n > 0$  (recursive case)

Sum of odd numbers,

sum\_of\_odd (n):

pseudocode

if  $n = 1$ :

return 1

$x^n$

$x^3, xx$

else:

return sum\_of\_odd ( $n-1$ ) +  $(2^n - 1)$ ;

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## Matrix Chain Multiplication : LCM Pseudocode

### Matrix

### Master Method

$$T(n) = 3T\left(\frac{n}{4}\right) + n^{2 \log 3}$$

$$f(n) = n^{2 \log n} = \Theta(n^2)$$

$$af\left(\frac{n}{b}\right) = 3\left(\frac{n}{4}\right)^{2 \log\left(\frac{n}{4}\right)}$$

$a f\left(\frac{n}{b}\right) \leq f(n)$  so,

time complexity of the algorithm

$$\Theta(n^{2 \log n})$$

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$$T(n) = 2T(n/3) + n/\log n$$

$$\Theta(\log n)$$

$$\log_b a = \log_3 2 = -$$

$$4T(n/2) + n^3 \quad f(n) = n^c = n^3$$

$$\log_2 4 = 2 \quad O(n^2)$$

$$\log_a b < 0 \quad n/\log n$$

$$5T(n/5) + n \quad n/\log n$$

sub problems = 5

each problem size =  $\frac{n}{5}$ 

$f(n) = n$  (costime complexity of dividing  
each problem and combining)

$f(n)$  with  $n^c$  compare

$$c = \log_5 5 = 1 \quad a = \log_5 5$$

$$\underline{\Omega(n \log n)}$$

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$$T(n) = 3T(n/4) + \underline{n^2/\log n}$$

$$\log_4 3 = 0.79 \quad f(n/b) = (9/16) \frac{n^2}{\cancel{\log n - \log 4}}$$

$$n^c = n^{\log_4 3}$$

$$cf(n) = c n^2 / \log n$$

$$T(n) = \overline{f}O(n) = O(n^2/\log n)$$

$$T(n) = 4T(n/2) + \frac{n^2}{\log_2 n}$$

$$\log_b a = \log_2 4 = 2$$

$$f(n) = n^2 / \log_2 n = \Omega(n^c)$$

$$\text{so } 2T(n/4) + \sqrt{n}$$

$$\log_b a = \log_4 2 = \frac{1}{2} \quad O(\sqrt{n})$$

$\frac{1}{2} < \sqrt{n}$ , so Case 3

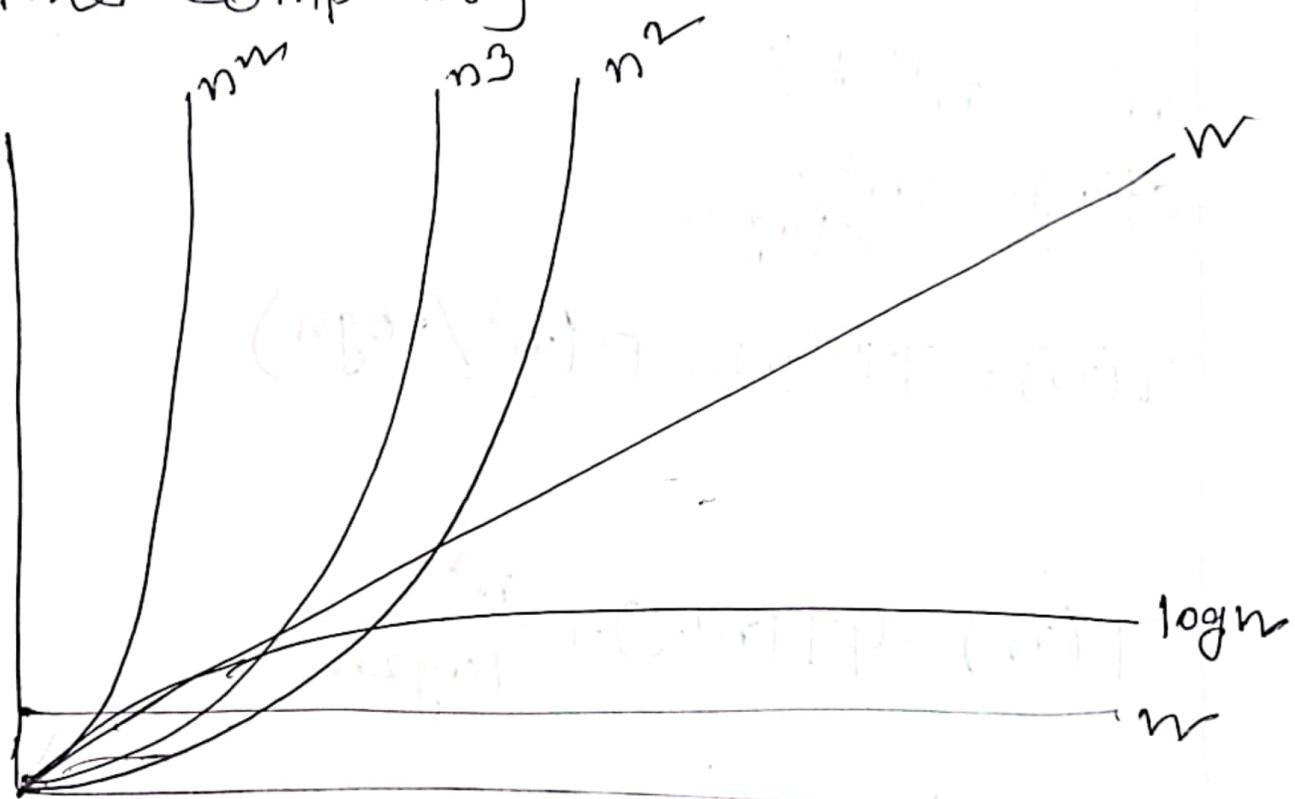
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$$f(n) = n^{\log_b a}$$

