

Final Topics

Assignment - 01

C211046

Serial	Age	Amount
A1	20	500
A2	40	100
A3	30	800
A4	18	300
A5	28	1200
A6	35	1400
A7	45	1800

Find out K-mean where $K=2$

Solⁿ: Assume that

$$\text{Centroid, } C_1 = A1 = (20, 500)$$

$$\text{Centroid, } C_2 = A2 = (40, 100)$$

$$\text{For, } A3 \Rightarrow d_1(A_3, A_1) = \sqrt{(30-20)^2 + (800-500)^2} = 300$$

$$d_2(A_3, A_2) = \sqrt{(30-40)^2 + (800-100)^2} = 700$$

As $[d_1 < d_2]$, it will go to Centroid C_1

$$\text{Now, } C_1 = \frac{30+20}{2}, \frac{500+800}{2} = (25, 650)$$

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$$\text{Centroid } C_1 = A_1 = (25, 650)$$

$$C_2 = A_2 = (10, 100)$$

$$\text{For } A_4 \Rightarrow d_1(A_4, C_1) = \sqrt{(18-25)^2 + (300-650)^2} \\ = 350$$

$$d_2(A_4, C_2) = \sqrt{(18-10)^2 + (300-100)^2} \\ = 201$$

As, $[d_2 < d_1]$, it will go to Centroid C_2

$$\text{So, } C_1 = A_1 = (25, 650)$$

$$C_2 = A_2, A_4 = \frac{40+18}{2}, \frac{100+300}{2} = (29, 200)$$

$$\text{For } A_5 \Rightarrow d_1(A_5, C_1) = \sqrt{(28-25)^2 + (1200-650)^2} = 550$$

$$d_2(A_5, C_2) = \sqrt{(28-29)^2 + (1200-200)^2} = 1000$$

As $[d_1 < d_2]$, it will go to Centroid C_1

$$\text{Now, } C_1 = A_1, A_5 = \frac{25+28}{2}, \frac{650+1200}{2} = (28, 925)$$

$$C_2 = A_2 = (29, 200)$$

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$$\text{For } A6 \Rightarrow d_1(A6, C1) = \sqrt{(35-26)^2 + (1400-925)^2} = 975$$

$$d_2(A6, C2) = \sqrt{(35-29)^2 + (1400-200)^2} = 1200$$

As $[d_1 < d_2]$, it will go to Centroid C1

$$\text{So, } C1 = A1, A6 = \frac{26+35}{2}, \frac{925+1400}{2} = (31, 1163)$$

$$C2 = A2 = (29, 200)$$

$$\text{For } A7 \Rightarrow d_1(A7, C1) = \sqrt{(45-31)^2 + (1800-1163)^2} = 637$$

$$d_2(A7, C2) = \sqrt{(45-29)^2 + (1800-200)^2} = 1600$$

As $[d_1 < d_2]$, it will go to Centroid C1

$$\text{Now, } M = A1, A7 = \frac{31+45}{2}, \frac{1800+1163}{2}$$

$$= (38, 1482)$$

$$C2 = A2 = (29, 200)$$

Assignment - 02

C211046

ID	A#1	A#2	Class
1	10	2	Yes
2	4	4	No
3	1	9	Yes
4	3	10	Yes
5	9	6	No
6	8	8	No
7	1	8	Yes

ID	A#1	A#2	Class
8	2	7	?
9	7	7	?
10	1	11	?

$$K = 3$$

Find $K\text{-nn}$?

Soln : Applying $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Euc. Dis.	8	9	10
1	9.43	5.83	12.72
2	3.61	9.29	7.62
3	2.2	6.32	2
4	3.16	5	2.29
5	2.23	3.16	5.83
6	6.08	1.41	7.62
7	1.41	6.08	3

Now, $3\text{NN}(8) = (7, 3, 5) = (\text{Yes}, \text{Yes}, \text{No}) = \text{Yes}$.

$3\text{NN}(9) = (6, 5, 2) = (\text{No}, \text{No}, \text{No}) = \text{No}$.

