CT Preparation

Segment - 2

Bisection Method (Algorithm):

1. Find Position a & b $[a<b,\ f(a).f(b)<0]$
2. $x_0 = \frac{a+b}{2}$
3. If $f(x_0)=0$ then, x_0 is the exact root.
   else if, $f(a).f(x_0)<0$ then $b=x_0$
   else if, $f(b).f(x_0)<0$ then $a=x_0$
4. Repeat step 2 and 3 until $f(x_i)\approx 0$.

Example: $f(x) = x^3 - x - 1$

Soln:

x	0	1	2
$f(x)$	-1	-1	5

Now,

$a = 1,\ b = 2$

$x_0 = \frac{1+2}{2} = 1.5$

$f(x_0) = 0.875 < 0$

2nd iteration:

$a = 1,\ b = 1.5$

$x_1 = \frac{1+1.5}{2} = 1.25$

$f(x_1) = -0.2969 < 0$

3rd iteration:

$a = 1.25$, $b = 1.5$ $\qquad f(x_2) = 0.2296 > 0$

$x_2 = \dfrac{1.25 + 1.5}{2} = 1.375$

---- And so On!

~~A~~ False Position (Algorithm):

$$\boxed{x = \frac{\cancel{0}\ a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}}$$

$$= \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

1. Find x_0 and x_1
   $[x_0 < x_1$ and $f(x_0) \cdot f(x_1) < 0]$

2. find, $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

3. If $f(x_2) = 0$, root is x_2
   else if, $f(x_2) \cdot f(x_0) < 0$, then $x_1 = x_2$
   else if, $f(x_2) \cdot f(x_1) < 0$, then $x_0 = x_2$

4. Repeat 2 and 3 ~~until~~ until $f(x_i) \approx 0$

Example: $f(x) = x^3 - x - 1$

Soln:

x	0	1	2
$f(x)$	-1	-1	5

1st iteration:

$x_0 = 1, \; x_1 = 2$ | $f(x_0) = -1$, $f(x_1) = 5$

$$x_2 = \frac{1*5 - 2*-1}{5-(-1)}$$

$$= \frac{7}{6} = 1.1667$$

$f(x_2) = -0.5787 < 0$

2nd iteration:

$x_0 = 1.1667 \; ; \; x_1 = 2$ | $f(x_0) = -0.5787$, $f(x_1) = 5$

$$x_3 = \frac{1.1667*5 - 2*(-0.5787)}{5-(-0.5787)}$$

$$= 1.2531$$

$f(x_3) = -0.2852 < 0$

3rd Iteration :

$x_0 = 1.2531 , x_1 = 2$

$f(x_0) = -0.2852$

$f(x_1) = 5$

$$x_4 = \frac{1.2531 * 5 - 2 * (-0.2852)}{5 - (-0.2852)}$$

$$= 1.3016$$

$f(x_4) = -0.0966 < 0$

———— —— And so ON !

Newton Raphson :

$$\boxed{x_1 = x_0 - f(x_0) / f'(x_0)}$$

Algorithm :

1. Find a and b, $[a < b$ and $f(a) . f(b) < 0]$
2. Find $f(x_0)$ and $f'(x_0)$ and x_1
3. If $f(x_1) \simeq 0$, then root $= x$, else if, $\boxed{x_0 = x_1}$
4. Repeat 2 and 3 stop until $f(x_i) = 0$

Example : $f(x) = x^3 - x - 1$

Sol^n : $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1 - 0$

$= 3x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	5

$$\boxed{x_0 = \frac{1+2}{2} = 1.5}$$

$f(x_0) = 0.875$

$f'(x_0) = 5.75$

1st iteration :

$$x_1 = 1.5 - \frac{0.875}{5.75}$$

$= 1.3478$

2nd iteration : $x_0 = 1.3478$

$f(x_0) = 0.1006$

$f'(x_0) = 4.4497$

$$x_2 = 1.3478 - \frac{0.1006}{4.4497}$$

$= 1.3252$

$f(x_2) = \cancel{0.6274}\ 0.0021$

3rd iteration :

