1. **Dice Game**: Let's play a dice game, there are three unusual six-sided dice. They are unusual because they do not have 1 to 6 numbers on their six sides. Each dice has the usual six sides, but opposite sides have the same number on them, and the numbers on the dice are different. First, you choose one of the dice, then your friend chooses another. Having made everyone's choices, you both roll your respective dice. You win the game if you roll a higher number than your friend.
   - **Dice Information:**
     - Dice A: 2, 6, 7
     - Dice B: 1, 5, 9
     - Dice C: 3, 4, 8
   - **Tasks:**
     - I. The probability of you winning the game if you choose Dice B and your friend chooses Dice A.
     - II. The probability of you winning the game if you choose Dice C and your friend chooses Dice A.
     - III. Overall probabilities of winning for each person in different dice combinations.
     - IV. Identify who has the higher overall chance of winning.

---

2. **Coin Flip Independence:** Suppose that we flip three fairs, mutually-independent coins and consider the following three events:
   - A is the event that coin 1 matches coin 2.
   - B is the event that coin 2 matches coin 3.
   - C is the event that coin 3 matches coin 1.
   - **Task:** Illustrate how pairwise independence does not necessarily imply mutual independence.

---

3. **Table Tennis Tournament:** Suppose there is a table tennis tournament where two players, A and B, compete in a best-of-five match series. Player A has a probability of 0.6 of winning any individual match, while player B has a probability of 0.4. The winner of the tournament is the first player to win three matches.
   - **Tasks:**
     - I. Probability that the first three matches decide the tournament champion.
     - II. Probability that player A wins the tournament, given that they win the first match.

4. **Let's Make a Deal:** Suppose that Let's Make a Deal is played according to different rules. There are four doors, with a prize hidden behind one of them. The contestant is allowed to pick a door. The host must then reveal a different door that has no prize behind it. The contestant is allowed to stay with their original door or to pick one of the other two that are still closed. If the contestant chooses the door concealing the prize in this second stage, they win.
   - **Assumptions:**
     - The prize is equally likely to be behind each door.
     - The contestant is equally likely to pick each door initially.
     - The host is equally likely to reveal each door that does not conceal the prize and was not selected by the player.
   - **Tasks:**
     - (a) Probability that the 1st contestant wins by staying with their original door.
     - (b) Probability that the 2nd contestant wins by switching to one of the remaining two doors with equal probability.

---

5. **Rat in a Maze:** A rat is trapped in a maze. Initially, it has to choose one of two directions. If it goes to the right, it will wander around in the maze for three minutes and then return to its initial position. If it goes to the left, then with probability 1/3 it will depart the maze after two minutes of traveling, and with probability 2/3 it will return to its initial position after five minutes of traveling.
   - **Task:** Find the expected number of minutes that the rat will be trapped in the maze.

---

6. **Burnout Probability of a Light Bulb:** Suppose a light bulb has a probability pp of burning out at the end of each hour of use.
   - **Task:** Formulate a procedure to estimate the expected time until the light bulb burns out.

---

7. **Gambler's Ruin:** In the Gambler's Ruin game with initial capital nn, target TT, and probability pp of winning each individual bet:
   - Albert starts with nn chips, and Eric starts with m=T−nm = T - n chips.
   - At each bet, Albert wins Eric's top chip with probability pp and loses his top chip to Eric with probability q=1−pq = 1 - p.
   - **Tasks:**
     - State the gambler's ruin problem.

        - Formulate a mathematical model to estimate the probability of avoiding ruin.

---

8. **Amusement Park Navigation:** At an unknown amusement park, a person arrives at a four-way intersection point and wants to find the exit gate. There are no signs available in the park. Initially, they must choose one of four directions:
    - North: Exit gate is found after 2 minutes of walking.
    - East: Wander for 5 minutes and return to the starting position.
    - South: Wander for 8 minutes and return to the starting position.
    - West: Exit gate is found after 3 minutes of walking.
    - **Task:** Estimate the expected number of minutes the person will wander in the park before finding the exit gate.

---

9. **Random Walk:** What is Random Walk?
    - **Tasks:**
        - Explain how random walk can be employed to calculate the ranks of the pages in a Web-graph.
        - Illustrate how the scheme works with a Web-graph of 4 webpages and at least 7 hyperlinks.

---

10. **Markov Chain Analysis:** Define Markov chain and Markov property.

- **Tasks:**
    - I. Draw the state diagram and determine its transition probability matrix.
    - II. If today is sunny, what is the chance of rain the day after the 3rd day?
    - III. Obtain the steady-state probability vector, if it exists, and calculate the percentage of sunny and rainy days after one year.

---

11. **Markov Chains in PageRank:** Markov chains play an important role in online search. "PageRank is an algorithm used by Google Search to rank websites in their search engine results."

- **Scenario:** Suppose we have 4 web pages that contain links to each other:
    - From page A1A_1, 30% link to A2A_2, 50% link to A3A_3, and 20% link to A4A_4.
    - From page A2A_2, 50% link to A1A_1 and 50% link to A4A_4.
    - From page A3A_3, 10% link to A2A_2, 70% link to A3A_3, and 20% link to A4A_4.

- From page A4A_4, 20% link to A1A_1, 40% link to A2A_2, 10% link to A3A_3, and 30% link to A4A_4.

- **Task:** Find the probability that a person viewing page A3A_3 will view page A4A_4 after three links.

---

12. **Slotted Aloha Protocol:** Following slotted Aloha protocol, nn contending nodes attempt to transmit via a shared channel.

- **Tasks:**
  - I. If there is a collision in a slot, what is the expected number of nodes involved in the collision?
  - II. What is the probability that a given slot sees at least one transmission?

---

13. **Single Cashier Queue:** Suppose there is a single cashier at a grocery store checkout counter where customers arrive in a Poisson fashion with an average of 10 customers per hour, and service time follows an exponential distribution with a mean of 3 minutes.

- **Tasks (M/M/1 Model):**
  - I. Average waiting time for a customer.
  - II. Average length of the queue.
  - III. Probability that a customer arriving at the checkout counter will have to wait.
  - IV. Utilization factor for the checkout counter.
  - V. Probability that the number of customers in the system is 2.

---

14. **Bikeshare Program:** In a bikeshare program with 3 bike stations (A, B, and C), people can borrow a bicycle at one station and return it to the same station or another. The transition matrix governs movement among stations.

- **Tasks:**
  - I. If a bicycle is initially at station A, what is the probability it will be at station C after 5 days?
  - II. If the initial distribution of bicycles is 50% at station A, 20% at station B, and 30% at station C, what will be the distribution after 2 days and after 5 days?

---

15. **College Choice and Happiness:** Farisha just graduated from high school and was accepted to three reputable colleges:

- With probability 4/12, she attends Yale.

