"Final Topics"

Numerical Method

Segment-(4) | System of Linear Equations :

Jacobi's Method :

Given,

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20$$

The given eqⁿ can be written as,

$$x = \cancel{12}\ \frac{1}{5}(12 - 2y - z)$$

$$y = \frac{1}{4}(15 - x - 2z)$$

$$z = \frac{1}{5}(20 - x - 2y)$$

We start the iteration by Putting,

$$x = 0,\ y = 0,\ z = 0$$

1st Iteration :

$$x^1 = \frac{12}{5} = 2.4$$

$$y^1 = 15/4 = 3.75$$

$$z^1 = \frac{20}{5} = 4$$

2nd iteration :

$$x^2 = \tfrac{1}{5}\left(12 - \overset{2y}{2*3.75} - \overset{z}{4}\right) = 0.10$$

$$y^2 = \tfrac{1}{4}\left(15 - \overset{x}{2.4} - \overset{2z}{2*4}\right) = 1.15$$

$$z^2 = \tfrac{1}{5}\left(20 - \overset{x}{2.4} - \overset{2y}{2*3.75}\right) = 2.02$$

The iteration Process is Continued and results are tabulated as follows :

	3	4	5	6	7	8	9	10
x	1.54	0.61	1.24	0.84	1.1	0.92	1.042	0.95
y	2.72	1.61	2.09	1.83	2.12	1.92	2.1	1.97
z	3.52	2.60	3.22	2.84	3.1	2.92	3.04	2.95

The values of x, y, z at the end of 10th iteration are,

$$x = 0.95,\quad y = 1.97,\quad z = 2.95$$

Gauss-Seidel Method:

Given,

$$2x - y = 7$$
$$-x + 2y - z = 1$$
$$-y + 2z = 1$$

We can write that,

$$x = \frac{1}{2}(7 + y)$$
$$y = \frac{1}{2}(1 + x + z)$$
$$z = \frac{1}{2}(1 + y)$$

Assume that, $x = 0$, $y = 0$, $z = 0$

1st Iteration:

$$x^1 = 7/2 = 3.5$$
$$y^1 = \frac{1}{2}(1 + 3.5 + 0) = 2.25$$
$$z^1 = \frac{1}{2}(1 + 2.25) = 1.63$$

Following iteration in tabular format:

	2	3	4	5	6	7	8	9
x	4.63	5.32	5.66	5.83	5.92	5.96	5.98	5.99
y	3.63	4.32	4.66	4.83	4.92	4.96	4.98	4.99
z	2.32	2.66	2.83	2.92	2.96	2.98	2.99	2.99

The value of x, y, z at the end of 9th iteration,

$$\left.\begin{aligned} x &= 5.99 \approx 6 \\ y &= 4.99 \approx 5 \\ z &= 2.99 \approx 3 \end{aligned}\right\}$$ Ans.

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Segment - 8 | Euler Method

Question: Find $y(0.2)$ for $y' = \frac{x-y}{2}$, $y(0)=1$, with step length 0.1 using EM.

Sol^n: Given, $y' = \frac{x-y}{2}$

$y(0) = 1$, $h = 0.1$

$y(0.2) = ?$

Formula:

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.1 * (-0.5)$

$= 0.95$

$\frac{0-1}{2}$

$= -0.5$

$y_2 = y_1 + h f(x_1, y_1) = 0.95 + 0.1 * f(0.1, 0.95)$

$= 0.9075$

So, $y(0.2) = 0.9075$ Ans

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Heun's Method:

$$\frac{dy}{dx} = f(x,y)$$

$$y_{i+1} = y_i + K*h \quad \Big| \quad K = \frac{K_1+K_2}{2} \quad \Big| \quad \begin{aligned} K_1 &= f(x_0, y_0) * h \\ K_2 &= f(x_0+h, y_0+K) * h \end{aligned}$$

Question: $\frac{dy}{dx} = \frac{x^2+y^2}{x+y}$; $y(0)=1$ find $y(1) = ?$ $[h=0.25]$

Soln:

x_i	y_i	K_1	K_2	K	y_{i+1}
0	1	0.25	0.27	0.26	1.26
0.25	1.26	0.27	0.32	0.29	1.56
0.5	1.55	0.32	0.38	0.35	1.91
0.75	1.91	0.39	0.47	0.44	2.35
1.00	2.35				

$y(1) = 2.35$ Ans

Runge-Kutta Method: (4th order)

$k_1 = hf(x_0, y_0)$

$k_2 = h * f(x_0 + h/2, y_0 + k_1/2)$

$k_3 = h * f(x_0 + h/2, y_0 + k_2/2)$

$k_4 = h * f(x_0 + h, y_0 + k_3)$

$y_1 = y_0 + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$ 4rt order

Question: $dy/dx = x^2 + y^2$, $y(1) = 1.2$ find $y(1.05) = ?$

$\underset{x_0}{1}$, $\underset{x}{1.05}$

Soln: $h = 0.05$

1 ←——→ 1.05

$0.05 = h$

$k_1 = 0.05 * f(1, 1.2) = 0.122$

$k_2 = 0.05 * f\left(1 + 0.05/2, 1.2 + \frac{0.12}{2}\right) = 0.1320$

$k_3 = 0.05 * f\left(1 + 0.05/2, 1.2 + \frac{0.132}{2}\right) = 0.1326$

$k_4 = 0.05 * f(1 + 0.05, 1.2 + 0.1326) = 0.1439$

So, $y_1 = y(1.05) = 1.2 + 1/6(0.122 + 2 * 0.132 + 2 * 0.1326 + 0.1439)$

$= 1.3325$ Ans

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Taylor Series Method:

Question: $y' = x^2 y \cancel{0} - 1$, $y(0) = 1$. Find $y(0.2) = ?$

Soln: $y' = x^2 y - 1$

$$y'' = 2xy + x^2 y'$$

$$y''' = 2xy' + 2y\cdot\overset{1}{\cancel{x}} + \cancel{2}x^2 y'' + 2xy'$$

$$= 4xy' + 2y + x^2 y''$$

$$y'''' = 4xy'' + 4y' + 2y' + x^2 y''' + y'' 2x$$

$$= 4xy'' + 6y' + x^2 y'''$$

Now,

$$y'_0 = x_0^2 y_0 - 1 = 0^2 . 1 - 1 = -1$$

$$y''_0 = 2x_0 y_0 + x_0^2 y'_0 = 2 * 0 * 1 + 0^2 * -1 = 0$$

$$y'''_0 = 2 \quad \text{and} \quad y'''' = -6$$

Taylor Series:

$$y_1 = y(0.1) = y_0 + h y'_0 + \frac{h^2}{2!} y''_0 + \frac{h^3}{3!} y'''_0 + \frac{h^4}{4!} y''''_0 + \ldots$$

$$= 0.90031 = y(0.1)$$

As like find $y(0.2) = ?$

~~#~~ Segment Inversion Method | Segment - 4

Linear Eq^n: Number of Variables is equal to number of equations.

Example:

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} = A$$

$$A^{-1} = \frac{1}{|A|} * A^T$$

$$|A| = 1 * \begin{vmatrix} 3 & 4 \\ 1 & 3 \end{vmatrix} - 1 * \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} + 1 * \begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= (9-4) - (9-8) + (3-6) = 5 - 1 - 3 = 1$$

$$A^T = \begin{bmatrix} +\begin{vmatrix} 3 & 4 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} \end{bmatrix}$$

$$A^T = \begin{vmatrix} 5 & -1 & -3 \\ -2 & 1 & 1 \\ 1\cancel{0} & -1 & 0 \end{vmatrix} = \begin{vmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ -3 & 1 & 0 \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} * A^T = \frac{1}{1} * \begin{vmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ -3 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ -3 & 1 & 0 \end{vmatrix} \text{ Ans}$$

Solution of Linear equation — 2 way

1. Direct Method
   - Gauss Elimination
   - Gauss Jordan
2. Indirect Method
   - Jacobi's Method
   - Gauss-Seidal "

11. Gauss Elimination | Seg-4

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & -5 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 3 \\ 5 \\ -13 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 3 & -4 & -5 & -13 \end{array}\right] \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -11 & -22 \end{bmatrix} \begin{array}{l} \\ \rightarrow R_2 = R_2 - R_1 \\ \rightarrow R_3 = R_3 - 3R_1 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -32 & -64 \end{bmatrix} \begin{array}{l} \\ \\ \rightarrow R_3 = 3R_3 + R_2 \end{array}$$

$$x - y + 2z = 3$$
$$3y + z = 2$$
$$-32z = -64$$

So,

$$\left.\begin{array}{l} z = 2 \\ y = 0 \\ x = -1 \end{array}\right\} \text{Ans}$$

Guass-Jordan Method | Seg-4

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

$$\begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$R_1 \rightleftharpoons R_3$

$$\begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ | $R_3 \to R_3 - 10R_1$

$$\begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix}$$

$R_1 \to 8R_1 - R_2$ | $R_3 \to 8R_3 + 9R_2$

$$\begin{bmatrix} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & -473 & -473 \end{bmatrix}$$

$R_3 \to R_3 / -473$

$$\begin{bmatrix} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1 - R_3 * 49$ | $R_2 \to R_2 + R_3 * 9$

$$\begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

So,

$$\left.\begin{aligned} 8x &= 8 \Rightarrow x = 1 \\ 8y &= 8 \Rightarrow y = 1 \\ z &= 1 \end{aligned}\right\} \text{Ans}$$

Cramer's Rule | Seg-4

$$x + 2y + 3z = -5$$

$$3x + y - 3z = 4$$

$$-3x + 4y + 7z = -7$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & 7 \end{vmatrix} - 2\begin{vmatrix} 3 & -3 \\ -3 & 7 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ -3 & 4 \end{vmatrix}$$

$$= -19 + 24 - 45 = |-40| = 40$$

To find the value of x, y & z:

$$x = \frac{|A_x|}{|A|}, \quad y = \frac{|A_y|}{|A|}, \quad z = \frac{|A_z|}{|A|}.$$

