Design of Experiments: STA6205
Term Project 2022

The Paper Helicopter Project

A project
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Contents

1. Introduction

Paper helicopters are easy to make (only paper, paper clip and scissors are required) and a lot of fun to play with. This is something most of us have played with in our childhood. Implementing our ideas and applying what we learned from STA 6205 course in such a familiar childhood toy made this project particularly exciting to play with.

1.1 Assumption

- The paper helicopter exhibits a linear trajectory path during its descend.

Since the paper helicopter does not descend in a straight line, its non-linear trajectory is bound to have some effect on the time for which the helicopter stays in the air. The descent time was found to have increased due to the presence of non-linear trajectory. Therefore, it is assumed for the experiment that the fall of the paper helicopter follows a straight path of descend.

- All the helicopters are launched by holding the center of the helicopter foot.

The paper helicopter was found to stay in the air for a longer duration when it was launched by holding the center of the foot. All the helicopters were launched with the same technique. Any other possible launching techniques, for example, holding the side of the helicopter foot or leg, were avoided to minimize the bias due to different launching styles.

- No external factors affect the helicopter's performance.

The experiment was performed indoors to avoid bias due to external factors such as wind or other possible climatic conditions that can potentially affect the descent time of the paper helicopter.

2. Objectives

The objective of the experiment is to find out the optimal design of a paper helicopter given the restriction of paper size.

The objective required the exploration of the best combination from the nine distinct selected design factors, which would assist the paper helicopter in staying for longest time in the air, given the paper size is 21cm x 28cm.

3. Description of Data

This section describes the nature of data. The data used for the experiment was collected by performing the experiment in various stages depending on the required model design. Therefore, this section covers a generic variable description.

The data consists of nine independent variables namely,

I. Rotor Length
II. Rotor Width
III. Body length
IV. Leg Length
V. Leg Width
VI. Foot length
VII. Paper type
VIII. Paper clip
IX. Numbers of rotors

Time of the descent is the dependent variable with a fixed drop height of 211 cm.

The paper type, paper clip and number of rotors are qualitative in nature.

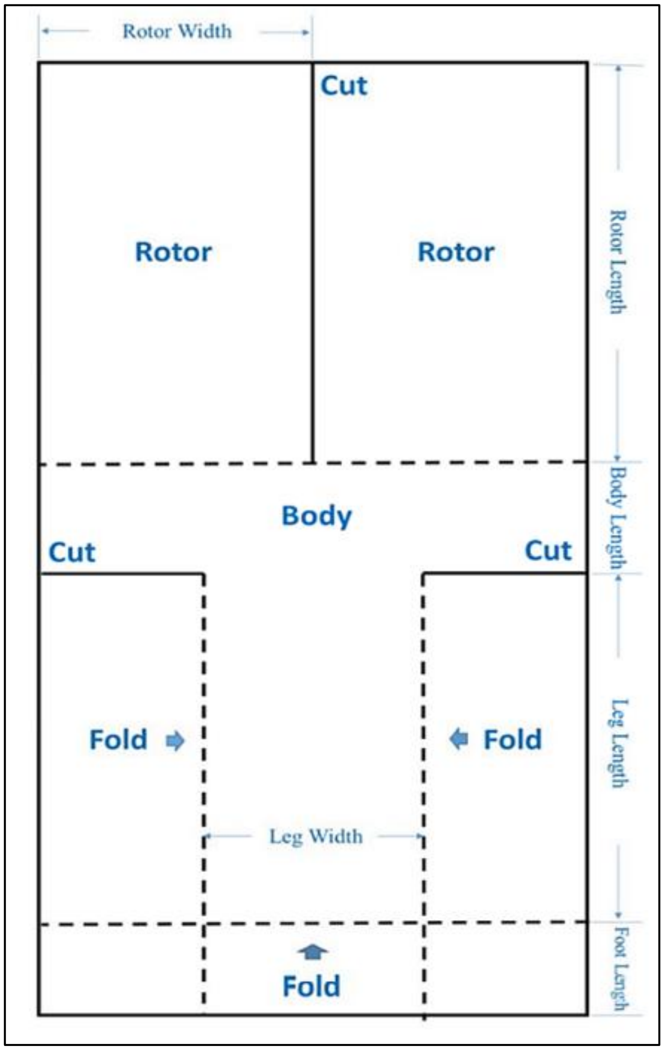
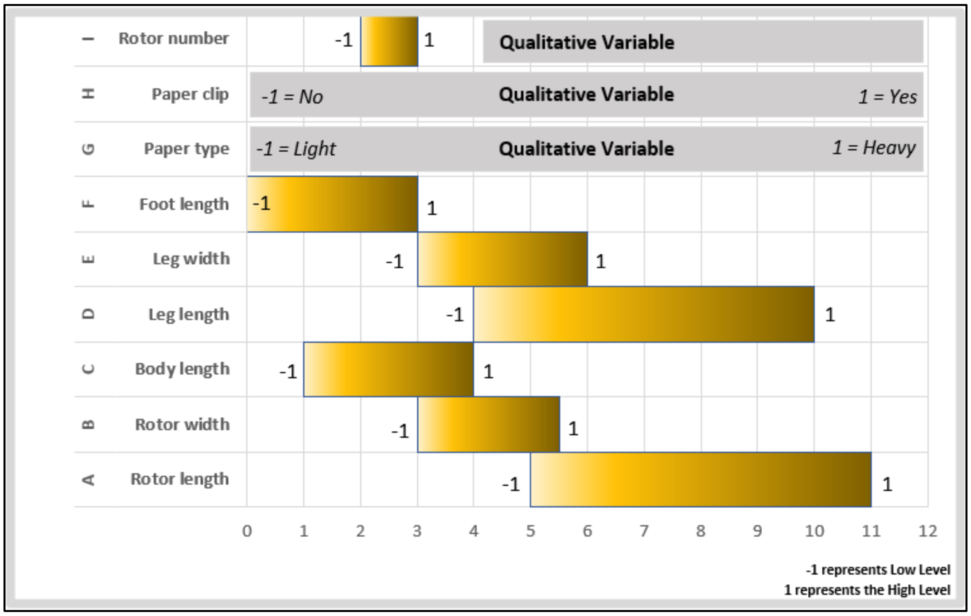


Figure 1. Labelled diagram of the paper helicopter



Graph 1. Levels of design factors

4. The Screening Experiment

4.1. Design

The design of the screening experiment, analysis and results are explained in this section.