- With probability 5/12, she attends MIT.
- With probability 3/12, she attends Little Hoop Community College.
- Happiness probabilities:
  - Yale: 4/12
  - MIT: 7/12
  - Little Hoop: 11/12
- **Tasks:**
  - I. Draw the tree diagram and fill in edge probabilities; write probabilities at each leaf.
  - II. Probability that Farisha is happy in college.
  - III. Probability that Farisha attends Yale, given that she is happy in college.

---

16. **Disk Failure in Data Centers:** Every minute a data center sees a single disk failure with probability pp. More than a single failure does not occur in a minute, and failures are independent across minutes.

- **Tasks:**
  - I. Probability of no disk failures in a given day.
  - II. Probability of no more than two disk failures in a given day.
  - III. Expected duration before encountering the first disk failure.

---

17. **Morning Commute:** The probability that you are ready on time is 0.7. If ready, you can take the university bus; otherwise, you take the local bus.

- University bus time: 41 to 50 minutes (uniform distribution).
- Local bus time: 46 to 55 minutes (uniform distribution).
- **Tasks:**
  - I. Expected time to get to campus.
  - II. Probability of reaching campus within 45 minutes.

---

18. **Coin Flip and Binomial Distribution:** Let's flip a fair coin 5 or 6 times.

- **Tasks:**
  - I. Find the probability distribution using binomial distribution.
  - II. Plot the probability distribution function.

19. **Rolling a Die Twice:** A fair 6-sided die is rolled twice.

- **Tasks:**
  - I. Determine the sample space and outcome probabilities.
  - II. Express and analyze events A, B, and C for independence.

---

20. **Local Area Network:** A local area network with nn nodes shares a single wireless channel. Each node transmits with probability pp.

- **Tasks:**
  - I. Probability that a node's transmission in a slot is successful.
  - II. Expected number of nodes refraining from transmission.
  - III. Probability of two successive idle slots.

---

21. **Radar Aircraft Detection:** A radar detects an aircraft with probability 0.99 (true positive). False alarms occur with probability 0.1.

- Aircraft presence probability: 0.04.
- **Task:** Find the probability that an aircraft is present given an alarm.

---

22. **Lie Detector Accuracy:** A lie detector is 90% effective in detecting lies but gives false positives for 1% of truthful persons.

- 15% of the population lies.
- **Task:** Find the probability a person is lying given the test indicates lying.

---

23. **Voting Transition Matrix:** The transition matrix for voters shifting among Democrats, Republicans, and Independents.

- **Tasks:**
  - I. Probability of a Democratic voter switching to Republican in one election.
  - II. Probability of a Democratic voter switching to Republican in two elections.

---

24. **PageRank Using Markov Chains:**

- **Tasks:**
  - Explain how to measure page importance using Markov chains.
  - Define when a Markov chain is ergodic.

---

25. **Web Graph Modeling:**

- **Tasks:**
  - Create a Markov chain-based model for ranking web pages.
  - Handle edge cases such as isolated pages.

---

26. **M/M/1 Queue System:**

- **Task:** Derive the steady-state probability of the M/M/1 queueing system.

---

27. **Network Gateway Analysis:** A network gateway forwards packets at a mean rate of 125 packets/second. The forwarding time is 2 milliseconds.

- **Tasks:**
  - I. Analyze gateway utilization, steady-state probability, and mean number of packets.
  - II. Determine the probability of buffer overflow with 12 buffers.
  - III. Calculate the number of buffers needed to keep packet loss below one per million.

---

28. **Disk Server Requests:** A disk server satisfies I/O requests in 100 milliseconds with an average rate of 100 requests/second.

- **Task:** Find the mean number of requests at the disk server.

Gambler's Ruin :

Question : State the gambler's ruin Problem, Formulate a mathematical model to estimate the Probability of avoiding ruin.

Solⁿ: Let,

T = Target Chips

Albert stats with a stack of n chips

Eric stats with a stack of $m = T-n$

At each bet, Albert wins Eric's top chip with

Probability $= P$

Albert Loss his top to Eric with Probability $q = 1-P$. Albert bottom Chip work, $r = q/P$

And, success chips, $-r^2, r^3$ ... up to his top chip with work $- r^n$

Eric's top chip gets assigned worth down of bottom chip $- r^{n+1}, r^{n+2}, r^{n+3} \ldots r^{n+m}$

The expected Payoff Albert's first bet is worth

$$r^{n+1}, P - r^n \cdot q = \left(r^n \frac{q}{P}\right) * P - r^n \cdot q = 0$$

When, Albert's win all of Eric's chips his total gain Worth is,

$$\sum_{i=n+1}^{n+m} r^i$$

When loss, $\sum_{i=1}^{n} r^i$

Letting be Albert's wining Probability,

$$0 = \underbrace{W_n \sum_{i=n+1}^{n+m} r^i}_{\text{win}} - \underbrace{(1-W_n)\sum_{i=1}^{n} r^i}_{\text{loss}}$$

Where, $r = 1$

$$0 = m W_n - n(1-W_n)$$

$$\Rightarrow 0 = mW_n - n + nW_n$$

[$n+n = T$ for $P = 1/2$]

$$\Rightarrow n = W_n(m+n)$$

$$\Rightarrow W_n = \frac{n}{m+n}$$

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Again, when $[n \neq 1]$ we have,

$$0 = r \cdot \frac{r^{n+m} - r^{n}}{r-1} \cdot W_n - r \frac{r^{n}-1}{r-1} (1 - W_n)$$

Solving for W_n given,

$$W_n = \frac{r^{n}-1}{r^{n+m}-1}$$

$$= \frac{r^{n}-1}{r^{t}-1} \quad (P \neq 1/2)$$

We have now Proved that,

$$Pr(\text{The gambler's Wins}) = \begin{bmatrix} n/T & \text{for } P = 1/2 \\ \frac{r^{n}-1}{r^{n}-1} & \text{for } P \neq 1/2 \end{bmatrix}$$

when, $r = q/p$ $\boxed{\text{Proved}}$

Question: Define Markov Chain and Markov Property.
- Assume that today is sunny, then there is 90% chance it will sunny tomorrow and 10% Chance of rain. If it doesn't sunny today, there is 50% Chance it ~~to~~ will sunny tomorrow and 50% Chance of rain.

(i) Draw state diagram and its transition Prob^y matrix.

(ii) If today is sunny, what is the Chance of rainy the day after 3rd day.

(iii) Now obtain the steady ~~stat~~ state Probability vector, if it exist, also Calculate the Percentage of sunny and rainy days after one year.

Soln:

Markov Chain: A system that can change over time according to given Probabilities & allow one to Predict future events.