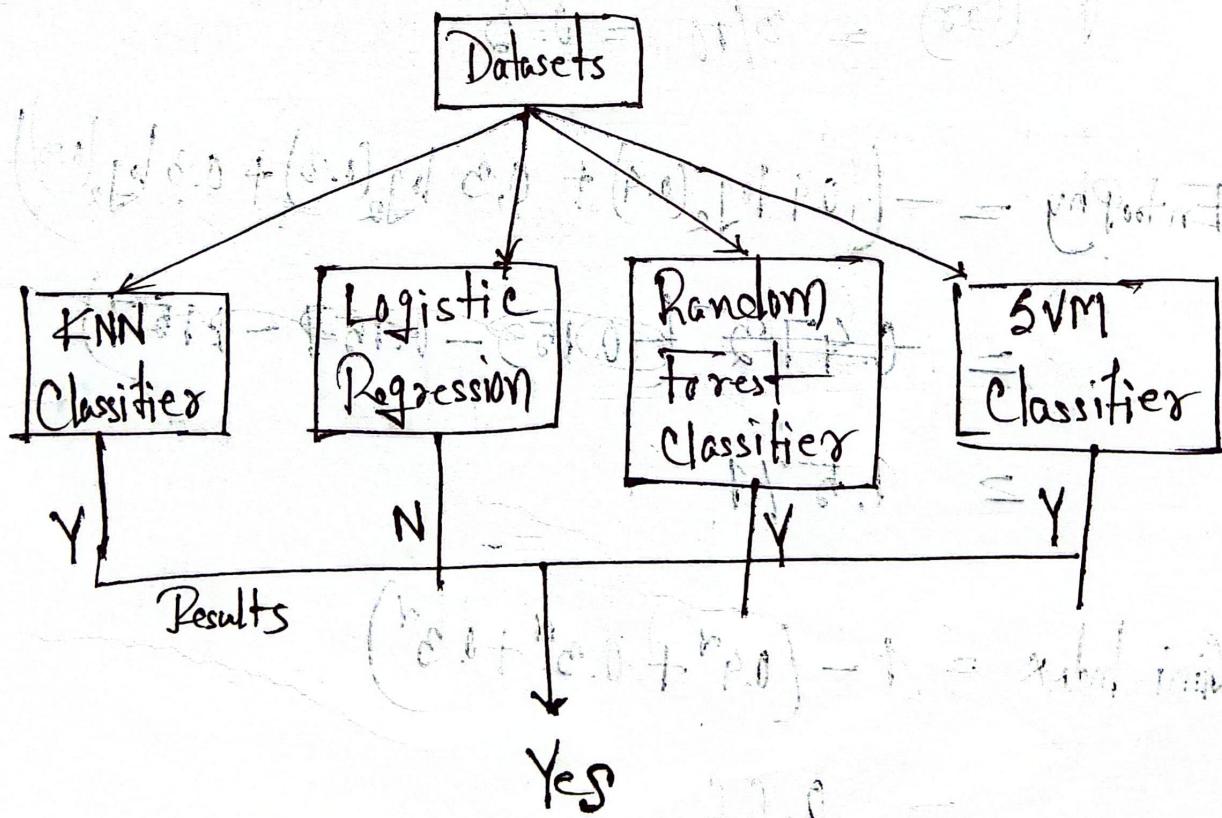
$3\text{-NN}(10) = (3, 4, 7) = (\text{Yes}, \text{Yes}, \text{Yes}) = \text{Yes}$.

~~#~~ Naive Bayes Classification:

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

$$P(Y|x_1, x_2, \dots, x_n) = \frac{P(x_1|Y) * P(x_2|Y) * \dots * P(x_n|Y) * P(Y)}{P(x_1) * P(x_2) * P(x_3) * \dots * P(x_n)}$$

~~#~~ Ensemble learning:



Decision Tree:

$$\text{Entropy} = - \sum P_j \log_2 (P_j)$$

$$\text{Gini Index} = 1 - \sum P_j^2$$

* SP23/1b

$$\text{Here, } P(\text{Bus}) = \frac{4}{10} = 0.4$$

$$P(\text{Train}) = \frac{3}{10} = 0.3$$

$$P(\text{Car}) = \frac{3}{10} = 0.3$$

$$\text{Entropy} = - (0.4 \log_2 (0.4) + 0.3 \log_2 (0.3) + 0.3 \log_2 (0.3))$$

$$= - (0.473 - (0.153 - 0.153 - 0.153))$$

$$\approx 1.571$$

$$\text{Gini Index} = 1 - (0.4^2 + 0.3^2 + 0.3^2)$$

$$= 0.66$$

$$\text{Classification Error} = 1 - \max(P_j)$$

$$= 1 - \max(0.4, 0.3, 0.3) = 1 - 0.4$$

Answer Part I: Degree of impurity (Entropy)

