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1 2ECC

```
struct graph {
  int n, t, sz;
  vector<vector<int>> adj;
  vector<int> tin, low, cmp;
  graph(int n): n(n),adj(n),tin(n),low(n),cmp(n){}
  void add_edge(int u, int v){
   adj[u].push_back(v);
    adj[v].push_back(u);
  void dfs(int u, int p){
    tin[u]=low[u]=t++;
    int cnt=0;
    for(int v: adj[u]){
      if(v==p and ++cnt <= 1) continue;</pre>
      if(tin[v]!=-1) low[u] = min(low[u], tin[v]);
        dfs(v,u);
        low[u] = min(low[u], low[v]);
  void dfs2(int u, int p){
   if(p!=-1 \text{ and } tin[p]>=low[u]) cmp[u] = cmp[p];
    else cmp[u] = sz++;
   for(int v: adj[u]){
      if(cmp[v]==-1) dfs2(v,u);
  void process 2ecc(){
   t = 0, sz = 0;
    for (int i = 0; i < n; ++i){
      tin[i] = low[i] = cmp[i] = -1;
    for (int i = 0; i < n; ++i){
      if(tin[i]==-1) dfs(i,-1);
    for (int i = 0; i < n; ++i){
      if(cmp[i]==-1) dfs2(i,-1);
```

2 2SAT

```
//CNF: (a | b) ^{\circ} (c | d) means (!a -> b) ^{\circ}
// (!a or \dot{b}) = (-a, \dot{b}), 1-based indexing
string two sat(int n, vector<array<int, 2>>

    clauses) {
  vector<int> adj[2 * n];
for (auto [a, b]: clauses) {
    if (a > 0) a = 2 * a - 2;
    else a = 2 * -a - 1:
    if (b > 0) b = 2 * b - 2;
    else b = 2 * -b - 1;
    adj[a ^ 1].push_back(b), adj[b ^
 \rightarrow 1].push back(a);
  vector<vector<int>>> sccs = get sccs(2 * n, adj);
  int tot scc = sccs.size();
  vector<int> scc no(2 * n);
  for (int i = 0; i < tot scc; ++i) {
    for (int u: sccs[i]) {
```

```
scc_no[u] = i;
}
string assignment;
for (int u = 0; u < n; u++) {
   if (scc_no[2 * u] == scc_no[2 * u + 1]) {
      return "";
   }
   if (scc_no[2 * u] < scc_no[2 * u + 1]) {
      assignment += '-';
   }
   else {
      assignment += '+';
   }
}
return assignment;
}</pre>
```

3 AHO_CORASICK

struct AC{

```
const int A = 26;
vector<vector<int>>> nxt, idx;
vector<int> lnk, out lnk, ans;
AC(){newNode();}
int newNode(){
  nxt.eb(A, 0), idx.eb(0);
  lnk.eb(0), out lnk.eb(0), ans.eb(0);
  return nxt.size()-1;
void clear(){
  nxt.clear(), idx.clear();
  nxt.clear(), idx.clear();
  lnk.clear(), out lnk.clear(), ans.clear();
  newNode();
// 0(|p|)
void add(string p, int i){
  int v=0;
  v = nxt[v][c-'a'];
  idx[v].eb(i);
// 0(|p1+p2+p3+..|)
void build(){
  queue<int> q; q.push(0);
  while (!q.empty()){
    int u=q.front(); q.pop();
for (int i = 0; i < A; ++i){</pre>
      int v = nxt[u][i];
      if(!v) nxt[u][i] = nxt[lnk[u]][i];
        lnk[v] = u? nxt[lnk[u]][i]: 0;
        out lnk[v] = idx[lnk[v]].empty()?
 out lnk[lnk[v]]: lnk[v];
        q.push(v);
// O(|T|+match)
void trav(string T){
  int v=0:
  for(char c: T){
```

```
if(!nxt[v][c-'a']) v = lnk[v];
    if(nxt[v][c-'a']) v=nxt[v][c-'a'];
    for(auto& i: idx[v]){
        ans[i]++;
    }
    int x = out_lnk[v];
    while(x){
        for(auto& i: idx[x]){
            ans[i]++;
        }
        x = out_lnk[x];
    }
    x = out_lnk[x];
}
//AC ac; ac.add(pi, i); ac.build(); ac.trav(T);
```

4 ARTICULATION_BRIDGE

```
vector<int> adj[N];
int t = 0:
vector<int> tin(N, -1), lo(N);
vector<array<int, 2>> ab;
void dfs (int u, int p) {
 tin[u] = lo[u] = t++;
 for (int v: adj[u]) {
   if (v != p) {
      if (tin[v] != -1)
        lo[u] = min(lo[u], tin[v]);
      else {
        dfs(v, u);
        if (tin[u] < lo[v]) +
          ab.push back({u, v});
        lo[u] = min(lo[u], lo[v]);
dfs(0, -1);
```

5 ARTICULATION_POINT

```
vector<int> adj[N];
int t = 0;
vector<int> tin(N, -1), low(N), ap;
void dfs (int u, int p) {
 tin[u] = low[u] = t++;
 int is ap = 0, child = 0;
 for (int v: adj[u]) {
   if (v != p) {
      if (tin[v] != -1) {
        low[u] = min(low[u], tin[v]);
      else {
        child++;
        dfs(v, u);
        if (tin[u] <= low[v]) {
          is ap = 1;
        low[u] = min(low[u], low[v]);
```

```
}
if ((p != -1 or child > 1) and is_ap)
    ap.push_back(u);
}
dfs(0, -1);
```

6 BCC

```
struct graph {
 int n, t=0, cno=0;
 vector<vector<int>> q;
 vector<int> tin, lo, bcomp;
 stack <int> st;
 graph(int n):n(n),g(n),lo(n),bcomp(n){}
 void add edge(int u, int v){
   g[u].push back(v);
    g[v].push_back(u);
 void dfs(int v, int p=-1){
   lo[v]=tin[v]=++t;
    st.push(v);
   for(int u:g[v]){
     if(u==p)
                  continue;
      if(!tin[u]){
        dfs(u, v);
        lo[v]=min(lo[v],lo[u]);
     } else{
        lo[v]=min(lo[v],tin[u]);
    if(tin[v]==lo[v]){
      while (!st.empty()){
        int tp=st.top(); st.pop();
        bcomp[tp]=cno;
        if(tp==v)
                     break:
      ćno++:
  vector<int> bcc(){
   tin.assign(n, 0);
   for (int i = 0; i < n; ++i){
     if(!tin[i])
       dfs(i);
    return bcomp;
```

7 BCC_EDGE

```
int ch = 0;
  tin[u] = lo[u] = t++;
  for(auto [v, e] : adj[u]) {
   if (v == p) continue;
if (tin[v] != -1) {
      if (tin[u] > tin[v]) {
        lo[u] = min(lo[u], tin[v]);
        stk.push(e);
    else {
      ch++:
      stk.push(e);
      dfs(v, u);
      if ((p != -1 \text{ or } ch > 1) \text{ and } tin[u] <= lo[v]) {
        is ap[u] = 1;
        pop bcc(e);
      lo[u] = min(lo[u], lo[v]);
void procces bcc(int n) {
  for (int i = 0; i < n; ++i) {
    tin[i] = -1, is ap[i] = 0;
    bcc ed[i].clear();
    bcc[i].clear();
  t = tot = 0;
  for (int u = 0; u < n; ++u) {
    if (tin[u] == -1) {
      dfs(u, -1);
      if (!stk.empty()) {
        while (!stk.empty()) {
          bcc ed[tot].push back(stk.top());
   stk.pop();
        tot++;
  for (int i = 0; i < tot; ++i) {
    for (auto e: bcc_ed[i]) {
      auto [u, v] = \overline{e}dges[e];
      bcc[i].push back(u);
      bcc[i].push_back(v);
  for (int i = 0; i < tot; ++i) {
    sort(bcc[i].begin(), bcc[i].end());
    bcc[i].erase(unique(bcc[i].begin(),
   bcc[i].end()), bcc[i].end());
```

8 BINARY_LIFTING

```
void dfs(int u, int p){
  d[u]=d[p]+1;
  tin[u] = t++;
  par[u][0]=p;
  for (int i = 1; i < LGN; ++i){</pre>
```

```
par[u][i] = par[par[u][i-1]][i-1];
}
for(int& v: tr[u])
    if(v!=p)    dfs(v,u);
tout[u] = t++;
}
bool is_anc(int u, int v){
    return tin[u]<=tin[v] and tout[u]>=tout[v];
}
int lca(int u, int v){
    if(is_anc(u, v))         return u;
    if(is_anc(v, u))         return v;
    for(int i=LGN-1; i>=0; i--){
        if(!is_anc(par[u][i], v))         u=par[u][i];
    }
    return par[u][0];
}
int dist(int u, int v){
    int w = lca(u, v);
    return d[u]-d[w]+d[v]-d[w];
}
```

9 BIT_TRICKS

10 BLOCK_CUT_TREE

```
vector<int> adi[N];
vector<int> tin(N, -1), lo(N), is ap(N), bcc[N];
|stack<int> stk;
int t = 0, tot = 0;
void pop bcc(int u, int v) {
 bcc[tot].push_back(u);
 while (bcc[tot].back() != v)
    bcc[tot].push back(stk.top());
    stk.pop();
  tot++;
void dfs (int u, int p) {
 tin[u] = lo[u] = t++;
 stk.push(u);
  int ch = 0;
 for (auto v: adj[u]) {
    if (v != p) {
      if (tin[v] != -1)
        lo[u] = min(lo[u], tin[v]);
```

```
5
```

```
else {
        ch++;
        dfs(v, u);
        if ((p != -1 \text{ or ch} > 1) \text{ and tin}[u] <=
   lo[v]) {
          // is ap[u] = 1;
          pop bcc(u, v);
        lo[u] = min(lo[u], lo[v]);
void process bcc (int n) {
  for (int u = 0: u < n: ++u) {
    tin[u] = -1;
    is ap[u] = 0;
    bcc[u].clear();
  t = tot = 0;
  for (int u = 0; u < n; ++u) {
    if (tin[u] == -1) {
      dfs(u, -1);
      if (!stk.empty()) {
        while (!stk.empty()) {
          bcc[tot].push back(stk.top());
          stk.pop();
        tot++;
vector<int> comp num(N), bct adj[N];
void build bct(int n) {
  process_bcc(n);
  int nn = tot;
  for (int u = 0; u < n; ++u) {
    if (is ap[u]) {
      comp num[u] = nn++;
  for (int i = 0; i < tot; ++i) {
    for (auto u: bcc[i]) {
      if (is ap[u])
        u = \overline{comp} num[u];
        bct adj[i].push back(u);
        bct adj[u].push back(i);
      else {
        comp num[u] = i;
```

11 CDQ

```
- cdq(l, m)
- handle influence of (l, m) to (m + 1, r)
- cdq(m + 1, r)
## Convert dynamic array problems to static array
- problem
```

12 CENTROID_DECOMPOSITION

```
void calc sz(int u, int p) {
  sz[u] = 1;
  for (auto v: adj[u]) {
    if (v != p and !is cen[v]) {
      calc sz(v, u);
      sz[u] += sz[v];
|int get cen(int u, int p, int n) {
 for (auto v: adj[u]) {
    if (v != p \text{ and } ! \text{ is cen}[v] \text{ and } 2 * \text{sz}[v] > n) 
      return get cen(v, u, n);
  return u;
|void decompose(int u=0, int p=-1, int d=0){
  calc sz(u, p);
  int c = get_cen(u, p, sz[u]);
  is cen[c] = 1, cpar[c] = p, cdep[c] = d;
  for(int v: adj[c]){
    if(!is cen[v]) {
      decompose(v,c,d+1);
decompose();
```

13 CLOSEST PAIR OF POINTS

```
ll min dis(vector<array<int, 2>> &pts, int l, int
→ r) {
 if (l + 1 >= r) return LLONG MAX;
  int m = (l + r) / 2;
  ll my = pts[m-1][1];
 ll d = min(min dis(pts, l, m), min_dis(pts, m,
 inplace merge(pts.begin()+l, pts.begin()+m,
   pts.begin()+r);
 for (int i = l; i < r; ++i) {
  if ((pts[i][1] - my) * (pts[i][1] - my) < d) {</pre>
      pts[j][0]) * (pts[i][0] - pts[j][0]) < d; ++j) {
        [i] dx = pts[i][0] - pts[j][0], dy =
   pts[i][1] - pts[j][1];
        d = min(d, dx * dx + dy * dy);
  return d:
|vector<array<int, 2>> pts(n);
|sort(pts.begin(), pts.end(), [&] (array<<mark>int</mark>, 2> a,
\rightarrow arrav<int. 2> b){
```

14 CONVEX_HULL

```
struct pt {
 int x, y;
ii cross(pt a, pt b, pt c) { //ab*ac
 return 111*(b.x-a.x)*(c.y-a.y) -
\rightarrow 1ll*(c.x-a.x)*(b.y-a.y);
vector<pt> convexHull(vector<pt>& p) {
 sort(p.begin(), p.end(), [\&] (pt a, pt b) {
    return (a.x==b.x? a.y<b.y: a.x<b.x);
  int n = p.size(), m = 0;
  vector<pt> hull(2*n);
  for (int i = 0; i < n; ++i){
    while (m>=2 and cross(hull[m-2], hull[m-1],
    p[i]) < 0) --m;
    hull[m++]' = p[i];
 for (int i = n-2, l = m; i >= 0; --i) {
    while(m>=l+1 and cross(hull[m-2], hull[m-1],
    p[i]) < 0) --m;
    hull[m++] = p[i];
  hull.resize(m-1);
  return hull:
```