The screening experiment entailed using a resolution IV design (2_{IV}^{9-4}). The design of the screening experiment is displayed in Table XXX. The design generators used for the same were F = BCDE, G = ACDE, H = ABDE, I = ABCE where,

A = Rotor Length
B = Rotor Width
C = Body length
D = Leg Length
E = Leg Width
F = Foot length
G = Paper type
H = Paper clip
I = Numbers of rotors.

The run order of the design was randomized to avoid selection and accidental bias. The design included thirty-two runs divided into two blocks of sixteen runs each. The variation in launching the paper helicopter and recording the time of decent by the two students performing the experiment was considered as the blocking factor. The nine design factors are coded into -1 and 1 to generate 32 unique combinations. The coded factors and the random order of the runs is displayed in Table 1.

Table 1. Coded design factors and the randomized run-order

S. NO	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**Run order**
						BCDE	**ACDE**	**ABDE**	**ABCE**	
1	-1	-1	-1	-1	-1	1	1	1	1	18
2	1	-1	-1	-1	-1	1	-1	-1	-1	4
3	-1	1	-1	-1	-1	-1	1	-1	-1	8
4	1	1	-1	-1	-1	-1	-1	1	1	9
5	-1	-1	1	-1	-1	-1	-1	1	-1	19
6	1	-1	1	-1	-1	-1	1	-1	1	13
7	-1	1	1	-1	-1	1	-1	-1	1	21
8	1	1	1	-1	-1	1	1	1	-1	14
9	-1	-1	-1	1	-1	-1	-1	-1	1	22
10	1	-1	-1	1	-1	-1	1	1	-1	10
11	-1	1	-1	1	-1	1	-1	1	-1	2
12	1	1	-1	1	-1	1	1	-1	1	23
13	-1	-1	1	1	-1	1	1	-1	-1	28
14	1	-1	1	1	-1	1	-1	1	1	15
15	-1	1	1	1	-1	-1	1	1	1	5
16	1	1	1	1	-1	-1	-1	-1	-1	11
17	-1	-1	-1	-1	1	-1	-1	-1	-1	30
18	1	-1	-1	-1	1	-1	1	1	1	7
19	-1	1	-1	-1	1	1	-1	1	1	31
20	1	1	-1	-1	1	1	1	-1	-1	20
21	-1	-1	1	-1	1	1	1	-1	1	29
22	1	-1	1	-1	1	1	-1	1	-1	3
23	-1	1	1	-1	1	-1	1	1	-1	17
24	1	1	1	-1	1	-1	-1	-1	1	32
25	-1	-1	-1	1	1	1	1	1	-1	1
26	1	-1	-1	1	1	1	-1	-1	1	6
27	-1	1	-1	1	1	-1	1	-1	1	27
28	1	1	-1	1	1	-1	-1	1	-1	24
29	-1	-1	1	1	1	-1	-1	1	1	26
30	1	-1	1	1	1	-1	1	-1	-1	16
31	-1	1	1	1	1	1	-1	-1	-1	12
32	1	1	1	1	1	1	1	1	1	25

The 2_{IV}^{9-4} design with two blocks using AEF, BEG, CEH, DEI as aliases is displayed in Table 2. The aliases AEF, BEG, CEH, DEI are simply generated by the product of the respective codes (-1 and 1) of the design factors whose coded letter appeared in the alias name.

Table 2. The 2_{IV}^{9-4} design with two blocks

	AEF	BEG	CEH	DEI	BLOCK
1	1	1	1	1	1
a	-1	-1	-1	-1	2
b	-1	-1	-1	-1	2
ab	1	1	1	1	1
c	-1	-1	-1	-1	2
ac	1	1	1	1	1
bc	1	1	1	1	1
abc	-1	-1	-1	-1	2
d	-1	-1	-1	-1	2
ad	1	1	1	1	1
bd	1	1	1	1	1
abd	-1	-1	-1	-1	2
cd	1	1	1	1	1
acd	-1	-1	-1	-1	2
bcd	-1	-1	-1	-1	2
abcd	1	1	1	1	1
e	1	1	1	1	1
ae	-1	-1	-1	-1	2
be	-1	-1	-1	-1	2
abe	1	1	1	1	1
ce	-1	-1	-1	-1	2
ace	1	1	1	1	1
bce	1	1	1	1	1
abce	-1	-1	-1	-1	2
de	-1	-1	-1	-1	2
ade	1	1	1	1	1
bde	1	1	1	1	1
abde	-1	-1	-1	-1	2
cde	1	1	1	1	1
acde	-1	-1	-1	-1	2
bcde	-1	-1	-1	-1	2
abcde	1	1	1	1	1

4.2 Data Collection

The data was collected by taking three readings for each of the thirty-two runs and averaging the time of decent for the final analysis. Both the students, Fahim and Divya, performed the experiment for the generated thirty-two runs which was followed by the blocking of thirty-two runs into two blocks of sixteen runs each. The 32 final runs considered for the analysis after blocking are highlighted and displayed in Table 3. The column T1, T2, T3 represent the three consecutive readings for each run and column Average_T represents the average time of descent of the paper helicopter for the run. There were three data points collected for calculating the average time of descent to avoid the error due to inconsistencies.