Now,

$$|A_x| = \begin{bmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{bmatrix}$$

$$= -5\begin{vmatrix} 1 & -3 \\ 4 & 7 \end{vmatrix} - 2\begin{vmatrix} 4 & -3 \\ -7 & 7 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ -7 & 4 \end{vmatrix}$$

$$= -95 \overset{14}{\cancel{+98}} + 69 = \cancel{|-40|} = 40$$

$$|A_y| = \begin{bmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & -7 & 7 \end{bmatrix}$$

$$= 1\begin{vmatrix} 4 & -3 \\ -7 & 7 \end{vmatrix} + 5\begin{vmatrix} 3 & -3 \\ -3 & 7 \end{vmatrix} + 3\begin{vmatrix} 3 & 4 \\ -3 & -7 \end{vmatrix}$$

$$= 7 + 60 - 27 = 40$$

$$|A_z| = \begin{bmatrix} 1 & 2 & -5 \\ 3 & 1 & 4 \\ -3 & 4 & -7 \end{bmatrix}$$

$$= 1\begin{vmatrix} 1 & 4 \\ 4 & -7 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ -3 & -7 \end{vmatrix} \cancel{+} -5\begin{vmatrix} 3 & 1 \\ -3 & 4 \end{vmatrix}$$

$$= -23 + 18 - 75 = -80$$

Now, $\cancel{|A_x|}$

$$\left.\begin{aligned} x &= \frac{40}{40} = 1 \\ y &= \frac{40}{40} = 1 \\ z &= -80/40 = -2 \end{aligned}\right\} \text{Ans}$$

~~LU Decomposition Method | seq-4~~

Curve fitting (least square method) : seq-5

x	1	2	3	4	5
y	1	5	11	8	14

For straight line

It's known that a relation of type $y = a + bx$ exist. Find the best possible values of a and b.

Soln: $y = a + bx$

$$\Rightarrow \sum y = na + b\sum x \quad \text{--- (i)}$$

$$\Rightarrow \sum xy = a\sum x + b\sum x^2 \quad \text{--- (ii)}$$

x	y	xy	x^2
1	1	1	1
2	5	10	4
3	11	33	9
4	8	32	16
5	14	70	25
15	39	146	55

From (i), $39 = 5a + 15b$ --- (iii)

From (ii), $146 = \cancel{a}\,15a + 55b$ --- (iv)

After calculation,

$a = -0.9$, $b = 2.9$

Now, $\boxed{y = -0.9 + 2.9x}$ Ans

Parabola Function :

Question : Fit the Parabola $y = a + bx + cx^2$ to the data.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Sol^n : $y = a + bx + cx^2$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad \text{——— (i)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{——— (ii)}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{——— (iii)}$$

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	1.8	1.8	1	1	1	1.8
2	1.3	2.6	4	8	16	5.2
3	2.5	7.5	9	27	81	22.5
4	6.3	25.2	16	64	256	100.8
10	12.9	38.1	30	100	354	130.3

(i) $\Rightarrow 12.9 = 5a + 10b + 30c$

(ii) $\Rightarrow 38.1 = 10a + 30b + 100c$

(iii) $\Rightarrow 130.3 = 30a + 100b + 354c$

$a = \cancel{0.65}\ 1.87$

$b = \cancel{0.173} -1.98$

$c = 0.77$

So, the eq^n is; $y = a + bx + cx^2$

$$\Rightarrow \boxed{y = 1.87 \cancel{a} - 1.98b + 0.77x^2}$$ Ans

Power Function:

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.1	6.8	7.5

Fit a best fitting curve in the form $y = ax^b$ for the folloing data.

Soln: $y = ax^b$

$\Rightarrow \log_e y = \log_e ax^b$

$\Rightarrow \log_e y = \log_e a + \log_e x^b = \log_e a + b \cdot \log_e x$

$\Rightarrow Y = \overset{A}{\cancel{\log_e a}} + bX$

$Y = \log_e y$

$A = \log_e a$

$X = \log_e x$

Normal eq^n to fit st. Line,

$\Sigma Y = nA + b\Sigma X$ — (i)

$\Rightarrow \Sigma XY = A\Sigma X + b\Sigma X^2$ — (ii)

x	y	x^2	xy	$X = \ln(x)$	$Y = \ln(y)$	X^2	XY
1	2.98	1	2.98	0	1.1	0	0
2	4.26	4	8.52	0.69	1.45	0.48	1
3	5.21	9	15.63	1.099	1.65	1.21	1.81
4	6.1	16	24.4	1.39	1.81	1.93	2.52
5	6.8	25	34	1.61	1.92	2.59	3.09
6	7.5	36	45	1.792	2.01	3.2	3.6
				6.581	9.94	9.41	12.02

$$\left.\begin{aligned} \text{(i)} &\Rightarrow 9.94 = 6A + \cancel{b}\ 6.581\, b \\ \text{(ii)} &\Rightarrow 12.02 = 6.581\, A + 9.41\, b \end{aligned}\right\} \quad \begin{aligned} A &= 1.09 \\ b &= 0.51 \end{aligned}$$

$$A = \log_e a = 1.09 \Rightarrow a = e^{1.09} = 2.97$$

$$b = 0.51$$

$$\cancel{Y = \log_e y = 9.9}$$

So, Required Curve:

$$\boxed{y = 2.97 * x^{0.51}}$$

Ans.