15 CONVOLUTION

```
## FFT
struct cplx {
 ld a, b;
  cplx(ld a=0, ld b=0):a(a), b(b) {}
  const cplx operator + (const cplx &z) const {
→ return cplx(a+z.a, b+z.b); }
 const cplx operator - (const cplx &z) const {

¬ return cplx(a-z.a, b-z.b); }

 const cplx operator * (const cplx &z) const {
\rightarrow return cplx(a*z.a-b*z.b, a*z.b+b*z.a); }
 const cplx operator / (const ld &k) const {
   return cplx(a/k, b/k); }
const ld PI=acos(-1);
vector<int> rev;
void pre(int sz){
 if(rev.size()==sz) return ;
  rev.resize(sz);
  rev[0]=0;
 int lg n = _builtin_ctz(sz);
for (int i = 1; i < sz; ++i) rev[i] = (rev[i>>1]
\rightarrow >> 1) | ((i&1)<<(lq n-1));
void fft(vector<cplx> &a, bool inv){
 int n = a.size();
 for (int i = 1; i < n-1; ++i) if(i<rev[i])

    swap(a[i], a[rev[i]]);
```

```
for (int len = 2; len <= n; len <<= 1){
    ld t = 2*PI/len*(inv? -1: 1);
    cplx wlen = {cosl(t), sinl(t)};
    int st = 0;
    for (int st = 0; st < n; st += len){</pre>
      cplx w(1);
      for (int i = 0; i < len/2; ++i){
        cplx ev = a[st+i];
        cplx od = a[st+i+len/2]*w;
        a[st+i] = ev+od;
        a[st+i+len/2] = ev-od;
        w = w*wlen;
  if(inv){
    for(cplx &z: a){
      z = z/n;
vector<ll> mul(vector<ll> &a, vector<ll> &b){
  int n = a.size(), m = b.size(), sz = 1;
  while (sz < n+m-1) sz <<= 1;
  vector<cplx> x(sz), y(sz), z(sz);
 for (int i = 0; i < sz; ++i){
  x[i] = cplx(i<n? a[i]: 0, 0);</pre>
    y[i] = cplx(i < m? b[i]: 0, 0);
  pre(sz);
  fft(x, 0);
  fft(y, 0);
  for (int i = 0; i < sz; ++i){
    z[i] = x[i] * y[i];
  fft(z, 1);
vector<ll> c(n+m-1);
  for (int i = 0; i < n+m-1; ++i){
   c[i] = round(z[i].a);
  return c;
## NTT
const int mod = 998244353;
const int root = 15311432;
const int k = 1 << 23;
int root 1;
vector<int> rev;
ll bigmod(ll a, ll b, ll mod){
  a \%= mod;
  ll ret = 1;
  while(b){
    if(b\&1) ret = ret*a%mod;
    a = a*a*mod;
    b >>= 1;
  return ret;
void pre(int sz){
  root 1 = bigmod(root, mod-2, mod);
  if(rev.size()==sz) return ;
  rev.resize(sz);
```

```
rev[0]=0;
  int lg n = builtin ctz(sz);
  for (int i = 1; i < sz; ++i) rev[i] = (rev[i>>1]
 \rightarrow >> 1) | ((i&1)<<(lg n-1));
void fft(vector<int> &a, bool inv){
  int n = a.size();
  for (int i = 1; i < n-1; ++i) if(i<rev[i])

→ swap(a[i], a[rev[i]]);

  for (int len = 2; len <= n; len <<= 1) {</pre>
    int wlen = inv ? root 1 : root;
    for (int i = len; i < k; i <<= 1){
      wlen = 1ll*wlen*wlen%mod:
    for (int st = 0; st < n; st += len) {
      int w = 1;
      for (int j = 0; j < len / 2; j++) {
        int ev = a[st+j];
        int od = 1ll*a[st+j+len/2]*w%mod;
        a[st+i] = ev + od < mod ? ev + od : ev + od
        a[st+j+len/2] = ev - od >= 0 ? ev - od : ev
    - od + mod;
        w = 1ll * w * wlen % mod;
  if (inv) {
    int n 1 = bigmod(n, mod-2, mod);
    for (int \& x : a)
      x = 111*x*n 1%mod;
vector<int> mul(vector<int> &a, vector<int> &b){
  int n = a.size(), m = b.size(), sz = 1;
  while (sz < n+m-1) sz <<= 1;
  vector<int> x(sz), y(sz), z(sz);
  for (int i = 0; i < sz; ++i){
    x[i] = i < n? a[i]: 0;
    y[i] = i < m? b[i]: 0;
  pre(sz);
  fft(x, 0);
  fft(y, 0);
  for (int i = 0; i < sz; ++i){
    z[i] = 111* x[i] * y[i] % mod;
 fft(z, 1);
  z.resize(n+m-1);
  return z;
## Any mod
const int N = 3e5 + 9, mod = 998244353;
|struct base {
  double x, y;
  base() { x = y = 0; }
  base(double x, double y): x(x), y(y) { }
inline base operator + (base a, base b) { return
\rightarrow base(a.x + b.x, a.y + b.y); }
```

```
inline base operator - (base a, base b) { return
→ base(a.x - b.x, a.y - b.y); }
inline base operator * (base a, base b) { return
    base(a.x * b.x - a.y * b.y, a.x * b.y + a.y *
\rightarrow b.x); }
inline base conj(base a) { return base(a.x, -a.y); }
int lim = 1;
vector<br/>vector<br/>vector<br/>se> roots = \{\{0, 0\}, \{1, 0\}\};
vector < int > rev = \{0, 1\};
const double PI = acosl(- 1.0);
void ensure base(int p) {
  if(p <= līm) return;</pre>
  rev.resize(1 << p);
  for(int i = 0; i < (1 << p); i++) rev[i] = (rev[i
\rightarrow >> 1] >> 1) + ((i & 1) << (p - 1));
  roots.resize(1 << p);</pre>
  while(lim < p) {</pre>
    double angle = 2 * PI / (1 << (lim + 1));
    for(int i = 1 << (lim - 1); i < (1 << lim);
      roots[i << 1] = roots[i];
      double angle i = angle * (2 * i + 1 - (1 <<
      roots[(i \ll 1) + 1] = base(cos(angle i),
    sin(angle i));
    lim++;
void fft(vector<base> \deltaa, int n = -1) {
 if(n == -1) n = a.size();
  assert((n \& (n - 1)) == 0);
  int zeros = builtin ctz(n);
  ensure base(zeros);
  int shift = lim - zeros;
  for(int i = 0; i < n; i++) if(i < (rev[i] >>
shift)) swap(a[i], a[rev[i] >> shift]);
  for(int k = 1; k < n; k <<= 1) 
    for(int i = 0; i < n; i += 2 * k) {
      for(int j = 0; j < k; j++) {
  base z = a[i + j + k] * roots[j + k];</pre>
        a[i + j + k] = a[i + j] - z;
        a[i + j] = a[i + j] + z;
//eg = 0: 4 FFTs in total
//eq = 1: 3 FFTs in total
vector<int> multiply(vector<int> &a, vector<int>
\rightarrow &b, int eq = 0) {
 int need = a.size() + b.size() - 1;
  int p = 0;
  while((1 << p) < need) p++;
  ensure base(p);
  int sz = 1 << p;
  vector<base> A, B;
  if(sz > (int)A.size()) A.resize(sz);
  for(int i = 0; i < (int)a.size(); i++) {</pre>
    int x = (a[i] \% mod + mod) \% mod;
    A[i] = base(x \& ((1 << 15) - 1), x >> 15);
  fill(A.begin() + a.size(), A.begin() + sz,
   base{0, 0});
```

```
fft(A, sz);
  if(sz > (int)B.size()) B.resize(sz);
  if(eq) copy(A.begin(), A.begin() + sz, B.begin());
  else -
    for(int i = 0; i < (int)b.size(); i++) {</pre>
      int x = (b[i] \% mod + mod) \% mod;
      B[i] = base(x \& ((1 << 15) - 1), x >> 15);
    fill(B.begin() + b.size(), B.begin() + sz,
\rightarrow base\{0, 0\});
    fft(B, sz);
  double ratio = 0.25 / sz;
  base r^{2}(0, -1), r^{3}(ratio, 0), r^{4}(0, -ratio),
\rightarrow r5(0, 1);
 for(int i = 0; i \le (sz >> 1); i++) {
    int j = (sz - i) \& (sz - 1);
    base a1 = (A[i] + conj(A[j])), a2 = (A[i] -

→ conj(A[j])) * r2;

    base b1 = (B[i] + conj(B[j])) * r3, b2 = (B[i])
- conj(B[j])) * r4;
    if(i != j) {
      base c1 = (A[j] + conj(A[i])), c2 = (A[j] -

    conj(A[i])) * r2;

      base d1 = (B[j] + conj(B[i])) * r3, d2 =
   (B[j] - conj(B[i])) * r4;

A[i] = c1 * d1 + c2 * d2 * r5;
      B[i] = c1 * d2 + c2 * d1;
    A[j] = a1 * b1 + a2 * b2 * r5;

B[j] = a1 * b2 + a2 * b1;
  fft(A, sz); fft(B, sz);
  vector<int> res(need);
  for(int i = 0; i < need; i++) {
    long long aa = A[i].x + 0.5;
    long long bb = B[i].x + 0.5;
    long long cc = A[i].y + 0.5;
    res[i] = (aa + ((bb \% mod) << 15) + ((cc \% mod))
 return res;
vector<int> pow(vector<int>& a, int p) {
  vector<int> res;
  res.emplace back(1);
  while(p) {
   if(p \& 1) res = multiply(res, a);
    a = multiply(a, a, 1);
   p >>= 1;
  return res;
int main() {
  int n, k; cin >> n >> k;
  vector<int> a(10, 0);
  while(k--) {
    int m; cin >> m;
    a[m] = 1;
  vector<int> ans = pow(a, n / 2);
  int res = 0:
  for(auto x: ans) res = (res + 1LL * x * x % mod)

→ % mod;
```

```
cout << res << '\n';
  return 0;
## Online NTT
void solve() {
  f[0]=1; // base case
  for(int i=0; i<=MAX; i++) {
    // Doing the part 1
    f[i+1]=(f[i+1]+f[i]*A[0])%mod;
    f[i+2]=(f[i+2]+f[i]*A[1])%mod;
    if(!i) continue;
    // part 2
    int limit=(i&-i);
    for(int p=2; p<=limit; p*=2) {</pre>
      convolve(i-p,i-1,p,min(2*p-1,MAX));
void convolve(int l1, int r1, int l2, int r2) {
  int n=max(r1-l1+1,r2-l2+1);
  int t=1:
  while(t<n) t<<=1;
  n=t;
  vector<ll> a(n), b(n);
  for(int i=l1; i<=r1; i++) a[i-l1]=f[i];</pre>
  for(int i=l2; i<=r2; i++) b[i-l2]=A[i];
  vector<ll> ret=fft::multiply(a,b);
    for(int i=0; i<ret.size(); i++) {</pre>
    int idx=i+l1+l2+1:
    if(idx>MAX) break;
    // adding to the appropriate entry
    f[idx]+=ret[i];
    f[idx]%=mod;
## FWHT (AND, OR, XOR)

    Time complexity: O(nlogn)

- AND, OR works for any modulo, XOR works for only

    size must be power of two

const ll mod = 998244353;
int add (int a, int b) {
  return a + b < mod? a + b: a + b - mod;
|int sub (int a, int b) {
  return a - b >= 0? a - b: a - b + mod;
ll poww (ll a, ll p, ll mod){
  a %= mod;
  ll ret = 1;
  while (p){
    if (p & 1) {
      ret = ret * a % mod:
    a = a * a % mod:
    p >>= 1;
  return ret:
void fwht(vector<int> &a, int inv, int f) {
  int sz = a.size():
```

```
for (int len = 1; 2 * len <= sz; len <<= 1) {</pre>
    for (int i = 0; i < sz; i += 2 * len) {
      for (int j = 0; j < len; j++) {
        int x = a[i + j];
        int y = a[i + j + len];
        if (f == 0) {
          if (!inv) a[i + j] = y, a[i + j + len] =
          else a[i + j] = sub(y, x), a[i + j +
\rightarrow len] = x;
        else if (f == 1) {
          if (!inv) a[i + j + len] = add(x, y);
          else a[i + j + len] = sub(y, x);
        else {
          a[i + j] = add(x, y);
          a[i + j + len] = sub(x, y);
vector<int> mul(vector<int> a, vector<int> b, int
\rightarrow f) { // 0:AND, 1:0R, 2:X0R
 int sz = a.size();
 fwht(a, 0, f); fwht(b, 0, f);
 vector<int> c(sz);
 for (int i = 0; i < sz; ++i) {
    c[i] = 111 * a[i] * b[i] % mod;
  fwht(c, 1, f);
 if (f) {
    int sz inv = poww(sz, mod - 2, mod);
    for (int i = 0; i < sz; ++i) {
     c[i] = 111 * c[i] * sz inv % mod;
 return c;
## subset convolution
vector<int> subset conv (vector<int> a, vector<int>
→ b) {
 int n = a.size();
 int lg = log2(n);
 vector<int> cnt(n);
 vector<vector<int>> fa(lg + 1, vector<int> (n)),
    fb(lq + 1, vector < int > (n)), q(lq + 1,
vector<int>(n));
 for (int i = 0; i < n; ++i) {
  cnt[i] = cnt[i >> 1] + (i & 1);
    fa[cnt[i]][i] = a[i] % mod;
    fb[cnt[i]][i] = b[i] % mod;
 for (int k = 0; k <= lq; ++k)
    fwht(fa[k], 0, 1); fwht(fb[k], 0, 1);
 for (int k = 0; k \le lg; ++k) {
    for (int j = 0; j \le k; ++j) {
      for (int i = 0; i < n; ++i) {
        g[k][i] = add(g[k][i], 1[l] * fa[j][i] *
\leftarrow fb[k - j][i] % mod);
```

```
for (int k = 0; k \le lg; ++k) {
 fwht(g[k], 1, 1);
vector<int> c(n);
for (int i = 0; i < n; ++i) {
 c[i] = g[cnt[i]][i];
return c;
```

16 CPP

```
## Ordered Set
#include <ext/pb ds/assoc_container.hpp>
using namespace qnu pbds;
typedef tree<int, null type, less<int>, rb tree tag,
tree order statistics node update> oset;
ost order of key(495): // return 0-based index of

→ lower bound

ost.find by order(5): // return iterator of 0-based

    index value

ost.erase(); ost.size(), ost.insert(2),

    st.lower bound(x);
## unordered map
struct chash{
  size_t operator()(const pair<int,int>&x)const{
    return hash<long long>()(((long

    long)x.first)^(((long long)x.second)<<32));</pre>
}; 
unordered map<pair<int, int>, int, chash> maf;
maf.reserve(max len);
maf.max load factor(0.25);
## qp hash table:
#include <ext/pb ds/assoc container.hpp>
using namespace gnu pbds;
struct chash{
 int operator()(ii p) const {
    return p.first*31 + p.second;
qp hash table<ii, int, chash> cnt;
```

17 DETERMINANT

```
const double EPS = 1E-9:
vector < vector<double> > a (n, vector<double> (n));
double det = 1;
for (int i=0; i<n; ++i) {
  int k = i:
  for (int j=i+1; j<n; ++j)</pre>
    if (abs (a[j][i]) > abs (a[k][i]))
  if (abs (a[k][i]) < EPS) {
    det = 0;
```

```
break;
swap (a[i], a[k]);
if (i != k)
  det = -det;
det *= a[i][i];
for (int j=i+1; j<n; ++j)
a[i][j] /= a[i][i];</pre>
for (int j=0; j<n; ++j)
  if (j != i \&\& abs (a[j][i]) > EPS)
     for (int k=i+1; k<n; ++k)
a[j][k] -= a[i][k] * a[j][i];</pre>
```

18 DINIC

```
V^2E, sqrt(E)E, sqrt(V)E(bpm)
// Effective flows are adi[u][3] where adi[u][3] > 0 | void dfs(int u) {
ll get max flow(vector<array<int, 3>> edges, int n,
int s, int t) {
vector<array<ll, 4>> adj[n];
  for (auto [u, v, c]: edges) {
    adj[u].push back({v, (int)adj[v].size(), c, 0});
    adj[v].push_back({u, (int)adj[u].size() - 1, 0,
   0});
  ll max flow = 0;
  while (true) {
    queue<int> q; q.push(s);
    vector<int> dis(n, -1); dis[s] = 0;
    while (!q.empty()) {
      int u = q.front(); q.pop();
      for (auto [v, idx, c, f]: adj[u]) {
        if (dis[v] == -1 \text{ and } c > f) {
          q.push(v);
          dis[v] = dis[u] + 1;
    if (dis[t] == -1) break;
    vector<int> next(n);
    function<ll(int, ll)> dfs = [&] (int u, ll
   flow) {
      if (u == t) return flow;
      while (next[u] < adj[u].size()) {</pre>
        auto &[v, idx, c, f] = adj[u][next[u]++];
        if (c > f \text{ and } dis[v] == dis[u] + 1) {
          ll bn = dfs(v, min(flow, c - f));
          if (bn > 0) {
            f += bn;
            adj[v][idx][3] -= bn;
            return bn:
      return Oll;
    while (ll flow = dfs(s, LLONG MAX)) {
      max flow += flow;
  return max flow;
```

19 DOMINATOR TREE

```
const int N = 2e5+5:
vector <int> g[N], rg[N], dtree[N], bucket[N];
int sdom[N], par[N], dom[N], dsu[N], lab[N],
→ arr[N], rev[N], dpar[N], n, ts, src;
void init(int n, int s) {
 ts = 0, n = _n, src = s;
for (int i = 1; i <= n; ++i) {
    g[i].clear(), rg[i].clear(), dtree[i].clear(),
   bucket[i].clear();
    sdom[i]=par[i]=dom[i]=dsu[i]=lab[i]=arr[i]=rev[_
   il=dpar[il=0;
   ts++; arr[u] = ts; rev[ts] = u;
   lab[ts] = sdom[ts] = dsu[ts] = ts;
  for(int &v : g[u]) {
   if(!arr[v]) {     dfs(v); par[arr[v]] = arr[u]; }
      rg[arr[v]].push back(arr[u]);
inline int root(int u, int x = 0) {
  if(u == dsu[u]) return x ? -1 : u;
  int v = root(dsu[u], x + 1);
  if(v < 0) return u;</pre>
  if(sdom[lab[dsu[u]]] < sdom[lab[u]]) lab[u] =</pre>
   lab[dsu[u]]:
  dsu[u] = v; return x ? v : lab[u];
void build() {
  dfs(src);
   for(int i=n; i; i--) {
      for(int j : rq[i]) sdom[i] =
    min(sdom[i],sdom[root(j)]);
      if(i > 1) bucket[sdom[i]].push_back(i);
      for(int w : bucket[i]) {
         int v = root(w);
         if(sdom[v] == sdom[w]) dom[w] = sdom[w];
         else dom[w] = v;
      f(i > 1) dsu[i] = par[i];
   for(int i=2; i<=n; i++) {
      int &dm = dom[i];
      if(dm ^ sdom[i]) dm = dom[dm];
      dtree[rev[i]].push back(rev[dm]);
      dtree[rev[dm]].push back(rev[i]);
      dpar[rev[i]] = rev[dm];
```

20 DP_ON_TREE

```
// Rerooting Technique
vector<array<ll, 2>> down(N), up(N);
void dfs() {
 // calculate down dp
void dfs2() {
```

```
9
```

```
ll pref = ?;
for (auto v: adj[u]) {
    // update up[v] and pref
}
reverse(adj[u].begin(), adj[u].end());
ll suf = ?;
for (auto v: adj[u]) {
    // update up[v] and suf
}
for (auto v: adj[u]) {
    dfs2(v)
}
```