Markov Property: It is a Property of a type of stochastic Process. It is a Characteristics of Certain

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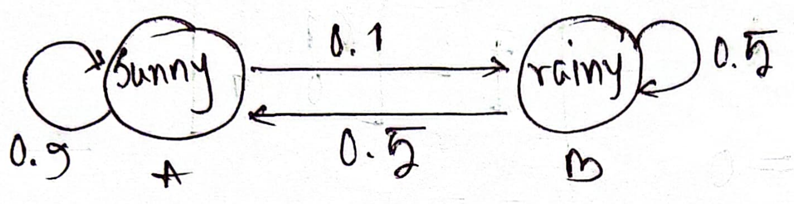
random Process. It means that to Predict the future state of the Process you only need to know the Current state. The Past state doesn't matter.

(i)



$$T = \begin{array}{c|cc} & A & B \\ \hline A & 0.9 & 0.1 \\ B & 0.5 & 0.5 \end{array}$$

(ii) If it's sunny, the initial state vector is, $A_0 = (1 \quad 0)$

$$T = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} \qquad T^2 = \begin{bmatrix} 0.86 & 0.14 \\ 0.7 & 0.3 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 0.844 & 0.156 \\ 0.78 & 0.22 \end{bmatrix}$$

$$A_3 = (1 \quad 0) \begin{bmatrix} 0.844 & 0.156 \\ 0.78 & 0.22 \end{bmatrix}$$

Ans

(iii) Steady State Vector,

$$Vt = V \qquad t = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

$$\Rightarrow Vt - V = 0$$

$$\Rightarrow V(t-1) = 0 \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow V(t-I) = 0$$

So,

$$V\left(\begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\Rightarrow V \begin{bmatrix} -0.1 & 0.1 \\ 0.5 & -0.5 \end{bmatrix} = 0 \quad \text{---- (i)}$$

Now, $V = [S \ \ R]$

~~Given~~ know that, $S + R = 1 \Rightarrow S = (1-R)$ ---- (ii)

$$\text{(i)} \Rightarrow [S \ \ R] \begin{bmatrix} -0.1 & 0.1 \\ 0.5 & -0.5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -0.1S + 0.5R & 0.1S - 0.5R \end{bmatrix} = 0$$

Now,

$$-0.1S + 0.5R = 0 \quad \text{——— (iii)}$$

and, $0.1S - 0.5R = 0$ ——— (iv)

~~(iii) + (iv) ⇒~~ Putting $S = (1-R)$ in (iii) ⇒

$$-0.1 + 0.1R + 0.5R = 0$$

$$\Rightarrow R = \frac{0.1}{0.6} = 0.167$$

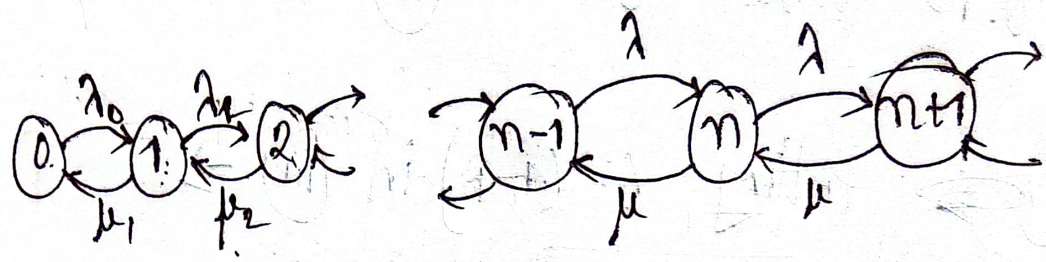
$R = 0.167$ Put (iv) ⇒ $S = 0.835$

So, After one year the Probability of,

$$\left.\begin{aligned} \text{Sunny} &= 83.5\% \\ \text{Rainy} &= 16.7\% \end{aligned}\right\} \text{Any}$$

Question: Derive the steady state Probability of the M/M/1 Queuing system.

Soln:



At state 0,

$$\lambda_0 P_0 = \mu_1 P_1 \Rightarrow P_1 / P_0 = \frac{\lambda_0}{\mu_1}$$

$$\Rightarrow \boxed{P_1 = \lambda_0/\mu_1 \cdot P_0} \text{ --- (i)}$$

$\lambda \rightarrow$ arrival rate

$\mu \rightarrow$ Service rate

At state 1,

$$\lambda_0 P_0 + \mu_2 P_2 = \mu_1 P_1 + \lambda_1 P_1$$

$$\Rightarrow \mu P_2 = P_1(\mu + \lambda) - \lambda P_0$$

$$\Rightarrow P_2 = \frac{\mu + \lambda}{\mu} \cdot P_1 - \frac{\lambda}{\mu} P_0$$

$$= \frac{\lambda}{\mu} P_1 + \frac{\mu}{\mu} P_1 - P_1$$

$$= \lambda/\mu \, P_1 + P_1 - P_1 = \lambda/\mu \, P_1$$

$$\therefore \boxed{P_2 = \lambda_1/\mu_2 \, P_1} \text{ ——— (ii)}$$

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Now,

$$P_1 = \lambda_0/\mu_1 \cdot P_0$$

$$P_2 = \lambda_1/\mu_2 \cdot P_1$$

$\boxed{\rho \rightarrow \text{Traffic Intensity}}$

So, $\boxed{P_n = \dfrac{\lambda_{n-1}}{\mu_n} \cdot P_{n-1}}$

Now ~~assume that~~, $\lambda/\mu = \rho$ [$\lambda < \mu$, then $\rho < 1$]

So,

$$P_1 = \rho P_0$$

$$P_2 = \rho P_1 = \rho^2 P_0$$

$$P_3 = \rho P_2 = \rho^3 P_0 \ldots$$

$$P_1 + P_2 + P_3 + \ldots P_n = (\rho + \rho^2 + \rho^3 + \ldots) P_0$$

$$\Rightarrow P_0 + P_1 + P_2 + P_3 + \ldots = (1 + \rho + \rho^2 + \rho^3 + \ldots) P_0$$

$$\Rightarrow 1 = \frac{P_0}{1-\rho}$$

$$\therefore P_0 = 1 - \rho$$

$$\Rightarrow P_1 = \rho(1-\rho)$$

$$\Rightarrow P_2 = \rho^2(1-\rho)$$

$$\therefore \boxed{P_n = \rho^n(1-\rho) = \rho^n P_0}$$

Question (13):