$f'(x_2) = 4.2685$

$$x_3 = 1.3252 - \frac{0.0021}{4.2685} = 1.3247$$

$f(x_3) = -0.000076 \ldots \approx 0$

Ans : 1.3247

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Secant Method (Algorithm):

① Find x_1 and x_2

② Compute $f(x_1)$ and $f(x_2)$

③ Compute $x_3 = \dfrac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$

④ If $|x_3 - x_2| > 0$ then set, $x_1 = x_2$

$x_2 = x_3$

Continue step ③ and ④

else,

set root $= x_3$

Example: $f(x) = x^2 - 4x - 10$

$x_1 = 2, \; x_2 = 4$

Soln: $f(x_1) = -14$

$f(x_2) = -10$

1st Iteration: $x_3 = \dfrac{2 * -10 - 4 * -14}{-10 - (-14)} = 9$

$|x_3 - x_2| = 9 - 4 = 5 > 0$

2nd Iteration: $x_1 = 4, \; x_2 = 9$

$f(x_1) = -10, \; f(x_2) = 35$

$x_4 = \dfrac{4 * 35 - 9 * -10}{35 - (-10)}$

$= 5.1111$

$|5.1111 - 9| = 3.88 > 0$

<u>3rd Iteration :</u>

$x_1 = 9, \; x_2 = 5.1111$

$$x_5 = \frac{9 * -4.3279 - 5.11 * 35}{-4.3279 - 35}$$

$f(x_1) = 35$

$f(x_2) = -4.3279$

$= 5.5381$

$|5.5381 - 5.1111| = 0.4269 > 0$

<u>4th Iteration :</u>

$x_1 = 5.1111, \; x_2 = 5.5381$

$f(x_1) = -4.3279$

$f(x_2) = \cancel{-0.8534} \; -1.4818$

$$x_6 = \cancel{\frac{19.6065}{-3.6934}} = \cancel{5.3085}$$

$$= \frac{5.1111 * -1.4818 - 5.5381 * -4.3279}{-1.4818 - (-4.3279)}$$

$= 5.7604$

$|5.7604 - 5.5381| = 0.22 > 0$

And so on !

Fixed Position:

Example: $f(x) = x^3 - x - 1$

Soln: $x^3 = x + 1$

$\Rightarrow \boxed{x = \sqrt[3]{x+1}}$

$= (x+1)^{1/3}$

$\phi' = \frac{1}{3}(x+1)^{-2/3} \cdot 1$

$\boxed{x = x^3 - 1}$

$= 3x^2$

x	0	1	2
$f(x)$	-1	-1	5

$\phi(x) = \sqrt[3]{x+1}$

$\frac{1+2}{2} = 1.5 = x_0$

$x_1 = \phi(1.5) = 1.3572$

$x_2 = \phi(1.3572) = 1.3309$

$x_3 = \phi(1.3309) = 1.3259$

$x_4 = \phi(1.3259) = 1.3249$

$x_5 = \phi(1.3249) = 1.3247$

$|x_5 - x_4| = 0.0002 \approx 0$

Ans: 1.3247

$$\Rightarrow u = \frac{5xy^2}{z^3} \ ; \ x=y=z=1 \ ; \ \Delta x = \Delta y = \Delta z = 0.001$$

$$u = \frac{5.1.1^2}{1^3} = 5$$

$$\Delta u = f(x, y, z)$$

$$= \frac{\delta f}{\delta x} \Delta x + \frac{\delta f}{\delta y} \Delta y + \frac{\delta f}{\delta z} \Delta z$$

$$= \left|\frac{5y^2}{z^3} \times 0.001\right| + \left|\frac{10xy}{z^3} \times 0.001\right| + \left|\frac{5xy^2}{3z^2} \times 0.001\right|$$

$$= \left|\frac{5 \times 1^2}{1^3} \times 0.001\right| + \left|\frac{10.1.1}{1^3} \times 0.001\right| + \left|\frac{5.1.1^2}{3.1^2} \times 0.001\right|$$

$$= |5 \times 0.001| + |10 \times 0.001| + \left|\frac{5}{3} \times 0.001\right|$$

$$= 0.03$$

Relative error, $= \frac{\Delta u}{u} = \frac{0.03}{5} = 0.006$ Ans

Forward Difference Operator:

$= E f(x) - f(x)$

$= f(x)(E-1)$

$$\boxed{\Delta f(x) = f(x+h) - f(x)} \Rightarrow \boxed{\Delta = E - 1}$$

$$\Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

$$\boxed{\Delta^n f(x) = a_0 \lfloor n \, h^n}$$

Backward Difference:

$$\boxed{\nabla f(x) = f(x) - f(x-h)}$$

$$\nabla^2 f(x) = \nabla f(x) - \nabla f(x-h)$$

Identity Operator: $\boxed{I f(x) = f(x)}$

Shifting Operator: $\boxed{E f(x) = f(x+h)} \Rightarrow E^n f(x) = f(x+nh)$

$$E^{-1} f(x) = f(x-h) \Rightarrow E^{-n} f(x) = f(x-nh)$$

Central difference:

$$\boxed{\delta f(x) = f(x + \tfrac{1}{2}h) - f(x - \tfrac{1}{2}h)}$$

$$\delta f(x) = E^{1/2} f(x) - E^{-1/2} f(x)$$

$$\Rightarrow \delta = E^{1/2} - E^{-1/2}$$

$$\delta = E^{1/2} - E^{-1/2} = E^{1/2} - \frac{1}{E^{1/2}}$$

$$= \frac{E^{1/2} \cdot E^{1/2} - 1}{E^{1/2}}$$

$$= \frac{E-1}{E^{1/2}} = \frac{\Delta}{E^{1/2}}$$

* Averaging Operator:

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \tfrac{1}{2}h\right) + f\left(x - \tfrac{1}{2}h\right)\right]$$

$$= \frac{E^{1/2} + E^{-1/2}}{2}$$

Horner's Rule: $x^3 - 6x^2 + 11x - 6$ at $x = 2$

1	-6	11	-6
$\downarrow$	$\xrightarrow{1\times 2}$ 2	-8	6
1	-4	3	0

$x+2=0 \Rightarrow x=-2$

5	3	-2	4	-6
	-10	14	-24	40
5	-7	12	-20	34

$\boxed{\text{Ant'22} \mid 2a}$ $x = 6$

~~1~~	~~-2~~	~~5~~	~~-16~~	~~5~~
	~~4~~	~~12~~	~~68~~	~~208~~
~~1~~	~~2~~	~~17~~	~~52~~	~~213~~

1	-2	5	-16	5
	6	24	174	948
1	4	29	158	952

$\cancel{(x^4 + 4x^3 + 29x^2 + 158x + 952)}(x-6)$

$$\frac{(x^3 + 4x^2 + 29x + 158)(x-6) + 952}{(x-6)}$$

Date : 29/01

Lagrang's InterPolation :

x	2	2.5	3
$f(x)$	.69315	.91629	1.0986

$x = 2.7$

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \times f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \times f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{(2.7-2.5)(2.7-3)}{(2-2.5)(2-3)} \times .69315 + \frac{(2.7-2)(2.7-3)}{(2.5-2)(2.5-1.0986)} \times .916$$

$$+ \frac{(2.7-2)(2.7-2.5)}{(3-2)(3-2.5)} \times 1.0986$$

$=$

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SP-23

3(a)

Interpolation: An operation of estimation the value of a function for any intermediate value of arguments when the value of the function corresponding to a number of the values of the argument are given is called interpolation.

Newton's forward and backward difference formulas are both used for polynomial interpolation, where the data point are assumed to be equidistant. They are typically employed when the data points are given in a tabular form with equal intervals between successive points. The choice between using Newton's forward or backword difference formulas depends on the specific requirement of the problem.