21 DP_OPTIMIZATION

```
## CHT
## Online CHT
const ll IS QUERY = -(1LL << 62);</pre>
struct line {
  ll m. b:
  mutable function <const line*()> succ;
  bool operator < (const line &rhs) const {</pre>
    if (rhs.b != IS QUERY) return m < rhs.m;</pre>
    const line *s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s -> b < (s -> m - m) * x;
struct CHT : public multiset <line> {
  bool bad (iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if(z == end()) return 0;
      return y \rightarrow m == z \rightarrow m \&\& y \rightarrow b <= z \rightarrow b;
    auto x = prev(y);
    if (z == end()) return y \rightarrow m == x \rightarrow m \& \& y \rightarrow
 \rightarrow b <= x -> b;
    return 1.0 * (x -> b - y -> b) * (z -> m - y -> b)
    m) >= 1.0 * (v -> b - z -> b) * (v -> m - x ->
 }
  void add (ll m, ll b)
    auto y = insert({m, b});
    y \rightarrow succ = [=] \{return \ next(y) == end() ? 0 :
 if (bad(y)) {erase(y); return;}
    while (next(y) != end() \&\& bad(next(y)))
   erase(next(y));
    while (y != begin() \&\& bad(prev(y)))
    erase(prev(y));
  ll eval (ll x) {
    auto l = *lower bound((line) {x, IS QUERY});
    return l.m * x + l.b;
// To find maximum
```

```
CHT cht;
cht.add(m, c);
y max = cht.eval(x);
// To find minimum
CHT cht;
cht.add(-m, -c);
y min = -cht.eval(x);
// Divide an array into k parts
// Minimize the sum of squre of each subarray
ll pref[N], dp[N][N];
|void compute(int l, int r, int j, int kl, int kr) {
  if (l > r) return ;
  int m = (l + r) / 2;
  array<ll, 2> best = {LLONG_MAX, -1};
  for (int k = kl; k \le min(m - 1, kr); ++k) {
    best = min(best, \{dp[k][j - 1] + (pref[m] -
   pref[k]) * (pref[m] - pref[k]), k});
  dp[m][j] = best[0];
compute(l, m - 1, j, kl, best[1]);
  compute(m + 1, r, j, best[1], kr);
## Knuth
// Divide an array into n parts.
// Cost of each division is subarray sum
 / Minimize the cost
ll dp[n][n], opt[n][n];
for (int i = 0; i < n; ++i)
  for (int j = 0; j < n; ++j) {
    dp[i][j] = LLONG MAX;
  opt[i][i] = i;
  dp[i][i] = 0;
for (int i = n - 2; i \ge 0; --i) {
  for (int j = i + 1; j < n; ++j) {
    for (int k = opt[i][j - 1]; k <= min(j - 1ll,
    opt[i + 1][j]); ++k) {
      if (dp[i][j] >= dp[i][k] + dp[k + 1][j] +
    (pref[j + 1] - pref[i])) {
        dp[i][j] = dp[i][k] + dp[k + 1][j] +
    (pref[j + 1] - pref[i]);
        opt[i][j] = k;
cout << dp[0][n - 1] << "\n";
## Lichao Tree
const int N = int(5e4 + 2);
const ll INF = ll(1e17);
vector<vector<ll> > tree(4*N, {0, INF});
ll f(vector<ll> line, int x){
return line[0] * x + line[1];
void insert(vector<ll> line, int lo = 1, int hi =
 \rightarrow N, int i = 1){
  int m = (lo + hi) / 2;
  bool left = f(line, lo) < f(tree[i], lo);</pre>
  bool mid = f(line, m) < f(tree[i], m);</pre>
  if(mid) swap(tree[i], line);
```

```
if(hi - lo == 1) return;
else if(left != mid) insert(line, lo, m, 2*i);
else insert(line, m, hi, 2*i+1);
}
ll query(int x, int lo = 1, int hi = N, int i = 1){
   int m = (lo+hi)/2;
   ll curr = f(tree[i], x);
   if(hi-lo==1) return curr;
   if(x<m) return min(curr, query(x, lo, m, 2*i));
else return min(curr, query(x, m, hi, 2*i+1));
}</pre>
```

22 **DSU**

```
struct DSU {
 int comps;
 vector<int> par, sz;
 DSU(int n): comps(n), par(n), sz(n,1) {
    iota(par.begin(), par.end(), 0);
 int find(int v) {
    return (par[v] == v)? v: (par[v] =
   find(par[v]));
 void unite(int u, int v)
    u = find(u), v = find(v);
    if(u != v){
     if(sz[u] < sz[v]) {
        swap(u, v);
     par[v] = u;
      sz[u] += sz[v];
comps--;
 int size(int v) {
   return sz[find(v)];
 bool same set(int u, int v) {
    return find(u) == find(v);
```

23 DSU_ON_TREE

```
void dfs(int u, int p) {
  node[tt] = u;
  tin[u] = tt++, sz[u] = 1, hc[u] = -1;
  for (auto v: adj[u]) {
    if (v != p) {
        dfs(v, u);
        sz[u] += sz[v];
        if (hc[u] == -1 or sz[hc[u]] < sz[v]) {
            hc[u] = v;
        }
    }
  }
  tout[u] = tt - 1;
}
void dsu(int u, int p, int keep) {
  for (int v: adj[u]) {
        if (v != p and v != hc[u]) {
            dsu(v, u, 0);
        }
}</pre>
```

```
DU_Devour, University of Dhaka
```

```
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```

```
if (hc[u] != -1) {
    dsu(hc[u], u, 1);
 for (auto v: adj[u]) {
   if (v != p and v != hc[u]) {
     for (int i = tin[v]; i <= tout[v]; ++i) {
        int w = node[i]:
        // get ans in case of ans is related to

→ simple path or pair

      for (int i = tin[v]; i <= tout[v]; ++i) {</pre>
        int w = node[i];
        // Add contribution of node w
  // Add contribution of node u
  // get ans in case ans is related to subtree
 if (!keep) {
   for (int i = tin[u]; i <= tout[u]; ++i) {</pre>
     int w = node[i];
      // remove contribution of node w
   // Data structure in initial state (empty
   contribution)
dfs(0, 0); dsu(0, 0, 0);
```

24 DSU_WITH_ROLLBACK

```
struct DSU {
 int comps;
 vector<int> par, rnk;
 stack<array<int, 4>> ops;
 DSU(){}
 DSU(int n): comps(n), par(n), rnk(n) {
   iota(par.begin(), par.end(), 0);
 int find(int u) {
   return (par[u] == u)? u: find(par[u]);
 bool unite(int u, int v)
   u = find(u), v = find(v);
   if (u == v) return false;
   if (rnk[u] > rnk[v]) swap(u, v);
   ops.push(\{u, rnk[u], v, rnk[v]\});
   par[u] = v;
   if (rnk[u] == rnk[v]) rnk[v]++;
   return true;
 void rollback() {
   if (ops.empty()) return ;
   auto [u, rnku, v, rnkv] = ops.top(); ops.pop();
   par[u] = u, rnk[u] = rnku;
    par[v] = v, rnk[v] = rnkv;
    comps++;
```

25 DS_TRICKS

26 DYNAMIC_CONNECTIVITY

```
const int 0 = 1e5+5;
|vector<array<<mark>int</mark>, 2>> t[4 * Q];
|vector<<mark>int</mark>> ans(0);
int q;
struct DSU {
  int n. comps:
  vector<int> par, rnk;
  stack<array<int, 4>> ops;
  DSU(){}
  DSU(int n): n(n), comps(n), par(n), rnk(n) {
    iota(par.begin(), par.end(), 0);
  int find(int u) {
    return (par[u] == u)? u: find(par[u]);
  bool unite(int u, int v)
    u = find(u), v = find(v);
    if (u == v) return false;
    comps - -;
    if (rnk[u] > rnk[v]) swap(u, v);
    ops.push({u, rnk[u], v, rnk[v]});
    par[u] = v;
    if (rnk[u] == rnk[v]) rnk[v]++;
    return true;
  void rollback() {
    if (ops.empty()) return ;
    auto [u, rnku, v, rnkv] = ops.top(); ops.pop();
    par[u] = u, rnk[u] = rnku;
    par[v] = v, rnk[v] = rnkv;
comps++;
|} dsu;
void add(int l, int r, array<int, 2> ed, int u = 1,
\rightarrow int s = 0, int e = q) {
  if (r < s or e < l) return ;</pre>
  if (l <= s and e <= r) {
    t[u].push back(ed);
    return ;
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  add(l, r, ed, v, s, m);
```

27 DnC

```
Divide an array into k parts
 Minimize the sum of squre of each subarray
ll pref[N], dp[N][N];
void compute(int l, int r, int j, int kl, int kr) {
 if (l > r) return ;
 int m = (l + r) / 2;
 array<ll, 2> best = {LLONG MAX, -1};
 for (int k = kl; k \le min(m - 1, kr); ++k) {
   best = min(best, \{dp[k][j-1] + (pref[m]) -
  pref[k]) * (pref[m] - pref[k]), k});
 dp[m][j] = best[0];
 compute(l, m - 1, j, kl, best[1]);
 compute(m + 1, r, j, best[1], kr);
for (int i = 0; i < N; ++i) {
 for (int j = 0; j < N; ++j) {
   dp[i][j] = 1e17;
dp[0][0] = 0;
for (int j = 1; j \le k; ++j) {
 compute(1, n, j, 0, n - 1);
cout << dp[n][k] << "\n";
```

28 DnC_SRQ

```
void dnc(int l, int r, vector<int> idx) {
    if (idx.empty())         return;
    if (l==r) {
        for (auto i: idx)         ans[i] = a[l];
            return ;
    }
    int m = (l + r) / 2;
    vectro<int> left(m), right(m + 1);
    for (int i = m - 1; i >= l; --i) {
        left[i] = min(a[i], left[i + 1]);
    }
    for (int i = m; i <= r; ++i) {
        right[i] = min(right[i - 1], a[i]);
    }
    vector<int> nxt[2];
    for (int i: idx) {
```

```
11
```

```
if (l[i] <= m and m <= r[i]) {
    ans[i] = min(left[l[i]], right[r[i]]);
}
else {
    nxt[i >= m].push_back(i);
}
dnc(l, m, nxt[0]);
dnc(m + 1, r nxt[1]);
}
```

29 EULER_WALK

```
## Directed graph
vector<int> euler cycle(vector<int> *adj, int s =
  vector<int> cycle;
  function\langle void(int) \rangle dfs = [&] (int u) {
    while (!adj[u].empty()) {
      int v = adj[u].back();
      adj[u].pop_back();
      dfs(v);
    cycle.push back(u);
  dfs(s);
  reverse(cycle.begin(), cycle.end());
  return cycle;
## Undirected graph
vector<int> euler cycle(vector<int> *adj,
    vector<int> *des idx, vector<int> *done, int s
 \stackrel{\frown}{\hookrightarrow} = 0) {
  vector<int> cycle;
  function<void(int)> dfs = [\&] (int u) {
    while (!adj[u].empty()) {
      int i = adj[u].size() - 1;
      if (done[u][i]) {
         adj[u].pop back();
         continue:
      int v = adj[u][i];
      adj[u].pop back();
      done[u][i] = 1;
done[v][des_idx[u][i]] = 1;
      dfs(v);
    cycle.push back(u);
  dfs(s);
  return cycle;
int n, m; cin >> n >> m;
vector<int> adj[n], des idx[n], done[n];
vector<int> deg(n);
for (int e = 0; e < m; ++e) {
  int u, v; cin >> u >> v; u--, v--;
des_idx[u].push_back(adj[v].size());
  des_idx[v].push_back(adj[u].size());
  adiTul.push bac\overline{k}(v);
```

```
adj[v].push back(u);
done[u].push_back(0);
done[v].push_back(0);
deg[u]++, deg[v]++;
}

for (int u = 0; u < n; ++u) {
   if (deg[u] & 1) {
      cout << "IMPOSSIBLE\n";
      return;
   }
}

vector<int> cycle = euler_cycle(adj, des_idx, done,
      0);
if (cycle.size() != m + 1) {
   cout << "IMPOSSIBLE\n";
   return;
}</pre>
```

30 FENWICK_TREE

```
## 2D BIT
int n;
|int ft[N][N];
void add(int x, int y, int val){
    x++, y++;
 int idx = x;
 while (idx<=n){</pre>
   int idy = y;
   while (idy<=n){
     ft[idx][idv] += val;
     idv += idv \& -idv;
   idx += idx \& -idx;
|int csum(int x, int y){
X++, Y++;
int ret = 0;
int idx = x;
 while (idx > 0){
   int idy = y;
   while (idy > 0){
     ret += ft[idx][idy];
     idy -= idy \& -idy;
   idx -= idx \& -idx;
return ret;
int rsum(int x1, int y1, int x2, int y2) {
return csum(x2, y2) - csum(x1-1, y2) - csum(x2, y2)
   y1-1)+csum(x1-1, y1-1);
```

31 GAUSSIAN_ELIMINATION

```
const double eps=1e-9;
const int INF=2;
typedef vector<double> vd;
```

```
int gauss(vector<vd>& a, vd& ans) {
 int n = a.size();
 int m = a[0].size()-1:
 vector<int> where(m, -1);
 for(int col=0, row=0; row<n and col<m; ++col,</pre>

→ ++row) {
   int sel = row;
   for(int i=row; i<n; ++i){</pre>
     if(abs(a[i][col]) > abs(a[sel][col])) sel = i;
   if(abs(a[sel][col]) < eps) continue;</pre>
   for(int j=col; j<=m; ++j) swap(a[sel][j],</pre>
   a[row][i]);
   where[col] = row;
   for(int i=0; i<n; i++){
     if(i!=row){
        double c = a[i][col] / a[row][col];
       for(int j=col; j<=m; ++j){</pre>
         a[i][j] -= a[row][j]*c;
 ans.assign(m, 0);
 for (int j = 0; j < m; ++j){
   if(where[j] != -1){
     ans[j] = a[where[j]][m] / a[where[j]][j];
 for (int i = 0; i < n; ++i){
   double sum = 0;
   for (int j = 0; j < m; ++j){
     sum += ans[j] * a[i][j];
    if(abs(sum-a[i][m]) > eps) return 0;
 for (int j = 0; j < m; ++j){
   if(where[j] == -1) return INF;
 return 1;
```

32 GRAPH_CYCLE

```
## Floyd cycle finding algorithm for successor
    graph:
int hare = x0, tort = x0;
do {
    hare = succ(hare);
    hare = succ(tort);
} while (hare != tort);

tort = x0;
while (tort != hare) {
    hare = succ(hare);
    tort = succ(tort);
}
// now hare and tort are the starting node of the
    cycle
```

```
int len = 0;
do {
   tort = succ(tort);
   len++;
} while (tort != hare);

## Number of basis simple cycle in an u-graph
E - V + number of connected components (using DSU)
Number of back-edge in dfs tree (using DFS)

## Number of eulerian subgraph in an u-graph
2^basis_cycle
```

33 GRAY_CODE

```
int gc(int n){
   return n^(n>>1);
}
int gc_to_dec(int g){
   int d=0;
   while (g){
      d ^= g;
      g >>= 1;
   }
  return d;
}
```

34 HALF PLANE INTERSECTION

```
// Redefine epsilon and infinity as necessary. Be

→ mindful of precision errors.

const long double eps = 1e-9, inf = 1e9;
// Basic point/vector struct.
struct Point {
    long double x, y;
    explicit Point(long double x = 0, long double y
= 0) : x(x), y(y) \{\}
    // Addition, substraction, multiply by

→ constant, dot product, cross product.

    friend Point operator + (const Point& p, const
   Point& q) {
        return Point(p.x + q.x, p.y + q.y);
    friend Point operator - (const Point& p, const
   Point& q) {
        return Point(p.x - q.x, p.y - q.y);
    friend Point operator * (const Point& p, const
   long double& k)
        return Point(p.x * k, p.y * k);
    friend long double dot(const Point& p, const
   Point& a) {
        return p.x * q.x + p.y * q.y;
    friend long double cross(const Point& p, const
   Point& q) {
        return p.x * q.y - p.y * q.x;
// Basic half-plane struct.
struct Halfplane {
```

```
is the direction vector of the line.
    Point p, pq;
    long double angle;
    Halfplane() {}
    Halfplane(const Point& a, const Point& b) :
    p(a), pq(b - a)
        angle = atan2l(pq.y, pq.x);
    // Check if point 'r' is outside this
   half-plane.
    // Every half-plane allows the region to the
    LEFT of its line.
    bool out(const Point& r) {
        return cross(pq, r - p) < -eps;</pre>
    // Comparator for sorting.
    bool operator < (const Halfplane& e) const {
        return angle < e.angle;</pre>
    // Intersection point of the lines of two
    half-planes. It is assumed they're never
    parallel.
    friend Point inter(const Halfplane& s, const
   Halfplane& t) {
        long double alpha = cross((t.p - s.p),
   t.pq) / cross(s.pq, t.pq);
        return s.p + (s.pq * alpha);
// Actual algorithm
vector<Point> hp intersect(vector<Halfplane>& H) {
   Point box[4] = { // Bounding box in CCW order Point(inf, inf), Point(-inf, inf), Point(-inf, -inf),
        Point(inf, -inf)
    for(int i = 0; i<4; i++) { // Add bounding box
        Halfplane aux(box[i], box[(i+1) % 4]);
        H.push back(aux);
    // Sort by angle and start algorithm
    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0:
    for(int i = 0; i < int(H.size()); i++) {</pre>
        // Remove from the back of the deque while
   last half-plane is redundant
        while (len > 1 && H[i].out(inter(dg[len-1],
   dq[len-2]))) {
            dq.pop back();
            --len;
        // Remove from the front of the deque while
   first half-plane is redundant
        while (len > 1 && H[i].out(inter(dq[0],
   dq[1]))) {
            dq.pop_front();
            --len:
        // Special case check: Parallel half-planes
```

// 'p' is a passing point of the line and 'pq'

```
if (len > 0 \& \& fabsl(cross(H[i].pg,
\rightarrow dq[len-1].pq)) < eps) {
           // Opposite parallel half-planes that
- ended up checked against each other.
           if (dot(H[i].pq, dq[len-1].pq) < 0.0)
               return vector<Point>();
           // Same direction half-plane: keep only
  the leftmost half-plane.
           if (H[i].out(dq[len-1].p)) {
               dq.pop back();
               --len:
           else continue;
       // Add new half-plane
       dq.push back(H[i]);
       ++len:
   // Final cleanup: Check half-planes at the
  front against the back and vice-versa
   while (len > 2 && dq[0].out(inter(dq[len-1],
   dq[len-2]))) {
       dq.pop_back();
       --len:
   while (len > 2 && dg[len-1].out(inter(dg[0],
   dq[1]))) {
       dq.pop_front();
       --len:
   // Report empty intersection if necessary
   if (len < 3) return vector<Point>();
   // Reconstruct the convex polygon from the
   remaining half-planes.
   vector<Point> ret(len);
   for(int i = 0; i+1 < len; i++)
       ret[i] = inter(dq[i], dq[i+1]);
   ret.back() = inter(dq[len-1], dq[0]);
   return ret;
```

35 HASHING