Given,

$\lambda = 10/hr$

$\mu = 1 \text{ customer}/3 \text{ minutes}$

$= 20/hr$

(i) Average waiting time for a Customer,

$\rho = \lambda/\mu = \frac{10}{20} = \frac{1}{2} = 0.5$

Waiting time $= \frac{\rho}{\mu(1-\rho)}$

$= \frac{0.5}{20 * (1-0.5)} = 0.05/hr$

$W_q = 3$ minutes

(ii) Avg. length of queue,

$L_q = \lambda W_q$

$= 10 * 0.05 = 0.5$ Customers

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(iii) Prob^y that an arriving customer has to wait,

$$P_w = \rho = 0.5 = 50\%$$

(iv) Prob^y that the number of customers in the system is 2,

$$P_2 = (1-\rho)\rho^2$$

$$= (1-0.5) * (.5)^2$$

$$= 0.125 = 12.5\%$$

* Avg waiting time of customer in ~~queue~~, ~~W_q~~ service,

$$\boxed{W_s = \frac{1}{\mu - \lambda}}$$

* Avg. waiting time of customer in queue,

$$\boxed{W_q = \frac{\lambda}{\mu} * W_s}$$

* Avg. number of customers in service, $\boxed{L_s = \lambda * W_s}$

* " " " in queue, $\boxed{L_q = \lambda * W_q}$

Solution (1) :

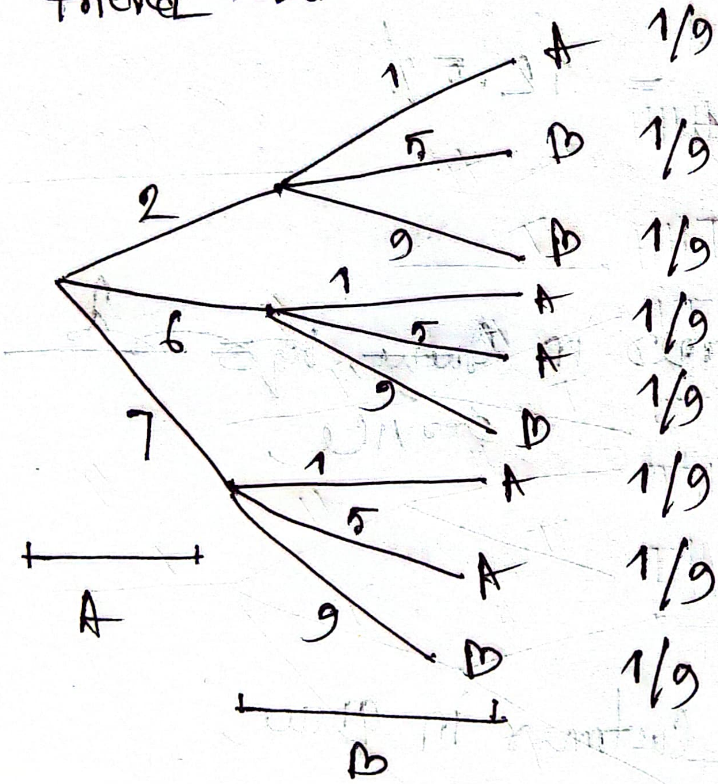
Dice A = 2, 6, 7

Dice B = 1, 5, 9

Dice C = 3, 4, 8

(i) I select = B

Friend select = A



Step (1) :

Sample space, S = {(2,1), (2,5), (2,9), (6,1), (6,5), (6,9), (7,1), (7,5), (7,9)}

Step (2):

When $(B>A) \Rightarrow \{(2,5),(2,9),(6,9),(7,9)\}$

$(A>B) \Rightarrow \{(2,1),(6,1),(6,5),(7,1),(7,5)\}$

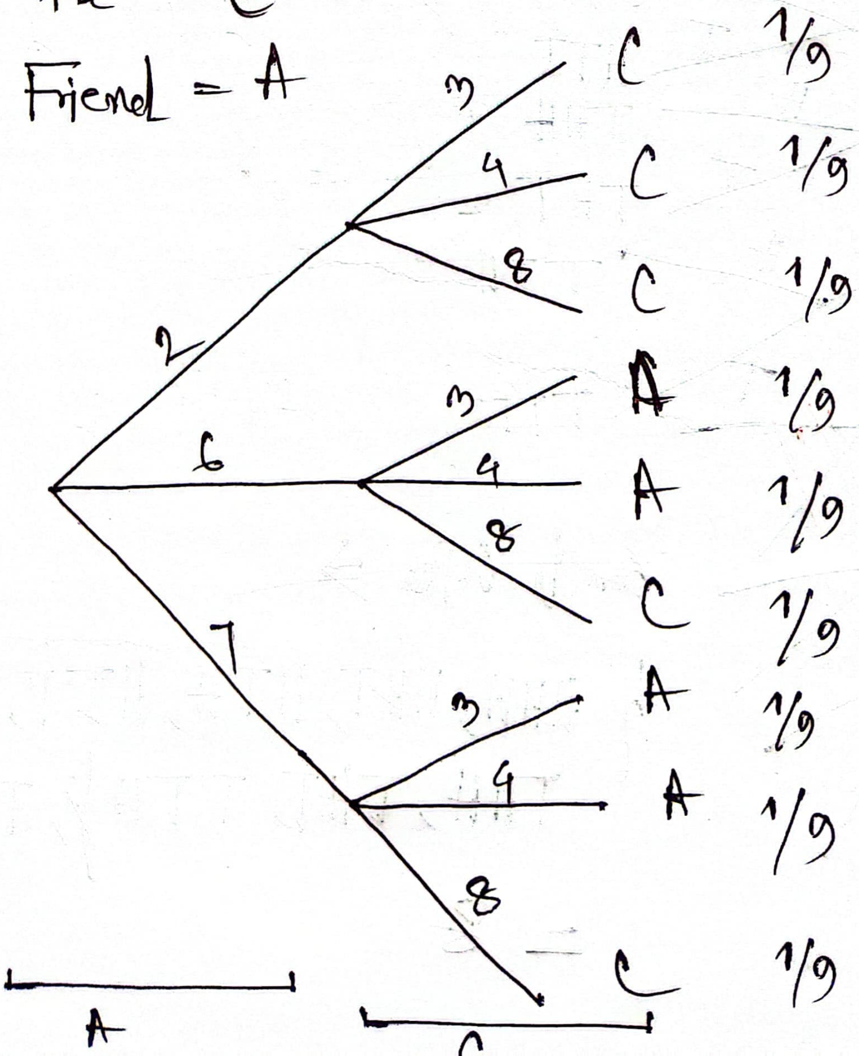
Step (3): Every outcome has Probability $= 1/9$