Derivatives using Newton's Forward | Seg-6

Question: Find dy/dx and d^2y/dx^2 for $x=1$

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
1	1					
		7				
2	8		12			
		19		6		
3	27		18		0	
		37		6		0
4	64		24		0	
		61		6		
5	125		30			
		91				
6	216					

$x_0 = 1$

$h = 1$

$x = 1$

$u = \dfrac{x-x_0}{h} = 0$

We know that,

$$\Rightarrow y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\Rightarrow dy/dx = \frac{1}{h}\left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \cdots\right)$$

$$= 1/1\left(7 - \frac{1}{2} * 12 + 1/3 * 6 - \cdots\right)$$

$$= 7 - 6 + 2 = 3$$

$$\Rightarrow d^2y/dx^2 = 1/h^2\left(\Delta^2 y_0 - \Delta^3 y_0 + 11/12\,\Delta^4 y_0 - \cdots\right)$$

$$= 1/1\,(12 - 6 + 0) = 6$$

So, for $x = 1 \Rightarrow dy/dx = 3$ and $d^2y/dx^2 = 6$ Ans

Derivation Using Newton's backward | Seq - 6

Question : For Previous table find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when, $x = 6$

Soln : We know that,

$$\Rightarrow y = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{h}\left(\nabla y_n + \frac{2u+1}{2!}\nabla^2 y_n + \frac{3u^2+6u+2}{3!}\nabla^3 y_n + \cdots\right)$$

$$= \frac{1}{1}\left(91 + \frac{1}{2} * 30 + \frac{2}{6} * 6\right)$$

$$= 108$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\nabla^2 y_n + (u+1)\nabla^3 y_n + \cdots\right)$$

$$= \frac{1}{1}(30 + 1 * 6) = 36$$

Ans : 108 and 36

$u^3 - 2u^2 - u^2 + 2u$

$\Rightarrow u^3 - 3u^2 + 2u$

$3u^2 - 6u + 2$

Max & Min Value of Tabulated Function | seg - 6

x	y	Δ	Δ^2	Δ^3	Δ^4
1.2	.932				
		.0316			
1.3	.963		-.0097		
		.0219		-.0002	
1.4	.985		-.0099		.0002
		.0120		0	
1.5	.997		-.0099		
		.0021			
1.6	.999				

Average value = Δ^3

For Central Value

$x_0 = 1.2$

$h = 0.1$

x, y is Maximum and find y

Soln: For maximum value of y

$$dy/dx = 0$$

~~After diff. Newton's forward interpolation formula with respect to u~~ :

$\cancel{dy/dx} =$ Newton's forward interpolation formula :

$$f(x_0 + uh) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \boxed{\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0}$$

$$= 0.932 + u * 0.316 * \frac{u^2 - u}{2} * 0.0097 - \boxed{\frac{(u^2-u)(u-2)}{6} * 0}$$

$$dy/du = 0 + 1 * 0.316 - \frac{2u-1}{2} * 0.0097 - \boxed{\frac{3u^2 - 6u + 2}{6} * .000}$$

$\Rightarrow 0 = 0.316 - \frac{2u-1}{2} * 0.0097$

$\Rightarrow u = 3.8$

Now, $x = x_0 + uh \Rightarrow x = 1.2 + 3.8 * 0.1$

$= 1.58$

The value of $x(1.58)$ is closer to $x = 1.6$. So, we use Newton's backward formula:

$y(1.58) = y_n + u \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n$

$= .999 + 3.8 * .0021 + \frac{3.8^2 + 3.8}{2} * (-0.0099)$

$= 0.954$ Ans

Derivatives in terms of Central difference

$\frac{dy}{dx} = \frac{1}{h}\left[\mu \delta y_0 - \frac{1}{6} \mu \delta^3 y_0 + \cdots\right]$

$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\delta^2 y_0 - \frac{1}{12} \delta^4 y_0 + \cdots\right]$

Here, $\boxed{\text{everage of } \Delta^2 y_0}$ Difference of $\Delta^4 y_0$ everage

SP-23 | 3(b)

x	y	Δ	Δ^2	Δ^3	Δ^4
0.4	1.584				
		0.213			
0.5	1.797		0.034		
		0.247		.003	
0.6	2.044		0.037		-.002
		0.284		.001	
0.7	2.328		0.038		
		0.323			
0.8	2.651				

$x = 0.6$

$h = 0.1$

We know that,

$$\frac{dy}{dx} = \frac{1}{h}\left[\mu\delta y_0 - \frac{1}{6}\mu\delta^3 y_0 + \cdots\right]$$

$$= \frac{1}{0.1}\left[\frac{0.247+0.284}{2} - \frac{1}{6} * \frac{0.003+0.001}{2}\right]$$

$$= \frac{1}{0.1}\left(0.266 - 3.3\times 10^{-4}\right)$$

$$= 10 * 0.266 = 2.66 \text{ (Ans)}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\delta^2 y_0 - \frac{1}{12}\delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(1)^2}\left(0.037 - \frac{-0.002}{12}\right)$$

$$= \cancel{0.0037}\ 0.387 \quad \text{Ans}$$

Trapezoidal Rule | Seg-7

$$\int_a^b y\,dx = h\left[\frac{y_0+y_n}{2} + y_1 + y_2 + \cdots y_{n-1}\right]$$

$$= \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+\cdots y_{(n-1)}) + y_n\right]$$

SP-22 | 4(b) or

Given, $f(x) = \frac{x}{1+x}$

$a=0,\ b=1,\ n=6$

$h = \frac{b-a}{n} = \frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	6/6
y	0	~~0.166~~ ~~0.142~~ 0.143	0.250	0.333	0.4000	0.455	.5000

NOW,

$$\int_0^1 \frac{x}{1+x}\,dx = \frac{1}{6}\left[\frac{0+.5}{2} + 0.143 + 0.25 + 0.333 + .4 + .455\right]$$

$$= 0.30512$$

Ans : 0.305 [3 significant figure]