```
// Hashvalue(l...r) = hsh[l] - hsh[r + 1] * base ^
#include<bits'/st'dc++.h>
using namespace std;
typedef long long li;
const int MAX = 100009;
ll\ mods[2] = \{10000000007, 10000000009\};
//Some back-up primes: 1072857881, 1066517951,
→ 1040160883
ll bases[2] = \{137, 281\};
ll pwbase[3][MAX];
void Preprocess(){
 pwbase[0][0] = pwbase[1][0] = 1;
 for(ll i = 0; i < 2; i++){
   for(ll j = 1; j < MAX; j++)
     pwbase[i][j] = (pwbase[i][j - 1] * bases[i])

→ % mods[i]:
```

```
struct Hashing
  ll hsh[2][MAX];
  string str;
  Hashing(){}
  Hashing(string str) {str = str; memset(hsh, 0,

¬ sizeof(hsh)); build();}

  void Build(){
    for(ll i = str.size() - 1; i >= 0; i--){
      for(int j = 0; j < 2; j++){
  hsh[j][i] = (hsh[j][i + 1] * bases[j] +</pre>

    str[i]) % mods[j];

        hsh[j][i] = (hsh[j][i] + mods[j]) % mods[j];
  pair<ll,ll> GetHash(ll i, ll j){
    assert(i <= j);
    ll tmp1 = (hsh[0][i] - (hsh[0][j + 1] *
   pwbase[0][j - i + 1]) % mods[0]) % mods[0];
    ll tmp2 = (hsh[1][i] - (hsh[1][j + 1] *
   pwbase[1][j - i + 1]) % mods[1]) % mods[1];
    if(tmp1 < 0) tmp1 += mods[0];
    if(tmp2 < 0) tmp2 += mods[1]:
    return make pair(tmp1, tmp2);
};
    * Everything is 0 based
    * Call precal() once in the program
    * Call update(1,0,n-1,i,j,val) to update the

→ value of position

      i to j to val, here n is the length of the
    strina
    * Call query(1,0,n-1,L,R) to get a node

→ containing hash

      of the position [L:R]
    * Before any update/query
        - Call init(str) where str is the string to
   be hashed
        - Call build(1,0,n-1)
***/
namespace strhash {
  int n;
  const int MAX = 100010;
  int ara[MAX]:
  const int MOD[] = {2078526727, 2117566807};
  const int BASE[] = {1572872831, 1971536491};
  int BP[2][MAX], CUM[2][MAX];
  void init(char *str) {
    n = strlen(str);
    for(int i=0;i<n;i++) ara[i] = str[i]-'0'+1; ///</pre>
 → scale str[i] if needed
  void precal() {
    BP[0][0] = BP[1][0] = 1;
    for(int i=1; i<MAX; i++)</pre>
      BP[0][i] = (BP[0][i-1] * (long long) BASE[0]
 → ) % MOD[0];
```

```
BP[1][i] = (BP[1][i-1] * (long long) BASE[1]
     % MOD[1];
  struct node {
    int sz;
    int h[2];
    node() {}
  } tree[4*MAX];
  int lazy[4*MAX];
  inline node Merge(node a, node b) {
    node ret;
    ret.h[0] = ( (a.h[0] * (long long) BP[0][b.sz]
   ) + b.h[0] ) % MOD[0];
    ret.h[1] = ((a.h[1] * (long long) BP[1][b.sz])
   ) + b.h[1] ) % MOD[1];
    ret.sz = a.sz + b.sz;
    return ret:
  inline void build(int n,int st,int ed) {
    if(st==ed) -
      tree[n].h[0] = tree[n].h[1] = ara[st];
      tree[n].sz = 1;
      return;
    int mid = (st+ed)>>1;
    build(n+n,st,mid);
    build(n+n+1.mid+1.ed):
    tree[n] = Merge(tree[n+n], tree[n+n+1]);
  inline void update(int n,int st,int ed,int id,int
    if(st>id or ed<id) return;</pre>
    if(st==ed and ed==id) {
      tree[n].h[0] = tree[n].h[1] = v;
      return;
    int mid = (st+ed)>>1:
    update(n+n,st,mid,id,v);
    update(n+n+1,mid+1,ed,id,v);
    tree[n] = Merge(tree[n+n], tree[n+n+1]);
 inline node query(int n,int st,int ed,int i,int
    j){
if(st>=i and ed<=j) return tree[n];</pre>
    int mid = (st+ed)/2;
    if(mid<i) return query(n+n+1,mid+1,ed,i,j);</pre>
    else if(mid>=j) return query(n+n,st,mid,i,j);
    else return Merge(query(n+n,st,mid,i,j),query(n)
   +n+1,mid+1,ed,i,j));
36 HLD
```

```
int tt, tin[N], tout[N], sz[N], par[N][LG], hvc[N];
void dfs(int u, int p) {
  tin[u] = tt++, sz[u] = 1, par[u][0] = p;
  for (int j = 1; j < LG; ++j) {
    par[u][j] = par[par[u][j-1]][j-1];
  }</pre>
```

```
int mx = 0;
 for (int \&v: adj[u]) {
    if (v != p) {
      dfs(v, u);
      sz[u] += sz[v];
      if (sz[v] > mx) {
        mx = sz[v];
        hvc[u] = v;
  tout[u] = tt-1;
int ch cnt, idx cnt, chno[N], chd[N], idx[N];
void hld(int u, int p) {
 if(chd[ch cnt] == -1) {
    chd[ch \overline{c}nt] = u;
  chno[u] = ch cnt, idx[u] = idx cnt++;
 if(hvc[u] != -1) {
    hld(hvc[u], u);
 for (int &v: adj[u]) {
    if (v != p and v != hvc[u]) {
      ch cnt++;
      hld(v, u);
void ?node update(int u, int x) {
  ?update(\overline{i}dx[u], x);
void ?pupdate up(int u, int anc) {
  if (chno[u] == chno[anc]) {
    return ?rupdate(idx[anc], idx[u]);
  ?rupdate(idx[chd[chno[u]]], idx[u]);
  ?pupdate up(par[chd[chno[u]]][0], anc);
void ?pupdate(int u, int v) {
 int l = lca(u, v);
  ?pupdate up(u, l);
  ?pupdate up(v, l);
ĺl ?node query(int u) {
  return ?query(idx[u]);
int ?pquery up(int u, int anc) {
 if (chno[\overline{u}] == chno[anc]) {
    return ?rquery(idx[anc], idx[u]);
 return f(?rquery(idx[chd[chno[u]]], idx[u]),
    ?pquery up(par[chd[chno[u]]][0], anc));
int ?rquery(int u, int v) {
 int l = lca(u, v);
  return f(?pquery up(u, l), ?pquery up(v, l));
adj[u].clear(); hvc[u] = -1;
tt = 0; dfs(0, 0);
chd[ch] = -1:
ch cnt = 0, idx cnt = 0; hld(0, 0);
```

37 HOPCROFT_KARP

```
// 1-based
const int N = 1e5+5, INF = 1e8 + 5;
vector <int> q[N];
int n, e, match[N], dist[N];
bool bfs() {
 queue <int> q;
for (int i = 1; i <= n; ++i) {</pre>
    if (!match[i]) dist[i] = 0, q.emplace(i);
    else dist[i] = INF;
  dist[0] = INF;
  while (!q.empty()) {
    int u = q.front(); q.pop();
    if (!u) continue;
    for (int v : g[u]) {
      if (dist[match[v]] == INF) {
        dist[match[v]] = dist[u] + 1,
        q.emplace(match[v]);
  return dist[0] != INF;
bool dfs (int u) {
 if (!u) return 1;
  for (int v : g[u]) {
    if (dist[match[v]] == dist[u] + 1 and

    dfs(match[v])) {

      match[u] = v, match[v] = u;
return 1;
  dist[u] = INF;
  return 0;
int hopcroftKarp() {
  int ret = 0;
  while (bfs()) {
    for (int i = 1; i \le n; ++i) {
      ret += !match[i] and dfs(i);
  return ret;
```

38 HUNGARIAN_ALGORITHM

```
auto residue = [&](int i, int j) { return c[i][j]
 - v[j]; };
for (int f = 0; f < n; ++f) {
  for (int j = 0; j < m; ++j) {
    dist[j] = residue(f, j); prev[j] = f;
  T w; int j, l;
  for (int s = 0, t = 0;;) {
    if (s == t) {
      l`= s; w'= dist[idx[t++]];
      for (int k = t; k < m; ++k) {
  j = idx[k]; T h = dist[j];</pre>
         if (h <= w) {
          if (h < w) \{ t = s; w = h; \}
           idx[k] = idx[t]; idx[t++] = j;
      for (int k = s; k < t; ++k) {
         j = idx[k];
         if (R[j] < 0) goto aug;
    int q = idx[s++], i = R[q];
    for (int k = t; k < m; ++k) {
      j = idx[k];
T h = residue(i,j) - residue(i,q) + w;
      if (h < dist[j]) {
         dist[j] = h; prev[j] = i;
        if (h == w) {
          if (R[j] < 0) goto aug;
           idx[k] = idx[t]; idx[t++] = j;
  for(int k = 0; k < l; ++k)
    v[idx[k]] += dist[idx[k]] - w;
  int i;
    R[j] = i = prev[j];
    swap(j, L[i]);
  } while (i != f);
f ret = 0;
for (int i = 0; i < n; ++i) {
  ret += c[i][L[i]]; // (i, L[i]) is a solution
return {ret, L};
```

39 JOSEPHUS

```
## k = 2
if n = 2^p + e; (p large as possible, e >= 0)
then survivor = 2 * e + 1;
for (int i = 1; i <= n; i++) {
   int x = k * i; // k = 2
   while (x > n) x = (k * (x - n) - 1) / (k - 1);
   cout << x << " ";
}
int kth(int n, int k, int f = 0) {
   if (n == 1) {
      assert(k == 1);
      return 1;</pre>
```

```
if (f == 0) {
   if (2 * k <= n) {
      return 2 * k:
   else {
     f = n \& 1;
     k -= n / 2;
     n -= n / 2;
      return 2 * kth(n, k, f) - 1;
 else {
   if (2 * k - 1 <= n) {
      return 2 * k - 1;
    else {
     f = n \& 1 ^ 1;
     k = (n + 1) / 2;
     n = (n + 1) / 2;
      return 2 * kth(n, k, f);
## k != 2
oset s;
for (int i = 1; i <= n; i++) s.insert(i);</pre>
for (int sz = n, ord = 0; sz > 0; --sz) {
 ord = (ord + k) % sz;
 auto it = s.find_by_order(ord);
 cout << *it << '-
 s.erase(it):
```

40 KMP

```
vector<int> get pi(string& s){
int n = s.size();
vector<int> pi(n);
for (int k = 0, i = 1; i < n; ++i){
  if(s[i] == s[k]) pi[i] = ++k;</pre>
   else if(k == 0) pi[i] = 0;
   else k = pi[k-1], --i;
return pi;
// Period = n % (n - pi.back() == 0)? <math>n - pi.back() == 0
   pi.back(): n
// Borders = pi.back(), pi[pi.back() - 1], ...
// Prefix palindrome: s + "#" + rev(s)
// Number of occurrences of each prefix:
vector<int> pref occur(vector<int> &pi) {
  int n = pi.size();
  vector<int> pref occur(n + 1);
 for (int i = 0; i < n; ++i) {
    pref occur[pi[i]]++;
  for (int len = n; len > 0; --len) {
    pref occur[pi[len - 1]] += pref occur[len];
    pref occur[len]++;
  return pref occur;
// Find the length of the longest proper suffix of
→ a suffix which also its prefix
```

```
15
```

```
// Reverse -> Find prefix function -> Reverse
// Find minimum length string such that given
-- strings occur as substring
```

41 KNUTH_OPTIMIZATION

```
// Divide an array into n parts.
// Cost of each division is subarray sum
// Minimize the cost
ll dp[n][n], opt[n][n];
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < n; ++j) {
    dp[i][j] = LLONG MAX;
 opt[i][i] = i;
dp[i][i] = 0;
for (int i = n - 2; i \ge 0; --i) {
  for (int j = i + 1; j < n; ++j) {
    for (int k = opt[i][j - 1]; k <= min(j - 1ll,
\rightarrow opt[i + 1][j]); ++k) {
      if (dp[i][j] >= dp[i][k] + dp[k + 1][j] +
    (pref[j + 1] - pref[i])) \{ dp[i][j] = dp[i][k] + dp[k + 1][j] +
    (pref[j + 1] - pref[i]);
         opt[i][j] = k;
cout << dp[0][n - 1] << "\n";
```

42 KRUSKAL

```
vector<Edge> mst(){
  vector<Edge> res;
  int cnt=0;
  sort(ed.begin(), ed.end());
  DSU dsu(n);
  for(Edge e: ed){
    if(!dsu.same_set(e.u, e.v)){
      res.push_back(e);
      dsu.unite(e.u,e.v);
      if(++cnt >= n-1) break;
  }
}
return res;
}
```

43 LICHAO_TREE

```
const int N = int(5e4 + 2);
const ll INF = ll(1e17);
vector<vector<ll> > tree(4*N, {0, INF});
ll f(vector<ll> line, int x){
  return line[0] * x + line[1];
}
void insert(vector<ll> line, int lo = 1, int hi =
  N. int i = 1){
```

```
int m = (lo + hi) / 2;
bool left = f(line, lo) < f(tree[i], lo);
bool mid = f(line, m) < f(tree[i], m);

if(mid) swap(tree[i], line);

if(hi - lo == 1) return;
else if(left != mid) insert(line, lo, m, 2*i);
else insert(line, m, hi, 2*i+1);
}

ll query(int x, int lo = 1, int hi = N, int i = 1){
   int m = (lo+hi)/2;
   ll curr = f(tree[i], x);
   if(hi-lo==1) return curr;
   if(x<m) return min(curr, query(x, lo, m, 2*i));
   else return min(curr, query(x, m, hi, 2*i+1));
}</pre>
```

44 MANACHER

```
// p[0][i] = half length of longest even palindrome
   around pos i-1, i and starts at i-p[0][i] and
   ends at i+p[0][i]-1
// p[1][i] = longest odd (half rounded down)
   palindrome around pos i and starts at i-p[1][i]
   and ends at i+p[1][i]
vector<vector<int>> manacher(string &s) {
 int n = s.size();
 vector<vector<int>>> p(2, vector<int>> (n));
 for (int z = 0; z < 2; ++z) {
    for (int i=0, l=0, r=0; i<n; ++i) {
      int t = r-i+!z;
      if (i<r) {
        p[z][i] = min(t, p[z][l+t]);
      int L = i-p[z][i], R = i+p[z][i]-!z;
      while (L>=1 \text{ and } R+1< n \text{ and } s[L-1]==s[R+1]) {
        p[z][i]++, L--, R++;
      if (R>r)
        l=L, r=R;
 return p;
```

45 MATRIX_EXPO

```
using row = vector<int>;
using matrix = vector<row>;
matrix unit mat(int n) {
    matrix I(n, row(n));
    for (int i = 0; i < n; ++i){
        I[i][i] = 1;
    }
    return I;
}
matrix mat_mul(matrix a, matrix b) {
    int m = a.size(), n = a[0].size();
    int p = b.size(), q = b[0].size();
    // assert(n==p);
    matrix res(m, row(q));
    for (int i = 0; i < m; ++i){</pre>
```

```
for (int j = 0; j < q; ++j){
    for (int k = 0; k < n; ++k){
        res[i][j] = (res[i][j] + a[i][k]*b[k][j]) %

    mod;
    }
}
return res;
}
matrix mat_exp(matrix a, int p) {
    int m = a.size(), n = a[0].size();
// assert(m==n);
    matrix res = unit_mat(m);
while (p) {
    if (p&1) res = mat_mul(a, res);
    a = mat_mul(a, a);
    p >>= 1;
}
return res;
}
```

46 MCF

```
struct MCF {
 int n:
 vector<vector<array<ll, 5>>> adj;
                                         // v, pos of

→ u in v, cap, cost, flow
 vector<ll> dis, par, pos;
 MCF(int n): n(n), adj(n), dis(n), par(n), pos(n)
→ {}
 void add edge(int u, int v, int cap, int cost) {
    adj[u] push back({v, adj[v].size(), cap, cost,
   adj[v].push back({u, adj[u].size() - 1, 0,
   -cost, 0});
 ll spfa(int s, int t) {
    dis.assign(n, INF);
   vector<ll> mn cap(n, INF), ing(n);
    aueue<int> a:
    q.push(s), inq[s] = 1, dis[s] = 0;
    while (!q.empty()) {
      int u = q.front(); q.pop();
      inq[u] = 0;
      for (int i = 0; i < adj[u].size(); ++i)</pre>
        auto [v, idx, cap, cost, flow] = adj[u][i];
if (cap > flow and dis[v] > dis[u] + cost) {
          dis[v] = dis[u] + cost;
          par[v] = u;
          pos[v] = i;
          mn cap[v] = min(mn cap[u], cap - flow);
          q.push(v);
          inq[v] = 1;
    return (mn cap[t] == INF? 0: mn cap[t]);
```

```
array<ll, 2> get(int s, int t, ll max flow = INF)
    til flow = 0, mc = 0;
while (ll f = min(spfa(s, t), max_flow - flow))
       flow += f;
      mc += f * dis[t];
       int u = t;
       while (u != s) {
         int p = par[u];
        adj[p][pos[u]][4] += f;
adj[u][adj[p][pos[u]][1]][4] -= f;
u = p;
    return {flow, mc};
MCF mcf(n);
for (int e = 0; e < m; ++e) {
  int u, v, r, c; cin >> u >> v >> r >> c; u--,
  mcf.add edge(u, v, r, c);
auto [f, mc] = mcf.get(0, n - 1, k);
```

47 MONOTONIC STACK