for Travel Cost:

$$\text{"Cheap" Travel Cost} \Rightarrow P(B) = \frac{4}{5} = 0.8$$

$$P(T) = \frac{1}{5} = 0.2$$

$$\text{Entropy} = - (0.8 \log_2(0.8) + 0.2 \log_2(0.2))$$

$$\text{Entropy} = 0.722$$

$$\text{"Expensive" Travel Cost} \Rightarrow P(C) = \frac{3}{3} = 1$$

$$\text{Entropy} = -1 \log_2 1 = 0$$

$$\text{"Standard" Travel Cost} \Rightarrow P(T) = \frac{2}{2} = 1$$

$$\text{Entropy} = -1 \log_2 1 = 0$$

Extra Part:

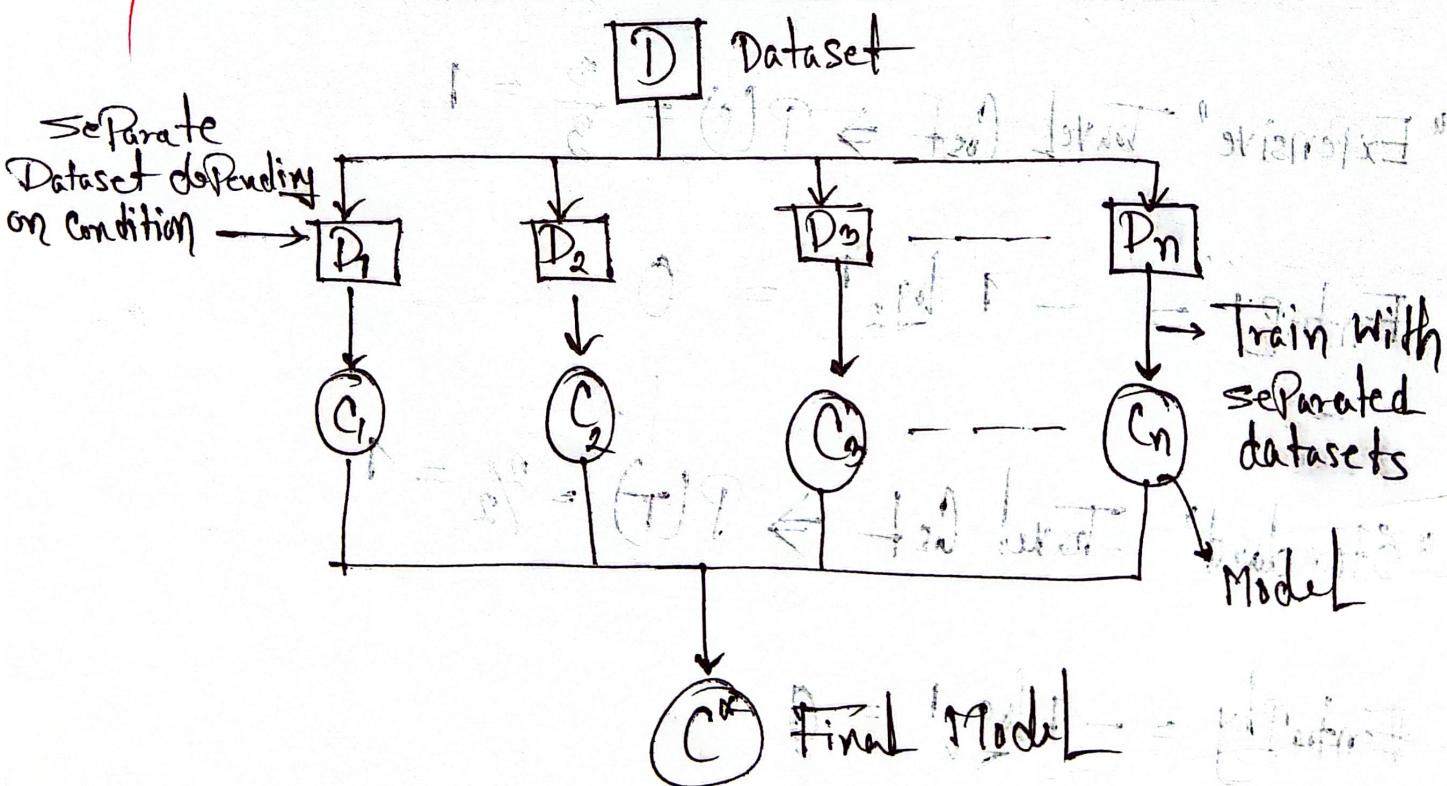
Information Gain (Travel Cost) = Main Parent Entropy