Table 3. Average time of descent

	Launch - Fahim				Launch - Divya			
Observation number	T1	T2	T3	Average_T	T1	T2	T3	Average_T
1	0.9	0.8	0.86	**0.85**	0.63	0.6	0.6	**0.61**
2	1.33	2.18	1.88	**1.80**	1.78	2.05	1.92	**1.92**
3	0.97	1.1	1.11	**1.06**	0.85	0.76	0.99	**0.87**
4	1.62	2.24	1.54	**1.8**	1.64	1.08	1.12	**1.28**
5	1.4	1.01	1.2	**1.20**	1.14	1.37	1.14	**1.22**
6	0.91	0.88	0.8	**0.86**	0.83	0.68	0.7	**0.74**
7	1.02	1.14	1.17	**1.11**	0.85	0.61	0.75	**0.74**
8	1.17	1	1.18	**1.12**	0.88	0.8	0.83	**0.84**
9	1.07	1.07	1.17	**1.10**	1.03	1.05	0.88	**0.99**
10	0.84	1.18	1.12	**1.05**	0.93	0.97	1.05	**0.98**
11	1.1	1.15	1.17	**1.14**	0.99	1	1.15	**1.05**
12	0.9	0.89	0.78	**0.86**	0.81	0.98	0.74	**0.84**
13	0.73	0.77	0.78	**0.76**	0.53	0.43	0.51	**0.49**
14	1.42	1.42	1.5	**1.45**	1.11	1.1	1.28	**1.16**
15	0.74	0.78	0.71	**0.74**	0.71	0.55	0.61	**0.62**
16	1.65	1.59	1.58	**1.61**	1.75	1.43	1.78	**1.65**
17	1.74	1.77	1.72	**1.74**	1.45	1.57	1.47	**1.50**
18	0.96	0.87	0.91	**0.91**	0.85	0.88	0.91	**0.88**
19	1.13	1.34	1.13	**1.2**	1.05	0.97	0.91	**0.98**
20	1.31	1.31	1.43	**1.35**	0.91	1.23	1.21	**1.12**
21	0.78	0.9	0.83	**0.84**	0.77	0.7	0.6	**0.69**
22	1.24	0.96	1.17	**1.12**	0.93	1.12	1.01	**1.02**
23	0.93	0.83	0.72	**0.83**	0.81	0.68	0.88	**0.79**
24	1.23	1.35	1.35	**1.31**	1.02	1.17	1.25	**1.15**
25	0.96	1	0.87	**0.94**	0.63	0.7	0.65	**0.66**
26	1.57	1.67	1.62	**1.62**	1.65	1.56	1.28	**1.50**
27	0.52	0.81	0.73	**0.69**	0.9	0.85	0.76	**0.84**
28	1.67	1.8	1.8	**1.76**	1.32	1.65	1.56	**1.51**
29	0.98	0.44	0.9	**0.77**	0.81	0.71	0.77	**0.76**
30	1.07	0.92	1.07	**1.02**	0.78	0.91	0.78	**0.82**
31	0.78	0.91	0.91	**0.87**	0.79	0.7	0.81	**0.77**
32	0.62	0.62	0.6	**0.61**	0.58	0.78	0.57	**0.64**

4.3 Result

The result of the analysis of the screening experiment is displayed in this section. The output from the ANOVA reflected that five out of the nine design factors were significant at 0.05 confidence level. Blocking factor was also significant at 0.1 level. The five significant factors are reflected in Figure XXX. The initial model used for the analysis can be written as,

$$time \ = \ 1.24 + 0.139A - 0.12C - 0.08D - 0.23G - 0.08I - 0.12block$$

where,

A = Rotor length

C = Body length

D = Leg length

G = Paper type (qualitative)

I = Rotor number (qualitative).

```
Analysis of Variance Table

Response: time
          Df  Sum Sq Mean Sq F value    Pr(>F)    
A          1 0.61883 0.61883 19.0619 0.0001927 ***
C          1 0.48758 0.48758 15.0190 0.0006818 ***
D          1 0.20963 0.20963  6.4572 0.0176324 *  
G          1 1.70663 1.70663 52.5697 1.345e-07 ***
I          1 0.20963 0.20963  6.4572 0.0176324 *  
block      1 0.12375 0.12375  3.8120 0.0621709 .  
Residuals 25 0.81160 0.03246                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.40781 -0.10109 -0.02188  0.10453  0.28031 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.24375    0.10072  12.348 3.88e-12 ***
A            0.13906    0.03185   4.366 0.000193 ***
C           -0.12344    0.03185  -3.875 0.000682 ***
D           -0.08094    0.03185  -2.541 0.017632 *  
G           -0.23094    0.03185  -7.250 1.35e-07 ***
I           -0.08094    0.03185  -2.541 0.017632 *  
block       -0.12437    0.06370  -1.952 0.062171 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1802 on 25 degrees of freedom
Multiple R-squared:  0.8053,	Adjusted R-squared:  0.7585 
F-statistic: 17.23 on 6 and 25 DF,  p-value: 8.635e-08
```

Figure 2. A representation of the r output for the ANOVA and summary for the screening experiment