```
vector<int> nsel(vector<int> &a) {
  int n = a.size();
  vector<int> nsel(n);
  stack<int> st;
 st.push(-1);
  for (int i = 0; i < n; ++i) {
   while (st.top() != -1 and a[st.top()] >= a[i]) +
      st.pop();
   nsel[i] = st.top();
    st.push(i);
  return nsel;
```

48 MO_ALOGO

```
vector<array<int, 4>> cu(m);
for (int i = 0; i < m; ++i)
  auto \&[b, l, r, idx] = cu[i];
  cin >> l >> r; l--;
  b = r / B:
  idx = i;
sort(cu.begin(), cu.end());
int s = 0, e = -1;
for (auto [b, l, r, i]: cu) {
  while (l < s) add(--s);
  while (e < r) add(++e);
  while (s < l) remove(s++);</pre>
  while (r < e) remove(e--);</pre>
  ans[i] = cur ans;
```

49 NOTE

Fibonacci Number: $gcd(F_m, F_n) = F_gcd(m, n)$

```
Lucas Number: L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1 - Number S_{(a,n)} = \frac{1}{a+1} [n^{a+1} - (-1)^a + \sum_{i=2}^a (-1)^i {a+1 \choose i} S_{(a-i+1,n)}]
 of edge cover of a cycle graph C_n is L_n
 Catalan Number: C(n+1) = C(0)C(n) + C(1)C(n-1) + ...
\begin{bmatrix} \text{Catchain Namber: } & \text{Catchain Figure 2} \\ \text{C(n)C(0) C(n)} & \text{rCr(2n, n)} & \text{rCr(2n, n + 1) C(n)} & \text{rCr(2n, n)} \end{bmatrix} \sum_{i=1}^{n} f_k(i) = \frac{1}{k+1} n(n+1)(n+2)...(n+k) = \frac{1}{k+1} \frac{(n+k)!}{(n-1)!}
  - The number of valid bracket sequences of length 2n when a
 prefix of the sequence is given (among f are first bracket and s Conditional Probability:
  are second bracket) C(2n, f, s) = ncr(2n-(f+s), n-f) - ncr(2n-(f+s), n-f)
  (n-f)+(f-s)+1
 Stirling Number of second Kind: "Number of ways to partition
 a set of n labelled objects into k nonempty unlabelled subsets"
 S(n, k) = S(n-1, k-1) + k * S(n-1, k) S(n, 1) = 1, S(n, n) = 1
  Number of ways to color a 1 * ngridwith k colors such that each color size a to the astonic =
 k! * sn2(n,k)
 Derangement: "A derangement is a permutation of the ele-
 ments of a set, such that no element appear in its original posi-
 tion." 1, 0, 1, 2, 9, 44, 265, ... D(n) = (n-1)(D(n-1) + D(n-2)) = n
  * D(n-1) + (-1)^n D(0) = 1, D(1) = 0
 Partition number: for (int i = 1; i \le n; ++i) pent[2 * i - 1] = i *
 (3 * i - 1) / 2; pent[2 * i] = i * (3 * i + 1) / 2; p[0] = 1; for (int i
 = 1; i \le n; ++i) p[i] = 0; for (int j = 1, k = 0; pent[j] \le i; ++j)
 if (k < 2) p[i] = add(p[i], p[i - pent[i]]); else p[i] = sub(p[i], p[i - l])
  pent[j]); ++k, k = 3;
  Ballot Theorem: Suppose that in an election, candidate A re-
  ceives a votes and candidate B receives b votes, where a kb
 for some positive integer k. Compute the number of ways the
 ballots can be ordered so that A maintains more than k times
 as many votes as B throughout the counting of the ballots. The
 solution to the ballot problem is ((a kb)/(a+b)) * C(a+b, a)
 Classical Problem F(n, k) = number of ways to color n objects
  using exactly k colors
 Let G(n, k) be the number of ways to color n objects using no
 more than k colors.
 Then, F(n, k) = G(n, k) - C(k, 1) * G(n, k-1) + C(k, 2) * G(n, k-2)
  - C(k, 3) * G(n, k-3) ...
  Determining G(n, k):
  Suppose, we are given a 1 * n grid. Any two adjacent cells can
 not have same color. Then, G(n, k) = k * ((k-1)^{(n-1)})
  If no such condition on adjacent cells. Then, G(n, k) = k^n
 /* 1/(1 - x) = 1 + x + x^2 + x^3 + \dots 1/(1 - ax) = 1 + ax + (ax)^2 + \dots
 (ax)^3 + \dots 1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + \dots 1/(1-x)^3 =
 C(2,2) + C(3,2)x + C(4,2)x^2 + C(5,2)x^3 + \dots + \frac{1}{(1-ax)^2}(k+1) = \frac{1}{(1-ax)^2}
 1 + C(1+k,k)(ax) + C(2+k,k)(ax)^{2} + C(3+k,k)(ax)^{3} + \dots x(x+1)(1-x)^{-3} = 1 + x + 4x^{2} + 9x^{3} + 16x^{4} + 25x^{5} + \dots e^{x} = 1 + x + 4x^{2} + 9x^{3} + 16x^{4} + 25x^{5} + \dots e^{x}
  (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots */
 /* Extended Binomial Theorem C(-n, r) = C(n + r - 1, r)
 |* (-1)^r//n > = 0 and r > = 0 C(n,r) = (n(n-1)(n-2)...(n-(r-1)(n-2)...(n-(r-1)(n-2)...(n-(r-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-(n-1)(n-2)...(n-2)...(n-(n-1)(n-2)...(n-2)...(n-(n-1)(n-2)...(n-2)...(n-(n-1)(n-2)...(n-2)...(n-(n-1)(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(n-2)...(
 1)))/r!//worksforanyn(x+a)^-n = a^(-n) + C(-n,1)x^1a^(-n-1) +
 C(-n,2)x^2a^{(-n-2)}+C(-n,3)x^3a^{(-n-3)}+\dots */
                                      \sum_{i=1}^{n} \gcd(i,n) = \sum_{d|n} d\phi(\frac{n}{d})
\begin{vmatrix} 1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n2+3n+-1)}{30} \\ S_{(a,n)} = 1^a + 2^a + 3^a + 4^a + \dots + n^a \end{vmatrix}
```

$$S_{(a,n)} = \frac{1}{a+1} [n^{a+1} - (-1)^a + \sum_{i=2}^a (-1)^i {a+1 \choose i} S_{(a-i+1,n)}]$$

$$1.2 + 2.3 + 3.4 + \dots = \frac{1}{3} n(n+1)(n+2)$$

$$\sum_{i=1}^n f_k(i) = \frac{1}{k+1} n(n+1)(n+2) \dots (n+k) = \frac{1}{k+1} \frac{(n+k)!}{(n-1)!}$$

$$\sum_{i=0}^n nix^i = 1 + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \dots + nx^n = \frac{(x-(n+1)x^{n+1} + nx^{n+2})}{(x-1)^2}$$
Condiational Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Time Complexity: $O(k \log n)$ ll $get_s n2(intn, intk)$ llsn2 = 0; for dinti = 0; i <= k; ++i) llnow = nCr(k,i) * powmod(k-i,n,mod) if

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

50 NUMBER_THEORY

```
Il mulmod(ll a, ll b, ll m){
 a%=m;
 ll res = 0;
 while(b > 0){
    if(b&1)
               res=(res+a)%m;
   a=(a<<1)%m;
    b >> = 1:
 return res:
ĺl powmod(ll a, ll b, ll m){
 ll res = 1;
 while(b > 0){
             res = mulmod(res, a, m);
   if(b&1)
    a = mulmod(a, a, m);
    b >>= 1;
 return res:
ll ext gcd(ll a, ll b, ll &x, ll &y){
 if(b = 0) \{ x=1, y=0;
    return a;
 Il q = ext qcd(b, a%b, y, x);
 v = a/b*x
 return g;
ll mod inv(ll a, ll m){
 ll g = ext gcd(a, m, x, y);
 if(g != 1) return -1; //no solution exists
 return (x%m+m)%m:
## Linear-sieve
int lpf[N], pm[N], pcnt = 0;
for (int i = 2; i < N; ++i) {
 if (!lpf[i]) lpf[i] = i, pm[pcnt++] = i;
 for (int j = 0; j < sz; ++j) {
    int p = pm[j];
```

```
if (lpf[i] = N)
                                    break;
    lpf[i * p] = p;
## Miller-Rabin
bool isp(ll n){
  if(n==2 || n == 3) return 1;
  if(n<=1 | n%2==0) return 0;
  for (int k = 0; k < 10; ++k){
   ll a = 2+rand()%(n-2);
   ll s = n-1;
    while(!(s&1)) s>>=1;
    if(powmod(a, s, n) == 1) continue;
    int iscomp = 1;
    while(s!=n-1){
      if(powmod(a, s, n)==n-1){
        iscomp = 0:
        break;
      s=s<<1;
    if(iscomp) return 0;
  return 1;
## Miller-Rabin Deterministic:
bool check composite(u64 n, u64 a, u64 d, int s) {
  u64 x = \overline{binpower}(a, d, n);
  if (x == 1 | | x == n - 1)
    return false:
  for (int r = 1; r < s; r++) {
    x = (u128)x * x % n;
   if (x == n - 1)
      return false:
  return true;
bool isp(u64 n) {
 if (n < 2)
    return false;
  int r = 0:
  u64 d = n - 1:
  while ((d \& 1) == 0) {
   d >>= 1;
    r++;
  for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
\rightarrow 31, 37\}) {
    if (n == a)
      return true;
   if (check composite(n, a, d, r))
      return false:
  return true:
## Prime Factorize of large number(Pollard Rho):
ll f(ll x, ll c, ll n){
  return (mulmod(x,x,n)+c)%n;
ll pollard rho(ll n){
  if(n == \overline{1}) return 1;
  if(n%2 == 0) return 2;
```

```
ll x = rand()%(n-2)+2;
  ll y = x;
  ll c = rand()%(n-1)+1;
  ll q = 1;
  while (g == 1){
   x = f(x, c, n);
    y = f(y, c, n);
   y = f(y, c, n);
    g = gcd(abs(x-y), n);
  return q;
vector<ll> prime factorize(ll n){
 if(n<=1) return vector<ll>();
  if(isp(n)) return vector<ll> ({n});
 ll d = pollard_rho(n);
vector<ll> v = factorize(d);
  vector<ll> w = factorize(n/d);
  v.insert(v.end(), w.begin(), w.end());
  sort(v.begin(), v.end());
  return v;
// auto pf = prime factorize(n);
## Number of divisors of n O(n^1/3):
int nod(ll n){
 sieve();
  int ret = 1;
  for (int i = 2; 1LL*i*i*i <= n; ++i){
    if(isp[i]){
      int e = 0;
      while(n\%i == 0){
        e++;
        n /= i;
      ret *= e+1:
  il sq = sqrt(1.0L*n);
  if(isprime(n)) ret *= 2;
  else if(n == sq*sq and isprime(sq)) ret *= 3;
  else if(n!=1) ret *= 4;
  return ret;
## Totient function of n:
|ll get phi(ll n) {
 ll phi = n;
  for (int i = 2; i * i <= n; ++i) {
    if (n % i == 0) {
      phi -= phi / i;
      while (n \% i == 0) {
        n /= i;
  if (n > 1) phi -= phi / n;
  return phi;
## Totient function from 1 to N:
int phi[N];
|iota(phi, phi + N, 0);
for(int i = 2; i < N; ++i){
 if(phi[i] == i){
    phi[i]--;
    for(int j = 2 * i; j < N; j += i){
```

```
phi[j] -= phi[j] / i;
## Smallest inverse phi
ll inv phi(ll phi, ll n, int pc) {
 if (phi == 1) return n;
if (pc == -1) return INF;
  ll ret = inv_phi(phi, n, pc - 1);
  if (phi % (p[pc] - 1) == 0) {
    phi /= (p[pc] - 1);
    n = n / (p[pc] - 1) * p[pc];
    while (phi % p[pc] == 0) {
      phi /= p[pc];
    ret = min(ret, inv phi(phi, n, pc - 1));
  return ret;
ll phi; cin >> phi;
if (phi & 1) {
 cout << (phi == 1) << "\n";
else {
  for (int i = 1; i * i <= phi; ++i) {
    if (phi % i == 0) {
      if (isp(i + 1)) {
        p.push back(i + 1);
      if (i * i != phi and isp(phi / i + 1)){
        p.push back(phi / i + 1);
  sort(p.begin(), p.end());
  ll ans = inv phi(phi, phi, p.size() - 1);
  cout \ll (ans == INF? 0: ans) \ll "\n";
## GCD sum function from 1 to N:
ll phi[N], g[N];
void pcqsm(){ //pre calculate qcd sum fucntion
  pcphi();
  for (int i = 1; i < N; ++i){
    for (int j = i; j < N; j+=i){
      g[j] += i*phi[j/i];
## All Pair gcd sum:
for (int i = 1; i < N; ++i) {
 for (int j = i; j < N; j += i) {
  gcd_sum[j] += 1ll * phi[i] * (j / i);</pre>
  qcd sum[i] -= i;
 pref gcd sum[i] = pref gcd sum[i - 1] +

    qcd sum[i];

## LCM sum function of n:
ll lsm(ll n){
  ll ret=0;
  for(ll d=1; d*d<=n; d++){
```

```
18
```

```
if(n%d==0){
      ret += d*phi(d);
      if(n/d!=d) ret += n/d*phi(n/d);
  return (ret+1)*n/2;
## LCM sum function from 1 to N
ll phi[N], l[N];
void pclsm(){ //pre calculate lcm sum function
  pcphi();
  for (int i = 1; i < N; ++i){
    for (int j = i; j < N; j+=i) {
      l[j] += i*phi[i];
  for (int i = 1; i < N; ++i){
    l[i] = (l[i]+1)*i/2;</pre>
## All pair lcm sum:
for (int i = 1; i < N; ++i) {
  for (int j = i; j < N; j += i) {
  lcm_sum[j] += i * phi[i];</pre>
  lcm sum[i]++;
  lcm_sum[i] /= 2;
lcm_sum[i] *= i;
  lcm sum[i] -= i;
  pref_lcm_sum[i] = lcm_sum[i];
  pref lcm sum[i] += pref lcm sum[i - 1];
## Number of co-prime pairs of an array:
vector<ll> cnt(A);
for (int xi: x) {
  for (int d = 1; d * d <= xi; ++d) {
    if (xi % d == 0) {
      cnt[d]++;
      if (xi / d != d) {
         cnt[xi / d]++;
ll ans = 0;
for (int i = 1; i < A; ++i) {
  if (!sq free[i]) continue;
  ll ways = cnt[i] * (cnt[i] - 1) / 2;
  if (pf[i].size() & 1 ^ 1) {
  ans += ways;
  else {
  ans -= ways;
## All pair gcd sum of an array:
vector<ll> cnt(A);
for (auto ai: a) {
  for (int d = 1; d * d <= ai; ++d) {</pre>
    if (ai % d == 0) {
      cnt[d]++;
      if (ai / d != d) {
```

```
cnt[ai / d]++;
      }
ll sum = 0:
vector<ll> left(A);
iota(left.begin(), left.end(), 0);
for (int i = 1; i < A; ++i) {
 ll add = left[i] * cnt[i] * (cnt[i] - 1) / 2;
  sum += add;
  for (int j = 2 * i; j < A; j += i) {
    left[j] -= left[i];
ll crt(ll r1, ll m1, ll r2, ll m2){
  if(m1<m2) swap(r1, r2), swap(m1, m2);</pre>
  ll p, q, g = bezout(m1, m2, p, q);
  if((r2-r1)%g !=0 ) return -1; //no solution
  ll x = (r2-r1)\%m2*p\%m2*m1/g + r1;
  return x<0? x+m1*m2/q: x;
ll crt(vector<ll>& r, vector<ll>& m){
 ll x = r[0], M=m[0];
  for (int i = 1; i < r.size(); ++i){</pre>
    x = crt(x, M, r[i], m[i]);
    ll q = qcd(M, m[i]);
    M = (M/\overline{q})*(m[i]/q);
  return x;
## Discrete Logarithm
ll discrete log(ll a, ll b, ll m) {
  a \% = m, b \% = m;
  if(a == 0){
    return (b == 0? 1: -1);
  ll k = 1, add = 0, g;
  while ((q = qcd(a, m)) > 1) {
    if (b = k)
      return add:
    if (b % g) {
      return -1;
      /= g, m /= g, k = (k * a / g) % m, ++add;
  int n = sart(m) + 1:
  unordered map<int, int> vals;
  for (ll a = 0, cur = b; a <= n; ++a) {
    vals[cur] = q;
    cur = (cur * a) % m;
  ll an = 1;
  for (int i = 0; i < n; ++i) {
  an = (an * a) % m;</pre>
```

```
for (ll p = 1, cur = k; p \le n; ++p) {
   cur = (cur * an) % m;
   if (vals.count(cur)) {
      return n * p - vals[cur] + add;
 return -1:
## Mobius
- mu[1] = 1.
 mu[n] = 0 if n has a squared prime factor,
- mu[n] = 1 if n is square-free with even number of
- mu[n] = -1 if n is square-free with odd number of

→ prime factors

"sum of mu[d] where d | n is 0 ( For n=1, sum is 1
int mu[MAX] = \{0\};
void Mobius(int N){
 int i, j;
 mu[1] = 1;
 for (i = 1; i \le N; i++)
   if (mu[i]){
     for (j = i + i; j \le N; j += i){
        mu[j] -= mu[i];
```

51 ONLINE_CHT