$$- \sum \left(\frac{n_k}{n} * \text{Entropy of } k \right)$$

$$= 1.571 - \left(\frac{5}{10} * 0.722 + \frac{3}{10} * 0 + \frac{2}{10} * 0 \right)$$

$$= 1.21$$

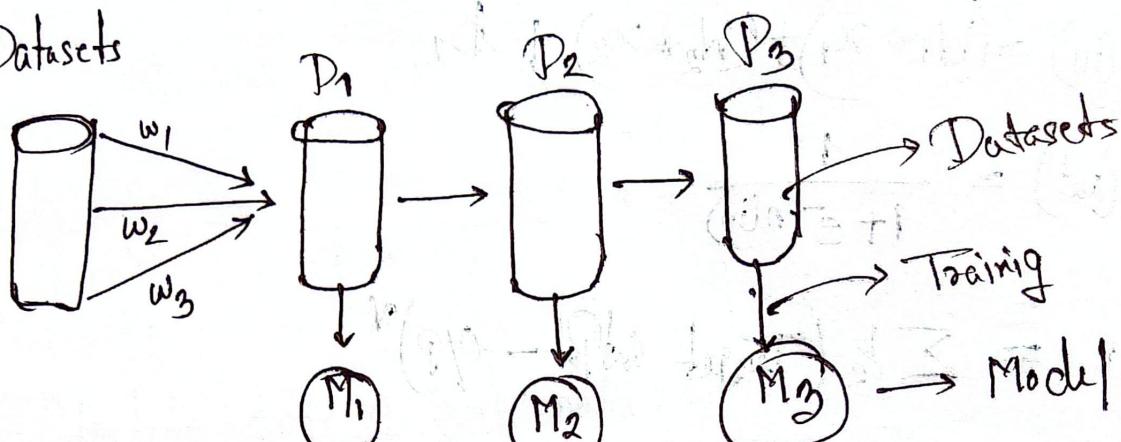
~~#~~ Bagging / Bootstrap Aggregation of Ensemble Learning:



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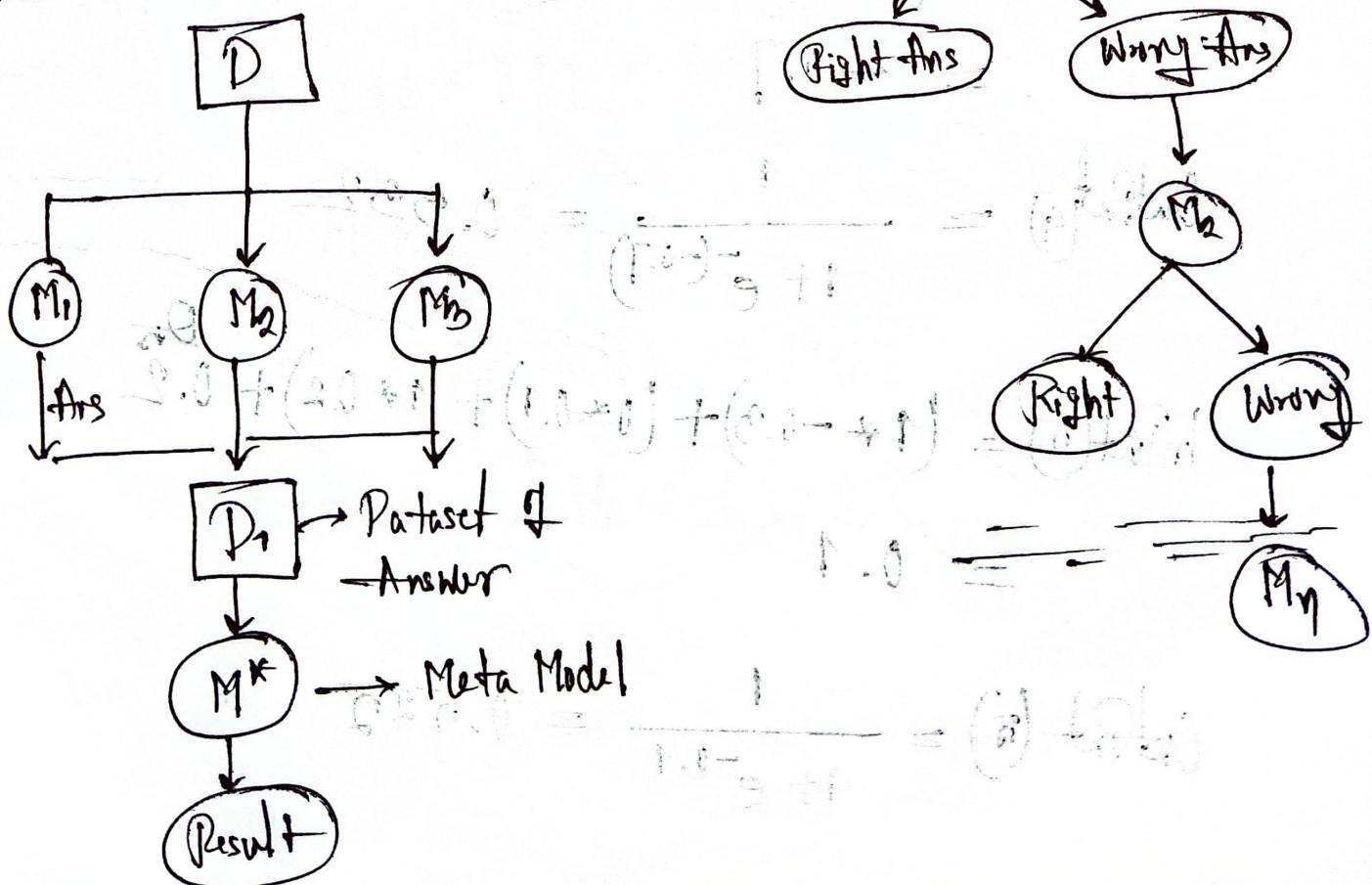
Boosting of Ensemble Learning:

Datasets



④

Stacking



~~#~~ Back Propagation:

$$h_1(\text{in}) = (w_1 * x_1) + (w_2 * x_2) + b_1$$

$$h_1(\text{out}) = \frac{1}{1 + e^{-h(\text{in})}}$$

$$\text{Error} = \sum k_2 (\text{Target OutPut} - O/P)^2$$

~~*~~ (3b, 0.3) | 5P'23

i) Input, Output Calculation:

$$\text{InPut}_4 = (x_1 * w_{14}) + (x_2 * w_{24}) + b_4$$

$$= (1 * 0.2) + (0 * 0.4) + (-0.4) + (1 * -0.5)$$

$$\approx -0.7$$

$$\text{OutPut}_4 = \frac{1}{1 + e^{-(-0.7)}} = 0.332$$

$$\begin{aligned} \text{InPut}_5 &= (1 * -0.3) + (0 * 0.1) + (1 * 0.2) + 0.2 \\ &= 0.1 \end{aligned}$$

$$\text{OutPut}_5 = \frac{1}{1 + e^{-0.1}} = 0.525$$

$$\text{Out}(6) = (0.332 * -0.3) + (0.525 * -0.2) + 0.7 \\ = -0.105$$

$$\text{Out}(6) = \frac{1}{1+e^{-0.105}} = 0.579$$

(ii) Calculate error of each node:

$$\text{Error}(6) = \frac{1}{2} (1 - 0.579)^2 \quad | \quad y=1 = \text{Target Output}$$

$$= 0.13$$

$$\text{Error}(4) = 0_4 * (1 - 0_4) * (E_6 * W_{46}) \\ = 0.33 * 0.67 * 0.13 * -0.3 \\ = -0.0086$$

$$\text{Error}(5) = 0_5 * (1 - 0_5) * (E_6 * W_{56}) \\ = 0.525 * 0.975 * 0.13 * (-0.2) \\ = -0.0065$$

(iii) Update of weight & Bias:

$$\begin{array}{l}
 \text{For Node } 6: \\
 N_{46} = \eta * E_6 * O_4 + W_{46} \\
 = 0.9 * 0.13 * 0.33 + (-0.3) \\
 = -0.26
 \end{array}
 \quad \left| \begin{array}{l} \eta = 0.9 \\ \text{Bias} = -0.3 \end{array} \right.$$

$$\begin{array}{l}
 N_{56} = \eta * E_6 * O_5 + W_{56} \\
 = 0.9 * 0.13 * 0.525 + (-0.2) \\
 = -0.19
 \end{array}$$

$$\begin{array}{l}
 O_6 = \eta * E_6 + \theta_6 = 0.9 * 0.13 + 0.1 \\
 = 0.277
 \end{array}$$