Step (4): $P(B>A) = 4/9$

$P(A>B) = 5/9$

(11) Me = C

Friend = A



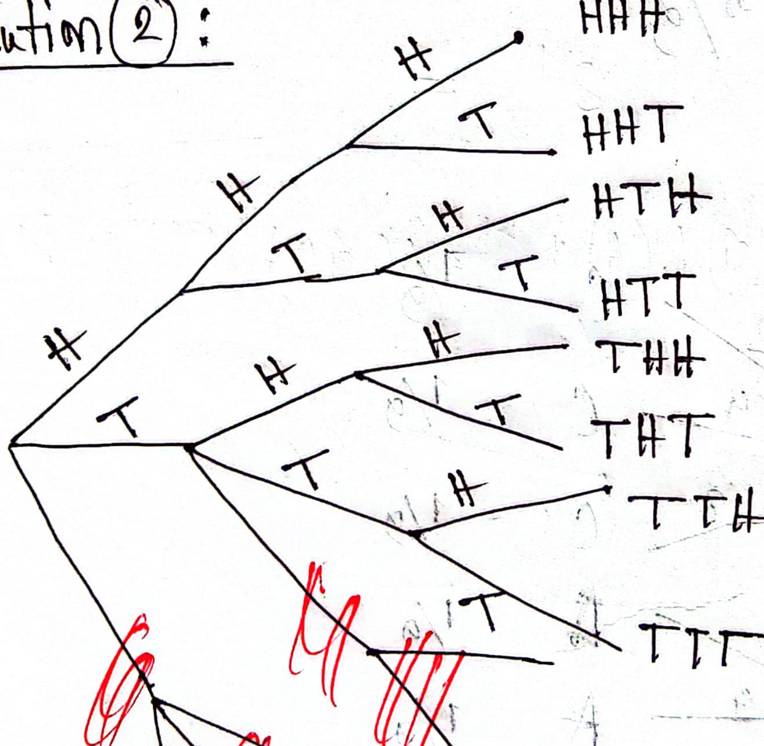
Sample Space $= \{(2,3), (2,4), (2,8), (6,3), (6,4), (6,8), (7,3), (7,4), (7,8)\}$

For $(A > C) = \{(6,3), (6,4), (7,3), (7,4)\} = 4$

$(C > A) = \{(2,3), (2,4), (2,8), (6,8), (7,8)\} = 5$

$P(A > C) = 4/9$ and $P(C > A) = 5/9$

Solution (2):



Sample SaPce $\Rightarrow$

$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$= 8$

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Case A: Coin 1 Match Coin 2,

$\Rightarrow \{HHH, HHT, TTH, TTT\} = 4$

$P(A) = 4/8 = 1/2$

Case B: Coin 2 Match Coin 3,

$B \Rightarrow \{HHH, HTT, THH, TTT\} = 4$

$P(B) = 4/8 = 1/2$

Case C: Coin 3 match Coin 1,

$C = \{HHH, HTH, THT, TTT\} = 4$

$P(C) = 4$

Checking Pairwise independence:

$P(A \cap B) = P(A) * P(B) = 1/2 * 1/2 = 1/4$

$P(B \cap C) = P(B) * P(C) = 1/4$

$P(A \cap C) = \text{- - - -} = 1/4$

Pairwise Independence

Mutual IndePendence Check :

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ [have to]

Now,

$P(A \cap B \cap C) = \{HHH, TTT\}/8$

$= 2/8 = 1/4$

$P(A) \cdot P(B) \cdot P(C) = 1/2 * 1/2 * 1/2$

$= 1/8$

So, $P(A \cap B \cap C) \neq P(A) \cdot P(B) \cdot P(C)$

[Not mutual IndiPendence]

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Solution ③ :

(i) Probability that, first 3 match decide the tournament ChamP,

Given, $P_A = 0.6 = 60\%$

$P_B = 0.4 = 40\%$

Now, the Probability $= P(AAA)^3 + P(BBB)^3$

$= (0.6)^3 + (0.4)^3$

$= 0.28$

(ii) Probability that, A win the tournament and A win first match given that

Case 1 :

Prob^y of A Win next 2 match,

$P(A \text{ win next 2 match}) = P_A \times P_A = 0.6 * 0.6 = 0.36$

Case 2 : Prob^y of A win 2 match out of next 3 matches,

$P(\cancel{A} \text{ win, Win, Lose}) = \cancel{0.6}\ P_A * P_A * P_B$

$= 0.6 * 0.6 * 0.4 = 0.144$

$P(\text{Win, Lose, Win}) = 0.6 * 0.4 * 0.6 = 0.144$

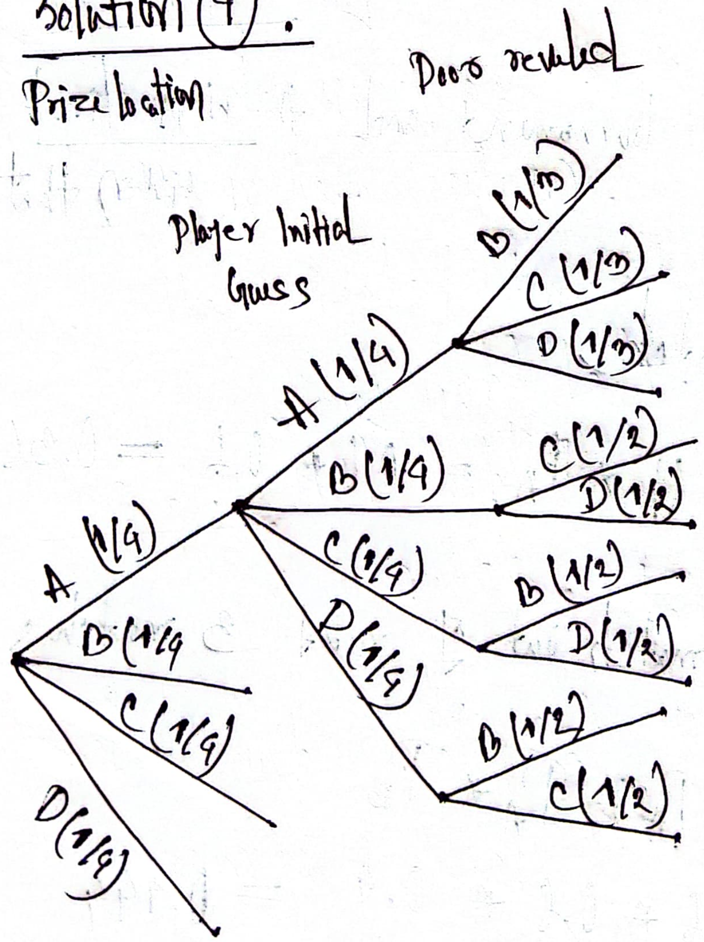
$P(\text{Lose, Win, Win}) = 0.4 * 0.6 * 0.6 = .144$

$\text{Total Probability} = 0.144 * 3 = 0.432$

Adding the Probability of both Case,

$(0.36 + 0.432) = 0.792$ Ans

Solution (4):



Outcomes	
AAB	1/48
AAC	1/48
AAD	1/48
ABC	1/32
ABD	1/32
ACB	1/32
ACD	1/32
ADB	1/32
ADC	1/32