Time Complexity



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## Time Complexity

$$T(n) = 2T(n/2) + n$$

$$\log_a b = 1$$

$$f(n) = n^{\log_a b}$$

So, time complexity  $O(n \log n)$

$$T(n) = 4T(n/2) + n^2$$

$$\log_b a = 2 \quad O(n \log n)$$

$$f(n) = \log_b a$$

$\log_3 3 < 1$ , polynomially larger

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## Dynamic Problem (Avoid Repetition)

$$\underbrace{T(n-1) + f(n)}_{\text{Recurrence relation}} \geq \underbrace{O(f(n)) \times n}_{\text{Time complexity}}$$

$$T(n) = 2T(n/2) + n$$

$$T(n) = 2^k T(n/2^k) + nk$$

$$\frac{n}{2^k} = 1 \quad \Rightarrow \quad n = 2^k$$

$$k = \log_2 n \quad \Rightarrow \quad k = \log n$$

$$k = \log n$$

$$T(n) = 2^k T(1) + kn$$

$$= n \times 1 + n \log n$$

$$= n \log n$$

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# Master Method

a

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$f(n) = O(n^k \log^p n)$$

$$\text{if } \log_b a > k \quad O(n^{\log_b a})$$

$$\log_b a = k \quad O(n \log$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$\log_b a = 1$$

$$f(n) = O(1)$$

$$= O(n^0 \log n^0)$$

$$O(n^1)$$

$$\log_2 4 = 2$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

$$f(n) = O(n^1, k=0, p=0)$$

$$O(n^{1+2})$$

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$$T(n) = 8T(n/2) + \text{constant}$$

$$T(n) \rightarrow n^k \log^p n$$

$$T(n) = 2T(n/2) + n$$

$$\log_2 2 = 1; \quad k=1, \quad p=0$$

$$O(n \log n)$$

if  $\log_b a = 1$

$$\underline{\underline{O(f(n) \log n)}}$$

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$$T(n) = 2T(n/2) + \frac{n}{\log n}$$

$$\log_2 2 = 1, k=1, \rho = -1$$

~~$$\frac{n}{\log n}$$~~

$$T(n) = 2T(n/2) + n^2$$

$$\log_2 1 = 0, k=2 \geq 2T(n/2) + n^2$$

$$O(n^2)$$

$$O(n^2 \log n)$$

$$T(n) = 2T(n/2) + n^2$$

$$T(n) = 2T(n/2) + n^2$$

$$T(n) = 2T(n/2) + n^2$$

$f(n)$  $n^0$ 

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(n)