For Node 4:

$$\begin{array}{l}
 E_5 \quad x \quad w_{1q} \\
 W_{1q} = 0.9 * (-0.0086) * 1 + 0.2 \\
 = -0.19
 \end{array}$$

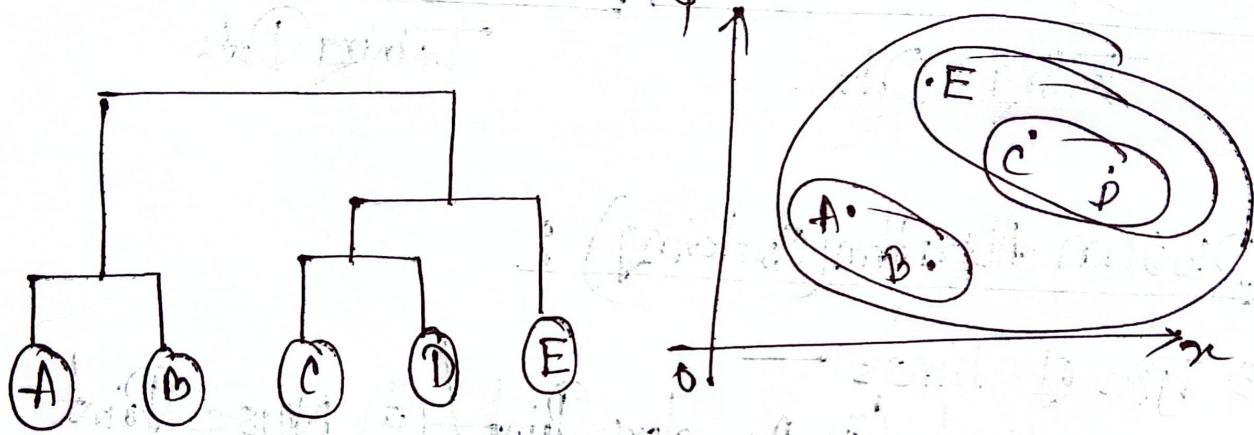
$$\begin{array}{l}
 x_2 \\
 W_{2q} = 0.9 * (-0.0086) * 0.0 + 0.29 = 0.29
 \end{array}$$

$$\begin{array}{l}
 x_3 \\
 W_{3q} = 0.9 * (-0.0086) * 1 + (-0.5) = -0.51
 \end{array}$$

$$\theta_4 = 0.9 * (-0.0086) + -0.4 \\ = -0.41$$

Same for Node (5)

(6) Hierarchical Clustering :



(7) Cross-validation :

2a/5P'23

Given,

(0,0,0,+), (0,0,1,-), (0,1,1,+), (0,1,1,-), (0,0,1,+), (1,0,1,-)

As $k=2$

Data = 6

\therefore Partition = $6/2 = 3$

Fold 1 :

$(0,0,0,+), (0,0,1,-), (0,1,1,+), (0,1,1,-), (0,0,1,+), (1,0,1,-)$

Training Data : $\{ (0,0,0,+), (0,0,1,-), (0,1,1,+), (0,1,1,-) \}$ Testing Data : $\{ (0,0,1,+), (1,0,1,-) \}$

Fold 2 :

$(0,0,0,+), (0,0,1,-), (0,1,1,+), (0,1,1,-), (0,1,1,+), (1,0,1,-)$

Testing Data .

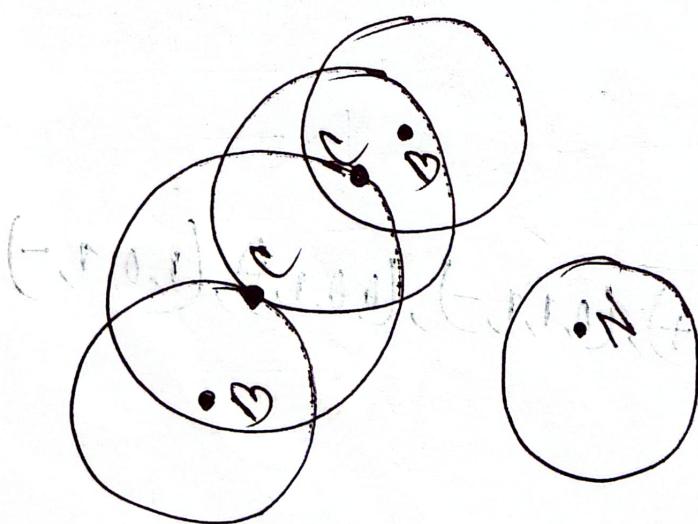
Training Data



DBSCAN Algorithm (Clustering) :