Simpson's 1/3 Rule | seg-7

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + y_5 + \cdots y_{n-1}) + 2(y_2 + y_4 + y_6 + \cdots y_{n-2})\right]$$

$$\boxed{n = \text{Multiple of } 2}$$

SP-23 | ~~2~~ 6 or :

Given, $h = 10$, $a = 0$, $b = 80$

NOW using 1/3 rule,

$$\frac{10}{3}\left[0 + 3 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14)\right]$$

$$= \frac{10}{3} \times 213 = 710 \text{ Ans}$$

Simson's 3/8 Rule | Seg-7 :

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[y_0+y_n+3(y_1+y_2+y_4+y_5+y_7+\cdots) + 2(y_3+y_6+y_9+\cdots)\right]$$

$$\boxed{n = \text{Multiple of } 3}$$

General Quadrature Formula | Seg-7

$$I = \int_a^b f(x)\,dx$$

For Newton's Forward InterPolation Formula,

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx$$

$$= \int_{x_0}^{x_0+nh} \left(y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots\right) dx$$

$$= \int_{x_0}^{x_0+nh} \left(y_0 + u\Delta y_0 + \frac{u^2-u}{2}\Delta^2 y_0 + \frac{u^3-3u^2+2u}{6}\Delta^3 y_0 + \cdots\right)$$

Now, $u = \dfrac{x-x_0}{h}$

$\Rightarrow x = x_0 + uh$

$\Rightarrow \dfrac{dx}{du} = h \Rightarrow dx = h.du$

At, $x = x_0 \rightarrow u = 0$

and, $x = x_0 + nh \rightarrow u = n$

Now,

$$I = \int_a^b f(x)\, dx$$

$$= \int_0^n f(x) \cdot h \cdot du$$

$$= h\int_0^n \left(y_0 + u\Delta y_0 + \frac{u^2-u}{2}\Delta^2 y_0 + \frac{u^3-3u^2+2u}{6}\Delta^3 y_0 + \cdots\right) du$$

$$= h\left[u y_0 + u^2/2\, \Delta y_0 + \left(u^3/3 - u^2/2\right)\Delta^2 y_0/2 + \left(u^4/4 - 3u^3/3 + 2u^2/2\right)\Delta^3 y_0/6 + \cdots\right]_0^n$$

$$\Rightarrow h\left[n y_0 + n^2/2\, \Delta y_0 + \left(n^3/3 - n^2/2\right)\Delta^2 y_0/2 + \left(n^4/4 - 3n^3/3 + 2n^2/2\right)\Delta^3\right.$$

Final Assignment

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Ans: 1a Gauss Elimination Method:

Let a system be,

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \text{------ (i)}$$

We first form a argment matrix of the system (i),

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix} \text{------ (ii)}$$

To eliminate x_1 from the second equation, we multiply the first equation by $-a_{21}/a_{11}$ and then add it to the 2nd eqn. Similarly, to elminate the x_1 from 3rd eqn, we multiply the first eqn by $-31/a_{11}$ and add it to the 3rd eqn. $-a_{21}/a_{11}$ and $-a_{31}/a_{11}$ are called the multipliers for the first stage of elimination. In this stage we assume that, $a_{11} \neq 0$. At the end of the first stage, the augment matrix becomes,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & a'_{32} & a'_{33} & b'_3 \end{bmatrix} \text{------ (iii)}$$

where, a'_{22}, a'_{23} --- are all changed elements.

Now, To eliminate x_2 from 3rd eqn, we need to —

$R_2 * -a'_{32}/a'_{22}$ and add this with 3rd eqn. Again we assume that, $a'_{22} \neq 0$. At the end of the second stage we have the upper triangle system —

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & 0 & a''_{33} & b''_3 \end{bmatrix} \text{------ (iv)}$$

From which the value of x_1, x_2 and x_3 Can be obtained by the back substitution.

Ans: ①a Or Gauss seidel Method :

Gauss seidel method is an improved version of Jacobi iteration method. In Jacobi method we begin with the value with the initial values $x_1^{(0)}, x_2^{(0)}, \ldots x_n^{(0)}$ and obtain new approximation $x_1^{(1)}, x_2^{(1)}, \ldots x_n^{(1)}$. Note that, in computing $x_2^{(1)}$, we used $x_1^{(0)}$ and not $x_1^{(1)}$ which just been computed. Since at that point, both $x_1^{(0)}$ and $x_1^{(1)}$ are available. We can use $x_1^{(1)}$ which is a better approximation. for computing $x_2^{(1)}$, similarly for computing $x_3^{(1)}$ we can use $x_1^{(1)}$ and $x_2^{(1)}$ along with $x_4^{(0)}, \ldots x_n^{(0)}$. This idea can execute to all subsequent computation. This approch is called the Guass-seidal method.