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$$2T\left(\frac{n}{2}\right) + 1 \rightarrow \Theta(n)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + 1 \rightarrow \Theta(n^2)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n \rightarrow \Theta(n^2)$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n^2 \rightarrow \Theta(n^3)$$

$$T(n) = 16T\left(\frac{n}{2}\right) + n^2 \rightarrow \Theta(n^4)$$

case - 3

$$T(n) = T\left(\frac{n}{2}\right) + n^{\frac{1}{2}} \Theta(n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^{\frac{1}{2}} \Theta(n^2)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \underline{n^{\frac{1}{2}} \log n} - \Theta(n^{\frac{3}{2}} \log n)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^{\frac{3}{2}} \log n - \Theta(n^3 \log n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \cancel{n^{\frac{3}{2}} \log n} - (n^{\frac{3}{2}} \log n)$$

 $f(n)$ 
 $\frac{n^{\frac{3}{2}} \log n}{\log n}$ 
 $n \log n$

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Case 2 =

$$T(n) = T(n/2) + \underline{1} \quad \Theta(\log n)$$

$$T(n) = 2T(n/2) + \underline{n} \quad \Theta(n \log n)$$

$$T(n) = 2T(n/2) + n \log n \quad (n^{\cancel{\log n}})$$

$$T(n) = 4T(n/2) + n^{\cancel{\log n}} \quad \Theta(n^{\cancel{\log n}})$$

$$T(n) = 4T(n/2) + \frac{n^{\cancel{\log n}}}{2} \quad \Theta(n^{\cancel{\log^3 n}})$$

$$T(n) = 2T(n/2) + \frac{n}{\log n} \quad \Theta(n \log \log n)$$

$$T(n) = 2T(n/2) + \frac{n}{\log n} \quad n(t+O(n))$$

\* Counting Sort

\* Repeated substitute method

\* bubble sort

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$$T(n) = T(\sqrt{n}) + 1$$

$$T(n) = T\left(n^{\frac{1}{2^k}}\right) + k$$

$$\frac{1}{n^{\frac{1}{2^k}}} = \frac{m}{2^k} = 1$$

$$m = 2^k \quad k = \log_2 m$$

$$m = \log_2 n \quad k = \log \log_2 n$$

$$O(\log \log_2 n)$$

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## Quick Sort

int partition (arr \*int arr, int low, int high);

int left, right, pivot;

~~arr~~ [left] pivot = arr [low];

left = low;

right = high;

{while ~~arr~~ (left < right) {

    while to arr [left] < pivot {

        left++; }

    while (arr [right] > pivot) {

        right--; }

    if (left < right) hswap (arr [left], arr [right]);

    arr [low] = arr [high];

    arr [right] = pivot;

    return right;

    }

void Quick\_Sort (int arr[],

                  int low, int high)

    pivot = partition (arr [], low, high);

    Quick\_Sort (arr [], low, pivot - 1);

    Quick\_Sort (arr

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Q4. Write Quick Sort (int arr, int left, int right) {

int low, high, pivot;

pivot = arr[low];

left = low;

right = high;

while (left < right) {

if arr[left] < pivot {

left++; }

while (arr[right] > pivot) {

right++; }

if (left > right) { swap(arr[left], arr[right]); }

arr[low] = arr[high];

arr[right] = pivot;

return right; }

```
void Merge (int arr[], int low, int mid, int high) {  
    int i = low;  
    int j = mid + 1;  
    int k = 0;  
    int arr2[] = arr[low:high-low+1];  
    while (i <= mid && j <= high) {  
        if (arr[i] < arr[j]) {  
            arr2[k] = arr[i];  
            i++;  
            k++;  
        } else {  
            arr2[k] = arr[j];  
            j++;  
            k++;  
        }  
    }  
    while (i <= mid) {  
        arr2[k] = arr[i];  
        i++;  
        k++;  
    }  
    while (j <= high) {  
        arr2[k] = arr[j];  
        j++;  
        k++;  
    }  
    for (int i = low; i <= high; i++) {  
        arr[i] = arr2[i - low];  
    }  
}
```