3 Types of Points —

- i Core Point
- ii Boundary Point
- iii Noise Point



~~#~~ Apriori Algorithm:

~~Aut'22 | 5b (Q)~~

Given,

$$\text{Threshold} = 55\%$$

$$\text{Confidence} = 75\%$$

$$\begin{aligned}\text{Support} &= \frac{55}{100} \times 5 \\ &= 2.75\end{aligned}$$

Items	Support
M	3
O	4
N	2
K	5
E	4
Y	3
D	1
V	1
C	2
T	1

Greater than support:

Items	Support
M	3
O	4
K	5
E	4

Generate Pair:

<u>Items</u>	<u>Support</u>	<u>Items</u>	<u>Support</u>
M,O	1		
M,K	3	MK	3
ME	2	OK	3
MY	2	OE	3
OK	3	KE	4
OE	3	KY	3
MY	2		
KE	4		
KY	3		
EY	2		

Triplets

	<u>Item</u>	<u>Support</u>
MKO	1	
MKF	2	→ OKE → 3
MKE	2	
OKE	3	
OKY	2	
OEM	1	
OEH	2	
KEY	2	

<u>Association Rule</u>	<u>Support</u>	<u>Confidence</u>	<u>Appears</u>
$O \wedge K \rightarrow E$	$2.75/3 = .91$	$.91$	91%
$O \wedge E \rightarrow K$	$2.75/3 = .91$	$.91$	91%
$K \wedge E \rightarrow O$	$2.75/4 = .687$	$.687$	69%
$E \rightarrow O \wedge K$	$2.75/4 = .687$	$.687$	69%
$K \rightarrow O \wedge E$	$2.75/5 = .55$	$.55$	55%
$O \rightarrow K \wedge E$	$2.75/4 = .687$	$.687$	69%

Compare this with min-Confidence = 75%.

<u>Rules</u>	<u>Confidence</u>	<u>Appears</u>
$O \wedge K \rightarrow E$	91%	$(A + B + C) = 100\%$
$O \wedge E \rightarrow K$	91%	$(A + B + D) = 100\%$

~~# Non-linear SVM:~~

Given,

$$B_1(1, 1)$$

$$R_1(2, 0)$$

$$B_2(-1, 1)$$

$$R_2(0, 2)$$

$$B_3(-1, -1)$$

$$R_3(-2, 0)$$

$$B_4(1, -1)$$

$$R_4(0, -2)$$

★
[9a] SP'23

$$\Phi(x_1, x_2) = \begin{cases} \left(\begin{array}{c} 6 - x_1 + (x_1 - x_2)^2 \\ 6 - x_2 + (x_1 - x_2)^2 \end{array} \right) & \text{if } \sqrt{x_1^2 + x_2^2} \geq 2 \\ \left(\begin{array}{c} x_1 \\ x_2 \end{array} \right) & \text{otherwise} \end{cases}$$

B Points are no change since $\sqrt{x_1^2 + x_2^2} \leq 2$ for all the vectors.

For R Points, ~~it's combination - min with diff. condition~~

$$\Phi(\vec{z}) = \begin{pmatrix} 6 - 2 + 4 \\ 6 - 0 + 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

$$\Phi(\vec{v}_2) = \begin{pmatrix} 6 - 0 + 4 \\ 6 - 2 + 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

$$\Phi(\vec{v}_0) = \begin{pmatrix} 6 + 2 + 4 \\ 6 - 0 + 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}$$

$$\Phi(\vec{v}_{-2}) = \begin{pmatrix} 6 - 0 + 4 \\ 6 + 2 + 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$$