3(b)

Newton's Forward Interpolation Formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\cdot\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\cdot\Delta^3 y_0 \text{------}$$

$$\frac{p(p-1)\text{---}(p-(n-1))}{n!}\cdot\Delta^n y_0$$

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Newton's Backword:

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\cdot\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$\text{------} \cdot \frac{p(p+1)\text{---}(p+n-1)}{n!}\cdot\nabla^n y_n$$

3(c)

Year	2008	2010	2012	2016	2020	2022
Sales	40	43	48	52	58	63

$x = 2018$

	x	y				
x_0	2008	40				
			$\frac{43-40}{2010-2008} = 1.5$			
x_1	2010	43		$\frac{2.5-1.5}{2012-2008} = .25$		
			$\frac{48-43}{2012-2010} = 2.5$		$-.08375$	
x_2	2012	48		$\frac{1-2.5}{16-10} = -0.42$		0.011
			$\frac{52-48}{16-12} = 1$		0.04825	
x_3	2016	52		$\frac{.5}{8} = 0.0625$		-0.002
			$\frac{6}{4} = 1.5$		.01045	
x_4	2020	58		$\frac{1}{6} = 0.167$		
			$\frac{5}{2} = 2.5$			
x_5	2022	63				

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Newton's Divided Difference:

$$f(x) = y_0 + (x-x_0) f[x_0,x_1] + (x-x_0)(x-x_1) f[x_0,x_1,x_2]$$

$$+ (x-x_0)(x-x_1)(x-x_2) \cdot f[x_0,x_1,x_2,x_3] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$\cdot f[x_0,x_1,x_2,x_3,x_4] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4) \cdot f[x_0,x_1,x_2,x_3,x_4,x_5]$$

$$f(2018) = 40 + (10\times 1.5) + (10\times 8\times .25) + (10\times 8\times 6\times -.08375)$$

$$+ (10\times 8\times 6\times 2\times .011) + (10\times 8\times 6\times 2\times -2\times -.00101)$$

$$= 40 + 15 + 20 - 40.2 + 10.56 + 1.9392$$

$$= 47.2992$$

0.00101

$\boxed{C211046}$

$$P(x) = x^5 - 4x^4 - 7x^3 + 11x - 46 \text{ at } (x+21) = 0$$
$$\Rightarrow x = -21$$

$$\begin{array}{cccccc}
1 & -4 & -7 & 0 & 11 & \cancel{-46}\ \ -46 \\
 & -21 & 525 & -10878 & 228438 & -4797429 \\
\hline
1 & -25 & 518 & -10878 & 228449 & -4797475
\end{array}$$

$$\frac{\underbrace{(1x^4 - 25x^3 + 518x^2 - 10878x + 228449)}_{Q}\cancel{(x+21)} \underbrace{- 4797475}_{R}}{\cancel{(x+21)}}$$

$$\Delta E^{-1} f(x) = (E-1)\, E^{-1} f(x)$$
$$= \frac{E-1}{E} f(x)$$
$$= (1 - E^{-1}) f(x)$$
$$= f(x) - E^{-1} f(x)$$
$$= f(x) - f(x-h)$$

Sources of Error:

(i) Inherent Error (Input errors)

- (i) Data Error (Empirical errors)
- (ii) Conversion Error (Representation errors)

(ii) Numerical Error (Procedural errors)

(i) Round-off error

- (i) Chopping
- (ii) Symmetric Roundoff

(ii) Trancation Error [e.g: $(0.1)^4 = 0.0001 \approx 0$
$(0.1)^5 = 0.00001 \approx 0$]

(iii) Modelling Error

(iv) Blunders (Is an error that cause due to human imperfection)

Absolute Error:

⇒ Difference between the true value and obtained value.