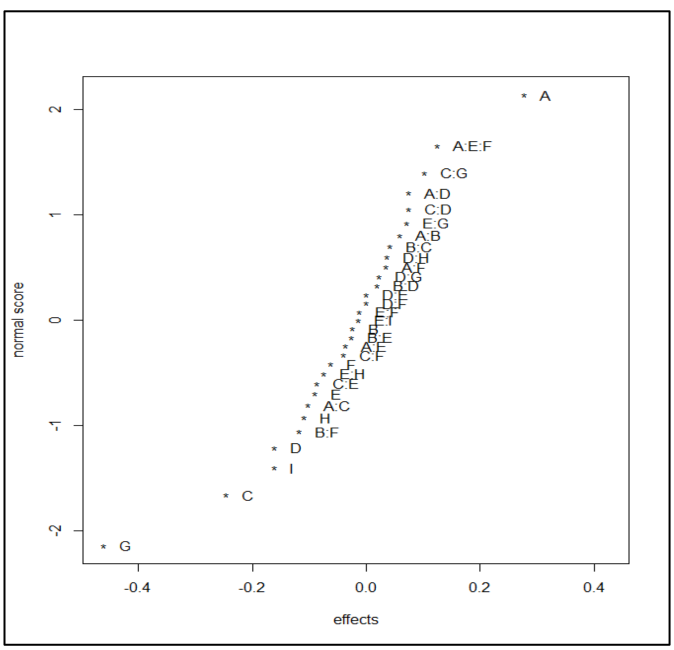


Figure 3. The half normality plot further backs the ANOVA table that, A, C, D, G, & I are the only significant main effects The two or three-way interaction terms are not significant

The studentized residual plot does not reflect any unusual pattern or obvious discrepancy and quantile-quantile plot is almost linear which shows normality conditions are met.

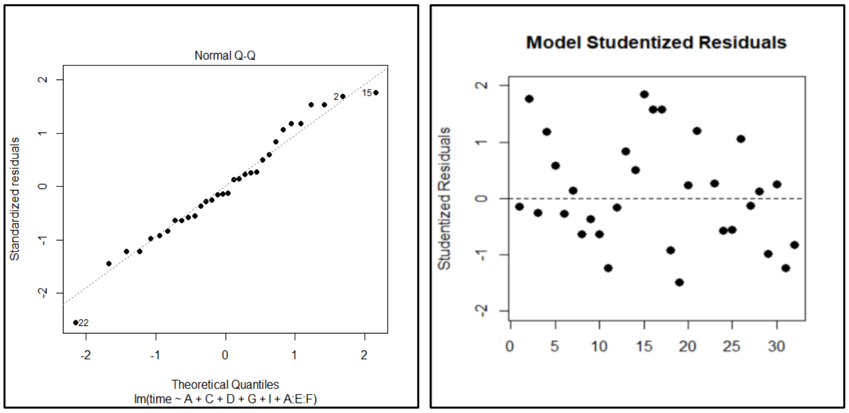


Figure 4. Residual plots from the screening experiment

5. Steepest Ascent Analysis

5.1 Design

From this part of the experiment blocking was no longer chosen. Moreover, out of the five significant variables, only three (Rotor length, Body length, and Leg length) were used for this part of experiment. The other two variables (Paper type and number of rotors) were considered as qualitative or categorical variables and kept constant. This was done because rotor number is a discrete variable while paper type was chosen as categorical variable due to limitation of resources. We only had 75 and 200 gsm paper type available for our experiment. From our initial screening experiment, we saw that lighter paper type (75 gsm) stays in the air longer than the heavier one(200gsm) while 2 rotors helicopters does marginally better than 3 rotor helicopters. The box plots below illustrate this point further.

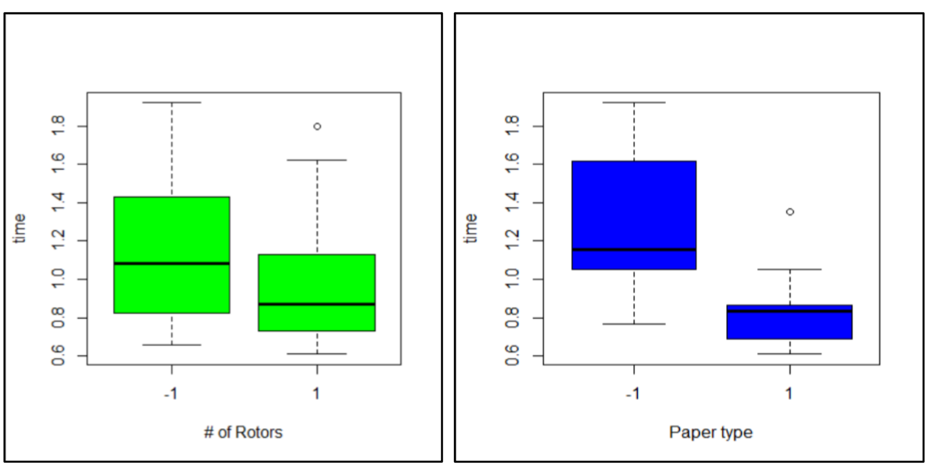


Figure 5. Box plots for the number of rotors and the paper type

Hence, number of rotors were fixed at 2 and the lighter paper type of 75 gsm was used. The other non-significant design factors were also kept constant since these does not have significant effect the paper helicopter's performance. Table 4 displays the value of each of the non-significant variables.

Table 4. Non-significant variables

Code	Factor	Measurement	Level
B	Rotor width	3 cm	low
E	Leg width	6 cm	high
F	Foot length	0 cm	low
G	Paper type	Light	low
H	Paper clip	No	low
I	Rotor number	2	low

The constant terms were kept like that throughout the rest of the project.

For the other three variables, we selected at Rotor length, A = 8cm, Body length, C = 2.5 cm, and Leg length, D = 7 cm. We choose these lengths because for our initial screening experiment low level of A was 5 cm and high level of A was 11 cm, we thought the average of the two values would be good place to start. We choose the other two lengths in a similar way. Now we needed to know how we change the variables we get those from our R-codes.