During the $(k+1)$th iteration of gauss seidel method, x_i taken from,

$$x_i^{(k+1)} = \frac{1}{a_{i1}} \left[b_i - a_{i1} x_1^{(k+1)} - \ldots - a_{in} x_n^{(k)} \right]$$

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Ans: 1b

Given,
$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array}$$

$R_1 * \frac{-3}{2}$ and add with R_2

$$\Rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & -1/2 & -1 & -3/2 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

$R_3 = R_1 * \frac{-4}{2} + R_3$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & -1/2 & -1 & -3/2 \\ 0 & -1 & -2 & -13 \end{bmatrix}$$

$R_3 = \frac{-1}{1/2} * R_2 + R_3$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & -1/2 & -1 & -3/2 \\ 0 & 0 & 1 & -23/2 \end{bmatrix}$$

Finally system becomes,

$2x_1 + 3x_2 + 4x_3 = 5$ ———— (i)

$-\frac{1}{2}x_2 - x_3 = -3/2$ ---- (ii)

$x_3 = -23/2$ ---- (iii)

From (ii) $\Rightarrow x_2 = \dfrac{\frac{-3}{2} - \frac{23}{2}}{-1/2}$

$= \frac{-3-23}{\cancel{2}} \times \frac{-\cancel{2}}{1}$

$= 26$

From (i) $\Rightarrow x_1 = \frac{5 - 78 + 46}{2} = -27/2$

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Ans: 1(c) Condition for Convergence

(i) Jacobi's Method: For each row, the absolute value of the diagonal element should be grather than the sum of absolute values of the other elements in the equation,

i.e:
$$|a_{ii}| > \sum_{i=1}^{n} |a_{ij}| \quad \text{for } i \neq j$$

Remember that this condition is sufficient, but not necessary for convergence. Some system may ~~Cover~~ Converge even if this condition is not satisfied.

(ii) Gauss Seidel Method:
Same as Jacobi's method. But the convergence in gauss seidel method is more rapid than in Jacobi's method.

Ans: (2a)

Curve fitting is the process of finding a mathematical model (Line, Curve, eq^n) the best represents the relationship between variables in data.

It's importance lies in identifying patterns, making prediction, modeling complex relationship etc.

Ans: 2(b)

Let the sum of squares of individual errors be expressed as,

$$Q = \sum_{i=1}^{n} q_i^2 = \sum_{i=1}^{n} [y_i - f(x_i)]^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

In the method of least square, we choose a and b such that Q is minimum. Since Q depends on a and b, a necessary condition for Q to be minimum is –

$$\frac{\delta Q}{\delta a} = 0 \text{ and } \frac{\delta Q}{\delta b} = 0$$

Then, $\frac{\delta Q}{\delta a} = -2 \sum_{i=1}^{n} (y_i - a - bx_i) = 0$

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$$\frac{\delta Q}{\delta b} = -2\sum_{i=1}^{n} x_i(y_i - a - bx_i) = 0$$

Thus, $-\sum y_i + na + b\sum x_i = 0$

$-\sum x_i y_i + a\sum x_i + b\sum x_i^2 = 0$

Rearranging we get,

$$na + b\sum x_i = \sum y_i$$

$$a\sum x_i + b\sum x_i^2 = \sum x_i y_i$$

These are called the normal eq^n. Solving for a and b -

$$a = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

Ans: 2b or   Fitting a Parabola

Let the Parabolic curve,

$$y = f(x) = a_0 + a_1 x + a_2 x^2$$

The sum of squares of individual errors be expressed,

$$S = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} [y_i - f(x_i)]^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$$

In the method of least squares, we have to choose a_0, a_1 and a_2 such that S is minimam. By this, we get the following normal eqn —

$$n a_0 + a_1 \sum x_i + a_2 \sum x_i^2 = \sum y_i$$

$$a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 = \sum x_i y_i$$

$$a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 = \sum x_i^2 y_i$$

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Ans: 2c

We know, $y = a + bx$

$\Rightarrow \sum y = na + b\sum x$ --- (i)

$\Rightarrow \sum xy = a\sum x + b\sum x^2$ --- (ii)

x	y	x^2	xy
10	600	100	6000
20	500	400	10000
40	300	1600	12000
50	200	2500	10000
120	1600	4600	38000

$n = 4$

Now, (i) $\Rightarrow 1600 = 4a + 120b \Rightarrow a + 30b = 400$ --- (iii)

(ii) $\Rightarrow 38000 = 120a + 4600b \Rightarrow 12a + 460b = 3800$ --- (iv)

Solving (iii) and (iv) we get,

$a = 700$

$b = -10$

Ans: 3(a) Derivation using Newton's forward interpolation formula —

$$y = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \quad \text{--- (i)}$$

where,

$$u = \frac{x - x_0}{h} \quad \text{--- (ii)}$$

Differentiating (i) with respect to u we get,

$$\frac{dy}{du} = \Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!}\Delta^3 y_0 + \cdots \quad \text{--- (iii)}$$

and eqn (ii) with respect to x,

$$\frac{du}{dx} = \frac{1}{h} \quad \text{--- (iv)}$$

Now from eqn (3) and (4) ⇒

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!}\Delta^3 y_0 + \cdots\right] \quad \text{--- (v)}$$

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$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{du}\left(\frac{dy}{dx}\right)\frac{du}{dx}$$

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For tabulated value of x, when $x = x_0$ and $u = 0$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

Differentiating (v) with respect to x,

$$\frac{d^2y}{d^2x} = \frac{1}{h}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2 - 18u + 11}{12}\Delta^4 y_0 + \cdots\right]$$

Ans: 3.b

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.4	1.58				
		0.21			
			0.04		
0.5	1.79			0	
		0.25			0
			0.04		
0.6	2.04			0	
		0.28			
			0.04		
0.7	2.33				
		0.32			
0.8	2.65				

$h = 0.1$

$$u = \frac{0.6 - 0.4}{0.1} = 2$$

Now,

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0\right]$$

$$= \frac{1}{0.1}\left[0.21 + 1.5 \times 0.04\right)$$

$$= 2.7$$

and,

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2-18u+11}{12}\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[\overset{0.04}{\cancel{0.04}} + 1 \times 0\right]$$

$$= 4$$

Ans: 3(c)

We know that, the maximum and minimum values of a function can be found by equating the first derivative to zero and solving for the variable. The same Procedure can be applied to determine the max and min of a tabulated function.