Given, 3 support vectors, $s_1 = \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix}$ $s'_1 = \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix}$
 $s_2 = \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix}$ $s'_2 = \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix}$
 $s_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Assume 3 linear eqn $(\alpha_1 s_1) + (\alpha_2 s_2) + (\alpha_3 s_3)$ $s'_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\alpha_1 s'_1 s_1 + \alpha_2 s'_2 \cdot s_1 + \alpha_3 s'_3 \cdot s_1 = 1$$

$$\alpha_1 s'_1 s_2 + \alpha_2 s'_2 \cdot s_2 + \alpha_3 s'_3 \cdot s_2 = 1$$

$$\alpha_1 s'_1 s_3 + \alpha_2 s'_2 \cdot s_3 + \alpha_3 s'_3 \cdot s_3 = -1$$

$$\Rightarrow 165\alpha_1 + 161\alpha_2 + 19\alpha_3 = 1$$

$$161\alpha_1 + 165\alpha_2 + 19\alpha_3 = 1$$

$$19\alpha_1 + 19\alpha_2 + 3\alpha_3 = -1$$

$$\therefore \alpha_1 = \alpha_2 = 0.859, \alpha_3 = -1.4219$$

Now,

$$\tilde{w} = \sum \alpha_i s'_i$$

$$= \alpha_1 \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.859 \\ 0.859 \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + \begin{pmatrix} 0.859 \\ 0.859 \end{pmatrix} \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + (-1.4219) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1293 \\ 0.1293 \\ -1.2501 \end{pmatrix}$$

$$y = w x + b \text{ where } w = \begin{pmatrix} 0.1293 \\ 0.1293 \end{pmatrix} = \begin{pmatrix} 0.1293/0.1293 \\ 0.1293/0.1293 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{and, } b \text{ (offset)} = -1.2501/0.1293 + \dots$$

$$\approx -10.057$$

4

~~#~~ Supervised vs UnSupervised:

- (i) Delta rule
- (ii) Back Propagation
- (iii) Linear Regression
- (iv) SVR
- (v) KNN
- (vi) Decision Tree
- (i) K-means
- (ii) Hierarchical Clustering
- (iii) K-medoids
- (vii) ANN/CNN

SP'23/2b

Here, Age: high, low = 2 values types

Income: Fair, Excellent = 2

Credit_Rating; Low, High = 2

Since each features has 2 Possible value type, the value of $J, k = 2$

Using k for Laplacian smoothing:

For Class "Yes":

Total "Yes" = 2

$$P(\text{Yes}) = \frac{2}{5} \geq 0.4$$

Age - Low:

$$P(\text{Low}|\text{Yes}) = \frac{\text{Count}(\text{low}, \text{Yes}) + 1}{\text{Count}(\text{Yes}) + k}$$

$$= \frac{0+1}{2+2} = 0.25$$

Income - fair:

$$P(\text{High}|\text{Yes}) = \frac{2+1}{2+2} = 0.75$$

Credit - Low :

$$P(\text{Low}|\text{Yes}) = \frac{1+1}{2+2} = 0.5$$

For Class No:

$$\cancel{P(\text{Low}/\text{No}) = 1} \rightarrow P(\cancel{\text{Yes}}/\text{Yes}) =$$

$$P(\text{No}) = \frac{3}{5} = 0.6$$

Age \rightarrow Low:

$$P(\text{Low}/\text{No}) = \frac{3+1}{3+2} = 0.8$$

Income \rightarrow Fair:

$$P(\text{Fair}/\text{No}) = \frac{1+1}{3+2} = 0.4$$

Credit. \rightarrow Low:

$$P(\text{Low}/\text{No}) = \frac{0+1}{3+2} = 0.2$$

Now, $P(\text{Yes}) =$

$$P(\text{Yes}/\text{Low, Fair, Low}) = \frac{P(\text{Low}/\text{Yes}) + P(\text{Fair}/\text{Yes}) \cdot P(\text{Low}/\text{Yes})}{P(\text{Yes})}$$

$$= \frac{0.25 \times 0.75 + 0.5 \times 0.4}{0.975} = 0.0975$$

$$P(\text{No} | \text{Yes, Low, Fair, Low})$$

$$= P(\text{Low} | \text{No}) \cdot P(\text{Fair} | \text{No}) \cdot P(\text{Low} | \text{No}) \cdot P(\text{No})$$

$$= 0.6 \cdot 0.8 \cdot 0.2 \cdot 0.6$$

$$= 0.384$$

Since ($\text{No} > \text{Yes}$), so for (Low, Fair, Low) sample

the Class label is "No"

~~5a) SP23~~

For Matrix 1,

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} = \frac{6959}{6959 + 41} = 0.99$$

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{6959}{6959 + 912} = 0.94$$

$$\text{F}_1 \text{ score} = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} = 0.96$$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = 0.95$$

Do for Matrix (ii) also - —