5.2 Analysis and result

From the output we can see the direction of steepest ascent in coded variables. We then changed the coded variable into natural variable and found that those units correspond to 2cm, -1cm, & -1 cm respectively. The following table show our steepest ascent experiment.

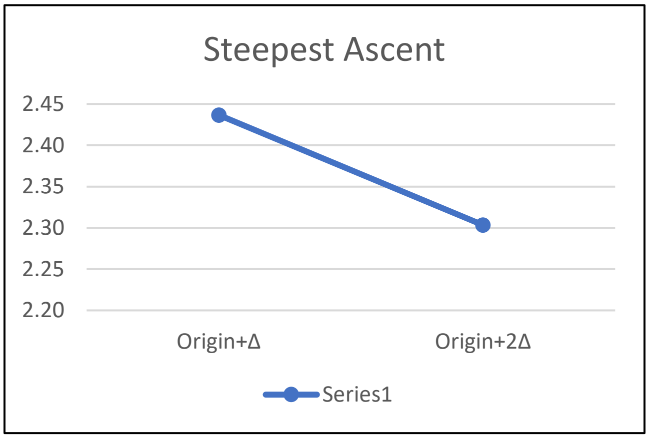
```
Analysis of Variance Table

Response: time
             Df  Sum Sq Mean Sq F value  Pr(>F)
FO(A, C, D)   3 1.31603 0.43868  4.3074 0.01282
Residuals    28 2.85161 0.10184
Lack of fit   4 0.21349 0.05337  0.4855 0.74618
Pure error   24 2.63813 0.10992

Direction of steepest ascent (at radius 1):
         A          C          D
 0.6857272 -0.6086792 -0.3991087
```

Figure 6. ANOVA output of the steepest ascent analysis

At origin the paper helicopter has a flight time of 1.92 seconds, once we move away from the center, the paper helicopter performs better. We could only do 2 runs since the third run would have body length, C = 0 cm and it is not physically possible to have lengths of 0 cm. Table 5 displays the data points for the Origin+Δ and Origin+2Δ for finding the peak time of descent.

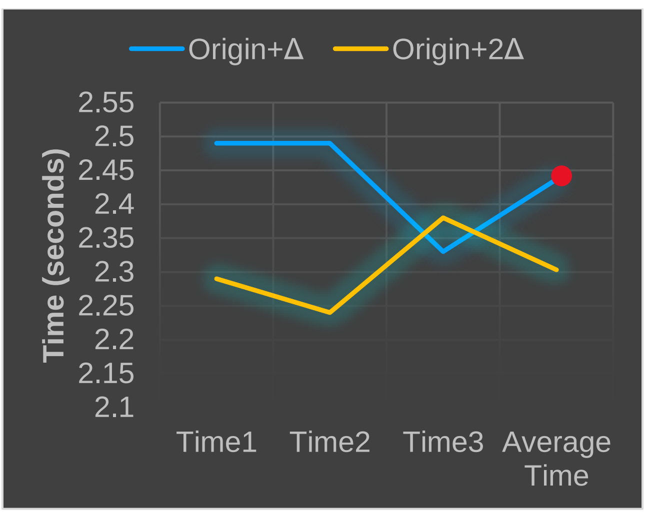
Table 5. Time of descent for finding the peak point

Steps	A	C	D	T1	T2	T3	Avg_T(sec)
Origin	8	2.5	7	1.95	1.81	1.99	1.92
Delta	2	-1	-1				
Origin+Δ	10	2	6	2.49	2.49	2.33	2.44
Origin+2Δ	12	1	5	2.29	2.24	2.38	2.30
Origin+3Δ	14	0	3				



Graph 2. Steepest Ascent

The graphs shows that the paper helicopter on average performs better at Origin+Δ but performance starts to regress as it moves to Origin+2Δ. So, this is good place to move on to the central composite design experiment as we have found what we think is an optimal region.



Graph 3. Average time of descent and the peak time of descent

6. Central Composite Design

The central composite design to carry out the identification of the presence of curvature is described in this section.

6.1. Part 1

Since there are 3 variables: (i) Rotor length, A (ii) Body Length, C, (iii) Leg Length, D. We choose 23 = 8 runs as well as 6 center runs.

Table XXX shows the design matrix. The Natural Variables are in the design region: A ∈ [8,12], C ∈ [1,3], D ∈ [5,7]. All 3 lengths are measured in cm. We then used the following formula to transform our natural variable into coded variable:

a = (A-8)/2, c= c-2, d= D-6.

Table 6. Design matrix for the central composite design (Part 1)

Natural Variables			Coded variables		
A	**C**	**D**	**a**	**c**	**d**
8	1	5	-1	-1	-1
12	1	5	1	-1	-1
8	3	5	-1	1	-1
12	3	5	1	1	-1
8	1	7	-1	-1	1
12	1	7	1	-1	1
8	3	7	-1	1	1
12	3	7	1	1	1
10	2	6	0	0	0
10	2	6	0	0	0
10	2	6	0	0	0
10	2	6	0	0	0
10	2	6	0	0	0
10	2	6	0	0	0

```
Analysis of Variance Table

Response: time
                    Df  Sum Sq Mean Sq F value    Pr(>F)
FO(A, C, D)          3 0.07695 0.02565  2.1375 0.1967581
TWI(A, C, D)         3 0.06460 0.02153  1.7944 0.2482481
I(A^2 + C^2 + D^2)   1 0.74134 0.74134 61.7786 0.0002244
Residuals            6 0.07200 0.01200
Lack of fit          1 0.03125 0.03125  3.8344 0.1075421
Pure error           5 0.04075 0.00815
```

Figure 7. ANOVA output for the CCD analysis (Part1)

For the quadratic terms we get a very high F-value of 61.77. the probability next to it indicates it is significant, so there is obvious curvature. Therefore, we needed to take axial runs.