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```
void Merge (int arr[], int low, int mid, int
            int i = low;
            int j = mid + 1;
            int k = 0
            int arr2[] = int [high - low + 1];
            while (i <= mid && j <= high) {
                while (arr[i] < arr[k]) {
                    arr2[k] = arr[i];
                    i++;
                    k++;
                }
                else {
                    arr2[k] = arr[j];
                    j++;
                    k++;
                }
            }
            while (j < high) {
                arr2[k] = arr[j];
                j++;
                k++;
            }
            for (int i = 0; i < high; arr[i]++) {
                arr[i] = arr2[i - low];
            }
        }
```

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start procedure Merge (arr[], low, kmid, high)  
initiate i=low, j=mid+1, K=0, arr2[k]=arr[high-low+1];  
do when  $i \leq \text{mid}$  &  $j \leq \text{high}$   
do when  $\text{arr}[i] < \text{arr}[j]$   
 $\text{arr2}[K] = \text{arr}[i]$ ;  
 $i++$ ;  
 $K++$ ;  
else do  $\text{arr2}[K] = \text{arr}[j]$ ;  
 $j++$ ;  
 $K++$   
when ( $j < \text{high}$ )  
do  $\text{arr2}[K] = \text{arr}[j]$ ;  
 $j++$ ;  
 $K++$   
for i=0 to  $i \leq \text{high}$   
do  $\text{arr}[i] = \text{arr2}[i - \text{low}]$   
end for  
end procedure

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## Asymptotic Notation

- It represents the simple form of time complexity.
- O-Big-oh notation represents the Upper bound notation of the function.
- Ω-Big Omega notation represents or waves as a lower bound of the function.
- Θ- Theta Notation works as a average bound of a function.
- Any functions either upper or lower bound or average.
- When finds the time complexity of any algorithm time complexity will be one among.

# Binary / linear search

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## Big Oh

$$1 < \log n < \sqrt{n} < n < n \log n < n^v < n^3 < \dots < 2^n < 3^n < \dots < n^w$$

let's say:

The function  $f(n) = O(g(n))$  if there exist positive constants  $c$  and  $n_0$ .

Such that  $f(n) \leq c * g(n)$  for all  $n > n_0$