```
const ll IS QUERY = -(1LL << 62);</pre>
struct line {
 ll m, b;
  mutable function <const line*()> succ;
  bool operator < (const line &rhs) const {</pre>
    if (rhs.b != IS QUERY) return m < rhs.m;</pre>
    const line *s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s -> b < (s -> m - m) * x;
struct CHT : public multiset <line> {
  bool bad (iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y \rightarrow m == z \rightarrow m \&\& y \rightarrow b <= z \rightarrow b;
    auto x = prev(y);
    if (z == end()) return y \rightarrow m == x \rightarrow m \& \& y \rightarrow
\rightarrow b <= x -> b:
    return 1.0 * (x -> b - y -> b) * (z -> m - y ->
    m) >= 1.0 * (y -> b - z -> b) * (y -> m - x ->
    m);
```

```
void add (ll m, ll b) {
    auto y = insert({m, b});
    y \rightarrow succ = [=] \{return \ next(y) == end() ? 0 :
 if (bad(y)) {erase(y); return;}
    while (next(y) != end() \&\& bad(next(y)))
 → erase(next(y));
    while (y != begin() \&\& bad(prev(y)))
 → erase(prev(v));
  ll eval (ll x) {
    auto l = *lower bound((line) {x, IS_QUERY});
    return l.m * x \mp l.b;
// To find maximum
CHT cht;
cht.add(m, c);
y max = cht.eval(x);
// To find minimum
CHT cht;
cht.add(-m, -c);
y min = -cht.eval(x);
```

52 PALINDROMIC_TREE

```
const int N = 1e5+10;
struct vertex
 int len, link, no of suf pal;
 map<char, int> next;
}pt[N];
int sz, at, cnt[N];
char s[N];
void pt init(){
 for (int i = 0; i < N; ++i){
   pt[i].next.clear();
 memset(cnt, 0, sizeof(cnt));
 pt[0].len = -1, pt[0].link = 0,
→ pt[0].no of suf pal = 0;
 pt[1].len = 0, pt[1].link = 0,
\rightarrow pt[1].no of suf pal = 0;
 sz = at = T;
void pt extend(int si){ //string index
 while (s[si - pt[at].len - 1] != s[si]) at =
→ pt[at].link;
 int x = pt[at].link, c = s[si]-'a';
 while (s[si - pt[x].len - 1] != s[si]) x =

    pt[x].link;
 if(!pt[at].next.count(c)){
   pt[at].next[c] = ++sz;
   pt[sz].len = pt[at].len + 2;
   // cnt[pt[at].len+2]++; //for finding number

→ of distinct palindrome of lenght k

   pt[sz].link = (pt[sz].len == 1)? 1 :
  pt[x].next[c]:
```

```
// pt[sz].no of suf pal = 1 +
   pt[pt[sz].link].no of suf pal; //for finding
   number of palindrome which last position is si
  // cnt[pt[at].len + 2]++; //for finding number

→ of palindrome of lenght k

  at = pt[at].next[c];
int num of pal(int ai){  //distinct palindrome,

→ arrav index

  int ret = pt[at].ans;
  for(auto x : pt[ai].next)
    ret += num of pal(x.second);
  return ret;
int main(){
  scanf("%s", s);
  pt init();
  for (int i = 0; s[i]; ++i){
    pt extend(i);
  int ans = num of pal(0) + num of pal(1) - 2;
  printf("%d\n", ans);
  return 0;
```

53 PERSISTENT_SEGMENT_TREE

```
## Point Addition & Range Sum:
struct node {
 ll sum:
  node *l, *r;
  node(ll \ s = 0, \ node \ *l = NULL, \ node \ *r = NULL):
\rightarrow sum(s), l(l), r(r) {}
|node* add(node *u, int i, int x, int s, int e) {
 if (s == e) return new node(u->sum + x);
  if (!u->l) u->l = new node(), u->r = new node();
  node *nu = new node(u->sum, u->l, u->r);
  int m = (s + e) / 2;
  if (i \le m) nu > l = add(nu - > l, i, x, s, m);
  else nu->r = add(nu->r, i, x, m + 1, e);
  nu->sum = nu->l->sum + nu->r->sum;
  return nu;
ll rsum(node *u, int l, int r, int s, int e) {
  if (!u) return 0;
  if (s > r or e < l) return 0;</pre>
  if (l <= s and e <= r) return u->sum;
  int m = (s + e) / 2;
  return rsum(u->l, l, r, s, m) + rsum(u->r, l, r, l)
\rightarrow m + 1. e):
|vector<node*> root(VER);
root[0] = new node(); // initialization
int ver = 1;
root[k] = add(root[k], i, x, 0, sz - 1);; //
\rightarrow Assign a[i] = x in version k
root[ver++] = root[k]; // Create a new version

→ from kth version

cout \ll rsum(root[k], l, r, 0, sz - 1) \ll "\n";
→ Range sum of version k
```

```
## count numbers > k in a range
root[0] = new node();
for (int i = 0; i < n; ++i) {
  root[i + 1] = add(root[i], a[i], 1);
while (q--) {
  int l, r, k; cin >> l >> r >> k; l--, r--;
  int ans = rsum(root[r + 1], k, E - 1) -
\rightarrow rsum(root[l], k, E - 1);
  cout << ans << "\n":
## kth number in a range: O(logn)
int kth(node *ul, node *ur, int k, int s = 0, int e
\rightarrow = E - 1) {
  if (s == e) return s;
  int m = (s + e) / 2;
  int cnt left = ur->left->sum - ul->left->sum;
  if (cnt<sup>-</sup>left >= k) return kth(ul->left,

    ur->left, k, s, m);

  else return kth(ul->right, ur->right, k -
\rightarrow cnt left, m + 1, e);
root[0] = new node();
for (int i = 0; i < n; ++i) {
  root[i + 1] = add(root[i], a[i + ], 1);
while (q--) {
  int l, r, k; cin >> l >> r >> k; l--, r--;
  int x = kth(root[l], root[r + 1], k);
```

54 PERSISTENT_TRIE

```
struct node {
 node *nxt[2]
 node() { fill(nxt, nxt + 2, nullptr); }
node* add(node *prev, int x) {
 node *new root = new node();
 node * cu\overline{r} = new root;;
 for (int idx = \overline{IDX} - 1; idx >= 0; --idx) {
    int f = (x >> idx) \& 1;
    if (prev and prev->nxt[!f]) cur->nxt[!f] =
   prev->nxt[!f];
    cur->nxt[f] = new node();
    cur = cur->nxt[f];
    if (prev) prev = prev->nxt[f];
  return new root;
int get max(node *root, int x) {
 if (!root) return 0:
 node *u = root;
  int ret = 0;
 for (int idx = IDX - 1; idx \geq 0; --idx) {
    int f = (x \gg idx) \& 1;
    if (u->nxt[!f]) ret += (1 << idx), u =
   u->nxt[!f];
    else u = u->nxt[f];
 return ret;
```

55 POINT

```
# point in convex poly: O(logn)
struct pt {
    double x, y;
    pt() {}
    pt(double x, double y) : x(x) , y(y) {}
    pt(const pt \& p) : x(p.x) , y(p.y)
    pt operator + (const pt &p) const { return pt(
\rightarrow x+p.x , y+p.y ); }
    pt operator - (const pt &p) const { return pt(
  x-p.x , y-p.y ); }
    pt operator * (double c) const { return pt( x*c
   , y*c ); }
inline double dot(pt u, pt v) { return u.x*v.x +

    u.y*v.y; }

inline double cross(pt u, pt v) {return u.x*v.y -

    u.v*v.x:}

inline double triArea2(pt a,pt b,pt c) { return

    cross(b-a,c-a); }

inline bool inDisk(pt a, pt b, pt p) { return
\rightarrow dot(a-p, b-p) <= 0; }
inline bool onSegment(pt a, pt b, pt p) { return
\rightarrow triArea2(a,b,p) == 0 && inDisk(a,b,p); }
// points of the polygon has to be in ccw order
// if strict, returns false when a is on the
→ boundarv
inline bool insideConvexPoly(pt* C, int nc, pt p,
→ bool strict) {
    int st = 1, en = nc - 1, mid;
    while (en - st > 1) {
        mid = (st + en) >> 1;
        if(triArea2(C[0], C[mid], p) < 0) en = mid;
        else st = mid;
    if(strict) {
        if(st==1) if(onSegment(C[0],C[st],p))
   return false;
        if(en==nc-1) if(onSegment(C[0],C[en],p))

→ return false;

        if(onSegment(C[st],C[en],p)) return false;
    if(triArea2(C[0], C[st], p) < 0) return false;</pre>
    if(triArea2(C[st], C[en], p) < 0) return false;</pre>
    if(triArea2(C[en], C[0], p) < 0) return false;</pre>
    return true;
# check point in polygon: O(n)
struct pt {
    double x, y;
    pt(double x, double y) : x(x) , y(y) {}
    pt(const pt \& p) : x(p.x) , y(p.y) \{ \}
    pt operator + (const pt &p) const { return pt(
\rightarrow x+p.x , y+p.y ); }
    pt operator - (const pt &p) const { return pt(
   x-p.x , y-p.y ); }
    pt operator * (double c) const { return pt( x*c
   , y*c ); }
inline double dot(pt u, pt v) { return u.x*v.x +
\rightarrow u.y*v.y; }
```

```
inline double cross(pt u, pt v) {return u.x*v.y -

    u.y*v.x;}

inline double triArea2(pt a,pt b,pt c) { return

    cross(b-a,c-a); }

inline bool inDisk(pt a, pt b, pt p) { return
    dot(a-p, b-p) \ll 0;
inline bool onSegment(pt a, pt b, pt p) { return
   triArea2(a,b,p) == 0 \&\& inDisk(a,b,p); }
// check if segment pg crosses ray from point a
inline bool crossesRay(pt a, pt p, pt q) {
    int mul = (q.y >= a.y) - (p.y >= a.y);
    return (mul * triArea2(a,p,q)) > 0;
// if strict, returns false when a is on the
boundary
inline bool insidePoly(pt *P, int np, pt a, bool
   strict = true) {
    int numCrossings = 0;
    for (int i = 0; i < np; i++)
        if (onSegment(P[i], P[(i+1)%np], a)) return
    !strict:
        numCrossings += crossesRay(a, P[i],
    P[(i+1)%np]);
    return (numCrossings & 1); // inside if odd
   number of crossings
# Polar sort:
|inline bool up (point p) {
  return p.y > 0 or (p.y == 0 and p.x >= 0);
sort(v.begin(), v.end(), [] (point a, point b) {
  return up(a) == up(b) ? a.x * b.y > a.y * b.x :
 \rightarrow up(a) < up(b);
});
# Convex Hull
struct pt {
 int x, y;
return 111*(b.x-a.x)*(c.y-a.y) -
   1ll*(c.x-a.x)*(b.y-a.y);
vector<pt> convexHull(vector<pt>& p) {
  sort(p.begin(), p.end(), [\&] (pt a, pt b) {
    return (a.x==b.x? a.y<b.y: a.x<b.x);
  int n = p.size(), m = 0;
  vector<pt> hull(2*n);
  for (int i = 0; i < n; ++i){
    while (m>=2 and cross(hull[m-2], hull[m-1],
    p[i]) < 0) --m;
    hull[m++] = p[i];
  for (int i = n-2, l = m; i >= 0; --i) {
    while (m>=l+1) and cross(hull[m-2], hull[m-1],
    p[i]) < 0) --m;
    hull[m++] = p[i];
  hull.resize(m-1);
  return hull:
```

56 POLAR_SORT

57 POLYNOMIAL_INTERPOLATION

```
P(x) = a0 + a1x + a2x^2 + ... + anx^n
   \frac{1}{2} \sqrt{|y|} = P(i)
ĺĺĺ evaĺ (vector<ll> y, ll k) {
          int n = y.size() - 1;
          if (k <= n) +
                     return y[k];
          vector<ll> L(n + 1, 1);
          for (int x = 1; x <= n; ++x) {
                     L[0] = L[0] * (k - x) % mod;
                     L[0] = L[0] * inv(-x) % mod;
          for (int x = 1; x <= n; ++x)
                    L[x] = L[x - 1] * inv(k - x) % mod * (k - (x - x) - x) % mod * (k - x) % mod * (x - x) % mod
              1)) % mod;
                     L[x] = L[x] * ((x - 1) - n + mod) % mod *
                  inv(x) % mod:
          ll yk = 0;
          for (int x = 0; x <= n; ++x) {
                     yk = add(yk, L[x] * y[x] % mod);
          return yk;
```

58 SCC

```
void dfs1(int u, vector<int> *adj, vector<int>
vis[u] = 1;
 for (int &v: adj[u]) {
   if (!vis[v]) {
     dfs1(v, adj, vis, order);
 order.emplace back(u);
void dfs2(int u, vector<int> *rev adj, vector<int>
  &vis, vector<int> &scc) {
 scc.emplace back(u);
 vis[u] = 1;
 for (int &v: rev adj[u]) {
   if (!vis[v]) {
     dfs2(v, rev adj, vis, scc);
vector<vector<int>> get sccs(int n, vector<int>
→ *adi) {
 vector<int> vis(n), order;
```

```
for (int u = 0; u < n; ++u) {
   if (!vis[u]) {
      dfs1(u, adj, vis, order);
  vector<int> rev adj[n];
  for (int u = 0; u < n; ++u) {
   for (int v: adi[u]) {
      rev adj[v].emplace back(u);
  vector<vector<int>> sccs;
  reverse(order.begin(), order.end());
  vis.assign(n, 0);
  for (int u: order) {
   if (!vis[u]) {
      sccs.emplace back(0);
      dfs2(u, rev adj, vis, sccs.back());
  return sccs;
vector<vector<int>> sccs = get sccs(n, adj);
int tot scc = sccs.size();
vector<int> scc no(n);
for (int i = 0; i < tot_scc; ++i) {</pre>
 for (int u: sccs[i]) {
    scc_no[u] = i;
```

59 SEGMENT_TREE

```
## Point Assign & Range Query
## Point Assign & Range Min/Max with frequency
## Point Assign & Range Maximum Sum
struct Node {
  ll sum, mx, pref, suf;
t[4 * N];
Node combine(Node l, Node r) {
  Node ret;
  ret.pref = max(l.pref, l.sum + r.pref);
  ret.suf = max(l.suf + r.sum, r.suf);
  ret.sum = l.sum + r.sum;
  ret.mx = max(\{l.mx, l.suf + r.pref, r.mx\});
  return ret;
void update(int idx, int x, int v = 1, int st = 0,
\rightarrow int ed = n - 1) {
  if (st > idx or ed < idx) return ;</pre>
  if (st == ed) {
    t[v].sum = t[v].mx = t[v].pref = t[v].suf = x;
  int lc = 2 * v, rc = 2 * v + 1;
  int mid = (st + ed) / 2;
  update(idx, x, lc, st, mid);
  update(idx, x, rc, mid + 1, ed);
  t[v] = combine(t[lc], t[rc]);
Node mx sum(int l, int r, int v = 1, int st = 0,
\rightarrow int ed = n - 1) {
 if (st > r or ed < l) return {0, ll(-1e15),
→ ll(-1e15), ll(-1e15)};
  if (l <= st and ed <= r) return t[v];</pre>
```

```
int lc = 2 * v, rc = 2 * v + 1;
  int mid = (st + ed) >> 1;
  Node lret = mx_sum(l, r, lc, st, mid);
Node rret = mx_sum(l, r, rc, mid + 1, ed);
  Node ret = combine(lret, rret);
  return ret;
## Point Assign & Range Composite:
- 0 p c d: f p : = cx + d
                    - 1 l r x: f r(f (r -
 \rightarrow 1)(...f l(x))) % mod
struct Node {
 int a, b;
\} t[3 * N];
void assign(int i, int ai, int bi, int u = 1, int s
 \rightarrow = 0, int e = n - 1) {
  if (s > i or e < i) return ;</pre>
  if (s == e) {
    t[u].a = ai;
    t[u].b = bi;
    return ;
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  assign(i, ai, bi, v, s, m);
  assign(i, ai, bi, w, m + 1, e);
t[u].a = 1ll * t[w].a * t[v].a % mod;
  t[u].b = (111 * t[w].a * t[v].b + t[w].b) % mod;
Node query(int l, int r, int u = 1, int s = 0, int
 \rightarrow e = n - 1) {
  if (l > e or r < s) return {1, 0};
  if (l <= s and e <= r) return t[u];</pre>
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  Node lret = query(l, r, v, s, m);
  Node rret = query(l, r, w, m + 1, e);
  Node ret:
  ret.a = 111 * rret.a * lret.a % mod;
  ret.b = (111 * rret.a * lret.b + rret.b) % mod:
  return ret:
## Range Affine & Range Sum
- 0 l r b c: a[i] := b * a[i] + c
                        - 1 l r: range sum
struct Node {
 int a = 1, b = 0, sum = 0;
\} t[3 * N];
void push(int u, int s, int e) {
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  t[v].sum = (1ll * t[u].a * t[v].sum + 1ll * (m - 1)
 \rightarrow s + 1) * t[u].b) % mod;
  t[v].a = 111 * t[u].a * t[v].a % mod;
  t[v].b = (111 * t[u].a * t[v].b + t[u].b) % mod;
  t[w].sum = (111 * t[u].a * t[w].sum + 111 * (e -

