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System of linear Equation

Solve the system of liner equations by Gaussion elimination method.

$$5x + 3y - 3z = -1$$
$$3x + 2y - 2z = -1$$
$$2x - y + 2z = 8$$

Consistent → at least one solution.

inconsistent → no solution.

$\boxed{\begin{aligned} r_2' &= 3r_1 - 5r_2 \\ r_3' &= 2r_1 - 5r_3 \end{aligned}}$

$$\Rightarrow \quad 5x + 3y - 3z = -1$$
$$-y + z = 2$$
$$11y - 16z = -42$$

$$15x + 9y - 9z = -3$$
$$15x + 10y - 10z = -5$$
$$\ominus \quad \ominus \quad \oplus \quad \oplus$$
$$-y + z = 2$$

$$\cancel{10x + 6y - 6z = -}$$
$$10x + 6y - 6z = -2$$
$$10x - 5y + 10z = 40$$
$$\ominus \quad \oplus \quad \ominus \quad \ominus$$
$$11y - 16z = -42$$

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$r_3'' = 11r_2 + r_3$

$$\begin{aligned} -11y + 11z &= 22 \\ 11y - 16z &= -42 \\ \hline -5z &= -20 \end{aligned}$$

$$\begin{aligned} 5x + 3y - 3z &= -1 \\ -y + z &= 2 \\ -5z &= -20 \end{aligned}$$

which is in echelon form

$-5z = -20$

$\therefore z = \frac{-20}{-5} = 4$

$-y + z = 2$

$\Rightarrow -y + 4 = 2$

$\Rightarrow -y = -2$

$\Rightarrow y = 2.$

$5x + 3y - 3z = -1$

$\Rightarrow 5x + 3.2 - 3.4 = -1$

$\Rightarrow 5x + 6 - 12 = -1$

$\Rightarrow 5x = -1 + 6$

$\Rightarrow 5x = 5$

$\Rightarrow x = 1$

$\therefore$ The solution of the system,

$x = 1, y = 2, z = 4$

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[4] Find the general Solution and a particulare solution of the system,

→ Homojnius Solution

$$x_1 - 2x_2 + 2x_3 = 0$$
$$2x_1 + x_2 - 2x_3 = 0$$
$$3x_1 + 4x_2 - 6x_3 = 0$$

$$2x_1 - 4x_2 + 4x_3 = 0$$
$$2x_1 + x_2 - 2x_3 = 0$$
$$-5x_2 + 6x_3 = 0$$

$\Rightarrow$
$$x_1 - 2x_2 + 2x_3 = 0$$
$$-5x_2 + 6x_3 = 0$$
$$-10x_2 + 12x_3 = 0$$

$$3x_1 - 6x_2 + 6x_3 = 0$$
$$3x_1 + 4x_2 - 6x_3 = 0$$
$$-10x_2 + 12x_3$$

$$-50x_2 + 60x_3 = 0$$
$$-50x_2 + 60x_3 = 0$$
$$\cancel{-120x_3 = 0}$$
$$0 = 0$$

$\Rightarrow$
$$x_1 - 2x_2 + 2x_3 = 0$$
$$-5x_2 + 6x_3 = 0$$
$$\cancel{-120x_3 = 0}$$
$$0 = 0$$

which is echelon System

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let, $x_3 = a$ be the free variable. $a \in \mathbb{R}$

$$-5x_2 + 6x_3 = 0$$

$$-5x_2 = -6x_3 = -6a$$

$$\Rightarrow x_2 = \frac{-6a}{-5} = \frac{6a}{5}$$

$$x_1 - 2x_2 + 2x_3 = 0$$

$$\Rightarrow x_1 - 2 \cdot \frac{6a}{5} + 2a = 0$$

$$\Rightarrow x_1 - \frac{12a}{5} + 2a = 0$$

$$\Rightarrow x_1 = \frac{12a}{5} - 2a = \frac{12a - 10a}{5} = \frac{2a}{5}$$

So, the general Solution of the system is $x_1 = \frac{2a}{5}$, $x_2 = \frac{6a}{5}$, $x_3 = a$

where $a \in \mathbb{R}$

For particular Solution

let, $x_3 = a = 1$

$$x_2 = \frac{6a}{5} = \frac{6 \cdot 1}{5} = \frac{6}{5}$$

$$x_1 = \frac{2a}{5} = \frac{2 \cdot 1}{5} = \frac{2}{5}$$

The Particular Solution is, $x_1 = \frac{2}{5}$, $x_2 = \frac{6}{5}$, $x_3 = 1$

H.W

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[4] Find the general Solution and Particular Solution of the system.

(i) $x_1 - 3x_2 - 2x_3 = 0$

$2x_1 + x_2 + 3x_3 = 0$

$3x_1 - 2x_2 + x_3 = 0$

(ii) $x_1 + 3x_2 + 2x_3 = 0$

$2x_1 - x_2 + 3x_3 = 0$

$3x_1 - 5x_2 + 4x_3 = 0$

$x_1 + 17x_2 + 4x_3 = 0$

(iii) $2x + 3y - z = 0$

$x - y + z = 0$

$x + 9y - 5z = 0$

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H.W # Find the rank of.

(i) $A = \begin{bmatrix} 3 & 0 & 2 & 1 \\ -1 & 2 & 3 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix}$

(ii) $B = \begin{bmatrix} 6 & -3 & 4 \\ 2 & 1 & 0 \\ 0 & 2 & 5 \end{bmatrix}$

If $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$, Then find A^{-1}

or, Find the inverse of A

$$\left\{ \begin{aligned} A^{-1} &= \frac{1}{|A|}(\text{Adjoint matrix of } A) \\ &= \frac{1}{|A|}(\text{Cofactor matrix})^T \end{aligned} \right\}$$

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$$|A| = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

cofactore of element $= (-1)^{r+c}$

$$|A| = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -1 & 2 \end{bmatrix} = 2\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= 2\{-2\cdot 2 - 1(-1)\} + 1(1\cdot 2 - 1\cdot 1) + 1\{1\cdot(-1) - 1\cdot(-2)\}$$

$$= 2(-4+1) + 1(2-1) + (-1+2)$$

$$= -6 + 1 + 1$$

$$= -4$$

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Cofactor matrix of A

$$
= \begin{pmatrix}
+\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} \\
-\begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} \\
+\begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
+(-4+1) & -(2-1) & +(-1+2) \\
-(-2+1) & +(4-1) & -(-2+1) \\
+(-1+2) & -(2-1) & +(-4+1)
\end{pmatrix}
$$

$$
\begin{pmatrix}
-3 & -1 & 1 \\
1 & 3 & 1 \\
1 & -1 & -3
\end{pmatrix}
$$

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$$A^{-1} = \frac{1}{|A|}\left(\text{Cofactor matrix}\right)^{T}$$

$$= \frac{1}{-4}\begin{pmatrix} -3 & -1 & 1 \\ 1 & 3 & 1 \\ 1 & -1 & -3 \end{pmatrix}$$

$$= -\frac{1}{4}\begin{pmatrix} -3 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & 1 & -3 \end{pmatrix}$$

HW find the inverse of

(i) $A = \begin{pmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{pmatrix}$

(ii) $B = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{pmatrix}$

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