if we have a few eg.:  $f(n) = 3n + 4$

$$\frac{3n+4}{f(n)} \leq \frac{5n}{c * g(n)} \text{ for all } n > n_0$$

$$f(n) = O(n),$$

$c$ -values we can write any numbers but make sure  $f(n)$  is less than the value of  $c * g(n)$  we can put any other value

$$3n+4 \leq 2n+3n \quad O(n)$$

$$3n+4 \leq 5n \text{ for all } n > n_0$$

\* If the function put the different

$$3n+4 \leq 3n+4n \quad f(n) = O(n)$$

$$3n+4 \leq 7n \quad \text{for all } n > 1$$

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### Omega notation, $\Omega$

$f(n) = \Omega(g(n))$  if exists positive constant  $C$  and  $n_0$  such that  $f(n) \geq Cg(n)$  for all  $n \geq n_0$ .

$$f(n) \geq 5n + 3$$

$$5n + 3 \geq 1^{\log n} \text{ for all } n \geq 1$$

$$f(n) = \Omega(n)$$

$$5n + 3 \geq \log n \text{ for all } n \geq 1 \quad f(n) = \Omega(\log n)$$

$$f(n) = \underline{5n + 3}$$

$f(n) = 5n + 3 = n$ , which is we can't write anything beyond the  $\log n$

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theta notation

$f(n) = \Theta(g(n))$  iff  $\exists$  positive constant  $C_1$  and  $C_2$ , and  $n_0$ .

$$C_1 * g(n) \leq f(n) \leq C_2 * g(n)$$

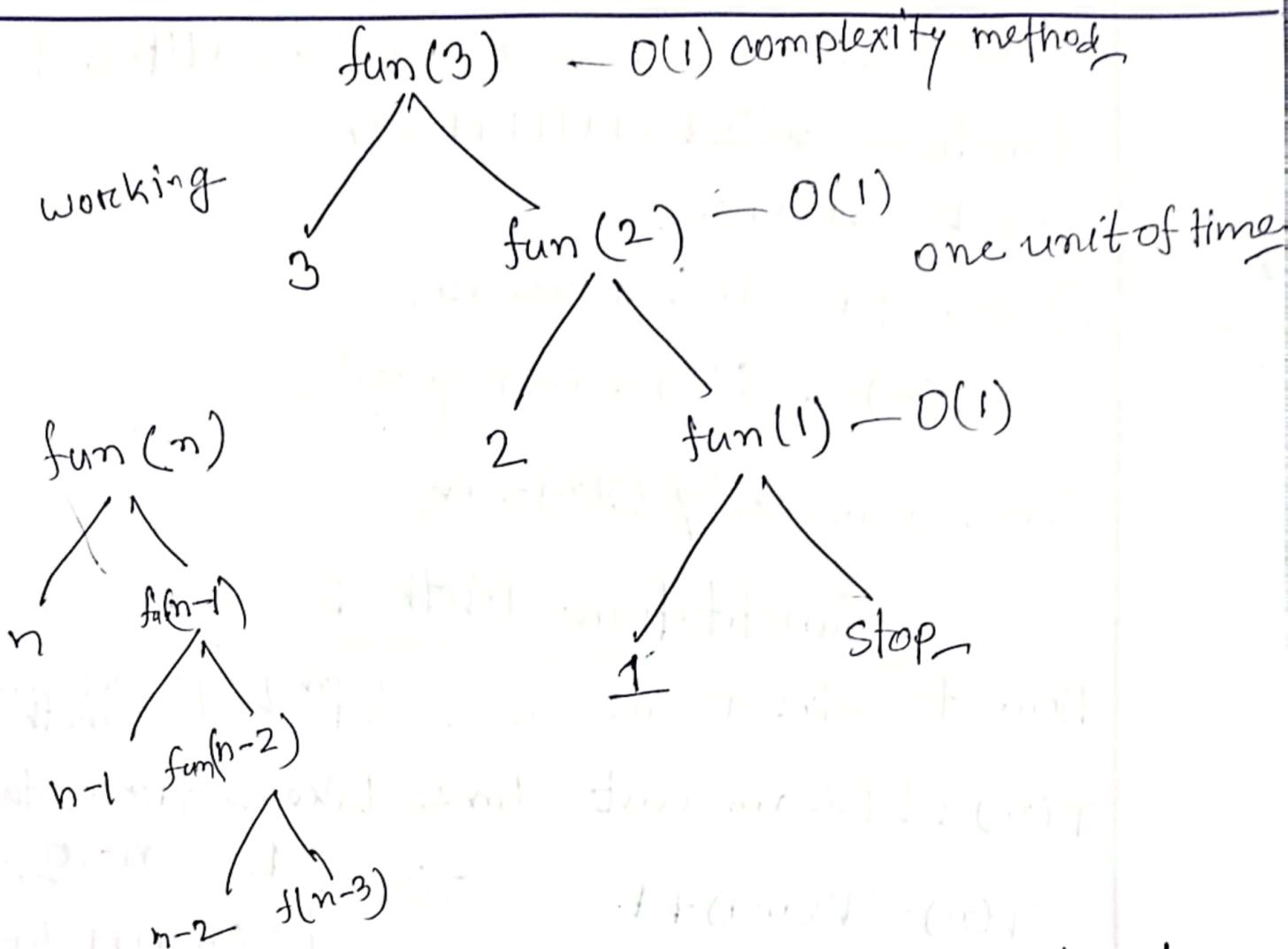
$$f(n) = 5n + 3$$

$$1 * n \leq 5n + 3 \leq 5n$$

$$C_1 = 1; f(n) = 5n + 3; C_2 = 5$$

$$f(n) = \Theta(n)$$

$f(n) = 5n + 3 = n$ , which is the true form less or equal to the function  $f(n)$ , for  $n$  class. Also is greater than or equal to the  $f(n)$  for  $n$  class.



It does not go further. It is called The tracing Tree; Recursive function for particular function.

- Amount of work is called just printing
- It is doing as 3 times as it's called the value of 4.
- First called and printing.

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→ Print the value and again call the function.

$$\text{function called} = 1+1+1+1 = 4$$

$$\text{print} = 1+1+1 = 3$$

if we pass the value  $n$ .

$$T(n) = f(n) = n+1 \text{ } \gamma \text{ calls}$$

Time complexity  $O(n) = n$

### Substitution Method

How to solve recurrence sol'n? [Recurrence method]

$T(n)$  = total amount time taken from statement

$$T(n) = T(n-1) + 1$$

$$T(n) \begin{cases} 1 & n=0; \\ T(n-1)+1 & \text{for all } n>0 \end{cases}$$