→ m) * t[u].b) % mod:

  t[w].a = 111 * t[u].a * t[w].a % mod;
  t[w].b = (111 * t[u].a * t[w].b + t[u].b) % mod;
  t[u].a = 1;
  t[u].b = 0;
void update(int l, int r, int a, int b, int u = 1,
   int s = 0, int e = n - 1) {
  if (l > e or r < s) return ;</pre>
  if (l <= s and e <= r) {
    t[u].sum = (111 * a * t[u].sum + 111 * (e - s +
 → 1) * b) % mod;
```

```
t[u].a = 111 * a * t[u].a % mod;
    t[u].b = (111 * a * t[u].b + b) % mod;
    return ;
  if (t[u].b != 0) push(u, s, e);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  update(l, r, a, b, v, s, m);
  update(l, r, a, b, w, m + 1, e);
  t[u].sum = (t[v].sum + t[w].sum) % mod;
int rsum(int l, int r, int u = 1, int s = 0, int e
\rightarrow = n - 1) {
  if (l > e \text{ or } r < s) return 0;
  if (l <= s and e <= r) return t[u].sum;</pre>
  if (t[u].b != -1) push(u, s, e);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  int lret = rsum(l, r, v, s, m);
  int rret = rsum(l, r, w, m + 1, e);
  return (lret + rret) % mod;
## Range Most Frequent for sorted array:
struct node {
  int pref, pref freq, suf, suf freq, max freq;
f(3 * MXN);
node combine(node lc, node rc) {
  node res;
  res.pref = lc.pref;
  res.pref freq = lc.pref freq;
  if (lc.pref == rc.pref) {
    res.pref freq += rc.pref freq;
  res.suf = rc.suf;
  res.suf freq = rc.suf freq;
  if (lc.\overline{s}uf = rc.suf)^{-}{
    res.suf freq += lc.suf freq;
  res.max freq = max({lc.max freq, lc.suf ==
    rc.pref ? lc.suf freq + rc.pref freq : 0, +

    rc.max freq});

  return res;
void build(int v = 1, int st = 0, int ed = n - 1) {
  if (st > ed) return ;
  if (st == ed) {
    t[v] = \{a[st], 1, a[st], 1, 1\};
    return ;
  int lc = v << 1;
  int rc = lc ^ 1;
  int m = (st + ed) >> 1;
  build(lc, st, m);
  build(rc, m + 1, ed);
  t[v] = combine(t[lc], t[rc]);
node query(int l, int r, int v = 1, int st = 0, int
\rightarrow ed = n - 1) {
 if (l > ed or r < st) return {INT MIN, 0,</pre>

→ INT MAX, 0, 0};

  if (l <= st and ed <= r) return t[v];</pre>
  int lc = v << 1;
  int rc = lc ^ 1;
  int m = (st + ed) >> 1;
  node lans = query(l, r, lc, st, m);
  node rans = query(l, r, rc, m + 1, ed);
```

```
node res = combine(lans, rans);
  return res:
## Point Update & kth non zero
## Point Update & first above
## Point Update & Range Max Prefix Sum
## Remove Point & Point Query
## Range Inversion Count
struct Node {
  int cnt[40];
  ll inv;
  Node(): inv(0) {memupdate(cnt, 0, sizeof(cnt));}
} t[4 * N];
void update(int i, int x, int y, int v = 1, int s =
\sim 0, int e = n - 1) {
  if (s > i or e < i) return ;</pre>
  if (s == e) {
    if (x != -1) t[v].cnt[x]--;
    t[v].cnt[v]++;
    t[v].inv = 0;
    return ;
  int lc = 2 * v, rc = 2 * v + 1;
  int m = (s + e) / 2;
  update(i, x, y, lc, s, m);
  update(i, x, ý, rc, m + 1, e);
t[v].inv = t[lc].inv + t[rc].inv;
  int now = 0;
  for (int j = 0; j < 40; ++j) {
  t[v].inv += 1LL * t[ic].cnt[j] * now;</pre>
    now += t[rc].cnt[j];
    t[v].cnt[j] = t[lc].cnt[j] + t[rc].cnt[j];
Node count inv(int l, int r, int v = 1, int s = 0,
\rightarrow int e = n - 1) {
 if (s > r or e < l) return Node();</pre>
  if (l <= s and e <= r) {
    return t[v];
  int lc = 2 * v, rc = 2 * v + 1;
  int m = (s + e) / 2;
  Node lret = count inv(l, r, lc, s, m);
  Node rret = count^inv(l, r, rc, m + 1, e);
  Node ret;
  ret.inv = lret.inv + rret.inv;
  int now = 0;
  for (int j = 0; j < 40; ++j) {
  ret.inv += 1LL * lret.cnt[j] * now;</pre>
    now += rret.cnt[j];
    ret.cnt[j] = lret.cnt[j] + rret.cnt[j];
  return ret;
## Point Update & Range Distinct Elements:
void add(int idx, int i, int x, int v = 1, int s =
\rightarrow 0, int e = n - 1) {
  if (s > i \text{ or } e < i) return:
  if (s == e) {
    t[idx][v] += x;
    return ;
  int lc = 2 * v, rc = 2 * v + 1;
  int m = (s + e) / 2;
  add(idx, i, x, lc, s, m);
```

```
add(idx, i, x, rc, m + 1, e);
  t[idx][v] = t[idx][lc] + t[idx][rc];
int rsum(int idx, int l, int r, int v = 1, int s =
 \rightarrow 0, int e = n - 1) {
  if (s > r or e < l) return 0;</pre>
  if (l <= s and e <= r) return t[idx][v];</pre>
  int lc = 2 * v, rc = 2 * v + 1;
  int m = (s + e) / 2;
  int lret = rsum(idx, l, r, lc, s, m);
  int rret = rsum(idx, l, r, rc, m + 1, e);
  return lret + rret;
## Range Addition & Range Weighted Sum:
struct Node {
 ll sum = 0, wsum = 0, p = 0;
} t[3 * N];
void f(int x, int u, int s, int e) {
 t[u].sum += 1LL * (e - s + 1) * x;
  t[u].wsum += 1LL * (e - s + 1) * (e - s + 2) / 2
 t[u].p += x;
|void push(int u, int s, int e) {
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  f(t[u].p, v, s, m);
  f(t[u], p, w, m + 1, e);
  t[u].p = 0;
void add(int l, int r, int x, int u = 1, int s = 0,
 \rightarrow int e = n - 1) {
  if (l > e or r < s) return;</pre>
  if (l <= s and e <= r) {
    f(x, u, s, e);
    return;
  push(u, s, e);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  add(l, r, x, v, s, m);
  add(l, r, x, w, m + 1, e);
  t[u].sum = t[v].sum + t[w].sum;
  t[u].wsum = t[v].wsum + (t[w].wsum + (m - s + 1)
    * t[w].sum):
// 1*a[l] + 2*a[l + 1] + 3*a[l+3] + ...
Node rwsum(int l, int r, int u = 1, int s = 0, int
 \rightarrow e = n - 1) {
  if (l > e or r < s) return {0, 0, 0};
  if (l <= s and e <= r) return t[u]:</pre>
  push(u, s, e);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  Node ln = rwsum(l, r, v, s, m);
  Node rn = rwsum(l, r, w, m + 1, e);
  return {ln.sum + rn.sum, ln.wsum + (rn.wsum +
 \rightarrow max(0, (m - max(l, s) + 1))*rn.sum), 0};
## Add Arithmetic Progression on Range & Range Sum
 → Query:
struct Node {    ll a = 0, d = 0, sum = 0;
t[3 * N];
|ll sum(ll a, ll d, ll n) {
 return n * (2 * a + (n - 1) * d) / 2;
void push(int v, int st, int ed) {
```

```
int lc = 2 * v, rc = 2 * v + 1, md = (st + ed)
 t[lc].a += t[v].a;
  t[lc].d += t[v].d;
  t[lc].sum += sum(t[v].a, t[v].d, md - st + 1);
  t[rc].a += t[v].a + 1LL* (md - st + 1) * t[v].d;
  t[rc].d += t[v].d;
  t[rc].sum += sum(t[v].a + 1LL * (md - st + 1) *

    t[v].d, t[v].d, ed - md);

 t[v].a = t[v].d = 0;
void add(int l, int r, int a, int d, int v = 1, int
\rightarrow st = 0. int ed = n - 1) {
 if (l > ed or r < st) return;</pre>
  if (l <= st and ed <= r) {
   t[v].a += a + 1LL * (st - l) * d;
    t[v].d += d;
    t[v].sum += sum(a + 1LL * (st - l) * d, d, (ed
\rightarrow - st + 1));
    return;
  int lc = 2 * v, rc = 2 * v + 1, md = (st + ed) /
 if (t[v].d) push(v, st, ed);
  add(l, r, a, d, lc, st, md);
  add(l, r, a, d, rc, md + 1, ed);
  t[v].sum = t[lc].sum + t[rc].sum;
ll rsum(int l, int r, int v = 1, int st = 0, int ed
\rightarrow = n - 1) {
  if (l > ed or r < st) return 0;</pre>
 if (l <= st and ed <= r) return t[v].sum;</pre>
  int lc = 2 * v. rc = 2 * v + 1. md = (st + ed) /

→ 2:

  if (t[v].d) push(v, st, ed);
  ll lret = rsum(l, r, lc, st, md);
  ll rret = rsum(l, r, rc, md + 1, ed);
  return lret + rret;
## Range Update & Number of Segment with only set
→ value & the total number of set value:
int t[3 * N], t2[3 * N], p[3 * N];
bool lb[3 * N], rb[3 * N]; //lb for left most bit
→ of interval corresponding to this node
void f(int u, int x, int l, int r) {
 t[u] = (r - l + 1) * x;
 t2[u] = x;
  p[u] = x;
  lb[u] = rb[u] = x;
void push(int u, int s, int e) {
 int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  f(v, p[u], s, m);
 f(w, p[u], m + 1, e);
  p[u] = -1;
void assign(int l, int r, bool x, int u = 1, int s
\rightarrow = 0, int e = N - 1) {
 if (l > e or r < s) return ;</pre>
  if (l <= s and e <= r) {
    f(u, x, s, e);
    return ;
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
```

```
if (p[u] != -1) push(u, s, e);
  assign(l, r, x, v, s, m);
 assign(l, r, x, w, m + 1, e);
t[u] = t[v] + t[w];
t2[u] = t2[v] + t2[w];
lb[u] = lb[v];
rb[u] = rb[w];
  if (rb[v] == 1 \text{ and } lb[w] == 1) t2[u]--;
Update: assign(l, r, 0 / 1);
Query: t[1]: = total number of set bit, t2[1]: =
→ total number of segment with only set value
## Make All Elements <= k and Make all elements >=

→ k on range & Point Query:
                const int I = 1e9 + 9;
int t[3 * N], pa[3 * N], pr[3 * N], ar[3 * N]; //pa
_ for propagate adding, pr for propagate remove,

¬ ar for check last on is adding(1) or remove(0)

void fg(int x, int u) { //function for

→ make greater

  t[u] = max(t[u], x);
  pa[u] = max(pa[u], x);
  pr[u] = max(pr[u], x);
  ar[u] = 1;
void fl(int x, int u) { //function for make less
  t[u] = min(t[u], x);
  pr[\bar{u}] = min(pr[u], x);
  pa[u] = min(pa[u], x);
  ar[u] = 0;
void push(int u) {
  int v = 2 * u, w = 2 * u + 1;
  if (ar[u] == 0) {
    if (pa[u] != -1) {
      fg(pa[u], v); fg(pa[u], w);
    if (pr[u] != I)
      fl'(pr[u], v); fl(pr[u], w);
  } else {
    if (pr[u] != I)
      fl(pr[u], v); fl(pr[u], w);
    if (pa[u] != -1) {
      fg(pa[u], v); fg(pa[u], w);
  pa[u] = -1; pr[u] = I;
void make greater(int l, int r, int x, int u = 1,
\rightarrow int s = 0, int e = N - 1) {
  if (l > e or r < s) return;</pre>
  if (l <= s and e <= r) {
    fg(x, u);
    return ;
  push(u);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  make greater(l, r, x, v, s, m);
  make\_greater(l, r, x, w, m + 1, e);
void make less(int l, int r, int x, int u = 1, int
\rightarrow s = 0, int e = N - 1) {
```

```
if (l <= s and e <= r) {
    fl(x, u);
    return;
  push(u);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  make less(l, r, x, v, s, m);
  make^{-less(l, r, x, w, m + 1, e)};
int at(int i, int u = 1, int s = 0, int e = N - 1) {
  if (s == e) return t[u]:
  push(u);
  int v = 2 * u, w = 2 * u + 1, m = (s + e) / 2;
  if (i <= m) return at(i, v, s, m);</pre>
  else return at(i, w, m + 1, e);
## Range Addition and Range Assign and Range sum
int n;
ll t[3 * N], p[3 * N], p2[3 * N]; //t for sum, p

→ for assign & p2. for add

void pull(int v) {
 t[v] = t[2 * v] + t[2 * v + 1];
void push(int v, int st, int ed) {
  int lc = 2 * v, rc = 2 * v + 1, md = (st + ed) /

→ 2;

  if (p[v] != -1) {
    t[[c] = p[v] * (md - st + 1);
t[rc] = p[v] * (ed - md);
    p[lc] = p[rc] = p[v];
    p2[lc] = p2[rc] = 0;
    p[v] = -1;
  if (p2[v]) {
    t[[c] += p2[v] * (md - st + 1);
    t[rc] += p2[v] * (ed - md);
    p2[lc] += p2[v];
    p2[rc] += p2[v];
    p2[v] = 0;
void assign(int l, int r, int x, int v = 1, int st
 \Rightarrow = 0, int ed = n - 1) {
  if (l > ed or r < st) return;</pre>
  if (l <= st and ed <= r) {
    t[v] = 1LL * (ed - st + 1) * x;
    p[v] = x;
    p2[v] = 0;
    return;
  int lc = 2 * v, rc = 2 * v + 1, md = (st + ed) /
  push(v, st, ed);
  assign(l, r, x, lc, st, md);
  assign(l, r, x, rc, md + 1, ed);
  pull(v);
void add(int l, int r, int x, int v = 1, int st =
   0. int ed = n - 1) {
  if (l > ed or r < st) return;</pre>
  if (l <= st and ed <= r) {
    t[v] += 1LL * (ed - st + 1) * x;
    p2[v] += x:
    return ;
```

if (l > e or r < s) return;</pre>

60 SHORTEST PATH

```
## Dijkstra
priority queue<array<ll, 2>> pq;
vector<ll> dis(n, INF), vis(n);
|while (!pq.empty()) {
  auto [d, u] = pq.top(); pq.pop();
  if (vis[u]) continue;
  vis[u] = 1;
  for (auto [v, c]: next[u]) {
    if (dis[v] > d + c) {
      dis[v] = d + c;
      pq.push({dis[v], v});
## Bellman-ford
vector<int> bellman ford(int s){
 vector<int> dis(n. I):
  dis[s]=0;
  while(1){
    int any=0;
    for (auto& e: ed){
      if(dis[e.u]<I){</pre>
        if(dis[e.u]+e.cost < dis[e.v]){</pre>
          dis[e.v] = dis[e.u]+e.cost;
          any=1;
    if(!any) break;
  return dis:
## Floy-Warshall
for (int k = 0; k < n; ++k) {
 for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      dis[i][j] = min(dis[i][j], dis[i][k] +
   dis[k][j]);
```

61 SOS_DP

```
## Count over subset
for (int i = 0; i < n; ++i) f[a[i]] = ?;
for (int i = 0; i < i; ++i) {
  for (int mask = 0; mask < (1 << n); ++mask) {
    if (\max k (1 << i)) {
      f[mask] += f[mask^(1<<i)];
## Count over superset
for (int i = 0; i < n; ++i) f[a[i]] = ?;
for (int i = 0; i < n; ++i) {
  for (int mask = (1 << n) - 1; mask >= 0; --mask)
    if (!(mask&(1<<i))) {
      f[mask] += f[mask^(1<<i)];
## How many pairs in ara[] such that (ara[i] &
\rightarrow ara[j]) = 0
/// N --> Max number of bits of any array element
const int N = 20:
int inv = (1 << N) - 1:
int F[(1 << N) + 10];
int ara[MAX];
/// ara is 0 based
long long howManyZeroPairs(int n,int ara[]) {
    CLR(F);
    for(int i=0:i<n:i++) F[ara[i]]++:</pre>
    for(int i = 0; i < N; ++i)
        for(int mask = 0; mask < (1<<N); ++mask){</pre>
            if(mask \& (1<<i))
                 F[mask] += F[mask^{(1<<i)}];
    long long ans = 0;
    for(int i=0;i<n;i++) ans += F[ara[i] ^ inv];</pre>
    return ans;
/// To get
    for(int mask = 0; mask < (1<<N); ++mask)
        for(int i = 0; i < (1 << N); ++i)
            if( (mask \& i) == mask ) { /// i is a
 F[mask] += A[i];
/// The code is the following
    for(int i = 0; i < (1 << N); ++i) F[i] = A[i];
    for(int i = 0;i < N; ++i)
for(int mask = (1<<N)-1; mask >= 0; --mask){
            if (!(mask & (1<<i)))
                `F[mask] += F[mask | (1<<i)];
## Number of subsequences of ara[0:n-1] such that
## sub[0] \& sub[2] \& ... \& sub[k-1] = 0
const int N = 20;
int inv = (1 << N) - 1;
int F[(1 << N) + 10];
int ara[MAX]:
int p2[MAX]; /// p2[i] = 2^i
```