Ans: 4(a)

~~He~~ Numerical method is essential when analytical methods are impractical, inefficient or impossible due to the complexity of the integrand, the nature of the domain or the form in which the function is Provided

Ans: 4(c) Given,

$a = 0, \quad b = 80$

$n = 9$

$$h = \frac{b-a}{n} = \frac{80}{9} = 8.89$$

(i) Trapezoidal rule:

$$\int_a^b y\,dx = h\left[\frac{y_0+y_n}{2} + y_1+y_2+y_3+\dots+y_8\right]$$

$$= 8.89\left[\overset{1.5}{\cancel{40}} + 4 + 7 + 9 + 12 + 15 + 14 + 8\right]$$

$$= 626.745$$

(ii) Simpson's 1/3 Rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_0+y_n+4\left(y_1+y_3+y_5+y_7+\dots\right) + 2\left(y_2+y_4+y_6+y_8+\dots\right)\right]$$

$$= \frac{8.89}{3}\left[0+3+4\left(4+9+15+8\right) + 2\left(7+12+14\right)\right]$$

$$= 631.19$$

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Ans: 5(a)

Advantages of Taylor and Euler's Method:

(i) High Accuracy
(ii) Systematic Approch
(iii) Local Error Control
(iv) Flexibility

(i) Simplicity
(ii) Low Computational Cost
(iii) Foundation for advance method
(iv) Ease of Use
(v) Initial step is Adaptive.

Or,

Problems of Taylor's and Heun's method:

(i) Complexity is higher
(ii) Computational difficulty
(iii) Implementation Challenge

(i) Accuracy
(ii) Stability

Ans : 5C

Given,

$$\frac{dy}{dx} = x + y^2$$

$$y(0) = 1, \; h = 0.1$$

$$x_0 = 0 \text{ and } y_0 = 1$$

1st iteration : $x = 0.1$

$$K_1 = h * f(x_0, y_0) = 0.1 * 1 = 0.1$$

$$K_2 = h * f(x_0 + h/2, y_0 + K_1/2)$$

$$= h * f(0.5, 1.05)$$

$$= 0.161025$$

$$K_3 = h * f(x_0 + h/2, y_0 + K_2/2)$$

$$= 0.1 \times f(0.5, 1.08) = 0.1667$$

$$K_4 = h \times f(x_0 + h, y_0 + K_3)$$

$$= 0.1 \times f(0.10, 1.6667)$$

$$= 0.1961$$

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$$y_1 = y(0.1) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + 1/6\,(0.1 + 0.3205 + 0.3334 + 0.1461)$$

$$= 1.15$$

Iteration 2 : $x = 0.2$ and $(x_0 = 0.1,\ y_0 = 1.15)$

$$k_1 = 0.1 \times f(0.1, 1.15) = 0.1423$$

$$k_2 = 0.1 \times f\left(0.1 + 0.1/2,\ 1.15 + \frac{0.1423}{2}\right) = 0.0155$$

$$k_3 = 0.1 \times f\left(0.1 + 0.1/2,\ 1.15 + \frac{0.0155}{2}\right) = 0.015$$

$$k_4 = 0.1 \times f(0.1 + 0.1,\ 1.15 + 0.015) = 0.1557$$

$$y_2 = y(0.2) = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.15 + \frac{1}{6}(0.3278)$$

$$= 1.2046$$

so, $y(0.1) \approx 1.15$

$y(0.2) \approx 1.2141$

Derivation of Matrix Inversion method:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$Ax = B$$

$$\Rightarrow x = A^{-1}B$$

Here, $A^{-1} = \dfrac{A^T}{|A|}$

Derivation of Cramer's Rule:

For above matrix A,

$$x_i = \frac{\det A_i}{\det A} = \frac{|A_i|}{|A|}$$

Curve fitting: The Process of establishing relation in the form of mathematical eq^n is known as curve fitting.

For Parabola:

$$f(x) = a_0 + a_1 x + a_2 x^2$$

Now,

$$\Sigma y_i = n a_0 + a_1 \Sigma x_i + a_2 \Sigma x_i^2$$

$$\Sigma x_i y_i = a_0 \Sigma x_i + a_1 \Sigma x_i^2 + a_2 \Sigma x_i^3$$

$$\Sigma x_i^2 y_i = a_0 \Sigma x_i^2 + a_1 \Sigma x_i^3 + a_2 \Sigma x_i^4$$

$$u(u-1)(u-2) = (u^2 - u)(u-2) = u^3 - 2u^2 - u^2 + 2u = u^3 - 3u^2 + 2u$$

$$d/du = 3u^2 - 6u + 2$$

$$6u - 6 + 8 = 6(u-1)$$

General Quadrature formula Using Newton's forward:

Applying Integral to Newton's forward,

$$I = \int_{x_0}^{x_0+nh} f(x)\, dx$$

$$= \int_{x_0}^{x_0+nh} \left[y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots\right] dx$$