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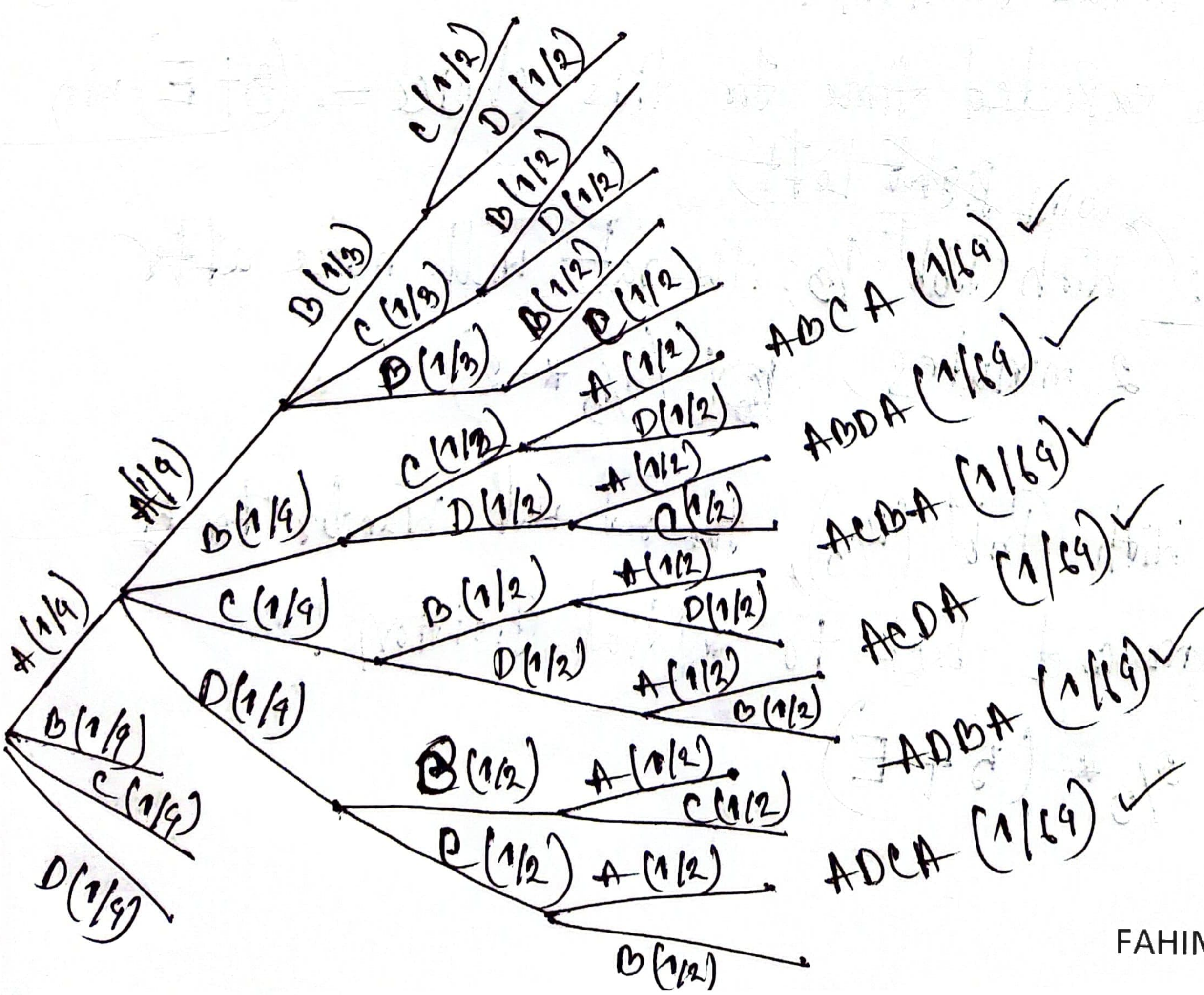
(i) The Prob^y that, 1st Contastant win,

$$\Rightarrow \left(\tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{48}\right) * 4 \quad [\,4 \text{ sub tree}\,]$$

$$\Rightarrow \tfrac{1}{4} \Rightarrow 25\%$$

(ii) Prob^y that, 2nd Contastant win after switching

$$\Rightarrow \left(\tfrac{1}{32} * 6\right) * 4 = 0.75 = 75\%$$



Prob^n of win after switching,

$\Rightarrow 4 * \left(\frac{1}{64} * 6\right)$

$\Rightarrow 3/8 = 37.5\%$

Solution (5):

Let, E be the expected time for the rat to escape the maze.

Case 1: Going to the right

The rat wanders for 3 minutes and then return to the initial Position.

So, the expected time for this choice = $(3 + E)$ min

Case 2: Going ~~Right~~ Left,
With Prob^n 1/3, the rat will maze after 2 minutes, $\left(\frac{1}{3} * 2\right)$

and, with Prob^n $\left(\frac{2}{3}\right)$, the rat will wander for 5 min and back to initial Position.

$\frac{2}{3} * (5 + E)$

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Therefore, the expected time for this Choice,

$$\frac{1}{3} * 2 + \frac{2}{3} * (5+E) = 4 + \frac{2}{3} E$$

Since, the rat has equal chance to go left and righ, the expected value E Can be,

$$E = \frac{1}{2} * (3+E) + \frac{1}{2} * (4 + \frac{2}{3} E)$$

$$\therefore E = 21 \text{ minutes}$$ Ans

Solution ⑥:

Let, E = The number of 1 hour untill the bulb burn out

A = ~~Batd~~ Bulb burn out on the first step.

$\bar{A}$ = Bulb didn't burn out on the first step.

$$\Rightarrow E = P(A) \cdot 1 + P(\bar{A}) \cdot (1+E)$$

$$= P(A) + P(\bar{A})(1+E)$$

Assume, $P(A) = P$

$P(\bar{A}) = q$

$$\Rightarrow E = p + q(1+E)$$

$$= p + q + qE$$

$$\Rightarrow E - qE = p + q$$

$$\Rightarrow E(1-q) = p + q$$

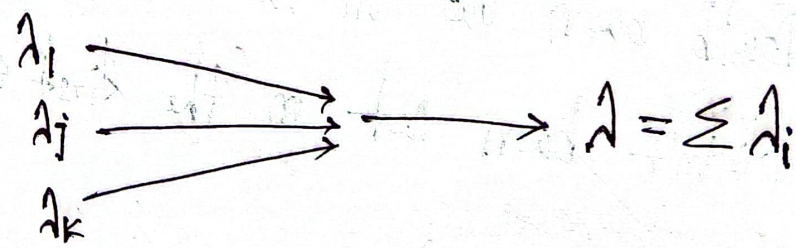
$$\Rightarrow E = \frac{p+q}{1-q} \qquad \left[\begin{array}{l} p+q = 1 \\ p = 1-q \end{array}\right]$$