6.2. Part 2

For the next part of the experiment, we used spherical CCD with α = √k. Since we have 3 design factors, α = √3 = 1.73. The following table shows the next part of the design matrix.

Table 7. Axial runs for the natural and coded variables

A	C	D	a	c	d
6.6	2	6	-1.7	0	0
13.4	2	6	1.7	0	0
10	0.3	6	0	-1.7	0
10	3.7	6	0	1.7	0
10	2	4.3	0	0	-1.7
10	2	7.7	0	0	1.7

We then proceeded with the next part of the analysis.

We got the following output: we can see that the interaction terms are not significant. So, I updated the rsm model in R by removing the two-way interaction terms. Then we get the following.

```
> summary(rs.ccd)

Call:
rsm(formula = time ~ SO(A, C, D), data = Paper.CCD)

             Estimate Std. Error t value  Pr(>|t|)
(Intercept)  2.281067   0.063815 35.7450 6.973e-12 ***
A           -0.023004   0.042126 -0.5461 0.5969753
C            0.039042   0.042126  0.9268 0.3758552
D           -0.030624   0.042126 -0.7270 0.4839138
A:C          0.025000   0.055287  0.4522 0.6607869
A:D          0.015000   0.055287  0.2713 0.7916708
C:D          0.085000   0.055287  1.5374 0.1552058
A^2         -0.162219   0.040533 -4.0021 0.0025097 **
C^2         -0.205471   0.040533 -5.0692 0.0004854 ***
D^2         -0.170869   0.040533 -4.2156 0.0017841 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared:  0.8422,    Adjusted R-squared:  0.7001
F-statistic: 5.929 on 9 and 10 DF,  p-value: 0.005124
```

```
> summary(rs.ccd2)

Call:
rsm(formula = time ~ FO(A, C, D) + PQ(A, C, D), data = Paper.CCD)

             Estimate Std. Error t value  Pr(>|t|)
(Intercept)  2.281067   0.062930 36.2480 1.908e-14 ***
A           -0.023004   0.041541 -0.5538 0.5891322
C            0.039042   0.041541  0.9398 0.3644338
D           -0.030624   0.041541 -0.7372 0.4740963
A^2         -0.162219   0.039971 -4.0585 0.0013547 **
C^2         -0.205471   0.039971 -5.1406 0.0001898 ***
D^2         -0.170869   0.039971 -4.2749 0.0009044 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared:  0.8005,    Adjusted R-squared:  0.7084
F-statistic: 8.693 on 6 and 13 DF,  p-value: 0.0006149

Analysis of Variance Table

Response: time
            Df  Sum Sq Mean Sq F value    Pr(>F)
FO(A, C, D)  3 0.04122 0.01374  0.5778    0.6398
PQ(A, C, D)  3 1.19905 0.39968 16.8078 9.252e-05
Residuals   13 0.30913 0.02378
Lack of fit  8 0.26838 0.03355  4.1163    0.0679
Pure error   5 0.04075 0.00815
```

Figure 8. Output for the CCD analysis (Part2)

```
Stationary point of response surface:
          A           C           D
-0.07090537  0.09500619 -0.08961266

Eigenanalysis:
eigen() decomposition
$values
[1] -0.1622187 -0.1708692 -0.2054713

$vectors
  [,1] [,2] [,3]
A    1    0    0
C    0    0    1
D    0    1    0
```

Figure 9. Output from the CCD analysis

We get our three stationary points: A = -0.071, C= 0.095, D = -0.0896. All three values are close to zero which indicates that our optimal model is very near. The eigen decomposition values are all negative which indicates that the stationary points are a maximum system. Then we plot the contour plots and do the canonical analysis.

7. Contour Plots

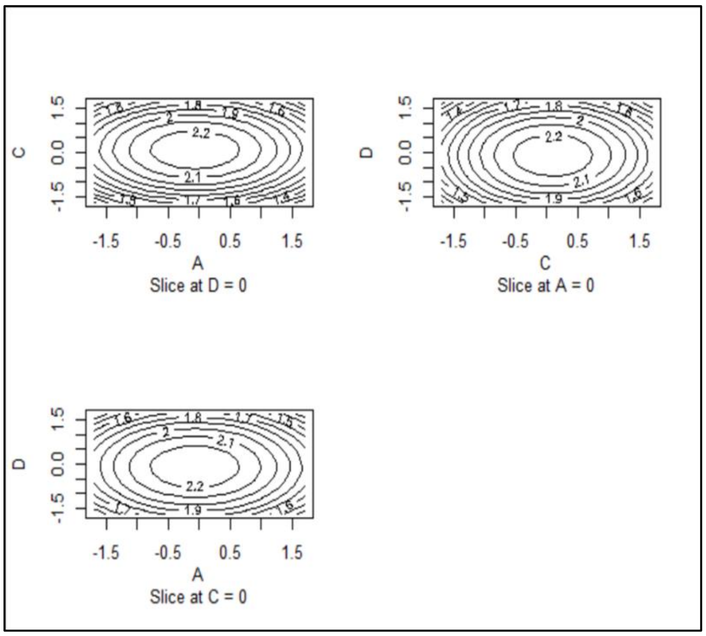


Figure 10. Contour plots of the significant variables

The contour plots suggest that maximum time is yield around the center, when A=0, C=0 and D = 0 in coded variables. This is what we expect since the stationary points from our rsm output also indicates that all three values are close to zero.