```
///0 based array
int howManyZeroSubSequences(int n,int ara[]) {
    CLR(F);
    for(int i=0;i<n;i++) F[ara[i]]++;</pre>
    for(int i = 0; i < N; ++i)
         for(int mask = (1 << N) - 1; mask >= 0; --mask){
             if (!(mask & (1<<i)))
                  F[mask] += F[mask | (1 << i)];
    int ans = 0;
    for(int mask=0; mask<(1<<N); mask++) {</pre>
         if( builtin popcount(mask) \& 1) ans =
    sub(ans, p2[F[mask]]);
         else ans = add(ans, p2[F[mask]]);
    return ans;
## Number of subsequences of ara[0:n-1] such that ## sub[0] | sub[2] | ... | sub[k-1] = Q
int F[(1 << 20) + 10], ara[MAX];
int p2[MAX]; /// p2[i] = 2^i
/// ara is 0 based
int howManySubsequences(int n, int ara[], int m,

    int Q) {

    CLR(F);
    for(int i=0;i<n;i++) F[ara[i]]++;</pre>
    if(Q == 0) return sub(p2[F[0]], 1);
    for(int i = 0; i < m; ++i)
         for(int mask = 0; mask < (1<<m); ++mask){</pre>
             if (mask & (1 << i))
                 F[mask] += F[mask ^ (1<<i)];
    int ans = 0:
    for(int mask=0; mask<(1<< m); mask++) {
         if(mask & Q != mask) continue;
         if( builtin popcount(mask \hat{Q}) & 1) ans =
    sub(ans, p2[F[mask]]);
         else ans = add(ans, p2[F[mask]]);
    return ans;
```

62 SPARSE_TABLE

```
int n, a[N], lg[N], st[N][K];
for (int i = 2; i < N; ++i) {
    lg[i] = lg[i / 2] + 1;
}
void build() {
    for (int k = 0; k < K; ++k) {
        for (int i = 0; i + (1 << k) <= n; ++i) {
            if (k == 0) st[i][k] = a[i];
            else st[i][k] = min(st[i][k - 1], st[i + (1 - (1 + 1))][k - 1]);
      }
}
int rmq (int l, int r) {
    int k = lg[r - l + 1];
    return min(st[l][k], st[r - (1 << k) + 1][k]);
}</pre>
```

63 SQRT_DECOMPOSITION

```
const int SZ2 = 2e5, SZ = sgrt(SZ2+.0)+1, N = SZ*SZ;
int n, a[N], b[SZ];
void build() {
 for (int i = 0; i < SZ; ++i){
    b[i] = INT MAX;
 for (int i = 0; i < n; ++i){
   b[i/SZ] = min(b[i/SZ], a[i]);
int rmq(int l, int r) {
 int lb = l/SZ, rb = r/SZ;
 int ret = INT MAX;
 if(lb==rb) {
    for (int i = l; i <= r; ++i){
      ret = min(ret, a[i]);
 } else {
    for (int i = l; i < (lb+1)*SZ; ++i){}
      ret = min(ret, a[i]);
    for (int i = lb+1; i < rb; ++i){
      ret = min(ret, b[i]);
    for (int i = rb*SZ; i <= r; ++i){
      ret = min(ret, a[i]);
 return ret;
```

64 STRESS TESTING

```
set -e
g++ -02 -static -std=gnu++17 gen.cpp -o gen
g++ -02 -static -std=gnu++17 main.cpp -o main
g++ -02 -static -std=gnu++17 brute.cpp -o brute
for((i = 1; ; ++i)); do
    echo $i
        ./gen $i > in
        # ./main < in > out
        # ./brute < in > out2
        # diff -w out out2 || break
        diff -w <(./main < in) <(./brute < in) || break
done</pre>
```

65 SUFFIX ARRAY

66 SUFFIX_AUTOMATON

```
struct state {
  int len, link, cnt tmp = 0, cnt = 0;
  map<char, int> next;
const int MAXLEN = 100000;
state st[2*MAXLEN];
int dp[2*MAXLEN];
int sz, last;
void sa init() {
  st[0].len = 0;
  st[0].link = -1;
  SZ++;
  last = 0;
  memset(dp, -1, sizeof(dp));
void sa extend(char c) {
  int cur = sz++;
  st[cur].len = st[last].len + 1;
  int p = last;
  while (p != -1 \&\& !st[p].next.count(c)) {
    st[p].next[c] = cur;
    p = st[p].link;
  if (p == -1) {
    st[cur].link = 0;
  } else {
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
      st[cur].link = q;
    } else {
      int clone = sz++;
      st[clone].len = st[p].len + 1;
      st[clone].next = st[q].next;
      st[clone].link = st[q].link;
      while (p != -1 && st[p].next[c] == q) {
   st[p].next[c] = clone;
        p = st[p].link;
      st[q].link = st[cur].link = clone;
  last = cur;
```

```
## Count Occurence
int occurence(string p){
  int at = 0;
  for (int i = 0; p[i]; ++i){
    if(st[at].next.count(p[i]) == 0){
    else{
        at = st[at].next[p[i]];
  return st[at].cnt tmp;
vector<int> used;
|void dfs(int x){
  used[x]=1:
  for(auto it:st[x].next) {
    if(!used[it.second]) dfs(it.second);
    if(it.first=='#') st[x].cnt tmp++;
    else st[x].cnt tmp+=st[it.second].cnt tmp;
    st[x].cnt+=st[it.second].cnt;
  st[x].cnt+=st[x].cnt tmp;
## number of distinct substring O(n)
long long disub(int at){
  if(dp[at] != -1)
    return 0;
  dp[at] = 0;
  long long ret = 0;
  for(auto x : st[at].next)
    ret += disub(x.second);
  if(at != 0)
    ret += (st[at].len - st[st[at].link].len);
  return ret;
## longest common substring: O(|T|)
int lcs (string S, string T) {
  sa init();
  for (int i = 0; i < S.size(); i++)
    sa extend(S[i]);
  int v = 0, l = 0, best = 0, bestpos = 0;
  for (int i = 0; i < T.size(); i++)
    while (v && !st[v].next.count(T[i])) {
      v = st[v].link;
      l = st[v].len;
    if (st[v].next.count(T[i])) {
      v = st [v].next[T[i]];
      l++;
    if (l > best) {
      best = l;
      bestpos = i;
  return best;
## Distinct Substring
|long long disub(int at){
```

```
long long ret = 1;
  for(auto x : st[at].next){
      ret += disub(x.second);
  return ret-1;
int main(){
 int T, caseno = 0;
scanf("%d", &T);
  while(T--){
    int q;
              cin >> q;
    sa init();
    string s; cin >> s;
    cout << s << endl;
    s += "#":
    for (int i = 0; s[i]; ++i){
      sa extend(s[i]);
    used.assign(sz,0);
    dfs(0);
    printf("Case %d:\n", ++caseno);
    while (q--){
      string p;
                   cin >> p;
      int ans = occurence(p);
      cout << ans << endl;
  return 0;
1. Finding Pattern
Frequency of each stat
3. First Occurrence
4. Last Occurrence
5. All Occurrenc
6. Longest Repeated substring:
7. Count number of different substring
8. Total length of different substring
   k-th smallest distinct substring
10. K-th smallest substring
11. Smallest Cyclic Shift
12. Find borders
13. Find Periods:
14. Longest Common Substring
```

67 TOP_SORT

```
int u = q.front(); q.pop();
order.push_back(u);
for (int v: adj[u]) {
   indeg[v]--;
   if (!indeg[v]) {
      q.push(v);
   }
}
has_cycle = order.size() < n;
return order;</pre>
```

```
68 TREAP
## Typical TEAP
struct node {
 ll val, prior, sz, sum;
node *l, *r;
  node(int val, int prior, int sz) : val(val),
    prior(prior), sz(sz), sum(0), l(nullptr),

¬ r(nullptr){}
};
using pnode = node*;
pnode root;
pnode new node(ll val){
  return new node(val, rand(), 1);
int get sz(pnode u){
  return u? u->sz: 0:
void update(pnode u){
  if (!u) return ;
  u->sz = get sz(u->l) + 1 + get sz(u->r);
  u->sum = u->val + (u->l? u->l->sum: 0) + (u->r?

    u->r->sum: 0):

void split(pnode u, pnode &l, pnode &r, ll val){
  if(!u) l = r = NULL;
  else if(val > u->val) split(u->r, u->r, r, val),
  else split(u \rightarrow l, l, u \rightarrow l, val), r = u;
  update(u);
void merge(pnode &u, pnode l, pnode r){
  if(!l or !r) u = l? l: r;
  if(l->prior > r->prior) merge(l->r, l->r, r), u
\rightarrow = l;
  else merge(r->l, l, r->l), u = r;
  update(u);
void insert(pnode &u, pnode it){
  if(!u) u = it:
  else if(it->prior > u->prior) split(u, it->l,

→ it->r, it->val), u = it;

  else insert(it->val <= u->val ? u->l: u->r, it);
  update(u);
void erase(pnode &u, ll val){
  if(!u) return
  if(val == u->val) merge(u, u->l, u->r);
  else erase(val < u->val ? u->l: u->r, val);
  update(u);
bool present(pnode u, int x){
```

```
if(!u) return false;
  if(u->val == x) return true;
  if(u->val < x) return present(u->r, x);
  return present(u->l, x);
ll <mark>kth</mark>(pnode u, int k){
  if(get sz(u) < k) return INT MIN;</pre>
  if(qet sz(u->l) == k-1) return u->val;
  if(qet sz(u->1) < k-1) return kth(u->r, k-1)
 \rightarrow get sz(u->l) - 1);
  return kth(u->l, k);
int cnt less(pnode u, ll x){
  if(!u) return 0;
  if(x <= u->val) return cnt less(u->l, x);
  return get sz(u->1) + 1 + c\overline{n}t less(u->r, x);
ll sum less(pnode u, ll x) {
  if (!u) return 0;
  if (x <= u->val) return sum_less(u->l, x);
  return u \rightarrow val + (u \rightarrow l? u \rightarrow l \rightarrow sum: 0) +
   sum less(u->r, x);
|insert(root, new node(4));
|insert(root, new<sup>-</sup>node(2));
insert(root, new node(7));
cout << cnt less(root, 5) << " " << sum less(root,
\rightarrow 5) << "\n";
root = NULL:
insert(root, new_node(8));
insert(root, new node(1));
insert(root, new node(5));
cout << cnt less(root, 7) << " " << sum less(root,</pre>
\rightarrow 7) << "\\n":
## Implicit TREAP
struct node {
  ll val, sum;
  int prior, sz, rev;
  node *l, *r;
  node(){}
  node(ll val): val(val), sum(val), prior(rand()),
 \rightarrow sz(1), rev(0), l(nullptr), r(nullptr) {}
using pnode = node*;
pnode root:
lint get sz(pnode t) {
  return t? t->sz: 0;
ll get sum(pnode t) {
  return t? t->sum: 0;
|void update(pnode &t) {
  if (!t) return ;
  t->sz = get sz(t->l) + 1 + get sz(t->r);
  t \rightarrow sum = ge\bar{t} sum(t \rightarrow l) + t \rightarrow va\bar{l} + get sum(t \rightarrow r);
void push(pnode t) {
  if (t and t->rev) {
    swap(t->l, t->r);
     t - rev = 0;
    if (t->l) {
      t->l->rev ^= 1;
```

```
if (t->r) {
      t->r->rev ^= 1;
void merge(pnode &t, pnode l, pnode r){
  push(l):
  push(r);
  if(!l or !r) t=l?l:r;
  else if(l->prior > r->prior) merge(l->r, l->r,
\rightarrow r), t=1;
  else merge(r->l,l,r->l) , t=r;
  update(t);
void split(pnode t, pnode &l, pnode &r, int pos,

    int add=0) {

  push(t);
  if(!t) return void(r=l=NULL);
  int cur pos = get sz(t->l)+add;
  if(pos > cur pos) split(t->r, t->r, r, pos,

    cur_pos+1), l = t;

  else split(t->l, l, t->l, pos, add), r=t;
  update(t);
void insert(pnode &t, pnode it, int i) {
  pnode t1, t2;
  split(t, t1, t2, i);
  merge(t1, t1, it);
  merge(t, t1, t2);
void reverse(pnode &t, int l, int r) {
  pnode lt, mt, rt;
  split(t, t, rt, r + 1);
  split(t, lt, mt, l);
  mt->rev = 1:
  merge(mt, mt, rt);
  merge(t, lt, mt);
ll rsum(pnode& t, int l, int r) {
  pnode lt, mt, rt;
  split(t, t, rt, r + 1);
split(t, lt, mt, l);
  ll ret = mt->sum;
  merge(mt, mt, rt);
  merge(t, lt, mt);
  return ret;
int n, q; cin >> n >> q;
vector<ll> a(n);
for (auto &ai: a) {
 cin >> ai;
for (int i = 0; i < n; ++i)
 insert(root, new node(a[i]), i);
while (q--) {
 int tp, l, r; cin >> tp >> l >> r; l--, r--;
  if (tp == 1) {
    reverse(root, l, r);
  else {
    cout << rsum(root, l, r) << "\n";
```

69 WAVELET TREE

```
typedef vector<int>::iterator itr;
const int N=1e6+7;
struct WT {
  int sigma=0;
  vector<int> a;
  vector<vector<int>> c;
  vector<vector<ll>>> pref;
  set<int> st;
  map<int, int> maf;
  int rmaf[N];
  WT(vector<int> a):a(a){
    for(int& x: a){
      st.insert(x):
    for(int x: st){
      maf(x)=sigma, rmaf(sigma++)=x;
    sigma=st.size();
    c.resize(2*sigma), pref.resize(2*sigma);
    build(a.begin(), a.end(), 0, sigma-1, 1);
  void build(itr st, itr ed, int lo, int hi, int v){
    if(lo==hi) return:
    int mid = (lo+hi)/2;
    c[v].reserve(ed-st+1), pref[v].reserve(ed-st+1);
    c[v].push_back(0), pref[v].push_back(0);
for(itr it=st; it!=ed; ++it){
  c[v].push_back(c[v].back()+(maf[*it]<=mid));</pre>
      pref[v].push back(pref[v].back()+(maf[*it]<=m_|</pre>
    id)*(*it));
    itr m = stable partition(st, ed, [=](int
 \rightarrow x){return maf[\overline{x}]<=mid;});
    build(st, m, lo, mid, 2*v);
    build(m, ed, mid+1, hi, 2*v+1);
  int count(int k, int idx){
    if(idx <1 or !maf.count(k)) return 0;</pre>
    k=maf[k];
    int v=1, lo=0, hi=sigma-1;
    while(lo<hi){</pre>
      int mid=(lo+hi)/2;
      int inlft=c[v][idx];
      if(k<=mid){
         idx=inlft, hi=mid, v=2*v;
      } else {
         idx-=inlft, lo=mid+1, v=2*v+1;
    return idx;
```

```
int count(int k, int l, int r){
  return count(k,r)-count(k,l-1);
int kth(int k, int l, int r){
  int v=1, lo=0, hi=sigma-1, mid, inlftl, inlftr;
  while(lo<hi){</pre>
    mid = (lo+hi)/2;
    inlftl = c[v][l];
    inlftr = c[v][r];
    if(k<=inlftr-inlftl){</pre>
      l=inlftl, r=inlftr;
      hi=mid, v=2*v;
    } else {
      k -= inlftr-inlftl, l-=inlftl, r-=inlftr;
      lo=mid+1, v=2*v+1;
  return rmaf[lo];
int lte(int k, int l, int r){
  if(!maf.count(k)) return 0;
  k=maf[k];
  r++:
  int v=1, lo=0, hi=sigma-1, ret=0;
  while(lo<hi){
    int mid=(lo+hi)/2;
    int inlftl=c[v][l]:
    int inlftr=c[v][r];
    if(k<=mid){</pre>
      l=inlftl, r=inlftr;
      hi=mid, v=2*v;
    } else {
      ret += inlftr-inlftl, l-=inlftl, r-=inlftr;
      lo=mid+1, v=2*v+1;
  ret += r-l:
  return ret;
ll sumlte(int k, int l, int r){
  auto it = st.upper bound(k);
  if(it==st.begin()) return 0;
  it--;
  k = maf[*it];
  r++;
  int v=1, lo=0, hi=sigma-1;
  ll ret=0:
  while(lo<hi){</pre>
    int mid=(lo+hi)/2;
    int inlftl=c[v][l];
    int inlftr=c[v][r];
    if(k<=mid){</pre>
```

```
l=inlftl, r=inlftr;
    hi=mid, v=2*v;
} else {
    ret += pref[v][r]-pref[v][l], l-=inlftl,
    r-=inlftr;
    lo=mid+1, v=2*v+1;
}
}
ret += (r-l)*rmaf[lo];
return ret;
}

void clear(){
    a.clear();
    for (int i = 0; i < c.size(); ++i){
        c[i].clear();
}
for (int i = 0; i < pref.size(); ++i){
        pref.clear();
}
st.clear(); maf.clear();
}
};</pre>
```

70 XOR_BASIS

```
// find rank of SLAE modulo 2 field
ll rnk, basis[D];
void insert vector(ll mask){
   for (int I = D-1; i >= 0; --i){
      if((mask & (1ll << i)) == 0) continue;
      if(!basis[i]){
        basis[i] = mask;
        rnk++;
      return;
    } else {
      mask ^= basis[i];
    }
}</pre>
```

71 Z ALGORITHM

```
vector<int> get_z(string s){
   int n=s.size(), l=1, r=0;
   vector<int> z(n); z[0]=n;
   s+='#';
   for (int i = 1; i < n; ++i){
      if(i<=r) z[i]=min(z[i-l], r-i+1);
      while(s[i+z[i]]==s[z[i]]) z[i]++;
      if(i+z[i]-1>r) l=i, r=i+z[i]-1;
   }
   return z;
}
```