$$\therefore E = \frac{\cancel{p+q}\;1}{p}$$

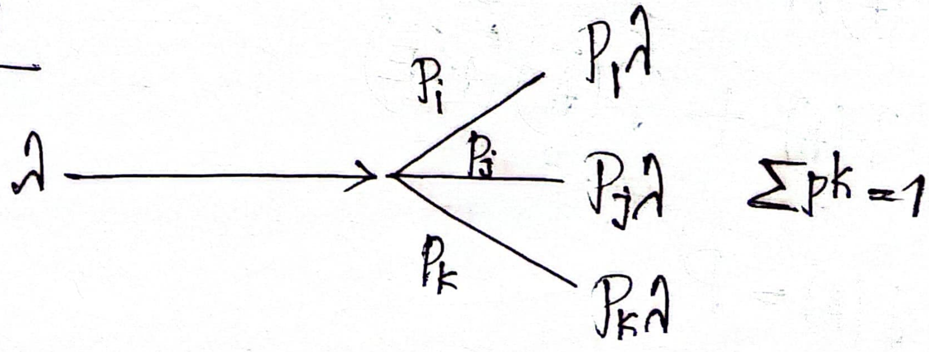
~~Ans~~

Poission Process :

Marging :



Splitting :



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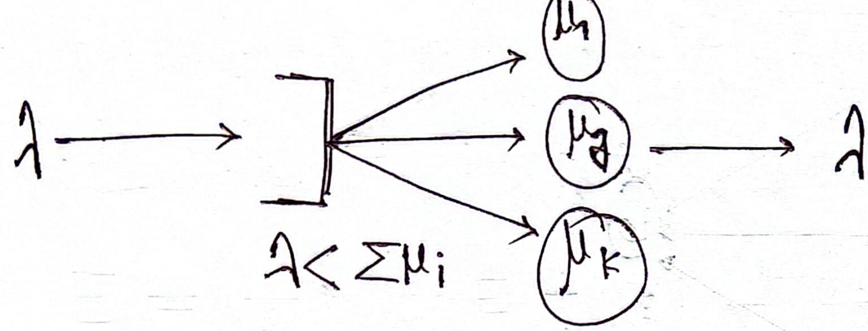
$\lambda < \mu$



Little's Law :

Mean number in the system = Arrival rate × Mean response Time

Example 30.3: A monitor on a disk server showed that the average time to satisfy an I/O request was 100 ms. The I/O rate was about 100 request per second. What was the mean number of request at the disk server?

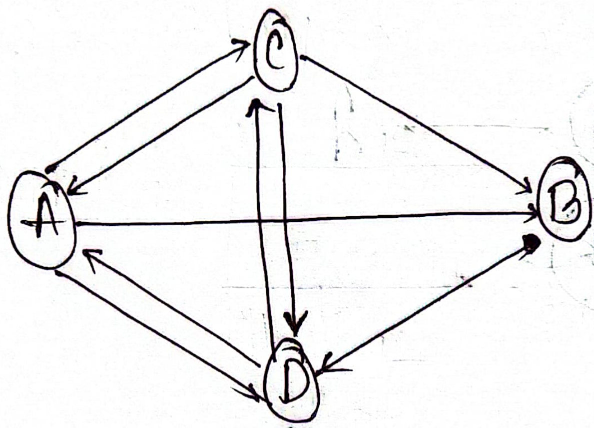
Soln: Mean number in disk server

= Arrivle time × response time

= $100 \times 0.1 = 10$ request Ans

Solution 9 :

Web ~~Page~~ graph of 4 Pages with at least 7 hyperlink.



The 9 hyperlinks are :

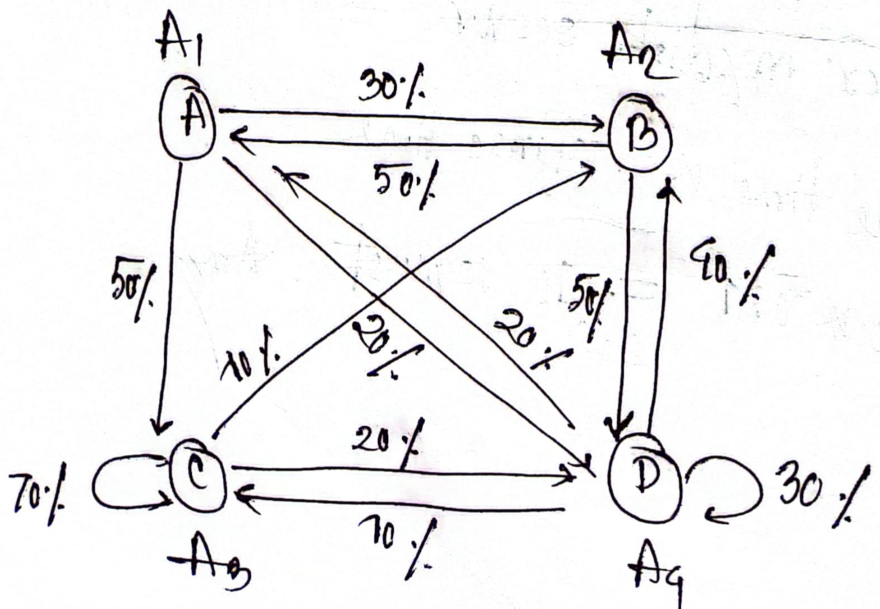
A ⟶ B, C, D

B ⟶ D

C ⟶ A, B, D

D ⟶ A, C

Solution 11 :



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Converting into matrix P,

$$P = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0 & 0.3 & 0.5 & 0.2 \\ A_2 & 0.5 & 0 & 0 & 0.5 \\ A_3 & 0 & 0.1 & 0.7 & 0.2 \\ A_4 & 0.2 & 0.4 & 0.1 & 0.3 \end{array}$$