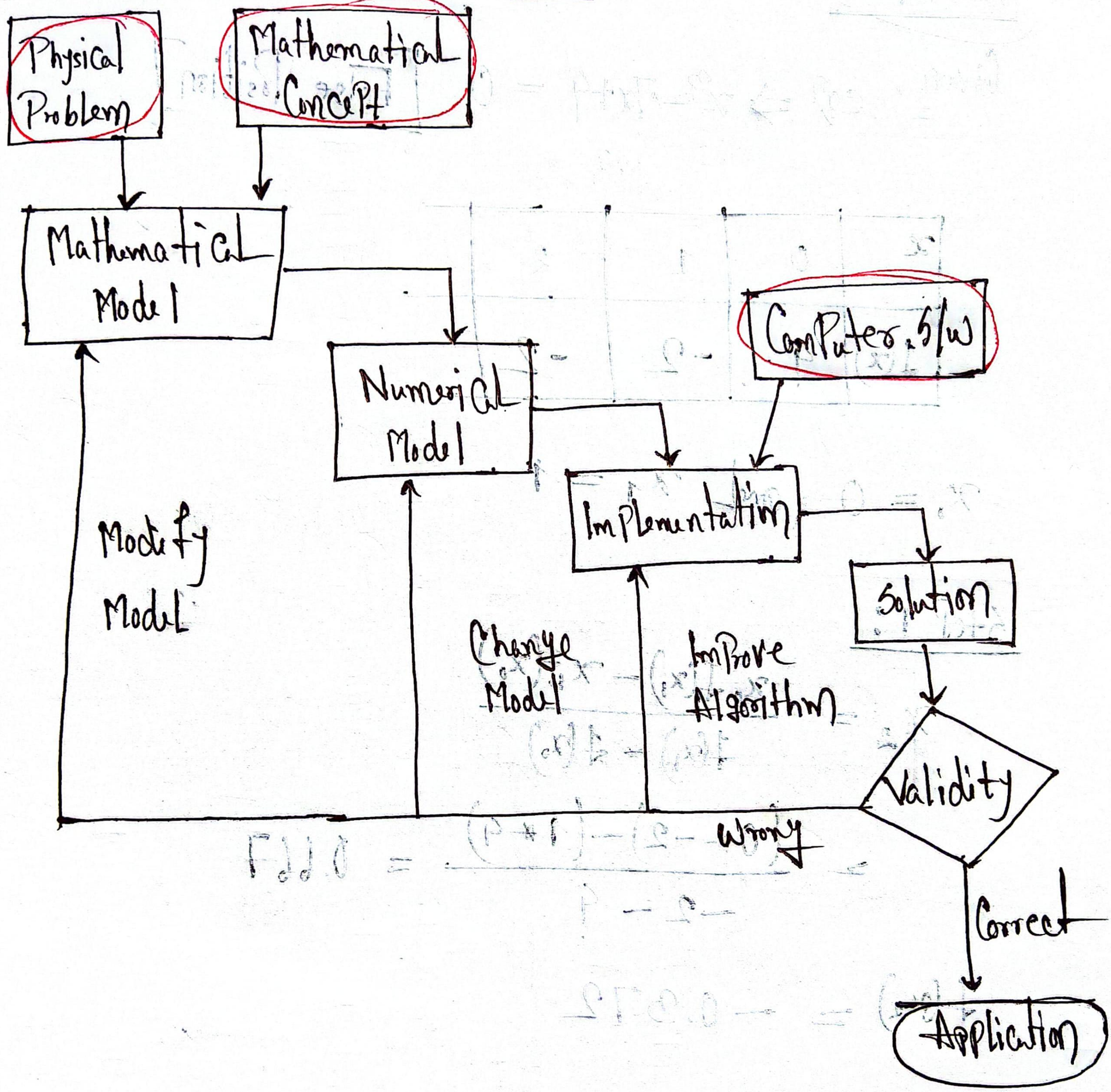
$$E_a = |x_t - x_a|$$

Relative Error:

$$\frac{\text{Absolute Error}}{|\text{True Value}|}$$

$$E_r = \frac{|x_t - x_a|}{|x_t|}$$

Process of Numerical Computing :