Here,

$$u = \frac{x-x_0}{h} \Rightarrow \frac{du}{dx} = \frac{1}{h} \Rightarrow dx = du.h$$

At, $x = x_0 \rightarrow u = 0$

and, $x = x_0 + nh \rightarrow u = n$

Hence,

$$I = \int_0^n \left[y_0 + u\Delta y_0 + \frac{u^2 - u}{2}\Delta^2 y_0 + \frac{u^3 - 3u^2 + 2u}{6}\Delta^3 y_0\right] du$$

$$= h\left[u y_0 + \frac{u^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2}\right)\Delta^3 y_0\right]_0^n$$

$$= h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y}{2!} + \left(\frac{n^4}{4} - \frac{3n^3}{3} + \frac{2n^2}{2}\right)\frac{\Delta^3 y}{3!} + \cdots\right]$$

~~#~~ Derivation of ~~Topo~~ Trapezoidal Rule:

From Gen. Quadrature formula —

$$I = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!} \cdots\right]$$

For $(n=1)$,

$$I_1 = h\left[y_0 + \frac{\Delta y_0}{2}\right]$$

$$= h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right]$$

$$= h/2\,[2y_0 + y_1 - y_0) = h/2\,(y_0 + y_1)$$

For, $(n=2)$,

$$\cancel{I_2 = h(2y_0 + 2\Delta y_0) = h\{2y_0 + 2(y_1 - y_0)\}}$$

$$\cancel{= 2h(y_0 + y_1 - y_0)}$$

$$I_2 = \frac{h}{2}(y_1 + y_2)$$

$I_3 = h/2\,(y_2 + y_3)$

$\vdots$

$I_n = h/2\,(y_{n-1} + y_n)$

Now, $I = I_1 + I_2 + I_3 + \dots + I_n$

$$= \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + \dots) + y_n\right]$$

For Simpson's 1/3 :

For $n = 2$,

$$I_1 = h\left[2y_0 + 2\Delta y_0 + \tfrac{1}{3}\Delta^2 y_0\right]$$

$$= h/3\,\left[6y_0 + 6\Delta y_0 + \Delta^2 y_0\right]$$

$$= \frac{h}{3}\left[6y_0 + 6(y_1 - y_0) + (y_2 - 2y_1 + y_0)\right]$$

$$= h/3\,(\cancel{6y_0} + 6y_1 - \cancel{6y_0} + y_2 - 2y_1 + y_0)$$

$$= h/3\,(y_0 + 4y_1 + y_2)$$

As usual,

$$I_2 = \frac{h}{3}\left(y_2 + 4y_3 + y_4\right)$$

$$I_3 = h/3\left(y_4 + 4y_5 + y_6\right)$$

$$\vdots$$

$$I_{n/2} = h/3\left(y_{n-2} + 4\,y_{n-1} + y_n\right)$$

Now,

$$I = I_1 + I_2 + I_3 + \text{---} + I_{n/2}$$

$$= h/3\Big(y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4 + y_4 + 4y_5 + y_6 + \cdots + \left(y_{n-2} + 4y_{(n-1)} + y_n\right)\Big)$$

$$= h/3\Big[y_0 + 4\left(y_1 + y_3 + y_5 + y_7 + \text{---}\, y_{n-1}\right) + 2\left(y_2 + y_4 + y_6 + \cdots\, y_{n-2}\right) + y_n\Big]$$

~~#~~ Taylor Series :

Let, $y = f(x)$

$$y(x_0) = y_0$$

$$f(x) \overset{= f(x_0)}{\cancel{\frac{dy}{dx}}} = f(x_0) + \frac{(x-x_0)}{1!} f'(x_0) + \frac{\cancel{2}(x-x_0)^2}{2!} f''(x_0)$$

$$+$$

$$\cdot\, y_0 = f(x) = y_0 + \frac{x-x_0}{1!} y_0' + \frac{\cancel{2}(x-x_0)^2}{2!} y_0'' + \dots$$

Putting $x = x_1 = x_0 + h$ we ge,

$$y_1 = f(x_1) = y_0 + \frac{\cancel{x_0}\, h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \dots$$

Where, $h = x - x_0$

Similarly,

$$f_{(n+1)} = y_{n+1} = y_n + \frac{h}{1!} y_n' + \frac{h^2}{2!} y_n'' + \dots$$

4

Minimum - maximum Value:

Newton's forward interpolation —

$$dy/dx = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \cdots\right]$$

Min and max value of $y = f(x)$ found if $dy/dx = 0$,

$$\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \cdots\right]$$

$y = f(x)$

$x = x_0$

$$y_0 = f(x_0) = f_{x_0} + \frac{(x-x_0)}{1!}y'_{(x_0)} + \frac{(x-x_0)^2}{2!}y''_{(x_0)} + \frac{(x-x_0)^3}{3!}y'''_{(x_0)}$$

$x = x_1 = x_1 + nh$

$$f(x_1) = f(x_1) + \frac{x_1 - x_0}{1!}f'(x_1) + \frac{(x_1-x_0)^2}{2!}y'(x_1) + \frac{(x_1-x_0)^3}{3!}y'''$$

$$= y_0 + \frac{h}{1!}y'_0 + \frac{h^2}{2!}y''_0 + \frac{h^3}{3!}y'''_0$$

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