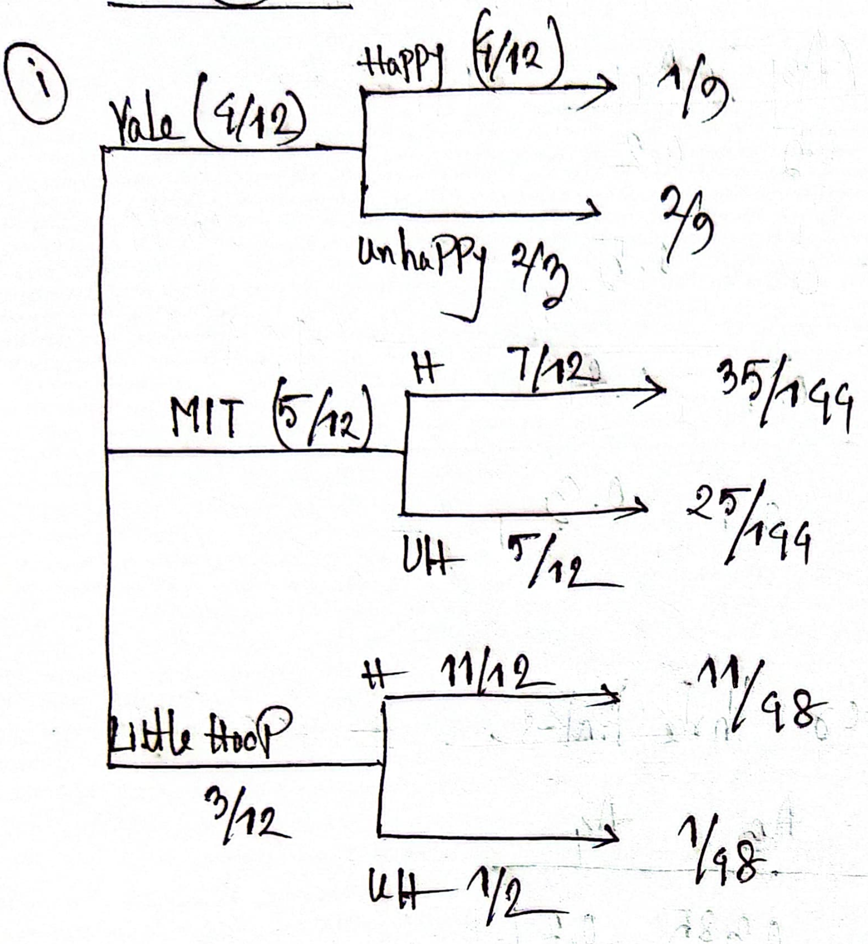
From A_3 to view any after three links.

$$P^3 = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 0.127 & 0.218 & 0.385 & 0.27 \\ A_2 & 0.225 & 0.160 & 0.285 & 0.33 \\ A_3 & 0.125 & 0.178 & 0.427 & 0.27 \\ A_4 & 0.165 & 0.238 & 0.305 & 0.292 \end{array}$$

$P(A_3 \text{ to } A_4 \text{ after } 3 \text{ link}) = 0.27 = 27\%$

Sol^n (15) :

(i)



(ii) $P(\text{Farisha is } \cancel{\text{un}}\text{happy}) = \left(1/9 + 35/144 + 11/48\right) = 7/12$

$$P(\text{Yale} \mid \text{Happy}) = \frac{P(\text{Yale} \cap \text{Happy})}{P(\text{Happy})}$$

$$= \frac{1/9}{7/12} = 4/27 \quad \text{Ans}$$

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Sol^n 19 :

Total number of sample space $= 6\times6 = 36$

Total Prob^y $= 36$

Prob^y of each outcome $= 1/36$

For Case A : 1st roll is 1, 2, or 3,

$$P(A) = \frac{3\times6}{36} = 1/2$$

Case B : 1st roll is 3, 4 or 5,

$$P(B) = \frac{3\times6}{36} = 1/2$$

Case C : The sum of two rolls is 9,

$\{(3,6), (4,5), (5,4), (6,3)\} = 4$

$$P(C) = 4/36 = \frac{1}{9}$$

For Pairwise independence,

$$P(x \cap y) = P(x) \cdot P(y)$$

Now,

$$P(A \cap B) = \frac{1 \times 6}{\cancel{1/2}\ 36}$$

$$= 1/6$$

$(A \cap B) = \{(1,2,3) \cap (3,4,5)\}$

$= \{3\}$

$$P(A \cap C) = \frac{\cancel{1 \times 6}\ 1}{36}$$

$(1,2,3) \cap (3,4,5,6)$

$= \{3\} \rightarrow (3,6)$

$$P(B \cap C) = \frac{3}{36} = 1/12$$

$P(A) \cdot P(B) = 1/4 \neq P(A \cap B)$

$P(A) \cdot P(C) = 1/18 \neq P(A \cap C)$

$P(B) \cdot P(C) = 1/18 \neq P(B \cap C)$

Not Pairwise Independence

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Sol^n 20 :

~~1~~ The Prob^y that, a Particular node transmit $= P$

a " " not " $= (1-P)$

If there is $(n-1)$ node, the Prob^y that all of them refrain from transmission $= (1-P)^{n-1}$

(i) The Probability that a Particular node transmission is successful, $P(\text{Success}) = P \times (1-P)^{n-1}$

(ii) Expected no. of node, that refrain from transmission in a given slot, $= n \times (1-P)$

(iii) Prob^y that, two successive slots go idle,

For one slot, $P(\text{idle}) = (1-P)^n$

Two slots, $P(\text{idle}) = (1-P)^{2n}$

Soln (21) :

Prob^y of aircraft being Present, $P(A) = 0.04$

not Present, $P(\neg A) = (1-0.04)$

$= 0.96$

Prob^y of radar detecting an aircraft when aircraft is Present, $P(R \cap A) = 0.99$

detecting an aircraft (False) when there is no aircraft Present,

$P(R \cap \neg A) = 1 - 0.99 = 0.01$

According to Bayes Theorem,

Aircraft being Present and gives radar alarm,

$$P(A \cap R) = \frac{P(R \cap A) \cdot P(A)}{P(R)}$$

Finding P(R),

Aircraft is Prasent and radar detected it,

$\Rightarrow$ ~~P(A∩R)~~ $P(R) = P(A) * P(R \cap A)$

Aircraft is not Present $(\neg A)$ and radar detected (False),

$P(R) = P(\neg A) . P(R \cap \neg A)$

$\therefore P(R) = \{P(A) * P(R \cap A)\} + \{P(\neg A) . P(R \cap \neg A)\}$

$= (0.04 * 0.99) + (0.96 * 0.01)$

$= 0.1356$

so, $P(A \cap R) = \dfrac{0.99 * 0.04}{0.1356} = 0.29 = 29\%$

Soln (7):

The senario for Gambler's ruin,

(i) Albert's initial Capital = n Chips

(ii) Eric's " " = $(T-n)$ "

(iii) Total no. of chips in game = T

Prob^y of Alber's winnig a single game = P

" " " lossing " ", $q = (1-P)$

The game Continue untill one Player is Completely ruined, meaning one Player losses all their Chips.

For Avoiding ruin:

Let, $P(n)$ = Prob^y that Albert ~~also~~ avoids ruin given, he ~~stand~~ starts with n chips.

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The recurrence relation:

$$P(n) = p.P(n+1) + q \cdot P(n-1)$$

Here,

$P(n+1)$ is the Prob^n of Albert win next bet and then avoide ruin starting with $(n+1)$ chips

$P(n-1)$ — — — — —

If, Albert has 0 cips, he is ruined, $P(0) = 0$

and, If Albert has all T chips, he win, $P(T) = 1$

Solving the recurrence:

$$r = \cancel{\frac{p}{q}} \; \frac{q}{p} = \frac{1-p}{p}$$

Now,

$$P(n) = p.P(n+1) + (1-p).P(n-1)$$

$$P(n) = A + Br^n$$

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