$\boxed{\text{Aut'22} \mid \text{2b}}$

Given, $eq^n \Rightarrow x^3 - 7x + 4 = 0$ [False Position]

x	0	1	2
$f(x)$	4	-2	-2

$x_0 = 0$ and $x_1 = 1$

Step 1:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(0 * -2) - (1 * 4)}{-2 - 4} = 0.667$$

$f(x_2) = -0.372$

Step 2: $x_0 = 0$, $x_1 = 0.667$

$$x_3 = \frac{0 * (-0.372) - (0.667 * 4)}{-0.372 - 4}$$

$$\simeq 0.610$$

$f(x_3) = -0.043$

Step 3: $x_0 = 0$, $x_1 = 0.610$

$$x_4 = \frac{(0 * -.043) - (0.61 * 4)}{-0.043 - 4} = 0.603$$

$f(x_4) = -0.0017$

so, the root is $= 0.603$ (Ans)

$\boxed{2b / or}$

$f(x) = x^3 - 7x + 4$ [Newton - Rapson]

$a_0 = 0$, $b_1 = 1$

$f'(x) = 2x^2 - 7 + 0$
$= 2x^2 - 7$

Step 1:

$x_0 = \frac{a+b}{2}$
$= \frac{1}{2}$
$= 0.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f(x_0) = 0.625$

$f'(x_0) = -6.5$

$= 0.5 - \frac{0.625}{-6.5}$

$= 0.596$

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$f(x_1) = 0.039$ & $f'(x_1) = -6.289$

<u>Step 2 :</u> $x_1 = .596$

$$x_2 = .596 - \frac{0.039}{-6.289} = 0.602 \text{ (Ans)}$$

$f(x_2) = 0.00416$

Aut'22 | 3(a)

x	1	2	3	4	5
y	7	a	13	21	37
	y_0	y_1	y_2	y_3	y_4

<u>Newton's Binomial Expansion :</u>

Here, $f(2) = a$ so, $\Delta^3 y_0 = 0$

$$\Rightarrow (E-1)^3 y_0 = 0 \quad [\because \Delta = E-1]$$

$$\Rightarrow \left(E^3 - {}^3C_1 E^2 . 1^1 + {}^3C_2 E^1 . 1^2 - {}^3C_3 E^0 . 1^3\right) y_0 = 0$$

$$\Rightarrow (E^3 - 3E^2 + 3E - 1) y_0 = 0$$

$$\Rightarrow E^3 y_0 - 3E^2 y_0 + 3E y_0 - y_0 = 0$$

$\Rightarrow y_3 - 3y_2 + 3y_1 - y_0 = 0$

$\Rightarrow 21 - 3*13 + 3a - 7 = 0$

$\Rightarrow 3a = 25$

$\Rightarrow a = 8.33$ Ans

3b

~~Net~~ Newton's Backward:

$$y(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\cdot\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\cdot\nabla^3 y_n + \frac{P(P+1)+\cdots(P+n)}{n!}\cdot\nabla^n y_n$$

~~Here~~,

$$P = \frac{x-x_n}{h} \qquad h = (x_1 - x_0)$$

Newton's Divided Difference:

$$f(x) = y_0 + (x-x_0)f[x_0,x_1] + (x-x_0)(x-x_1)f[x_0,x_1,x_2]\cdots$$

Ant'22 | 3C $x = 2018$ [Newton's Divided Difference]

x	y						
1961	29						
		1.2					
1971	36		−0.01				
		1		0.00032			
1981	46		0		−0.000015		
		1		−0.000306		9.6×10^{-7}	
1992	57		−0.0095		0.000034		-4.47×10^{-8}
		0.8		0.0011		-1.76×10^{-6}	
2002	65		0.025		−0.000056		
		1.3		−0.0012			
2012	78		−0.01				
		1.1					

$$y(2018) = y_0 + (x-x_0) f[x_0, x_1] + (x-x_0)(x-x_1) f[x_0, x_1, x_2]$$
$$+ (x-x_0)(x-x_1)(x-x_2) f[x_0, x_1, x_2, x_3] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)$$
$$f[x_0, x_1, x_2, x_3, x_4] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4) f[x_0, x_1, x_2,$$
$$x_3, x_4, x_5] + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5) f[x_0, x_1, x_2, \ldots$$
$$\ldots x_6]$$

$$= 29 + 68.4 + -26.79 + 31.72 - 38.66 + 39.58 - 11.06$$

$$= 87.19$$

For this Math, Lagrange's formula:

$$y(2018) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)(x_0-x_6)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)(x_1-x_6)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)(x_2-x_6)} y_2 +$$