8. 3-D Plots

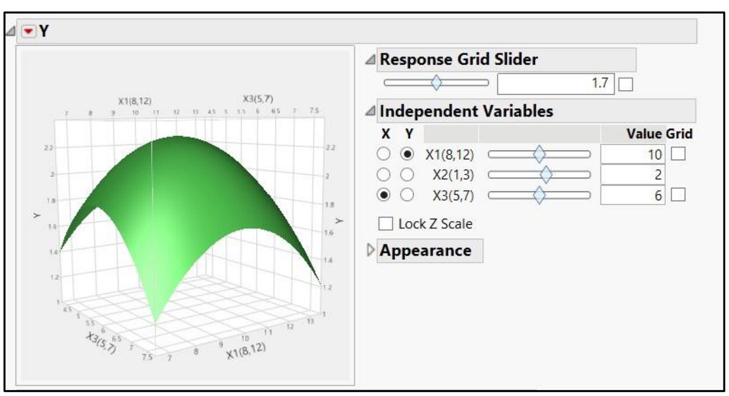


Figure 11. Response surface plot 1

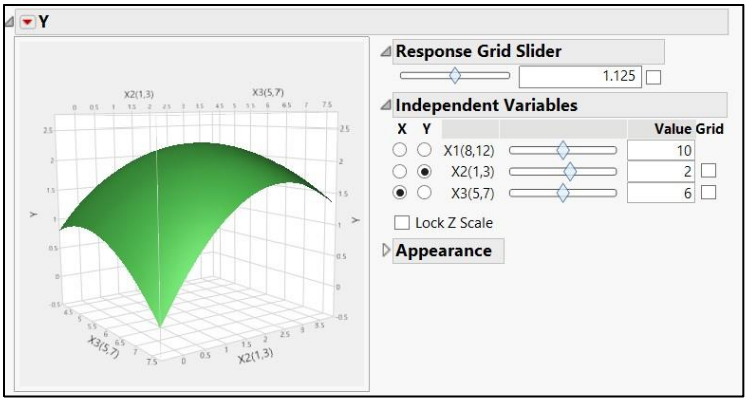


Figure 12. Response surface plot 2

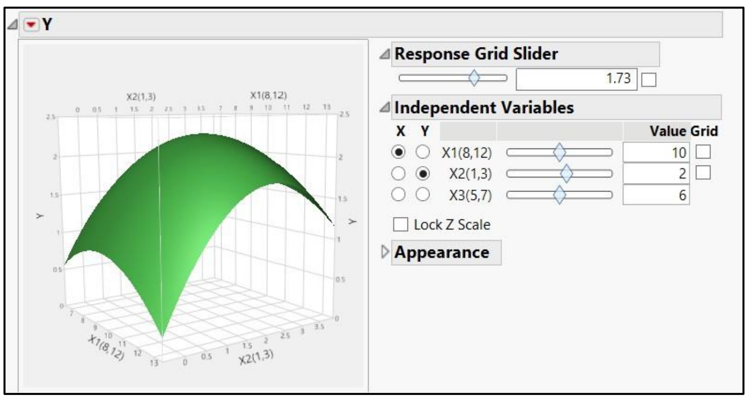


Figure 13. Response surface plot 3

To better get a sense of the visualization, we plotted the 3-dimensional response surface plot with the help from JMP software. The result further illustrates what we expect, which is the stationary point is indeed a maximum system and that there is a maximum around the center.

9. Canonical Analysis

$\hat{y}$ = $\hat{y_s}$ + $\lambda_1 w_1^2$+ $\lambda_2 w_2^2$+ $\lambda_3 w_3^2$

where, $\lambda_1 = -0.162$, $\lambda_2 = -0.171$, $\lambda_3 = -0.205$

$$\hat{y_s} = 2.28 - 0.5\begin{bmatrix} -0.023 & 0.039 & -0.0306 \end{bmatrix}\begin{bmatrix} -0.0709 \\ 0.095 \\ -0.0896 \end{bmatrix} = 2.27596$$

$\hat{y}$ = 2.28 $-0.16(x_1 + 0.071)^2 - 0.17(x_2 - 0.095)^2 - 0.21(x_3 + 0.09)^2$

This tells us about the sensitivity of the contour plots, how much does our response variable, time change depending on which direction in the contour plot we move.

10. Confirmation Run

Once, we selected the best model, we ran 9 confirmation runs of that best model. The following table summarizes how it performed.

Table 8. The data points from the confirmation run

Observation	Time/s
1	2.49
2	2.49
3	2.33
4	2.37
5	2.25
6	2.14
7	2.29
8	2.27
9	2.39
Average	2.34

11. Hypothesis Testing

Original Screening Design: $\overline{x_1}$= 1.05 and s_1 = 0.37, n_1=32

H_0: $\mu_1 = \mu_2$ H_1: $\mu_2 > \mu_1$

Final Model: $\overline{x_2}$ = 2.34 n_2=9, s_2=0.107

$$t = \frac{|\overline{x_2} - \overline{x_1}|}{\sqrt{\frac{s_2^2}{n_2} + \frac{s_1^2}{n_1}}} = \frac{|2.34 - 1.05|}{\sqrt{\frac{0.107^2}{32} + \frac{0.37^2}{9}}} = 10.34$$

df = $n_1 + n_2 - 2$ = 32+9-2 =39

$t_{39,0.05}$ = 1.7 < 10.34

We have sufficient evidence to reject the null hypothesis

The two means are significantly different, and the final model of paper helicopter performs significantly better than a typical paper helicopter.

12. Confidence Interval

Confidence Interval: $\bar{x} \pm t_{n-1,\alpha}\sqrt{s^2}$

95% Confidence Interval: 2.34 $\pm\, t_{9-1,0.05}$(0.107)

2.34 $\pm$ (1.833)(0.107) = (2.14,2.54)

We are 95% confident that the average flight time of our best model of paper helicopter will be between 2.14s and 2.54s. This is what we found in the confirmation run as well as the average flight time over there ranged from 2.14s to 2.49s which is within our 95% confidence interval region.