— — — — — — —

*

⇒ Guess Forward → $0 < u < 1$

" Backward → $-1 < u < 0$

Stirling Formula → $-0.25 \leq u \leq 0.25$

Bessel's Formula → $\frac{1}{4} \leq u \leq \frac{3}{4}$

$\boxed{\text{SP'22} | 1 \text{ a}}$ *

Numerical Computing : It's an approch of solving Complex mathematical Problems which can not be solved easily by analytical mathematics ~~or~~ by using simple arithmetic operations. ~~and~~

4 - Characteristics :

(i) Accuracy

(ii) Efficiency

(iii) Numerical Instability

(iv) Iterative

SP'22 | 2b

Derivation of Newton-Raphson Formula:

Let x_0 is an approximate root of $f(x_0) = 0$

and, x_1 is the correct root of $f(x_1) = 0$, $x_1 = (x_0+h)$

Applying Taylor's series on $f(x_0+h) = 0$,

$$f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots = 0$$

Neglecting 2nd and higher order,

$$f(x_0) + h f'(x_0) = 0$$

$x_1 = (x_0+h)$

$\Rightarrow h = x_1 - x_0$

$$\Rightarrow h = -f(x_0)/f'(x_0)$$

$$\Rightarrow x_1 - x_0 = -f(x_0)/f'(x_0)$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

so,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$\boxed{2C}$ $f(x) = x^3 - 9x + 1$ [Bisection Method]

x	0	1	2	3
$f(x)$	1	-7	-9	1

$x_0 = 2$, $x_1 = 3$

Step 1 : $x_2 = \frac{2+3}{2} = 2.5$

$f(x_2) = -5.875$

~~As~~ Step 2 ; ~~As~~ $f(x_2) \cdot f(x_1) < 0$

$x_0 = 2.5$, $x_1 = \cancel{2.5}\ 3$

$x_3 = \frac{3+2.5}{2} = 2.75$

$f(x_3) = \cancel{7.875}\ -2.953$

Step 3 : $x_0 = 2.75$, $x_1 = 3$

$x_4 = 2.875$ | $f(x_4) = -1.111$

Step 4 : $x_0 = 2.875$ $x_1 = 3$

$x_5 = 2.938$ | $f(x_5) = -0.095$

$\boxed{2(c)\ \text{or}}$ $f(x) = x^3 - 6x + 4$ [Secant Method]

x	0	1	2	3
$f(x)$	4	-1	0	13

$x_0 = 0,\ x_1 = 1$

Step 1: $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(0 * -1) - (1 * 4)}{-1 - 4} = 0.8$

$f(x_2) = -0.288$

$|x_2 - x_1| = 1.28 > 0$, so, $x_0 = 1$
$x_1 = 0.8$

Step 2: $x_3 = \dfrac{(1 * -0.288) - (0.8 * -1)}{-0.288 + 1}$

$= 0.719$

$f(x_3) = 0.058$

Step 3 : $x_0 = 0.8$, $x_1 = 0.719$

$$x_4 = \frac{(0.8 \times \cancel{-0.288}\ 0.058) - (0.719 \times -.288)}{0.058 + 0.288}$$

$= \cancel{0.733}\ \cancel{-1.112}\ .733$ // Ans

$f(x_4) = -0.009167$

$\boxed{2(d)}$

Synthetic Devision: Synthetic division can be defined as a simplified way of dividing a Polynomial with another Polynomial equation of degree 1 and is generally used to find the zeroes of Polynomials.

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