13. Conclusion

Our best design has the following design factors. Our model equation looks as follows: time = 2.28 -0.023A+0.039C-0.03D. From response surface methodology we have Rotor length, A of 10 cm, Body length, C =2cm, and Leg length, D = 6cm. The rest of design factors are the same constants we mentioned earlier.

14. Future Work

- Test the performance of a comparatively smaller design: If we built a paper helicopter of different size but the ratio of the lengths is proportional, we may want to know, how will that perform.
- Much lighter paper type (less than 75 gsm) can be utilized: Paper which are much lighter would probably stay in the air much longer, further analysis can be done on that.
- Experimenting with the role of drop height: The same experiment can be done with height of 1.5m, 2m, 2.5 and 3m and see if different model performs best from different height. Such comparative study might be beyond the scope of this course. But it would be interesting to see such an analysis.
- Outdoor vs Indoor to study the effect of external factors: Air resistance might be a factor outdoor; it will be interesting to see which model of paper helicopter performs better when there is air resistance.
- Experimenting with the number of rotors: We noticed that 3 rotors is unbalanced. Perhaps experiment with even number of rotors such as 4 could be used, it would probably be much more balanced than 3 rotors.

Appendix

R-codes

```
Paper=read.csv("Screening_data.csv",header=T)
f = lm(time~(A)*(B)*(C)*(D)*(E)*(F)*(G)*(H)*(I), data =Paper)
library(faraway)
f2 = lm(time~A+C+D+E+F+G+I+C:G+A:E:F, data=Paper)
library(BsMD)
DanielPlot(f)
anova(f2)
summary(f2)
f3 = lm(time~A+C+D+E+F+G+I+C:G+A:E:F +block, data=Paper)
anova(f3)
summary(f3)
fs = lm(time~A+B+C+D+E+F+block, data=Paper)
anova(fs)
summary(fs)
fs2 = lm(time~A+B+C+D+E+F, data = Paper)
anova(fs2)
summary(fs2)
DanielPlot(fs3)
fs3 = lm(time~A+B+C+D+E+F + A:B+A:C+A:D+A:E+B:C+B:D+B:E+B:F+C:D+C:E+C:F+D:E+D:F+E:F, data =
Paper)
anova(fs3)
summary(fs3)

fs4 = lm(time~A+B+C+D+E+F + A:B+A:C+A:D+A:E+B:C+B:D+B:E+B:F+C:D+C:E+C:F+D:E+D:F+E:F
+D:G+D:I+D:H+D:G:I, data = Paper)
anova(fs4)
summary(fs4)
f4 = lm(time~A+C+D+G+I+block, data=Paper)
anova(f4)
summary(f4)
f5 = lm(time~A+C+D+G+I, data = Paper)
anova(f5)
summary(f5)
f6 = lm(time~A+C+D+block, data=Paper)
anova(f6)
summary(f6)
f7 = lm(time~A+C+D, data = Paper)
anova(f7)
summary(f7)
#Extras
#Studentised Residuals
library(MASS)
plot(studres(f4),main="Model Studentized Residuals",xlab="",ylab="Studentized
Residuals",pch=16,cex=1.2,ylim=c( -2,2))
abline(h=0,lty=2)
library(car)
# check normality
qqnorm(residuals(f4),pch=16,main="Normal Plot of Residuals",xlab="NormalScores",ylab="Ordered Residuals")
#Descriptive Statistics
boxplot(Paper$time~Paper$G, ylab = "time", xlab = "Paper type", col="blue")
boxplot(Paper$time~Paper$I, ylab = "time", xlab = "# of Rotors", col="green")
#Other boxplots
par(mfrow=c(2,2))
```

```
boxplot(Paper$time~Paper$B, ylab = "time", xlab = "Rotor Width")
boxplot(Paper$time~Paper$E, ylab = "time", xlab = "Leg Width")
boxplot(Paper$time~Paper$F, ylab = "time", xlab = "Foot length")
boxplot(Paper$time~Paper$H, ylab = "time", xlab = "tape vs clip")
par(mfrow=c(1,1))
#Steepest Ascent
library(rsm)
rs_mod = rsm(time~FO(A,C,D)+TWI(A,C,D)+I(A^2 +C^2 +D^2),data = Paper)
summary(rs_mod)

rs_mod2 = update(rs_mod, .~., - I(A^2 +C^2 +D^2), data= Paper )
summary(rs_mod2)
rs_mk = rsm(time~FO(A,C,D), data = Paper)
summary(rs_mk)
rs_mk2 = rsm(time~FO(A,C,D,G,I), data = Paper)
summary(rs_mk2)
#CCD part
New_Paper=read.csv("Paper_new.csv",header=T)
rs.new = rsm(time~FO(A,C,D)+TWI(A,C,D)+I(A^2 +C^2 +D^2), data= New_Paper)
summary(rs.new)
#After Curvature
Paper.CCD = read.csv("Curvature.csv",header=T)
rs.ccd = rsm(time~SO(A,C,D), data= Paper.CCD)
summary(rs.ccd)
rs.ccd2=update(rs.ccd,.~.-TWI(A,C,D),data=Paper.CCD)
summary(rs.ccd2)

#Contour Plots
par(mfrow=c(2,2))
contour(rs.ccd, ~ A+C)
contour(rs.ccd, ~ C+D)
contour(rs.ccd, ~ A+D)
par(mfrow=c(1,1))
par(mfrow=c(2,2))
contour(rs.ccd2, ~ A+C)
contour(rs.ccd2, ~ C+D)
contour(rs.ccd2, ~ A+D)
par(mfrow=c(1,1))
```

***Reference

Montgomery, D. C. (2017). *Design and analysis of experiments*. John wiley & sons.